Production Technology: Theory and Practice
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Lecture – 08
Forces in Machining

Hello and welcome back to the discussion sessions of the course on production technology: theory and practice. Let me remind you that in the last session, we started discussing the mechanics of metal cutting. I told you that the the first treatment to mechanics of metal cutting is given way back in 1944 by Merchant and Ernst.

Since then the Merchant's theory and the Merchant's circle diagram are popularly used in metal cutting to understand the mechanics of machining, to understand the inside of the metal cutting.

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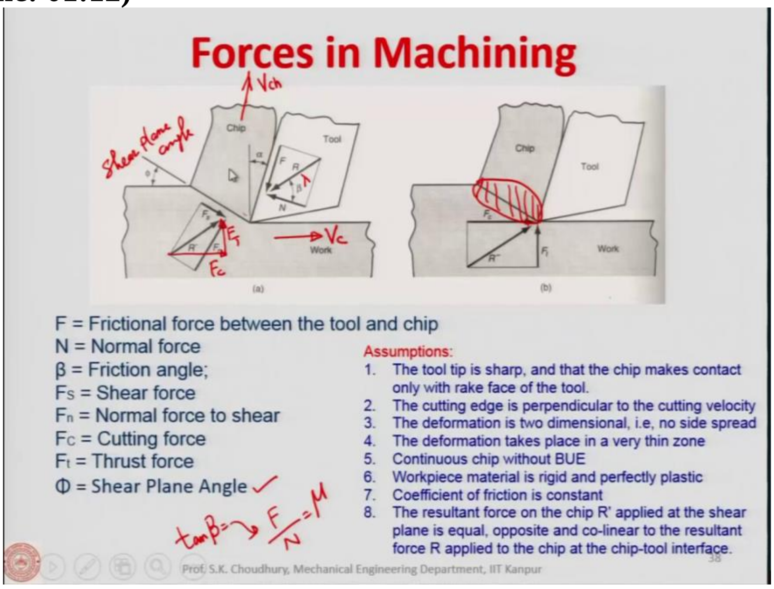

Here I will remind you that what we said is that for imparting shape, size, finish and accuracy to the workpiece, the excess material is removed from the workpiece in terms of small chips. When the chip moves over the rake face of the tool at a constant velocity, let us say V_{ch}, then the work piece material has to be rigid and perfectly plastic, i.e. homogeneous so that the chip could move at a constant velocity when the resultant forces acting on the chip from the tool side through the rake face and from the work piece side through the shear plane are equal opposite and collinear.

Then we have resolved the R and R' into the following components. R is resolved into 2 components, namely F and N, F is parallel to the rake face of the tool N is perpendicular to that. Friction force F is created because of the normal force, N. Similarly, the R' which is the resultant force acting on the chip from the work piece through the shear plane we have resolved into 2 more components, F_s and F_N.

F_s is parallel to the shear plane and F_N is normal to the shear plane. Then we said that if α or ϕ change, in that case all these components 4 components that is F, N, F_s and F_N will change the direction. So, we do not have any unique direction, they always change with the change in the ϕ and α.

Therefore, we resolve the R' into 2 more components which will not change their direction with the change in ϕ and α. One component, which is parallel to the V_c direction. That component since it will have the responsibility of power consumption, is called the cutting force because it is parallel to the cutting velocity and perpendicular to that is the thrust force which is also a resolved component of the R'.

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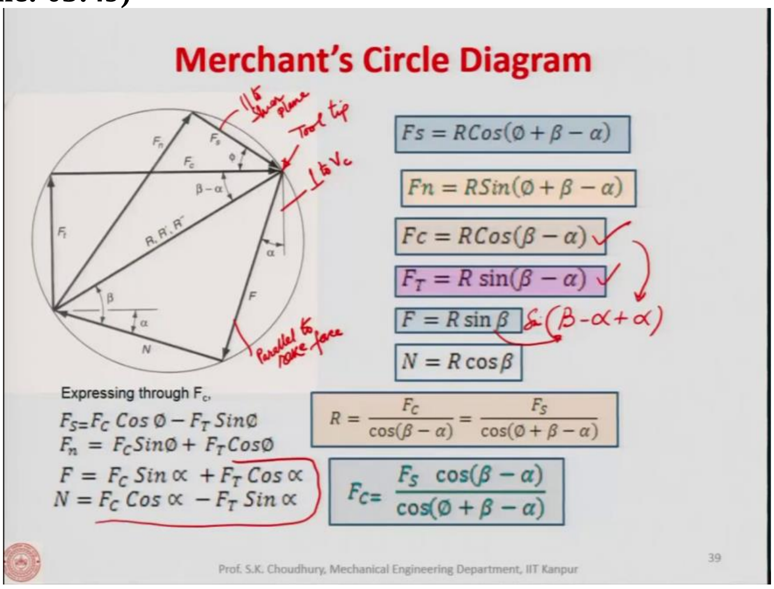

So, altogether we have 6 components and those 6 components can then be related keeping in mind these 8 assumptions that I have already told you. All those assumptions indicate that this treatment given or this model, Merchant's model, is 2-dimensional and this is for the orthogonal cutting not for the oblique cutting because all these assumptions are valid for only orthogonal cutting. In oblique cutting these assumptions are not valid.

They are correlated with the 3 angles, namely shear plane angle, rake angle and the friction angle and we said that all the forces could be converted through the F_c and F_t because we measured the forces through F_c and F_t only. then we said that the F_c can be found out through the F_s.

