Production Technology
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Lecture – 07
Selection of Tools Angles

Hello and welcome back to the discussion sessions of production technology course theory and practice.

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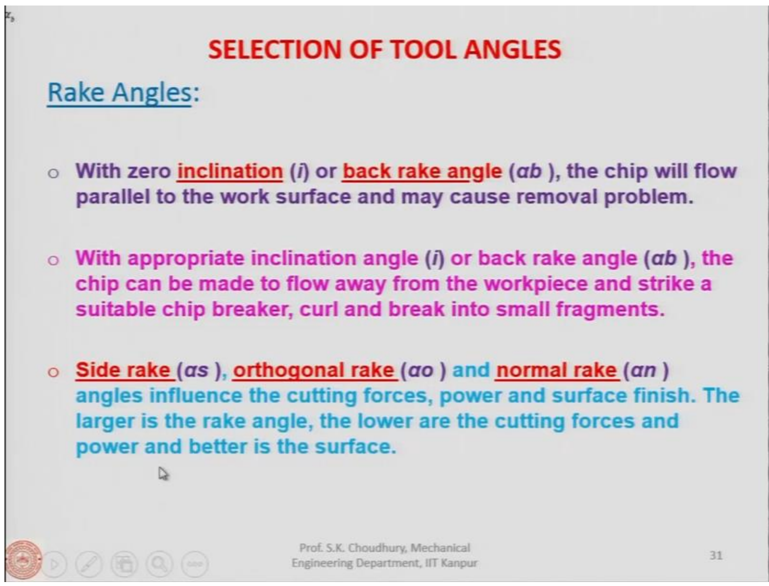

Let me remind you that last time what we discussed are the tool angles. We said that we have 3 sets of angles, namely one set of rake angles, one set of flank angles and one set of cutting edge angles apart from the tool nose radius. So, we have altogether 7 parameters, i.e. 2 rake angles, 2 flank angles, 2 side cutting edge angles and the nose radius by which a tool in any system is nomenclatured.

And then we have seen that there are different systems of tool nomenclature. Depending on the country adopting the system it is either coordinate system, or orthogonal rake system, or normal rake system and so on. It was also said that normal rake system is the one which is the international system and most of the countries are now trying to adopt that system so that everywhere the same standard is used. Afterwards we said that however right now we have different systems.

So, if we are purchasing tool from one system, the angles have to be converted to the system adopted by us and I have shown it to you that conversion of the angles is not so difficult because all the views could be taken in one place and they can be correlated. Then geometrically you can find out the relationship between the angles of one system and the another system, then we started discussing the selection of tool angles.

Let us see this slide. With zero inclinations or back rake angle we said that the chip will flow parallel to the workpiece surface and it may cause removal problem. So, we have to have some sort of inclination angle and some back rake angle. Now with appropriate inclination or back rake angle the chip can be made to flow away from the workpiece and strike a suitable chip breaker, curl and break into small fragments.

I talked to you about the chip breaker. Continuous chip is not desirable all the time because it rubs, entangles and so on. Therefore, we have to have some sort of provision, some kind of a provision to break the chip. Now these provisions are the chip breakers and chip breakers are of different types. Chip breaker can be a protrusion on the rake face and indentation or the crater on the rake face.

Now sometimes the research went on for imparting some sort of small amplitude vibration to the tool so that with that vibration the tool can vibrate and break the chip. However, that research did not go well because along with that vibration which is imparted to the tool and the tool imparts in its turn to the workpiece, the feed marks or vibration marks will be there on the workpiece surface.

So that spoils the surface finish. Therefore, that research really did not go well. Now the side rake, orthogonal rake and then normal rake that is α_s, α_0, α_n if you remember that side rake was in the orthogonal rake system and the normal rake was in the normal rake system that is the international system. These angles particularly influence the cutting forces power and surface finish. The larger is the rake angle the lower at the cutting forces and better is the surface.

