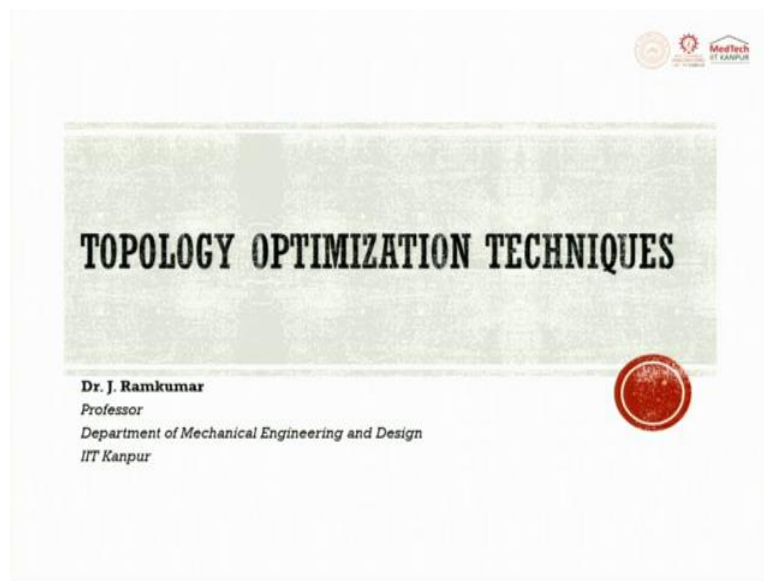


**Metal Additive Manufacturing**  
**Professor J. Ramkumar & Dr. Amandeep Singh**  
**Department of Mechanical Engineering and Design**  
**Indian Institute of Technology, Kanpur**  
**Lecture 22**  
**Topology Optimization Techniques**

Welcome to the next lecture in the series of Metal Additive Manufacturing. This topic is quite vast. So if I go exclusively in metallurgy you might get little boredom; so I have also added the other advancements which are happening in Metal Additive Manufacturing.

(Refer Slide Time: 0:40)



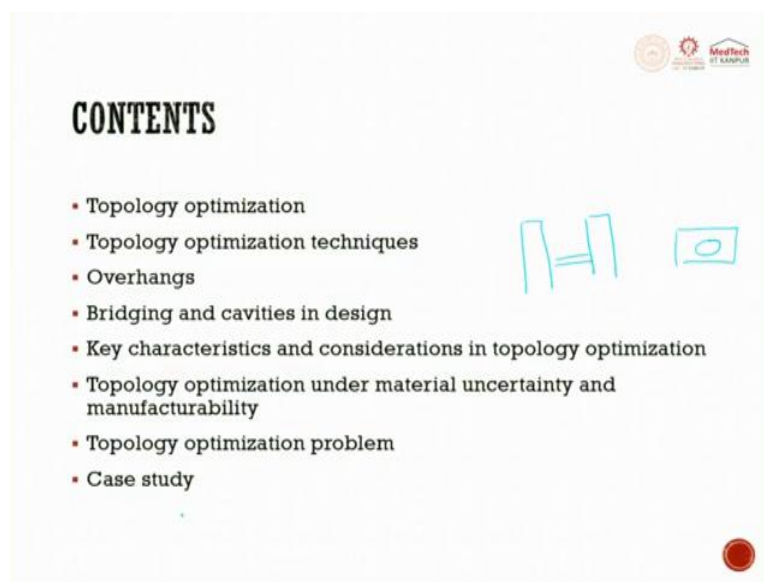
Today there is a big drive which is given to Topology Optimization Techniques. So this optimization technique is a tool which is effectively used to reduce the material and strengthen the material as and when required in some spaces. For example, if you wanted to strengthen a place where there is two moving parts or there is a hole and then you have a shaft which is going, the interface between these two we wanted to strengthen it, so now what we do is we try to do topology optimization and try to strengthen those parts.

And wherever there is material which is redundant, which was used only for connectivity purpose, those material is also getting removed. So by and large topology optimization technique should be used for any part which is getting printed in Metal Additive Manufacturing because it moves directly to the final application. It gives weight reduction, it also gives you material consumption, so by and large the cost goes down drastically.

So now let us try to focus on this topic of Topology Optimization Techniques. In this lecture we will try to cover some basics of topology optimization and then we will try to move towards Topology Optimization Techniques. There are multiple, I have picked only 2 or 3 so that you can try to briefly have an introduction.

And today there is lot of advancements are happening in this Topology Optimization Techniques. Next, overhangs, which are a part and parcel of Metal Additive Manufacturing parts, which have a projection outside the main body, overhangs, we will try to see. Then we will try to see bridging and cavities in design.

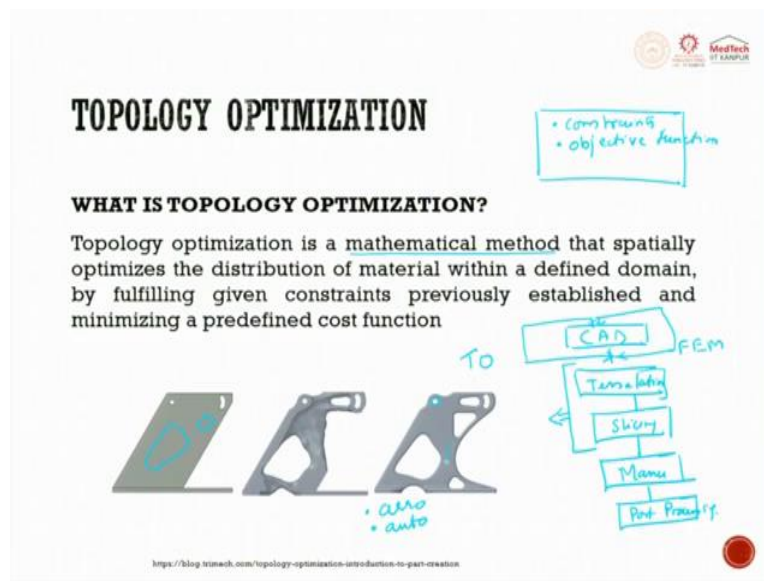
(Refer Slide Time: 3:00)



So, when you talk about bridging, means when there are two small features and how do we connect these two features such that they do not break while manufacturing or they do not warp during manufacturing. The next one is cavities, when they are designed how are we going to go about taking this in topology optimization. So, bridging and cavities in design. Next, we will try to see key characteristics and consideration in Topology Optimization.

Topology optimization under material uncertainty and manufacturing. Keeping uncertainty happening in with respect to powder or with respect to defects, which are happening during the process, how can we accommodate that in topology optimization. Then we will try to see topology optimization problems, which are available. And finally, a single case study so that you try to have a better understanding what we discuss in this lecture.

(Refer Slide Time: 4:06)



Topology optimization - Topology optimization is a mathematical method. This is integrated in CAD itself. Now let me go back to the initial five steps which are involved in additive manufacturing. First, we do CAD, then we do tessellation, then slicing, then manufacturing, then post processing. Where does this topology optimization come? It comes here itself. Topology optimization comes here. Where does finite element come here? It also comes here.

So, all these things happen at the design stage itself, it undergoes back and forth iteration, so that you try to get the best part and then you move for tessellation, slicing, manufacturing and post processing. Today, these two processes are also joined together. We do not have a separate step called as tessellation and slicing. We directly take the CAD, do slicing. And there we try to integrate it with tessellation.

But the only problem is, there are lot of errors which happen in this process, which a lot of companies are trying to develop exclusive library functions to sort out those problems. So, they are trying to develop patch algorithms to solve these problems. So, topology optimization happens here in CAD. What is topology optimization? Topology optimization is a simple mathematical method that spatially optimizes the distribution of material within a defined domain. This domain can be size, shape, weight optimization.

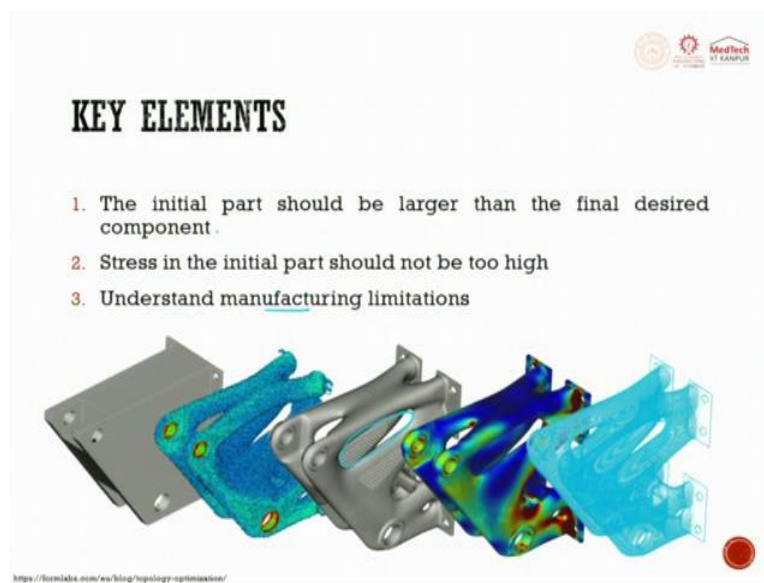
So, it works very nicely in optimizing the distribution of material within a defined domain by fulfilling given constraints previously established and minimizing a predefined cost function. So here what we do is, we try to define the domain space and in this domain space we define

the constraints and then we try to have an objective function. We try to solve this objective function such that we try to get the benefit in whatever application we want.