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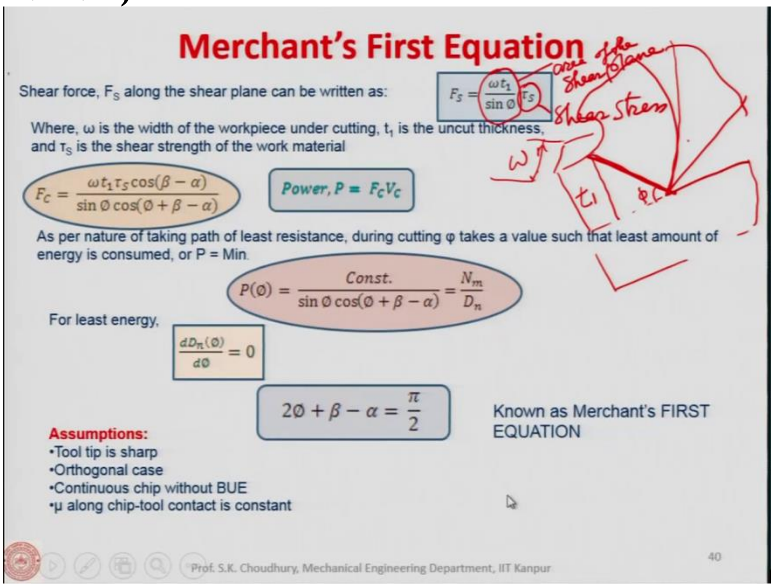

F_s is the shear force which is parallel to the shear plane. Therefore, shear force $F_s$along the shear plane can be written as the area into the shear plane area into the stress and this is the shear stress. The area is given by the width of cut, uncut thickness divided by $\sin\phi$. if you go back to this, In this exaggerated view, let us say this is the work piece. This is the tool.

Perpendicular to that is t_1, this angle is ϕ. If you see this is the width of the work piece. let us say this is the width of cut w. So, $\frac{wt_1}{\sin\phi}$ is the area of the shear plane. $\frac{t_1}{\sin\phi}$ is the length of the shear plane and this is the stress. Stress multiplied by area is the F_s. So, we are expressing the F_s through the area of the shear plane.

Let me write down here that this is the area of the shear plane and this is the shear stress which is applied to the workpiece during the cutting process where w is the width of the work piece under cutting this is the width and t_1 is uncut thickness, τ_s is the shear strength of the work material. This is equivalent to the shear stress that I said to you earlier also during our discussion.

In the expression of F_c, we are putting the value of $F_s = \frac{wt_1}{\sin\phi}$ to get this equation as shown in the slide. The power P, as I said, is equal to the product of F_c and the V_c, F_c is the cutting force component V_c is the cutting velocity which is πDN. Nature always takes the path of least resistance. Similarly, during the cutting process, ϕ takes the value such that least amount of energy is consumed or P is minimum.

Let me explain it to you that in nature everything always goes to the least resistance. If you remember we said that in case of the lattice structure for example, the movement of the defects, dislocations etc., everywhere the movement is to the least resistance where the resistance is less. So, in the case of metal cutting also during the metal cutting as per the nature, the cutting process itself adjusts the ϕ in such a way that minimum power is consumed for the process.

Like in nature, this phenomenon of using least energy happens in metal cutting as well. That means the power P will be minimum. Now, if we take the value of cutting force, F_c from the expression as shown in the slide and multiply that with the V_c, we will get the power, P. In this expression of power, all the terms in the numerator will be constant if you take the P as a function of ϕ. Meaning that as the ϕ is changing or α is changing, let us say here the ϕ is changing; we have taken as a function of ϕ.

So, here in the numerator it will be constant but the denominator will not because here we have the ϕ. So, if we take the power, P as a function of the ϕ, then this equation can be considered as the numerator divided by denominator, numerator being constant and the denominator is $\sin\phi\cos(\phi+\beta-\alpha)$. Now for the least energy, for P to be minimum, the first derivative of this equation has to be equal to 0.

And if you take the first derivative of this, because the numerator is constant, and for the power to be minimum, the denominator has to be maximum. Therefore, the first derivative of this expression should be equal to 0 and if you take that it will give you $2\phi+\beta-\alpha=\frac{\pi}{2}$ and this equation is known as the Merchant's first equation.

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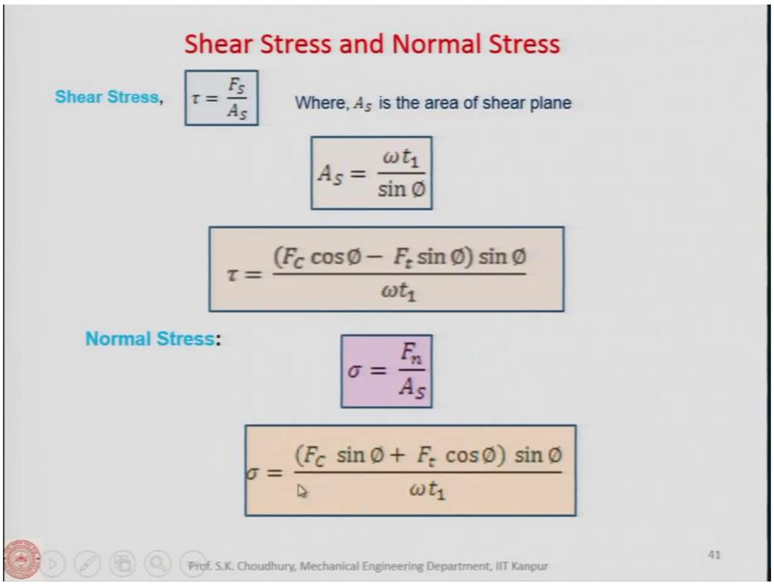

Now, from that shear stress, we know that this is the shear force divided by area of the shear plane and the area of shear plane we have already seen that this area of the shear plane is $\frac{wt_1}{\sin\phi}$. Therefore, the shear stress is equal to $\frac{F_s}{A_s}$. F_s we already found out as $F_s = F_c \cos\phi - F_T \sin\phi$. This expression divided by the area of the shear plane which is $\frac{wt_1}{\sin\phi}$ gives the expression of shear stress as shown in the slide.

Now, if you measure F_c and F_T and if you have the value of the ϕ, you can find out the value of the shear stress which is required because the w and the t_1 are the physical parameters; this is the width of the work piece and t_1 is uncut thickness; these are physical parameters. similar to shear stress, we can also find out the normal stress which is if it was $\frac{F_s}{A_s}$ in case of shear stress this will be $\frac{F_N}{A_s}$.