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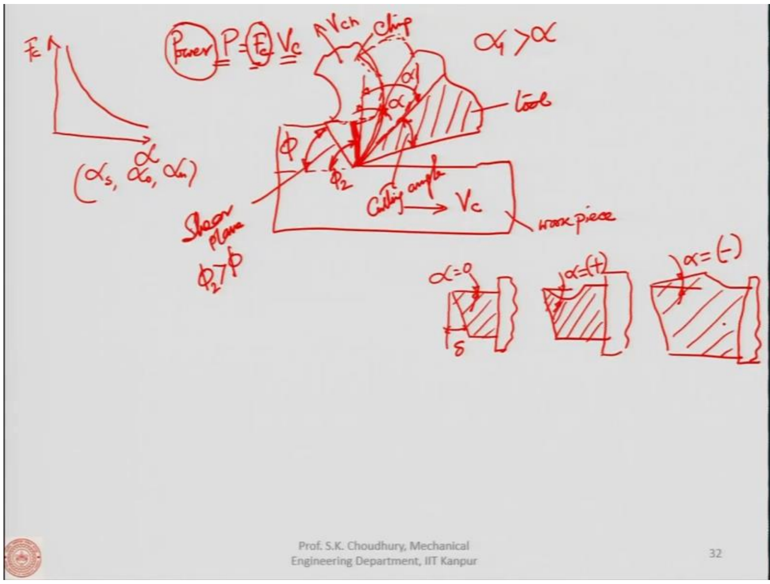

I showed it to you last time probably that if it is the cutting force and if it is one of the rake angles, so it goes like this meaning that as the α increases, the F_c decreases. We always want the cutting force to be less. Power P is equal to F_c and the V_c. So, if we can reduce F_c, V_c being constant, in that case the power is reduced.

This is what is our aim that to reduce the F_c and the power as much as possible, so that less power is consumed for the process and the process could be cheaper that way because last time I have discussed that once the power consumption is less in that case the prime mover of the machine could be less powerful and that is cheaper. Finally, the product that you are making using that machine will be cheaper.

And it can compete in the market with that cheaper product - that is the idea. Among side rake angle, orthogonal rake angle or the normal rake angle, if one of these rake angles is increased, then the cutting forces is decreased. Why it happens? The physics behind this can be explained in the following way. Suppose we have the cutting process viewed as I am showing in an exaggerated form.

Because otherwise it will be difficult to explain. Let us say this is the workpiece, this is the chip flowing, let us say chip velocity is V_{ch}. This is the tool and here we have a certain rake angle, which is one of these rake angles, namely α_s, α_0 or α_n. This is the cutting velocity V_c and this line is perpendicular to the cutting velocity vector.

Now if this rake angle is increased, then the cutting force will decrease. The physics behind this is that suppose we are increasing the rake angle and the tool becomes like this. So, now the angle is α_1 which is more than α initial. Then the chip as it was flowing with the α will now flow like this. I will show it in dotted line.

This is the new position of the chip and the chip will be flowing along this rake face. Now from this point of tool tip to the point where the plastic deformation starts this is called the shear plane. And this angle that it makes with the direction of the cutting velocity vector, this is the shear plane angle. Now with the shear plane as the α is increased it becomes from here to this way, this is the new shear plane.

As you can see, the length of the shear plane is decreased as the rake angle is increased as a result what happens is that less effort is required to remove that area and the less force is required. Because the length of the shear plane is decreased, now it is the ϕ_2 and as you can see that the ϕ_2 is more than ϕ_1 because of the α increasing because rake angle goes from α to α_1 .

Therefore, the length of the shear plane is decreased, area is decreased and less effort is required. The force is decreasing and if the F_c is decreasing, I showed that the power consumption will be less in the cutting process. The surface finish will be better because when the cutting force less the surface finish also improves, the surface roughness becomes less.

These are the reasons that we should consider to control the process so that the force could be less. Now if you see in this diagram that when we are increasing the rake angle from α here to α_1 in that case the tool becomes weaker, the material here in the tool will be less as we are increasing the α.

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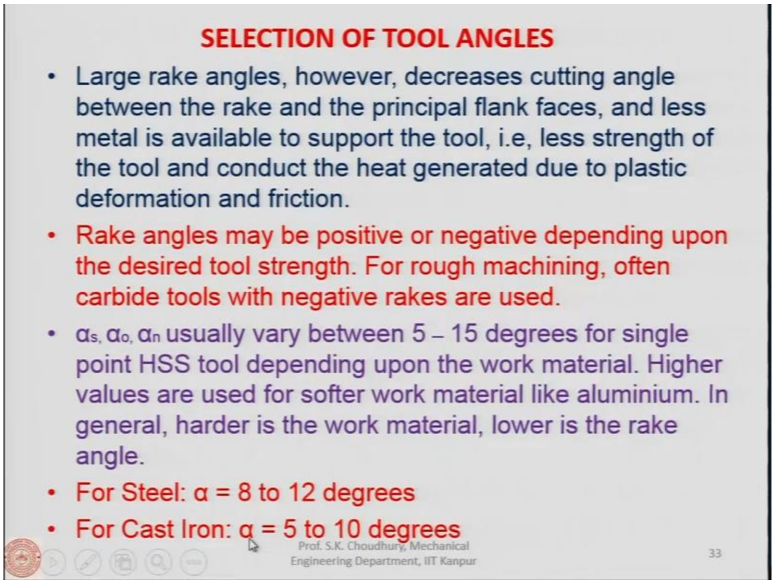