So, this is the initial version, maybe of a frame. Thus keeping size, shape, weight, cost optimization, it gets transformed into this, but still gives a better yield or a better performance in this component. So, we have removed those areas where we do not need material and we have done it. Today, aerospace industry and automotive industry are doing this topology optimization in a very big way.

So, you can see here, these are constraints. This is a mounting hole constraint, so they have maintained the constraint all through. They have only removed the material. When they remove the material also, you try to see here, at certain portions where there is a load component coming they try to strengthen it.

(Refer Slide Time: 8:18)



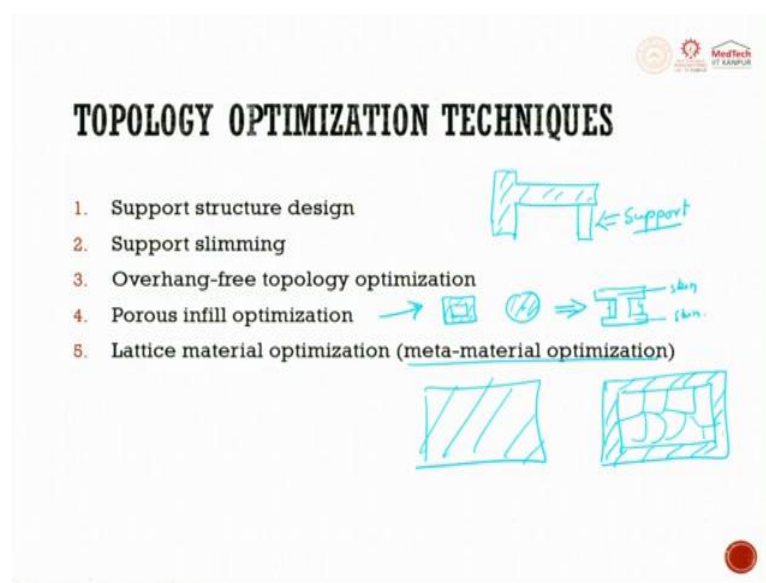
The initial part should be larger than the final desired component. Stress in the initial part should not be too high. Understand the manufacturing limitations. These are the elements which we try to consider when doing topology optimization. You see here, this is a bracket or this is a holder, where there is grouting happening.

So, from here, we try to transform into this and we try to develop a CAD model, do finite element analysis, find out the stress concentration points, add more material in those areas and try to develop simpler design keeping manufacturing also in the consideration. Earlier

design for manufacturing and today design for manufacturing, particularly in additive manufacturing, is completely different.

Earlier we used to talk about the material flowing. We used to talk about continuous material in a component, but today in additive manufacturing that is not required, what was full of material here you see it is avoid here, but still it tries to do the function very nicely. Look at this, there is a void, we have removed material, we find out wherever material is not required or it does not play a role, we try to remove it.

(Refer Slide Time: 9:51)



So, there are many topology optimization techniques. We are taking only five of them for our discussion and in this lecture, we will not go deeper into mathematics, in solving the topology optimization problem. I am only going to tell you that how do we use this topology optimization for our benefit and today there are so many softwares which can help you in doing topology optimization.

First is support structure design. What are the support structure design? These support structure designs exist because you generally have a free hanging structure. This free hanging structure, we try to support material such that it does not sag and while it is getting built also we try to support it to withstand the thermal distortion or shrinkage.

This is a supporting structure. The supporting structure whatever we use, this material will be removed when the final component is taken out or when it is put into good use. That means to say supporting material, whatever you invest, it is used to aid in manufacturing, but nothing

more than that. So, if you can reduce the supporting structure material to a large extent, then you have a huge financial saving and the supporting structure also gets built when the component is getting built.

So, your manufacturing time also reduces drastically. When we talk about polymers you can try to polymer additive manufacturing processes, you can have different materials for supporting and the part, original part what you want, but in metals what happen, we try to use the same raw material for both. The only difference will be the density distribution, what happens inside the supporting structure.

We will have a very small filling factor here as compared to that of this original part, so support structure if we optimize that itself is a huge saving with respect to time and money. So, support structure design we will try to have one technique. Next is support slimming. Then overhang-free topology optimization.

If you are a very smart person what happens is, you should try to think of a way such that this supporting structure itself is not used while developing because this itself tries to reduce the cost and money. So, overhanging-free topology optimization. How do I do it? You try to orient the part in such a fashion such that the support structure is eliminated. Next is porous infill optimization. Lattice material optimization.

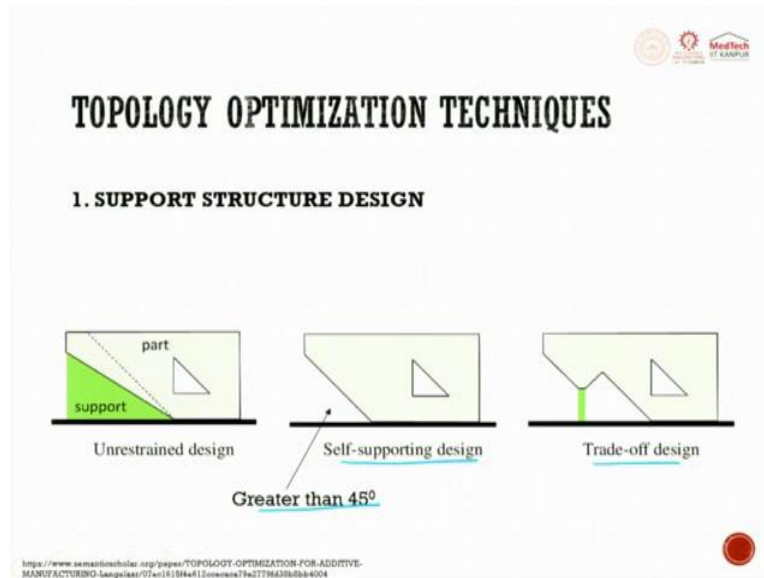
See this porous infill optimization; let us try to go back to the box type, you had a shaft (or a round shaft), is replaced by I frame, so shaft is used for rotating, assuming that there is a stationary load you wanted to do it, so then you can try to have an I beam in which the top skin layer and the bottom skin layer takes the load. The in-between I, whatever is there, web, so this is only used as a spacer between the two skins. So, these two are skin.

So now what we are trying to talk about is to do such thing when we are trying to do in a single layer, so now you take a single layer where there was full of material. Now what we are trying to talk about is try to make a skin and this will be where densely material will be filled, in one layer I am talking about. And then you have infilling in between. This tries to give the strength component, but the skin alone is given lot of material.

So, this type of optimization is called as porous infill optimization. You can decide the size, you can decide the shape, etcetera. The last one is going to be lattice material optimization for

an application called meta-material application. Meta-materials are materials, which are artificially made, which does not follow the nature's law.

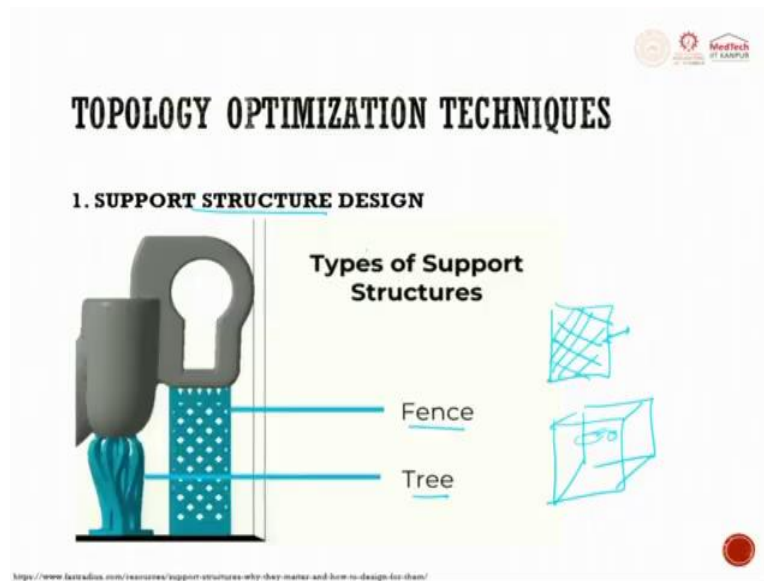
(Refer Slide Time: 14:42)



So, this is the support structure design. In support structure design, you have three different types, so the first one is called as unrestrained design. This is the part, this is the support you give and it is completely supporting the object. The other one is called as self-supporting design where in which the support is removed and the object itself while making tries to support itself.