Because it is a normal stress divided by the same area of the shear plane since we are measuring along the shear plane or we are determining analytically what will be the shear stress and the normal stress along the shear plane. Therefore, here it will be the area of the shear plane. Now, put the value of the F_N from the Merchant's equations through the F_c and F_T and this results to this equation, as shown in the slide. So, by measuring F_c and F_T you can find out the value analytically by knowing the physical parameters and the shear plane angle.

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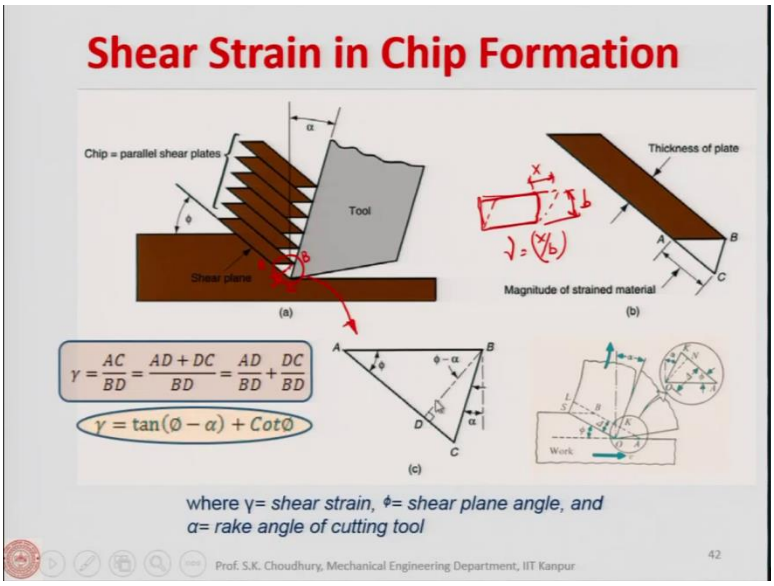

Shear strain in the chip information can be found out in the following way. Shear strain I have already shown it to you. I will once again show you that if this work piece has been strained to this way, let us say, this much is *x* and this much is *b* then the shear strain was equal to *x / b*. This I already told you earlier. In case of metal cutting the shear strain therefore, could be found out using this concept.

Chip which is flowing along the rack face of the tool can be assumed to be segregated, as if we are taking this into different slices. So, the entire chip we are dividing into various slices each one having a particular thickness. This is the kind of playing card pack. Each playing card has some thickness. That thickness is the same for all the playing cards.

Now when the force is applied, let us say you we have a stack of playing cards and you hold it like this and push from the top gently. Then what happens is that the whole stack will be sliding in the following way as it is shown here. Let us assume that it is equivalent to that stack of the playing card.

So, if we draw this and if we take this triangle, let us say we will consider this triangle here, then let us say this is *A*, this point is *A*, this point is *B* and this point is *C*. Now, from point *B*, we can take a perpendicular and this we have called as the *D*. If you see this first slice for example, this slice has been deformed up to this *AC*, if that tool had not been there, this chip could have slided up to this.

So, AC is the deformation which is equivalent to x in this diagram. AC is the amount of deformation that has taken place as this shear force is applied to the work piece. This is equivalent to x. Then D is the perpendicular from the point B on the AC Therefore, this is the shortest distance and the distance between these 2 layers will be this value, this is the BD.

So, BD will be equivalent to b here. Then the shear strain from this diagram could be said as equal to $\frac{AC}{BD}$ because AC here in this case x and BD is the b. So, shear strain is x / b which is $\frac{AC}{BD}$ and then you can find out that AC is equal to $CD + AD$. So, it will be $\left(\frac{AD}{BD}+\frac{DC}{BD}\right)$. From the geometry, finally we get the value of strain as, $\gamma = \tan(\phi-\alpha)+\cot\phi$.

Overall, we will find out that the shear strain is equal to $\gamma = \tan(\phi-\alpha)+\cot\phi$. This concept is the same as shown here that if this is the layer with the thickness of delta and this layer could have gone to this much had the tool not been there. We are considering this triangle. This is the triangle and from here we are putting a perpendicular to the AK. Here also you will see that the AK is equivalent to this AC or equivalent to x.

In this case the shear strain will be $\gamma = \frac{AK}{ON} = \frac{AN+KN}{ON} = \tan(\phi-\alpha)+\cot\phi$. So, it will be ultimately the same expression. This is just to understand that how we can make the equivalent of this diagram in case of metal cutting.

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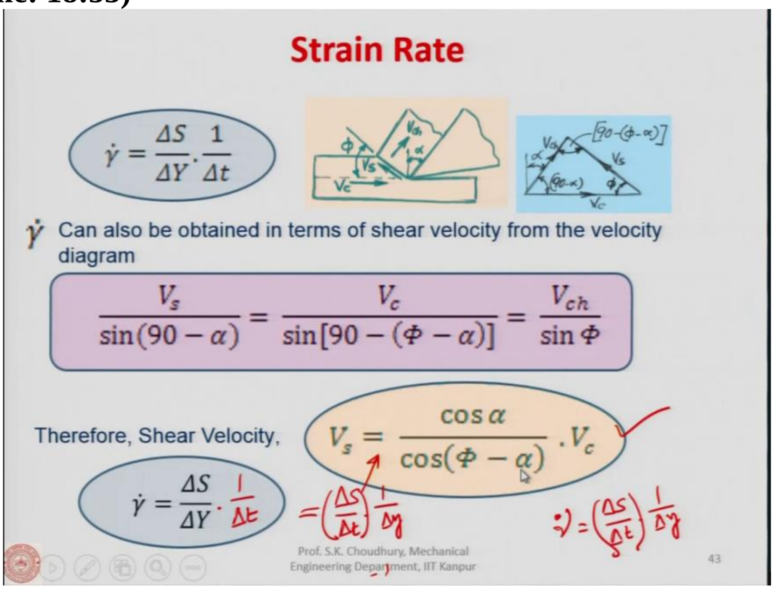

The strain rate involves time that I told you during the discussion on material. Therefore, the strain rate is $\dot{\gamma} = \frac{\Delta s}{\Delta y}\frac{1}{\Delta t}$ Shear strain rate can also be obtained in terms of shear velocity from the velocity diagram. If we see from this diagram, this is the diagram. This is the shear plane angle and this is the cutting velocity. This is the shear velocity and this is chip velocity.