The third one is going to be a trade-off between these two, it is called as trade-off design where in which you try to build the object with a small support structure. Whenever you use for self-supporting design, you try to have this angle which is always greater than 45.

(Refer Slide Time: 15:40)



So when we try to see the support structure design, so we have two important things which you should note down, one is called as the fence and the other one is called as a tree type. These two are used for building; there are many more but these two are very common. So, when we try to use fence, it is almost a 2d plane. In a 2d plane you try to have the supports.

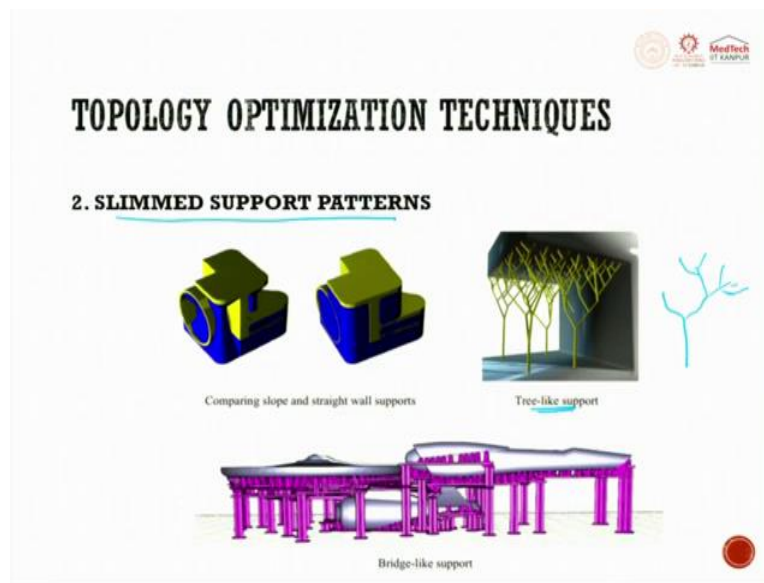
You have material removed; so these are infills which are removed and this will try to reduce the weight and it tries to support the component. And here also along the z direction it will be completely filled, so if I draw in a 3 dimensional fashion, this fence what will happen is, it will have parts which runs through to through.

But here also it uses lot of material, when we go to tree structure it is much lesser and you see here it gives a very small line which supports from the bottom to the top, it is almost like a tree structure which supports the object. So, this is why even today we call bio mimicking manufacturing. We try to look at nature, understand nature, how nature supports, we try to take that as an inspiration in manufacturing.

Many successful manufacturing techniques, technologies and parts are developed by bio mimicking. For example, solar panels they got the idea from leaves. So, these are the two different types of support structure design.



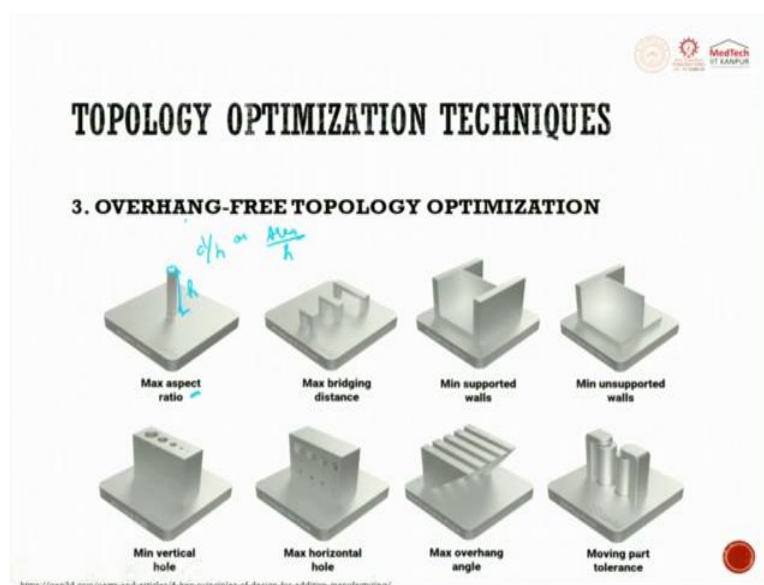
(Refer Slide Time: 17:23)



Next moving on to slimmed support patterns. You can see here how is it, when trying to do it comparing slope and straight wall support you are trying to build in slim supports patterns. In slim support patterns what we do is we try to follow a tree like support.

What is a tree like support? You have a shoot, then you have branches and from these branches you try to have leaves. So now you see you have further reduced the amount of material which is used for support structure. So, if you want to build a huge component you can try to optimize your support structure by following slimmed support pattern.

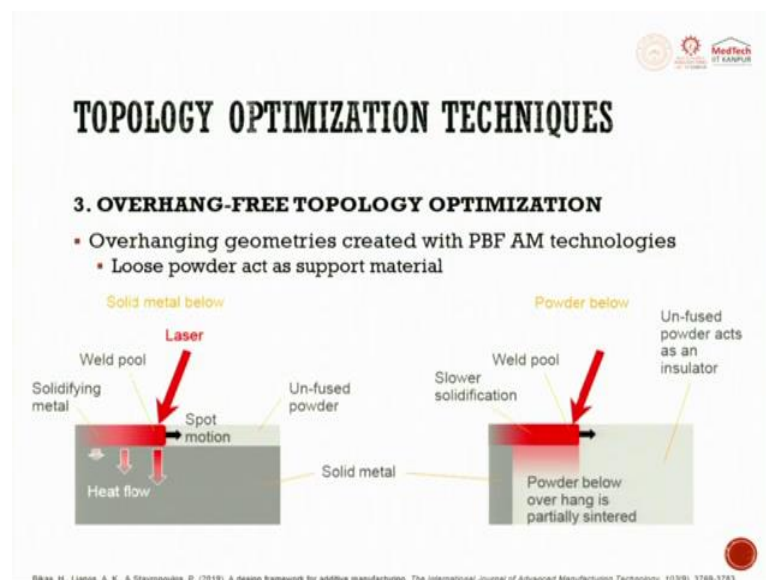
(Refer Slide Time: 18:19)



These are all some overhang-free topology optimization technique used. So, you see here this is maximum aspect ratio topology optimization method which is used. Then you can have max bridging distance. The next one is minimum supported wall. Minimum unsupported wall, very thin output. Then you can have minimum vertical hole, maximum horizontal hole, max overhang angle and moving part tolerance.

These are all different overhang-free topology optimization used, approach for trying to reduce the support structure. So maximum aspect ratio, aspect ratio is diameter to height,  $d$  to  $h$  or it can be area to  $h$ ,  $h$  is the height. So here you can see these are the bridges, this is a bridge, so minimum unsupported, minimum supported, vertical hole, horizontal hole, overhanging and moving part tolerance.

(Refer Slide Time: 19:48)

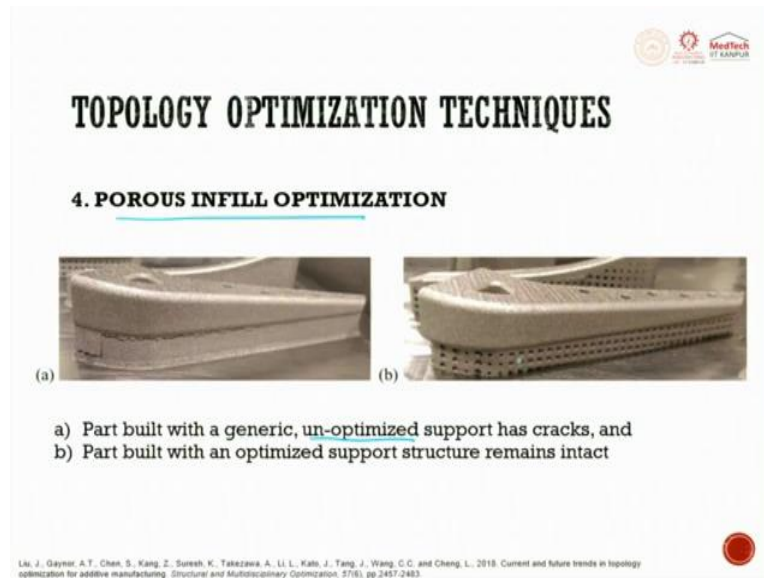


In overhang-free topology optimization the overhanging geometries created with powder bed fusion additive manufacturing technologies looks like this. So, you can see here this is the solidified metal, this is how the laser hits, this is the weld pool or the melt pool, this is the spot moving direction, this is unfused powder, this is where is the heat flow which happens on a solid metal.

So here this complete thing is getting replaced into this, so where in which you have this whole structure converted into a small strip like structure, so the powder below overhang is partially sintered, it is partially sintered, so that you can try to easily break, so slower solidification happens. And then here it is weld pool which moves in the front and here it is

unfused powder acts as an insulator. So here it is, this is with minimum support, you can try to have this overhang structures.

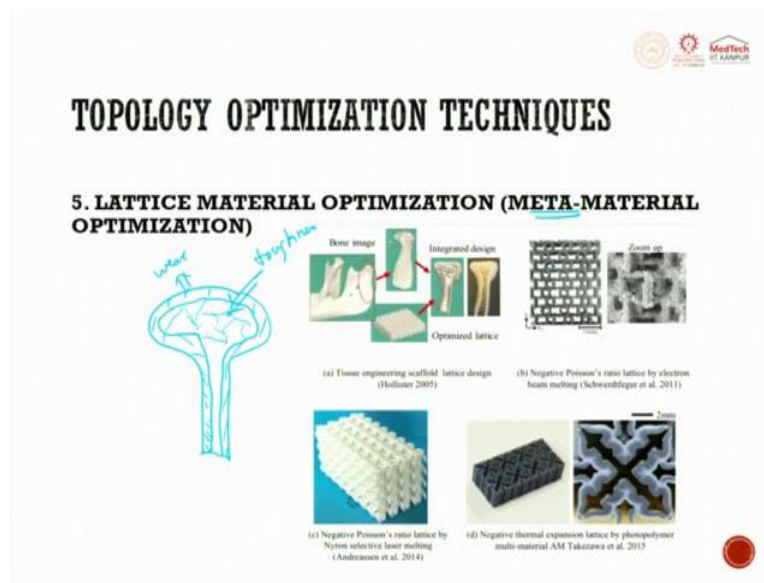
(Refer Slide Time: 20:56)



Next one is porous infill optimization. So porous infill optimization, you can see here in a part built with generic un-optimized supports have cracks, these are cracks which are getting build because of the un-optimized support. So, when we try to do porous infill optimization, because of this porous there was a heat exchange and because of this heat exchange the residual stresses are reduced.

So, there is no crack which is formed in the part built with optimized support structure remains intact. This is called as porous infill optimization. This is as far as the support structure is concerned.

(Refer Slide Time: 21:44)



The next one is the lattice material optimization. The lattice material optimization as I told you it is copying from nature. So, it is bio mimicking, so we have a bone, when we try to take a bone the bone, if you have a structure like this as a bone, you will have densely populated cells which are just around the skin, just around the skin you have this thin densely populated one. So here what happens is in between portion is loosely packed cells are there.

Why, because the surface takes the load and the loosely packed cell, this takes care of the toughness, this takes care of the wear resistance. So almost the same thing is followed here in the real component. So, we try to make negative Poisson's ratio, when you pull it expands. So negative Poisson's ratio lattice by electron beam melting we do it and this one is trying to give lot of new properties.

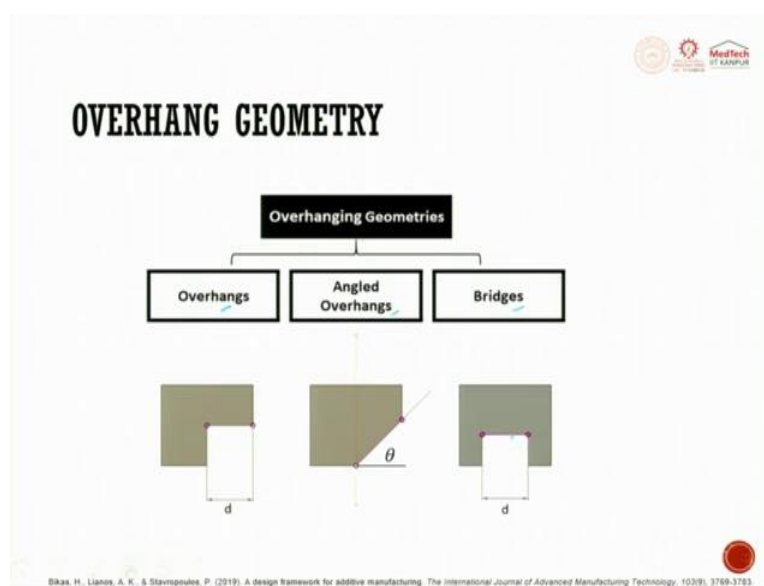
When it is getting loaded the cells get expanded. Once the cells get expanded it can take more load or it tries to go for a larger surface area, so this can help another sensor to get activated. So, this is made out of metal, people use laser bed powder fusion method or they use electron beam. So here is a unit a packed cell which is used for where in which you get negative Poisson's ratio lattice when we use electron beam melting method.

So, we can also generate negative Poisson's ratio lattice by neuron selective laser melting or we can try to have negative thermal expansion lattice by photopolymer multi-material additive manufacturing technique. This is a new area where lot of people have started

working in meta-materials. They are trying to develop such structures through and through or on the surface for various applications.

It can be for infrared, it can be for electromagnetic, it can be for load application also, acoustic application, it can also be used for mechanical load application. People have started using it. Through this what happens your amount of material used is reduced and they also get the enhanced property, so all these things are done through topology optimization techniques.

(Refer Slide Time: 24:32)

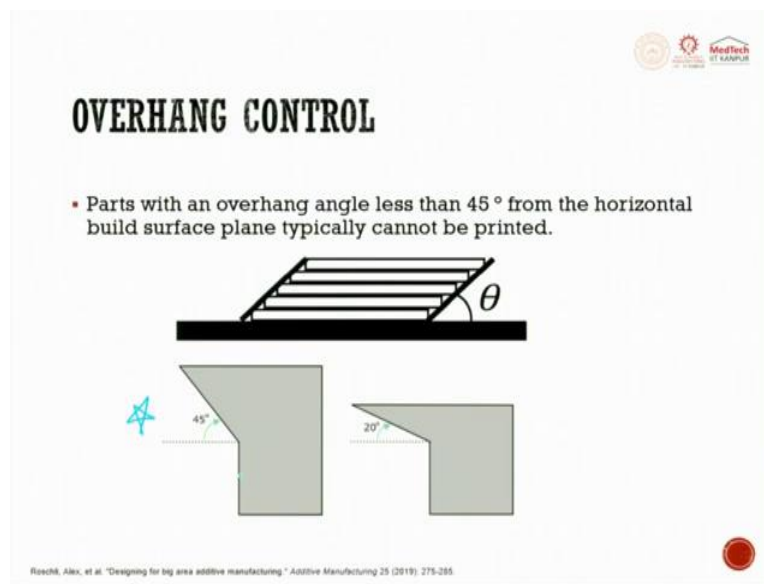


When we talk about overhang geometry there are three types of overhangs. Why is overhang important because the support structure come into existence, the moment you have an overhang. One is called as overhangs, the other one is called as angular overhangs, and the third one is called as bridges. So, this is what we saw in the previous case.

Here we saw maximum bridge distance. So, this is again a topology optimization technique used for getting the reduced amount of support which is used, so bridges. So, you have a overhang, you have overhang like this, then you have an angular overhang. So, what we do, we make a component or we design a component like this such that the overhang can be reduced, so no overhang support structure is required.

So, the third one is you can also try to have bridges. These bridges rather than having a complete fill if you can have a bridge like this your material required is less.

(Refer Slide Time: 25:32)



The overhang controls the part with an overhang angle less than 45 degrees from the horizontal built surface plane typically cannot be printed. This is something which you should understand. So, this is a very, very important point for you when we try to have features which are getting built in our final part, you also have this in mind, because your layer thickness is extremely low.

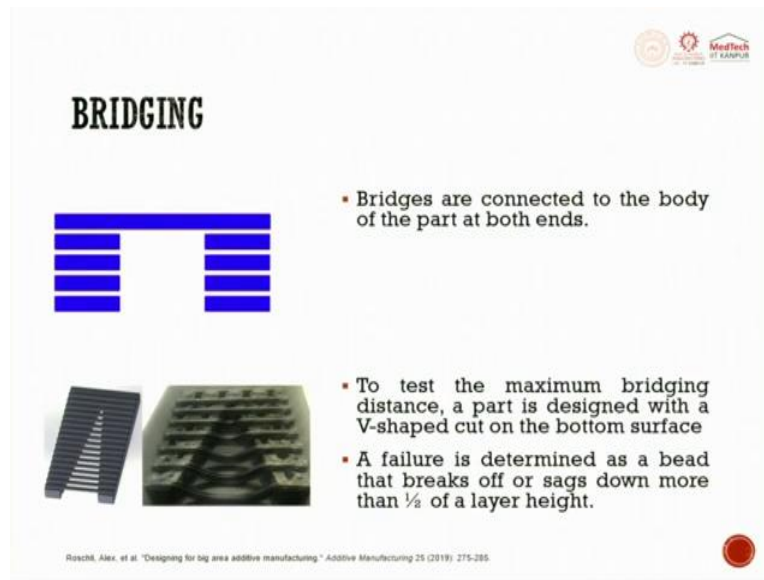
Part with an overhang structure angle less than 45 degrees, from the horizontal built surface plane typically cannot be printed, so this also you should know. So, if you have this less than 45 degrees.