It will be difficult to find out the value of the Δs, Δy and $\frac{1}{\Delta t}$. So, we take the help of the velocity diagram. Here, the V_{ch} ,V_c and the V_s they can be coupled in this way because you see that the V_c is in the direction of cutting velocity, V_s is in the direction of shear plane and V_{ch} is in the direction of chip flow and these are the angles. This angle will be α that is the rake angle, this angle will be the shear plane angle between the V_s and the V_c and so on.

Using the sine rule we can write the following, $\frac{V_s}{\sin(90-\alpha)} = \frac{V_c}{\sin\left[90-(\phi-\alpha)\right]} = \frac{V_{ch}}{\sin\phi}$ From here we can find out the shear velocity, $V_s = \frac{\cos\alpha}{\cos(\phi-\alpha)}V_c$. This value can be determined since V_c and α will be known to us and the value of shear plane angle, ϕ can be analytically determined.

The shear strain rate we can express as $\dot{\gamma} = \frac{\Delta s}{\Delta t}\frac{1}{\Delta y}$, where $\frac{\Delta s}{\Delta t}$ is the shear velocity, V_s. So, for any given layer thickness, Δy what will be the $\frac{\Delta s}{\Delta t}$ or V_s will define the strain rate.

Therefore, if we know the shear plane angle, ϕ, α will be known because we are using a tool and for that tool the rake angle we have given according to our choice. I have already discussed with you that depending on the work piece material hardness or the tool material, we have to find out the value of the rake angle, α .

So, if we can find out the ϕ which is the shear plane angle, we can analytically find out the value of the shear strain, as you can see, we can find out the value of all these factors, i.e. shear stress, normal stress and the analytical values of the F_s, F_N etc.

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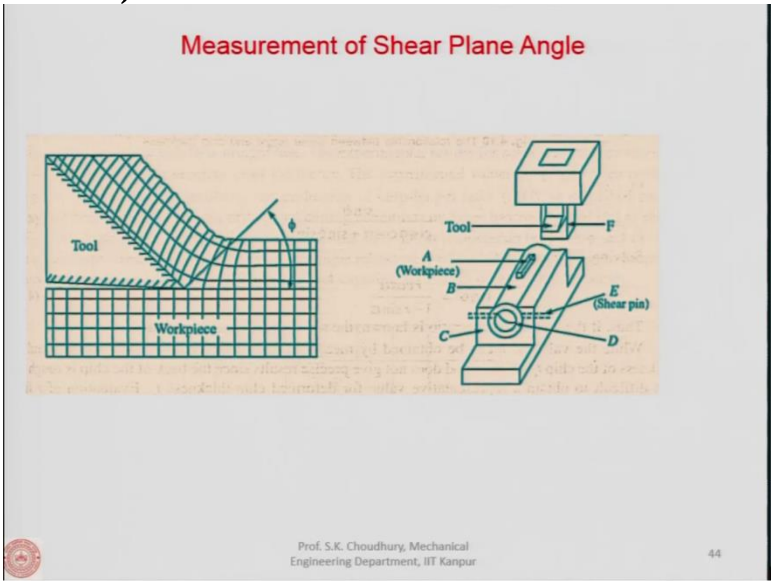

There are 2 ways to find out the shear plane angle; one way is shown here. This is experimental procedure, that means there is a device which is called the quick stop mechanism. And on the quick stop mechanism there is a cylinder which can actually move freely and the movement is stopped from the front by a shear pin and on this cylinder work piece is mounted here on this slide.

At the back of the workpiece there is a tool with the tool post and it moves forward. The tool is at the another level which will remove the work piece material from here, but this is the tongue behind the tool. When the tool is removing the material, after sometime, this tongue, located at the back of the tool, will starts contacting with the cylinder.

The cylinder is loosely fitted on this semi-circular groove. it can be stopped from moving forward by the shear pin. So, when the tool has started removing material then this tongue will come from behind and it will hit this cylinder and the cylinder will move forward breaking the shear pin. As a result, what will happen is that the machining process will be stopped and we will get the work piece with partially made chip.

This is important for further measurement of the shear plane angle to get a clear picture of the of the chip. The work piece is graduated as shown in the slide. These grids are painted on the work piece. These grids, these lines will be deformed which are painted on the work piece.

Then whenever it started getting deformed, these are not deformed because the plastic deformation not taking place here, but at this point that is along the shear plane it will be deformed and you will clearly see the points along which the material started deforming. If you join them with the tip of the tool, you will get a straight line and that straight line makes an angle with the cutting velocity vector which is equal to the shear plane angle.

Of course, , there are difficulties and is inaccuracy in this measurement. You have to exactly stop the machining process so that a clear picture can be found out when the chip is being formed. The second error or the inaccuracy creeps in when these points are to be connected with the tool tip because otherwise you will not get the accurate value of this angle. Because of that fact the measurement of shear plane angle is not used popularly in practice and the value of the shear plane angle is determined analytically.