(Refer Slide Time: 26:21)



So, if you have an overhang of 20 degrees, 25 degrees and 30 degrees, if you see at 35, 40 and 45, so you can get the curved one at 40 and 50 it is straight. So, this is what we were trying to talk about the overhang angle less than 45 degrees from the horizontal build surface plane typically cannot be printed. Keep that in mind because when you are trying to make topology optimization this also should be added as one of the constraints.

(Refer Slide Time: 26:58)

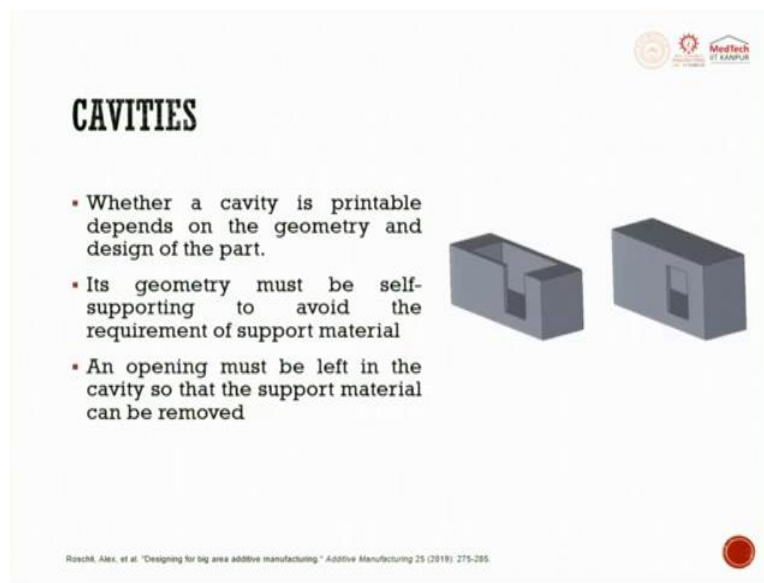


Bridging or bridge - Bridges are connected to the body of the part at both ends, to test the maximum bridging distance a part is designed with a V shape cut on the bottom surface. If you wanted to use bridge type and you have a part within it, so now what you do is in order to understand what is the load, it can take both in terms of mechanical and in terms of thermal, so we try to create this V structure.

This V structure will try to maintain, we will try to keep increasing the V structure design. So, to test the maximum bridging distance a part is designed with V shape cut on the bottom surface. A failure is determined as the bead that breaks off or sags down more than half of the layer height, more than half of the layer height is considered as a failure. Now you decide your support structure.



(Refer Slide Time: 28:02)



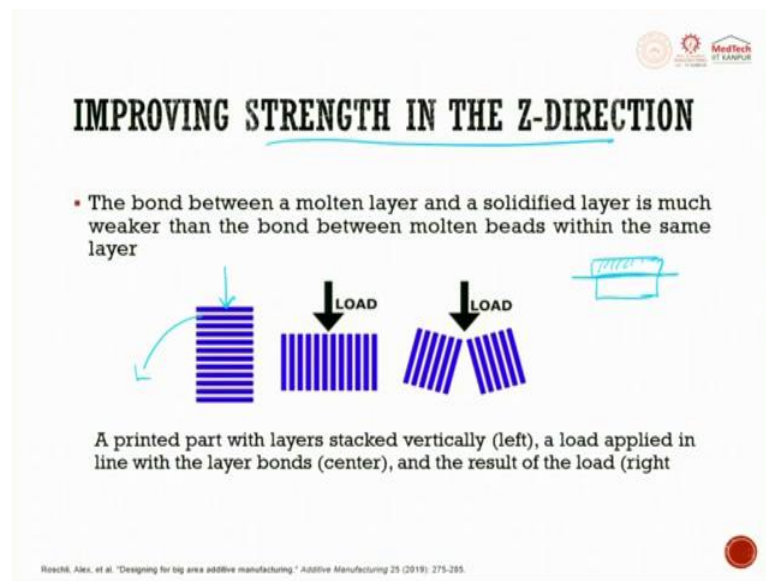
Next are cavities. When a cavity is printable depends on the geometry and the design of the part, when a cavity is to be printed. You cannot have a structure like this, rather than having a structure like this, you make a house, house like structure, this is much more advisable and easy and it is produced to meet out to your dimensional requirement. The geometry must be self-supporting to avoid the requirement of supporting material.

If you have a top surface just a bridge and later you try to remove it that is a different story, or try to have, do not try to have a large free area on the top, so that there is always a warping or distortion happening because of residual stresses. And opening must be left in the cavity so that the support material can be removed.

Suppose you try to have a closed one, you try to have something close, so then inside it if you have supporting material it is too difficult for you to scoop out and remove the supporting material, so it is always a good idea you keep open surface, so you remove the supporting structure once your component is built.



(Refer Slide Time: 29:09)




So, improving the strength in the Z direction, this is a direction loading, so the bond between a molten layer and a solidified layer is much weaker than the bond between molten beads within the same layer. I think you should be able to visualize, one layer, second layer, between this layer the bond is very weak as compared to within this layer.

So, the bond between a molten layer and a solidified layer is much weaker than the bond between molten beads within the same layer. A printed part with the layers stacked vertical, a load applied in the line with the layer bond and the result of the load, so when it is this direction nothing will happen, when you try to rotate it and apply load you can see there is a sharing happen. So, improving strength in the Z direction is also very important. This tries to help in orienting the part while printing.

(Refer Slide Time: 30:14)

TOPOLOGY OPTIMIZATION KEY CHARACTERISTICS	
Process-specific characteristics	Component-specific characteristics
•Material	•Geometry
•Component orientation	•Dimensional accuracy
•Fiber orientation	•Roughness
•Build strategy	•Density
•Support strategy	•Strength
•Layer thickness	•Stiffness
•Nozzle/beam diameter	•Color
•Temperature/power	•Transparency



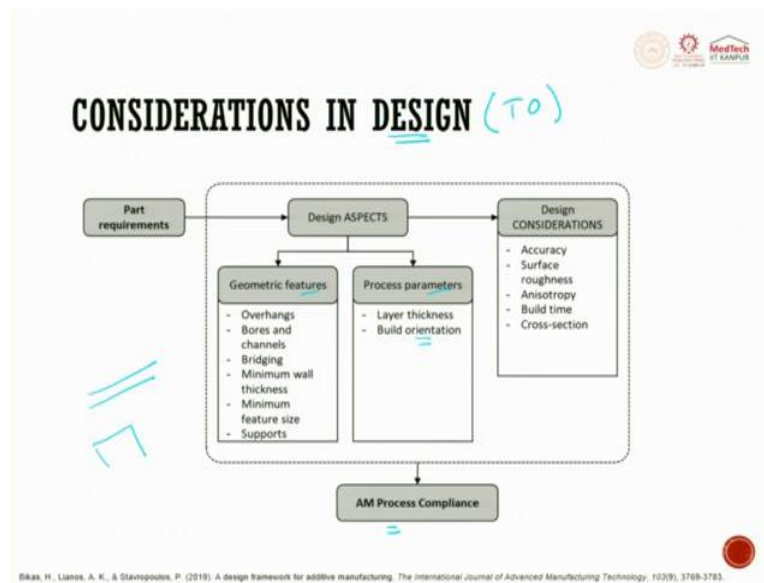
So, some of the topology optimization key characteristics are material, component orientation, fiber orientation, build strategy, support strategy, layer thickness, nozzle beam diameter, temperature/power. So, all these things are process specific characteristics where in which topology optimization can be done or while doing topology optimization these constraints can be added.

When we talk about component specific characteristics, the geometry of the component we can do topology optimization. Next is dimensional accuracy. The Moment, you wanted to have a higher dimensional accuracy, the infill within the layer and the supporting structure both help in playing a very important role.

For example, if you have a component like this, the first layer you are printing immediately, what will happen is, you will have a sag which happens in this direction. In order to avoid you try to do, from the table level you, this is a table, with a support structure, this is the part. You try to have this such that this fellow does not sag.

In order to have better dimensional accuracy, better roughness, better density, strength, stiffness, color and transparency we do lot of topology optimization. So, as I told, you should define a domain space, in that domain space you will define the constraints and with this constraints you will try to have an objective function. You start working on the objective function to solve the optimization techniques.

(Refer Slide Time: 32:12)



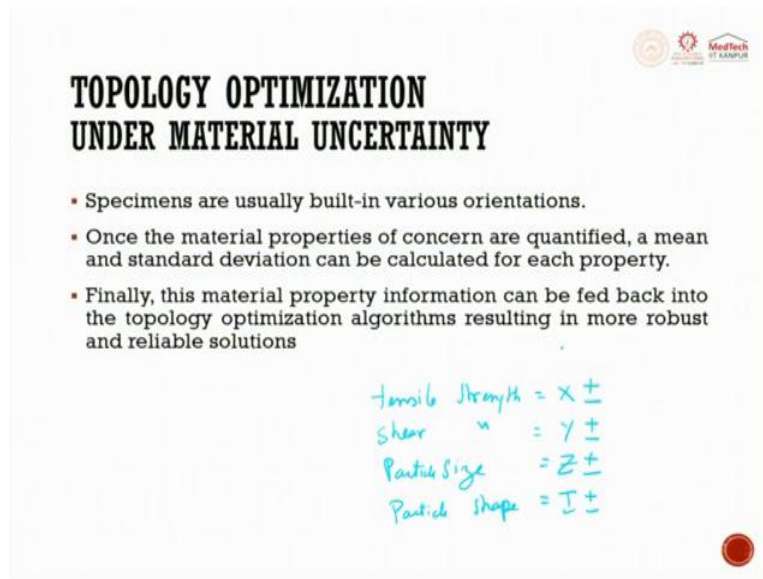
So, some of the considerations in design are the part requirement in the final use, the design aspects and the design consideration. When we talk about design consideration, accuracy, surface roughness anisotropy, build time, cross section; all these things come into existence. When we try to talk about design aspects we have two classifications, one is geometric feature, the other one is process parameter.

In the geometric features, you have overhangs, bores and challenge, bores means holes, holes and straight channels, then we have bridges, minimum wall thickness which we saw, minimum feature size and supports. These are all the geometric features where it has to be considered in design. And here I mean to say in topology optimization. When we talk about process it is going to be the layer thickness and build orientation.

Build orientation, this is very important. Build orientation affects the accuracy, build orientation affects the surface roughness and anisotropy, build time, cross section; layer thickness affects the accuracy, surface roughness, anisotropic behavior, build time and cross section area. So, if we do all these things, we try to take this into AM process compliance and we try to produce it.

The requirements from the customer will not change, he will say a shaft has to be made, he will say a shaft is to be rotated and a bracket has to be made. So now taking that requirement, we have to do all these things such that we make a cost effective good quality part.

(Refer Slide Time: 34:02)



**TOPOLOGY OPTIMIZATION  
UNDER MATERIAL UNCERTAINTY**

- Specimens are usually built-in various orientations.
- Once the material properties of concern are quantified, a mean and standard deviation can be calculated for each property.
- Finally, this material property information can be fed back into the topology optimization algorithms resulting in more robust and reliable solutions

Handwritten notes in blue ink:

$$\begin{aligned}\text{Tensile Strength} &= X \pm \\ \text{Shear} &= Y \pm \\ \text{Particle Size} &= Z \pm \\ \text{Particle Shape} &= I \pm\end{aligned}$$

The topology optimization under material uncertainty. The specimens are usually built in various orientation. Once the material properties of concern are quantified, a mean and a standard deviation can be calculated for each property. For example, tensile strength will be  $X (+/-)$ , so that is what they said, mean and standard deviation. They will say shear strength,  $Y (+/-)$ , then they will say particle size  $Z (+/-)$ .

And then, they try to say particle shape, they say some  $I (+/-)$ , so when they say it as a quantitative value, but really speaking you can try to see whether it is an ellipse sphere or something like, the aspect ratio they define. So once the material properties of concern are quantified within this limit a mean and a standard deviation can be calculated for each property.

Finally, the material property information can be fed back into the topology optimization algorithms resulting in more robust and reliable solutions. This is very, very important. So, we try to take it and feed it into topology optimization algorithms.

(Refer Slide Time: 35:39)

**TOPOLOGY OPTIMIZATION  
UNDER MANUFACTURING UNCERTAINTY**

- Manufacturing process will always deviate from the design, even if that deviation is minimal.
- There is a need to develop approaches to tackle material shrinkage during the AM build.
- Significant residual stress buildup during the build.
- Significant “spring back” distortions while removing parts from the build platform.
- In many cases, topology optimized geometries are inaccessible for post-machining

*hole = 6 mm (design)  
= 6 ± 0.1 mm (manuf.)*

Manufacturing process will always deviate from the design, even if that deviation is minimal, it is accepted. We always try to make a hole of 6 millimeter diameter, but when you manufacture, this is designed, but when you manufacture it always becomes plus or minus 0.1 millimeter. This is manufacturing. Manufacturing process will always deviate from the design, even if that deviation is minimal it is accepted.

There is a need to develop approaches to tackle material shrinkage during the AM build. So let it be support structure or let it be the original part, the layer thickness, all these things take care by the material shrinkage. Significant residual stress built up during the period is very important. Significant spring back distortion which is quite common in metal forming is a feature also here.

This spring back is because of the thermal stresses which are getting induced and this can be released while you do in the post processing heat treatment. Significant spring back distortion while removing parts from the built platform is very, very important. After building it, it gets destroyed. In many cases topology optimized geometries are inaccessible for post machining.

Topology optimization can be done only in the CAD state and getting all the material manufacturing uncertainty, you try to pitch it into and then try to get the output.


(Refer Slide Time: 37:25)

**LAYER TIME LIMITATIONS**

- The thermal properties of a material dictate layer time limitations and some aspects of design
- Can lead to thermal stresses and geometric distortions.

Feed rates  $\rightarrow \uparrow$   
need a time for layer cooling.

TO  $\rightarrow$  design  
T<sub>0</sub>  $\rightarrow$  design + manufacturing.



Layers drooping and collapsing caused by insufficient layer cooling time.

Layer Time Limitation - The thermal properties of a material dictate layer time limitation and some aspects of design. Layers drooping and collapsing caused by insufficient layer cooling time. This is very important. So what I am trying to say is the feed rates, you cannot keep on be increasing, because you need a time for layer cooling, you need a time for layer cooling and so this is also put in the optimization.

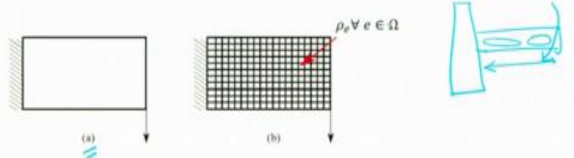
So topology optimization, you were thinking the topology optimization is only for design, no, topology optimization in the constraints you can have design and manufacturing. So what is manufacturing? This is manufacturing layer time limitation. What should be the layer thickness, what should be the way in which the layer is built? That means to say time and the solidification, otherwise what will happen?

This will be, if you give a very small solidification time the layer will never become a solid. If the layer does not become a solid, no point in further printing because it will sag.

(Refer Slide Time: 38:53)

## TOPOLOGY OPTIMIZATION PROBLEM FORMULATION

- **2D EXAMPLE FOR TO OF A CANTILEVER BEAM**
  - a) Design domain  $\Omega$  with kinematic loading conditions, i.e. point load at lower right vertex, while left edge remains fixed.
  - b) Discretised domain (schematic, the actual mesh is finer) with each finite element assigned a design variable  $\rho_e$  (subscript 'e' denotes the element number).
- **Aim is to find the optimal layout with densities that are either zero or one, such that the structure has maximum stiffness**




So, topology optimization problem formulation, one simple example I have taken. Let us do that for a 2D example for a topology optimization of a cantilever beam. So, design domain with kinematic loading condition, that is point load at lower vertex, while the left edge remains fixed. Next, discretize domain, the schematic is shown here, discretize the domain, the actual mesh is finer, with each finite element assigned a design variable  $\rho_e$  where subscript 'e' denotes the element number.

So, the aim is to find the optimal layout with densities that are either 0 or 1, such that the structure has maximum stiffness. So, what I am trying to do here is I wanted to make a cantilever. In this cantilever whatever is the projection, this projection should not sag and parallelly it should also be made in such a way, such that the material is removed, with respect to shape optimization.

The aim is to find the optimal layout with densities that are either 0 or 1 such that the structure has maximum stiffness. So, I remove the material, but still it has a maximum stiffness. So, this is the design domain with kinematic loading condition is this, and the discretized domain is this,  $\rho_e$  for every  $e$  in the, within the domain  $\Omega$ .