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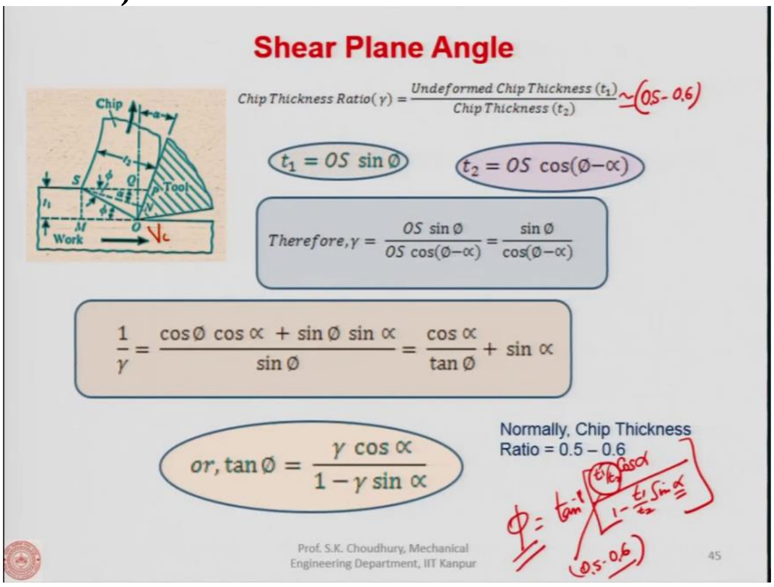

Let us see how this shear plane angle can be analytically found out. If you see this diagram, there is a concept of chip thickness ratio that is given by the ratio of $\left(\frac{t_1}{t_2}\right)$. where t_1 is the undeformed thickness. t_2 is the chip thickness which can be measured from the chip itself. This one is uncut thickness.

This is the chip thickness. The ratio of the $\left(\frac{t_1}{t_2}\right)$ is given as the chip thickness ratio. In practice it has been seen that this is roughly about 0.5 to 0.6. That means from here the conclusion is that first of all, after the plastic deformation along the shear plane, OS, the chip thickness ratio is 0.5 - 0.6.

That means, the chip is expanded by 50 to 60%, than the uncut thickness. From this triangle shown in the slide, let us say, SN is perpendicular to the rake face of the tool and SP is the extension of this line. This will be then equivalent to the or parallel to the V_c.

This is the direction of the cutting velocity vector, V_c . Therefore, if this angle is α between the rake face and the line perpendicular to cutting velocity vector, then perpendicular to this line, that means, this line SN is perpendicular to rake face of the tool and SP is perpendicular to this line that means, it is parallel to the V_c. Therefore, between SP and the SN the angle will also be the rake angle. Between the SO and the direction of the cutting velocity vector is the shear plane angle.

. So, from this triangle, you can see that the t_1 is equal to $OS \sin\phi$. From this triangle *SNP* you can find out that know from SON this triangle SO and the N you can find out that OS is equal to $\frac{t_2}{\cos(\phi-\alpha)}$.

Therefore, the chip thickness ratio will be $\frac{t_1}{t_2} = \frac{\sin\phi}{\cos(\phi-\alpha)}$ Then to get the reverse of this chip thickness ratio, we are inverting them and opening that $\cos(\phi-\alpha)$ as shown in the slide. So, this will be then $\frac{\cos\phi\cos\alpha+\sin\phi\sin\alpha}{\sin\phi}$ = $\frac{\cos\alpha}{\tan\phi}+\sin\alpha$ because $\sin\phi$ is getting cancelled here. from here you can find out the $\tan\phi$ analytically that this is equal to $\frac{\gamma\cos\alpha}{1-\gamma\sin\alpha}$.

So, the ϕ can be then analytically determined that this is $\phi=\tan^{-1}\left[\frac{\gamma\cos\alpha}{1-\gamma\sin\alpha}\right]$. α is known to us because it is the rake angle. This is the tool that we are using. If we take the value of $\gamma=\frac{t_1}{t_2}=0.5-0.6$ which is experimentally found out, in that case analytically we can find out the value of the ϕ.

This is important because once we find out analytically the value of the shear plane angle in that case many of the other parameters we can also find out analytically without having the actual machining process or actual machine. This is the idea here that how to estimate the shear plane angle and through that how to estimate the stress strain, strain rate and so on.

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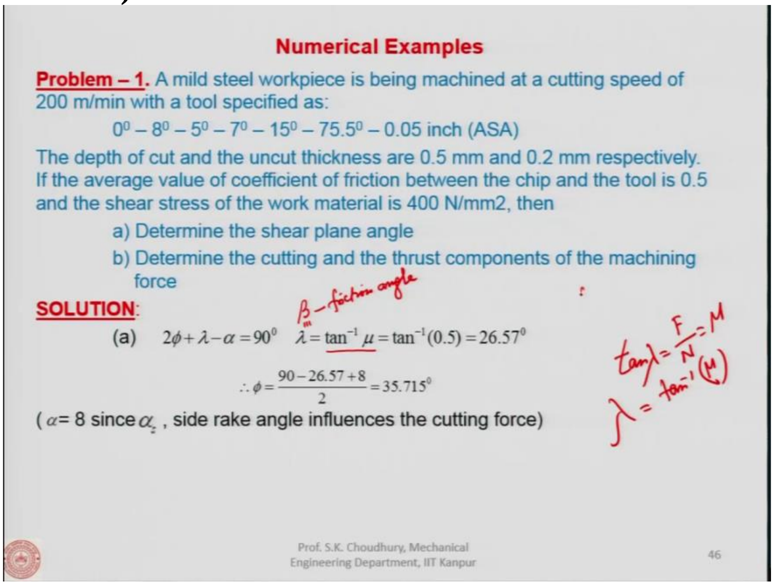

Let us discuss few numerical examples which will be the direct implementation of whatever we have discussed. Let us say we have a problem that a mild steel workpiece is being machined at a cutting speed of 200 *m/min* with the tools specified as 0^0 - 8^0 - 5^0 - and so on. it is in ASA. In ASA, I will remind you that if it is given in ASA you have to understand that this is back rake angle, followed by side rake angle, next are the flank angles and so on.

This is the flank angle and these are the cutting edge angles, γ_s and γ_e. The depth of cut and uncut thickness are 0.5 mm and 0.2 mm respectively; if the average value of coefficient of friction between the chip and the tool given is 0.5 and the shear stress of the work material is 400 *N/mm²*, then determine the shear plane angle and determine the cutting and the thrust components of the machining force. So, first we have to find out the ϕ.

To find out the ϕ, we have the formula Merchant's formula $2\phi+\lambda-\alpha=\frac{\pi}{2}$. Here, the λ can be found out because this is given as 0.5, the average value of coefficient of friction. Mind it, λ we have shown as β when we derived the Merchant's equation. These are the same. This is the friction angle, friction angle either λ or what we have seen as β. So, this will be the $\tan^{-1}\mu$ because $\tan\lambda = \frac{F}{N} = \mu$.