(Refer Slide Time: 40:55)



## TOPOLOGY OPTIMIZATION PROBLEM FORMULATION

- Solid Isotropic Material With Penalization (SIMP) approach in combination with finite element analysis (FEA) is used for numerical computation of the optimization objective and constraints.


$$\min : c(\rho) = F^T U = U^T K U = \sum_{e=1}^N (\rho_e)^p u_e^T k_e u_e, \quad (1)$$

Which is subjected to  $KU = F, \quad (2)$

Equilibrium equation for the discretized FE problem is given by equation (2)

The SIMP penalization is imposed as a power law on the density values given by the penalty factor  $p > 1$ , as shown in equation (1)

- $\rho$  is the vector of all the design variables  $\rho_e$ .
- $U$  and  $F$  are the nodal degrees of freedom and load vectors respectively.
- $K$  is the global stiffness matrix.
- $u_e$  and  $k_e$  are the element displacement vector and stiffness matrix respectively.
- $N$  is the total number of elements.




The solid isotropic material with penalization, it is called as a SIMP approach. Several approaches, I have taken one for this example is SIMP - Solid Isotropic Material with Penalization approach in combination with finite element analysis is used for numerical computation of the optimization objective and constraint. This is the objective, this is a constraint which is given.

So, when the equilibrium equation for the discretized finite element problem is given by the equation, the SIMP penalization is imposed as a power law on the density values given by the penalty factor  $p$  is greater than 1 as shown in the equation one, where  $\rho$  is a vector of all the design variables  $\rho_e$ . I have said what is  $e$  in this.  $U$  and  $F$  are the nodal degrees of freedom and load vectors respectively.

$K$  is a global stiffness matrix,  $U$  and  $K_e$  are the element displacement vector and stiffness matrix respectively.  $N$  is the total number of elements. By solving this equation by this manner you will try to get the final equation.



(Refer Slide Time: 42:26)




## TOPOLOGY OPTIMIZATION PROBLEM FORMULATION

- Next step is the calculation of sensitivities i.e. the design derivatives of objective  $c$  and constraint  $g(\rho)$  with respect to design variable  $\rho_e$ .

$$g(\rho) = \frac{V(\rho)}{V_0} - f \leq 0, \quad (3)$$
$$0 < \rho_{\min} \leq \rho_e \leq 1, \quad \forall e \in \Omega, \quad (4)$$

- $\rho_{\min}$  is the minimum physical density (non-zero to avoid singularity of  $K$ ).
- $V$  and  $V_0$  are the volume of solid region and design domain respectively
- $f$  is the prescribed maximum volume fraction of material allowed in the design domain.

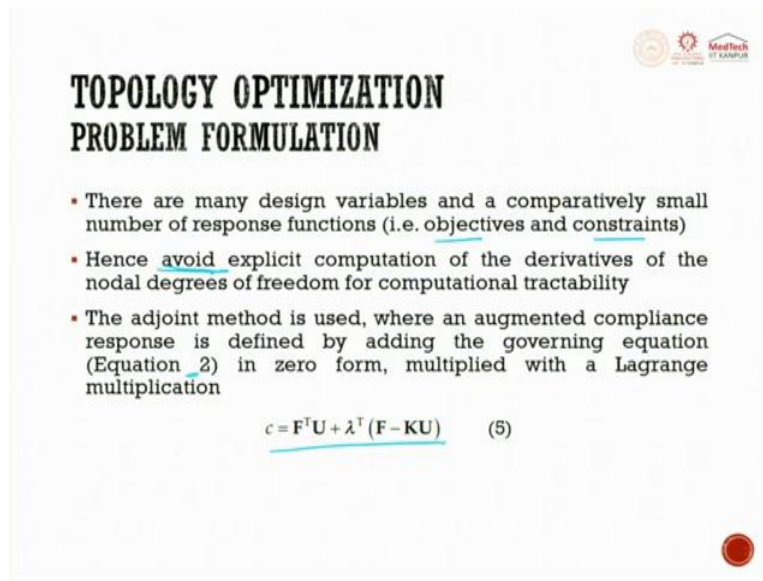
The design derivative of Equation (3) can be easily found as it depends directly on design variable  $\rho_e$ , but for Equation (1), the derivatives of the nodal degrees of freedom with respect to the design variables are required when direct differentiation is applied



Now once you do that step the next step is the calculation of sensitivity that is the design derivatives of the objective  $c$  and the constraint  $g(\rho)$  with respect to a design variable  $\rho_e$ . So this is equation number three and equation number four, where  $\rho_{\min}$  is the minimum physical density,  $V$  and  $V_0$  are the volume of the solid regions and design domain respectively,  $f$  is the prescribed maximum volume fraction of material allowed in the design domain. So, this is a maximum material.

You can put a constraint on the material also. So, the design derivatives of equation three can be easily found out as it depends directly on the design  $\rho_e$ . Put from the equation one which we saw in the previous slide, the derivatives of the nodal degree of freedom with respect to the density variables are required, when direct differentiation is applied.

(Refer Slide Time: 43:39)




The slide is titled "TOPOLOGY OPTIMIZATION PROBLEM FORMULATION". It contains three bullet points and an equation. The first bullet point states: "There are many design variables and a comparatively small number of response functions (i.e. objectives and constraints)". The second bullet point states: "Hence avoid explicit computation of the derivatives of the nodal degrees of freedom for computational tractability". The third bullet point states: "The adjoint method is used, where an augmented compliance response is defined by adding the governing equation (Equation 2) in zero form, multiplied with a Lagrange multiplication". Below the bullet points is the equation: 
$$c = F^T U + \lambda^T (F - KU) \quad (5)$$

There are many design variables and a comparatively smaller number of response functions, so that is objectives and constraints. Writing the objective equation and the constraint equation is a challenge. If you truly understand the problem only you can easily write, otherwise if you make a wrong choice I am sure you will not land up with the good output what you require.

Hence, avoid explicit computation of the derivatives of the nodal degrees of freedom for computational traceability, avoid. So the adjoint method is used when an augmented compliance response is defined by adding the governing equation 2 in 0 form. Let us see 0 form, 2 in 0 form and then in the 0 form multiplied with the Lagrange multiplication you get this,

$$c = F^T \cdot U + \lambda^T (F - KU)$$

(Refer Slide Time: 44:40)



## TOPOLOGY OPTIMIZATION PROBLEM FORMULATION

- Next, Equation (5) is differentiated with respect to the design variable  $\rho_e$  and the terms containing the derivative with respect to displacements are collected

$$\frac{\partial c}{\partial \rho_e} = (\mathbf{F}^T - \lambda^T \mathbf{K}) \frac{\partial \mathbf{U}}{\partial \rho_e} - \lambda^T \frac{\partial \mathbf{K}}{\partial \rho_e} \mathbf{U} \quad (6)$$

- Equation (6) can be simplified to

$$\frac{\partial c}{\partial \rho_e} = -\lambda^T \frac{\partial \mathbf{K}}{\partial \rho_e} \mathbf{U}, \quad (7)$$


- such that  $\lambda$  satisfies the adjoint equation KV

$$\mathbf{F}^T - \lambda^T \mathbf{K} = 0. \quad (8)$$

Equation (8) resembles the form of equilibrium Equation(2)

Next, the equation five is differentiated with respect to the design variable rho e and the term containing the derivatives with respect to the displacement are given in this. So, it is a simple math, so I am not getting into it. So, equation six, then you simplify equation six, then you try to put the lambda which satisfies the adjoining equation and then you try to get equation eight resembles the form of an equilibrium equation which we discussed in the stiffness matrix.