Therefore, λ is the friction angle. Since μ is given, so, you can find out the value of λ, friction angle which is in this case it is 26.57^0. Then ϕ can be found out because α is given from here, if you see this. In this we have to actually take the alpha value as 8^0 .Well let us see, why?

The first angle is given as 0^0 which is the back rake angle and the second angle is the side rake angle. If you remember I told you that the side rake angle influenced the cutting force and the power whereas, back rake angle decides whether the movement of the chip will be appropriate, not parallel to the workpiece, and not entangling the workpiece. So, α_s is the angle which works here in this example; α_s is the one that we have to take because it is responsible for the power consumption and the force.

So, we are taking this value 8^0 but not the 0^0 here; it is the alpha this is we found out that is the λ and therefore, we can find out the ϕ = 35.715. So, this will be the value of the shear plane angle. Now determine the cutting and the thrust components F_c and F_T. These are the two force components that we have to finally find out.

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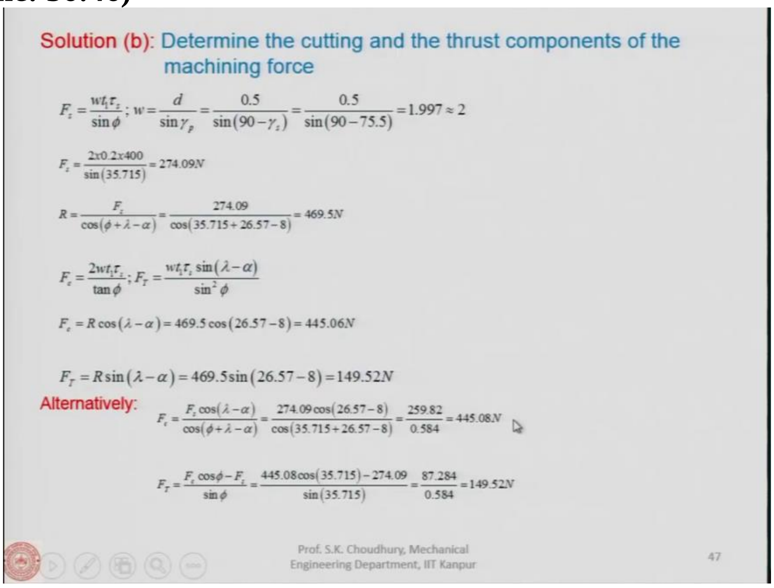

First we will determine the value of shear force, F_S through the width of cut, uncut thickness and shear stress in the following way as shown in the slide:

$$F_s = \frac{w t_1 \tau_s}{\sin\phi}\,;\; w = \frac{d}{\sin\gamma_p} = \frac{0.5}{\sin(90-\gamma_s)} = \frac{0.5}{\sin(90-75.5)} = 1.997 \approx 2$$

From here, the value of F_s would be 274.09 N. Depth of cut, d is given as 0.5 mm; γ_p is $(90-\gamma_s)$. γ_s is the side cutting edge angle; γ_p is principle cutting edge angle.

Now, we can find out the resultant force, R through the F_s in the following way.

$$R = \frac{F_s}{\cos(\phi+\lambda-\alpha)} = \frac{274.09}{\cos(35.715+26.57-8)} = 469.5N$$

Knowing the value of R, we can determine the values of cutting force component, F_c and thrust component, F_T using the formula derived in merchant's equation. This will be equal to

$$F_c = R\cos(\lambda-\alpha) = 469.5\cos(26.57-8) = 445.06N \quad F_T = R\sin(\lambda-\alpha) = 469.5\sin(26.57-8) = 149.52N$$

So, you can see that this is a practical implementation of the Merchant's equation. This could be a practical problem that you have the tool you have these values. Thenhow to find out this shear plane angle analytically or how to find out the cutting and the thrust force components. Another advantage of this is that you may not have the cutting process, but you can analytically find out the values of F_c and F_T.

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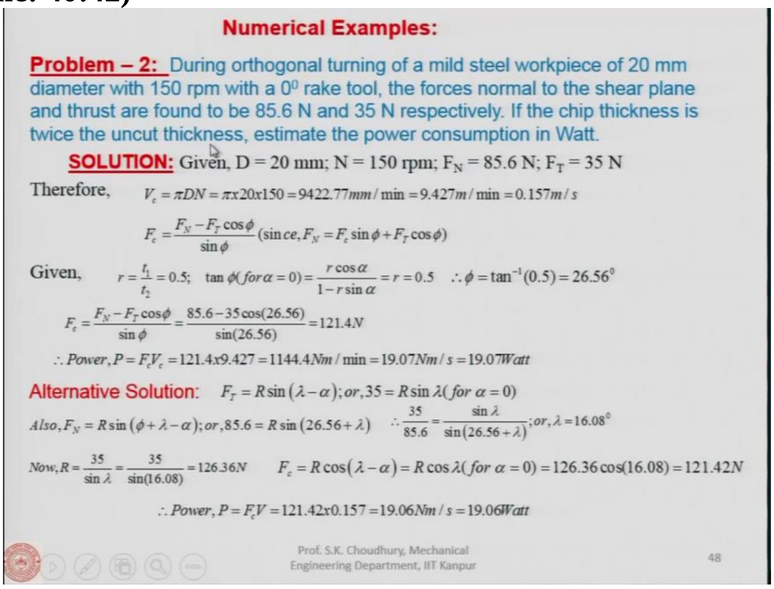

Second problem is the following: during orthogonal turning of a mild steel workpiece of 20 mm diameter with 150 rpm with a 0^0 rake tool the forces normal to the shear plane and thrust are found to be this. If the chip thickness is twice the uncut thickness estimate the power consumption in watt. Here also you can find out the V_c because V_c is πDN all other things are given, F_c we also found out from the Merchant's relationships $F_N = F_c\sin\phi + F_T\cos\phi$.