(Refer Slide Time: 45:12)




## TOPOLOGY OPTIMIZATION PROBLEM FORMULATION

- For the choice  $\lambda = \mathbf{U}$ , the displacement derivatives vanish from Equation (6)
- Hence, the sensitivity of the objective with respect to the design variable can be efficiently evaluated using

$$\frac{\partial c}{\partial \rho_e} = -\mathbf{U}^T \frac{\partial \mathbf{K}}{\partial \rho_e} \mathbf{U}$$

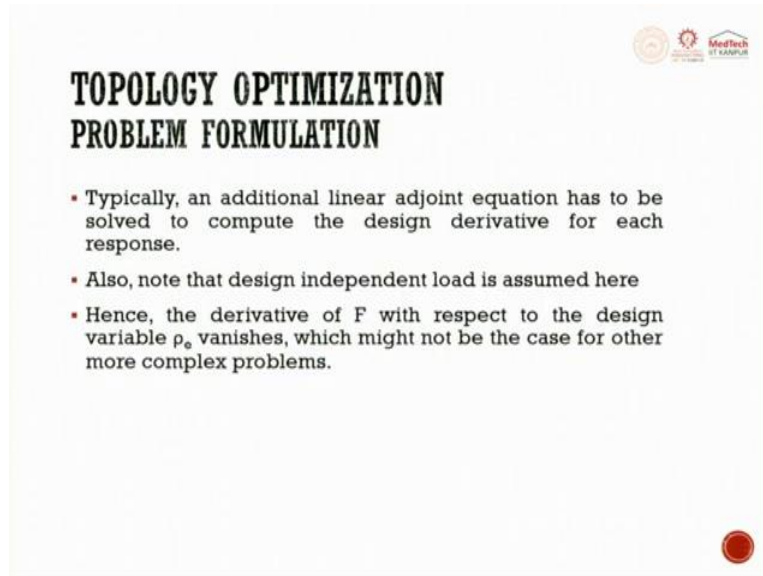
- The final solution to the problem is shown in figure given below



From the choice  $\lambda = \mathbf{U}$ , the displacement derivatives vanishes from the equation 6. Hence, the sensitivity of the objective with respect to the design variable can be efficiently evaluated using this equation. So, the final solution to the problem is shown in this figure given below.

What was thought of like this, now it is getting defined like this, to do the same function reduced material better performance.

(Refer Slide Time: 45:45)

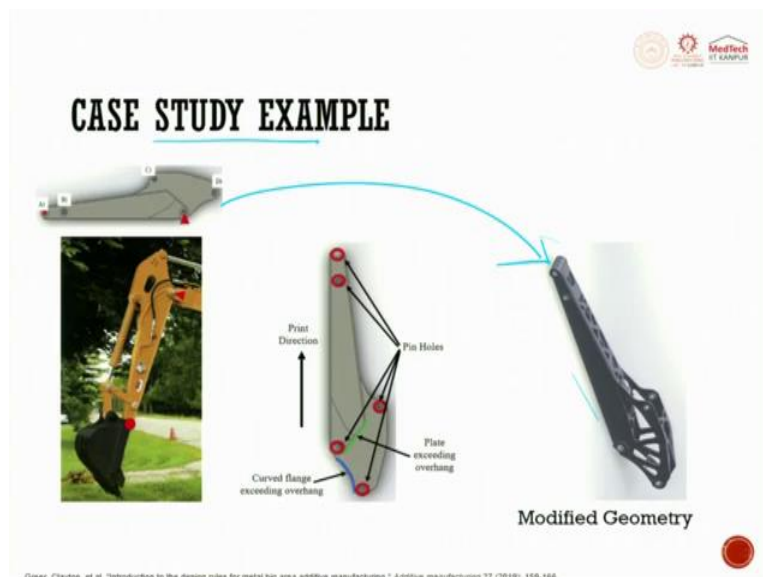


**TOPOLOGY OPTIMIZATION  
PROBLEM FORMULATION**

- Typically, an additional linear adjoint equation has to be solved to compute the design derivative for each response.
- Also, note that design independent load is assumed here
- Hence, the derivative of  $F$  with respect to the design variable  $\rho_e$  vanishes, which might not be the case for other more complex problems.

Typically, an additional linear adjoining equation has to be solved to compute the design derivatives for each response. Also, note the design independent load is assumed here. Hence, the derivative of  $F$  with respect to the design variable  $\rho_e$  vanishes,  $e$  is the element, which might not be the case for other more complex problems.

(Refer Slide Time: 46:13)



**CASE STUDY EXAMPLE**

The slide illustrates a design optimization process for an excavator arm. It shows a standard excavator arm on the left, a middle diagram with annotations for 'Print Direction', 'Pin Holes', 'Flare exceeding overhang', and 'Curved flange exceeding overhang', and a 'Modified Geometry' on the right. A blue arrow indicates the transition from the standard arm to the modified geometry.

Greer, Clayton, et al. "Introduction to the design rules for metal big area additive manufacturing." Additive manufacturing 27 (2019): 155-166.

So, whatever was done like this now which was very commonly used like this finds an application, this can be replaced like this. So, you have pin holes, print direction, you have

curved flange exceeding overhang, exceeding over, it also tells you the red portion, green portion, where you will have manufacturing difficulty, plate exceeding overhang.

It also tells all the information and then what you do is you try to take all these things into consideration and the modified geometry looks like this. You see here this geometry is converted into this. If you do not do topology optimization for your component the fullest success of Metal Additive Manufacturing cannot be visualized or enjoyed.

Today, people just replace the sheet metal part or the manufactured part, the complete part with additive manufacturing. So here the cost benefit is not achieved in a big way. Now when you do this topology optimization you see lot of material getting removed and the arm getting stiffened properly, such that it tries to take the toughness, deflection and the loads very easily.

(Refer Slide Time: 47:38)

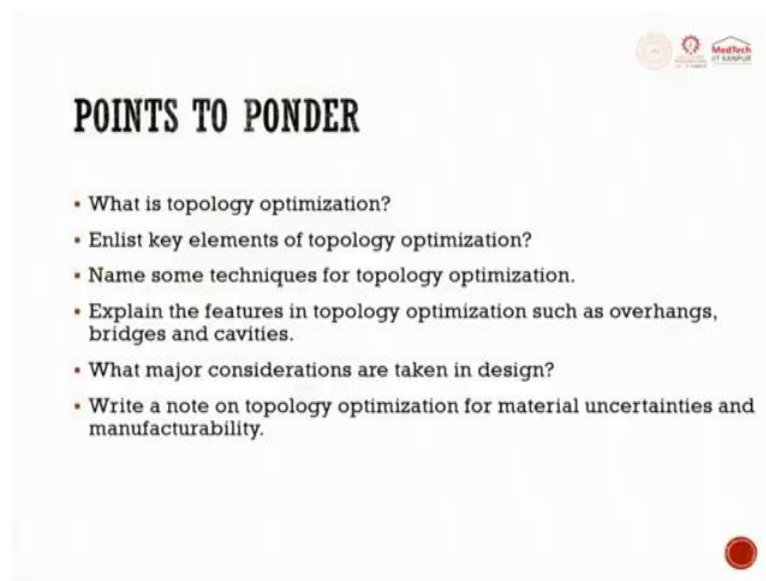


So, what are all the challenges? Though it is said easy, practicing is extremely difficult. It needs lot of understanding. So, challenges in topology optimization are, effective properties prediction of the lattice structure is a big challenge. Effective property means, so you have various lattice structures, like this, then you can have hexagon, their effective property prediction is not available, that means to say that data is not available.

Material anisotropy in additive manufacturing, it also leads to a difficult challenge while doing topology optimization. This is okay when you use a simple geometry, when you have an overhanging or a slant overhang, so then it is very difficult and on top of it anisotropic component getting inbuilt with respect to orientation.

Structural fatigue performance in additive manufacturing, that information is not available in public domain because it is still in a research stage. The design and manufacturing of functionally graded materials is also immaterial, the knowledge is not available or the information is not available, these are the challenges which are faced in topology optimization for today. Because of this difficulty, people try to bypass this topology optimization while doing Metal Additive Manufacturing.

(Refer Slide Time: 49:02)



**POINTS TO PONDER**

- What is topology optimization?
- Enlist key elements of topology optimization?
- Name some techniques for topology optimization.
- Explain the features in topology optimization such as overhangs, bridges and cavities.
- What major considerations are taken in design?
- Write a note on topology optimization for material uncertainties and manufacturability.

Points to Ponder - In this lecture we saw, what is topology optimization and list of key elements of topology optimization. Some of the techniques for topology optimization with respect to support and material. Then explain the feature of topology optimization such as overhang, bridge and cavities. Next, what major considerations are to be taken in design; then write a note on topology optimization of material uncertainty and manufacturing.

(Refer Slide Time: 49:36)

The slide is titled "ASSIGNMENT" in a bold, black, serif font. It contains two bullet points: "▪ Design a topology-optimized bracket for aerospace application and show the difference from initial to the final design." and "▪ List major benefits of design." In the top right corner, there are three logos: a circular one, a square one, and a red one with the text "MedTech". In the bottom right corner, there is a red circle.

## ASSIGNMENT

- Design a topology-optimized bracket for aerospace application and show the difference from initial to the final design.
- List major benefits of design.

So, assignments, like every other lecture you have to do the assignment, need not submit, but try to do the assignment such that you can enjoy this lecture whatever you attended. Design a topology optimized bracket for an aerospace application and show the difference from the initial and final. Try to draw a small simple bracket which is used in aerospace or in automobile, draw it and then you try to look at topology optimization.

If you find it difficult in doing this problem try to look forward for research publication, which is available in research gate or some other thing, try to pick up. Understand the complete research paper, so that will try to give more insight about topology optimization. The next one is list down major benefits of design while using topology optimization. Thank you very much.