From here F_c can be found out as the chip thickness ratio is is given as 0.5, here twice the uncut thickness.

Therefore, $r = \frac{t_1}{t_2} = 0.5; \quad \tan\phi(\,for\alpha = 0) = \frac{r\cos\alpha}{1 - r\sin\alpha} = r = 0.5$

From here we can find out the value of the shear plane angle as, $\tan^{-1}(0.5)$ which is 26.56^0. Then we can find out F_c as well, because all other parameters now are known, as shown in the slide.

If we know the F_c and the power is $F_c V_c$ V_c you calculated, and you multiply them to find out the power which will be 19.07 *second-m/sec* which is equivalent to watt. So, this is 19.07 watt. This is an alternative solution given in the slide which is self-explanatory. So, you can see that the problem can be solved in many ways.

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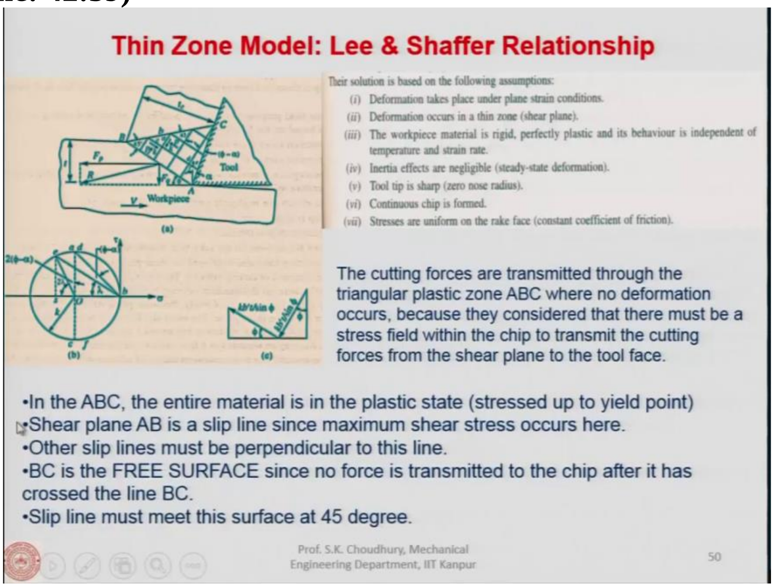

Next, we will discuss another model. This is also a thin zone model and I already said that the thin zone we assume. Normally the deformation takes place within an area. So, it is not really a plane in practice. It is a zone. The zone is thin and we are assuming that to be so thin that it becomes a plane. Here also this is a thin zone model and this model is called the Lee and Shaffer relationship.

This is another treatment given to the metal cutting and metal removal process. But the principle used here is different. Let me remind you that Merchant's principle is the minimum power consumption. During the machining process, the shear plane angle changes itself in such a way that power consumption remains minimum.

In the Lee and Shaffer model, the principle is very different. The principle that they are using is a slip line field theory. We are not going to discuss the slip line field theory in details, but we will be using the concepts only. Let us see in this slide. This is the workpiece; here is the tool and the chip is flowing along the rake face of the tool which is shown here.

Now the cutting forces are transmitted through the triangular plastic zone ABC where no deformation occurs. The cutting forces will be transmitted only when they will go through a plastic zone where no deformation occurs because of this force because if the force while transmitted from the shear plane to the rake face if it further deforms the material then it will not be transmitted.

Therefore, for the forces to be transmitted from here to here, it will go through this plastic deformation where no deformation occurs, it is already deformed. In that plastic zone the entire material is plastically deformed maximally and there will be no deformation. So, we are saying that force is going through this plastic zone where the deformation has already taken place.

This is the assumption that Lee and Shaffer have made because they consider that there must be a stress field within the chip to transmit the cutting forces from the shear plane to the tool face. I already explained to you that the force cannot be deteriorated, it cannot be reduced or lost. Therefore, it has to go through a plastic zone where already deformation has taken place. In the ABC the entire material is in the plastic state this is stressed up to the yield point.

Shear plane AB, AB is the shear plane here, the maximum shear stress occurs. Other slip lines are perpendicular to this line. Let us say this line, this line, this line and so on - they are perpendicular to AB and we have also drawn some parallel lines which are parallel the AB. BC is the surface which is called a free surface because no force is transmitted to the chip beyond this line.

That is, no force is transmitted to the chip after it has crossed the line BC. Once the force is transmitted then the chip is formed and the chip is being removed along the rake face. The chip is formed here. Slip line must meet the surface at 45^0. This is the theory.

We are not deriving or describing but according to the theory the maximum stress where it occurs that is the shear plane that makes an angle of 45^0 with the free surface. BC is the free surface because no forces are transmitted beyond this line. We are designating the parallel lines as *k*; then here this line is designated as *d* and similarly, this line is designated by a certain letter, it is not given here.

And this line which is parallel to the AB is designated as *c* and this line is parallel to the AB is given us let us say some value. This is *d*; this we have designated as *e*. This surface is *d* and so on. This surface is *a* and this surface is *d* the surface is *c* and this there are parallel. altogether what I am saying is that there are parallel lines of the AB and there are perpendicular to that. all perpendicular lines are also the slip lines.

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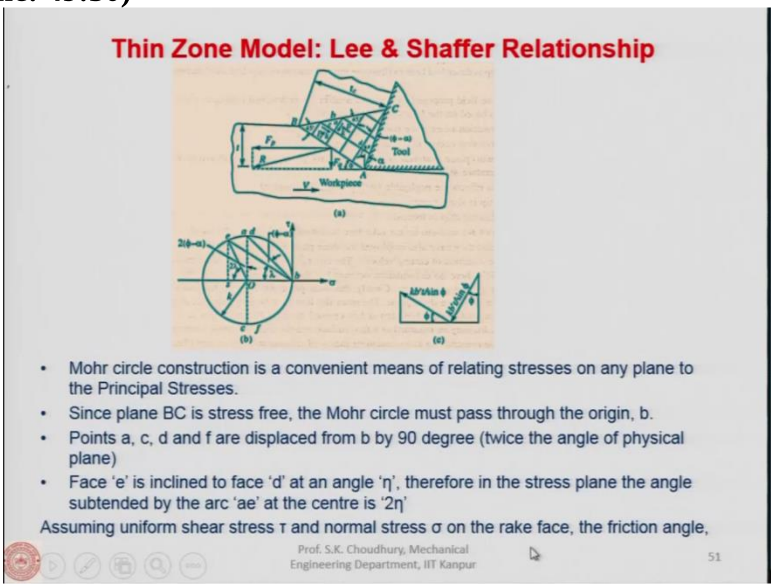

Mohr circle construction is convenient means of relating the stresses on any plane on the principal stress. If you remember I mentioned during our discussion on the stress and strain that a body which is already stressed has 9 stresses, 3 normal stresses and 6 shear stresses. For some reason the system of axis can be selected in such a way that shear stresses are 0.

Then all the normal stresses which are acting are called principal stresses. Mohr circle diagram, is a convenient means to relate the stresses on any plane to the principal stress. Let us say this is the plane *BC*, designated as *b* meaning that this is the plane and the stresses on the plane can be designated by point *b*. This is B point will be located on the periphery of the Mohr circle.

Since *BC* is stress-free, Mohr circle must pass through the origin *b* where the stresses would be 0. Points *a, c, d* and *f* are displaced from *b* by 90^0. These points are nothing but a designation of parallel and vertical planes.

These are the parallel planes, and these are the vertical planes. *a* and *d* should be the parallel ones and *a* and *d* are parallel to each other. *c* and *f* should be the horizontal ones. This is *c* and this is *f* this is *d* this is *d* and this is *a*. *a, d* and *c* and *f* which will be at a 90^0 displaced from the point *b* because the *b* and these lines you can see that they are at a 45^0 angle. So, when it is in the Mohr circle, it will be twice the angle of the physical plane.

Physical plane angle is 45^0 and in the Mohr circle it will be 90^0, i.e. twice. This is displaced by 90^0 from the point *b*. Point *b* is the *BC* plane. Face *e* is inclined to the face *d* by an angle of $(\phi-\alpha)$. This angle we already found out earlier because this angle is α and this angle is ϕ. So, you can find out that this angle is $(\phi-\alpha)$.

Face *e* is a rake face inclined from the *d*; *d* is perpendicular to *AB* and that angle $(\phi-\alpha)$ we denote as η. Therefore, in the stress plane the angle subtended by the arc *ae*, will be 2η as shown in the slide.

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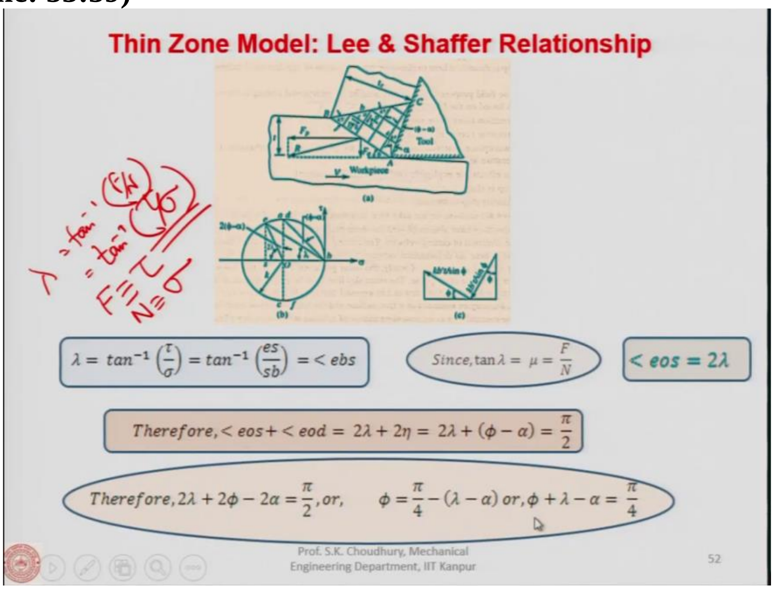

Assuming uniform shear stress τ and the normal stress σ on the rake face, the friction angle can be given as $\tan^{-1}\left(\frac{\tau}{\sigma}\right)$. This is shown here in the slide. This is the Mohr circle; here the vertical axis is the shear stress, horizontal axis is normal stress. Therefore, this is the τ and *sb* is the normal stress with respect to this *e*. *e* is the rake face of the tool. From here to this would be the *es*. The λ is equal to $\tan^{-1}\left(\frac{\tau}{\sigma}\right)$.

This is $\tan^{-1}\left(\frac{es}{sb}\right)$ and this is the angle *ebs*. You understand that λ which is the friction angle, is expressed as $\tan^{-1}\left(\frac{F}{N}\right)$. *F* is the friction force, which is equivalent to shear stress τ and the normal force *N* is equivalent to normal stress σ. So, the λ which is the $\tan^{-1}\left(\frac{F}{N}\right)$ is equal to $\tan^{-1}\left(\frac{\tau}{\sigma}\right)$. τ here is the *es* and σ is the *sb*.

Therefore, it can be expressed as $\tan^{-1}\left(\frac{es}{sb}\right)$ and this means that the angle *ebs* would be λ which is the friction angle. if this is the friction angle λ in that case this angle *eos* will be twice that because it is at the center.

Therefore, the *eos* and the *eoa* or *eod* these 2 angles together will give you the 90^0 angle. This angle is a 90^0 angle then this angle *eos* would be twice the friction angle λ and *eod* would be twice the η because η here is this angle and at the center it will be twice of that.

η is the angle at which this face is inclined with respect to the *d*. Therefore, you can find out that these 2 angles together will be $2\lambda+2\eta=2\lambda+(\phi-\alpha)=\frac{\pi}{2}$. Ultimately, what we are getting is that $\phi+\lambda-\alpha=\frac{\pi}{4}$. So, you can see that this is very similar to the Merchant's equation, but the principle at how they arrive to that is different. Rest of the material we will discuss in our next session of discussion. Thank you for your attention.