This is what it is written in this slide that the large rake angle however decreases the cutting angle, cutting angle is this angle, this is the cutting angle. Now this cutting angle is between the rake face and the flank face, this cutting angle decreases as we are increasing the α that is the rake angle and the tool is becoming weaker because the tool is becoming thinner and less material is available.

Less material is available means that material will not be sufficient to conduct the heat because when there is more material the heat conduction is more. So, less material will be available to conduct the heat. Large rake angles, however decrease the cutting angle between the rake and the principal flank faces.

And less metal is available to support the tool, that is the less strength of the tool because tool is becoming weaker, tool may not be able to withstand that much force as in case of the earlier tool when the rake angle was less. Less strength of the tool and conduct the heat generated due to plastic deformation and the friction, the heat that is generated because of the plastic deformation and because of the friction.

Between the chip and the rake face of the tool that heat will not be conducted as much as it used to be for the lower value of the rake angle. Therefore, we cannot really infinitely increase the rake angle to get the benefit of reduced cutting force or reduced cutting power because there is a constraint that if we increase more than a certain value in that case the tool will be weaker and less heat will be conducted.

What does it mean that less his heat will be conducted is that the heat will be accumulated in the tool and the tool will have the thermal deformation. As a result, the tool geometry may be lost, rake angle may be lost, flank angle may be lost and overall, the tool may be thermally deformed and the tool life will be less. Because of those constraints that with increasing the α that is a rake angle we cannot infinitely increase it.

Now the rake angles may be positive or negative depending upon the desired tool strength. For rough machining often carbide tools with negative rakes are used. When we discussed the carbides in the material chapter, I told you that carbides are used for making the tool but carbides are very brittle, so when carbide tools with negative rake angle is used then the tool becomes fatter.

The tool may be like this, let us say this is a turning tool it may be like this or it may be like this. I mean to say the shape of the tool or it may be like this so in all these tools I am just showing the rake angle because that this is the flank angle but this is the rake angle, here this is the rake angle. As you can see the rake angle can be that α is equal to 0^0.

Here the α is positive this is called a positive rake angle when it is in this way and when it is in the opposite direction this is the negative rake angle. Therefore, it is said that tool with 0^0 or with the negative rake angle can be used when the tool strength required will be more, because as that if it is 0^0 or if it is negative rake angle, then the tool material available will be more.

In case of the tool with the positive rake angle, the tool strength is less than in case of tool with 0^0 rake angle or with a negative rake angle. When we use let us say ceramic tools, as the ceramic tools are brittle, they cannot withstand the shock or any kind of fluctuation in the force. So, in that case the tool has to have more strength and hence we need to have the tool fatter.

In that case either 0^0 of the rake angle or negative rake angle can be used to strengthen the tool and to have the less thermal deformation because more material will be available to conduct the heat from the tool. Often carbide tools with negative rakes are used that is what I said. Now side rake, orthogonal rake, and the normal rake usually vary between 5 and 15^0 for a single point HSS tool.

Now of course this range is quite a wide range like 5 to 15^0 so now it depends on the work material, for example if the work material is hard then of course you need more force and the tool has to be stronger in that case the rake angle has to be less. And vice versa, if the workpiece is softer then you can get more rake angle, since in that case the tool does not have to be that much strong as in case of the hard workpiece material.

And suppose your tool is weaker, made of a material which is weaker material, the strength of the material is not very high, in that case you need to have the low rake angle because in that case the tool has to be fatter, tool strength has to be more and so on. This is the judgment of a good engineer to select the right kind of rake angle depending on the material combination given, depending on the situation, that means that depending on the work environment, on the cutting parameters and so on.

Overall, we said that the range is between 5 and 15^0 however we can have the rake angle as 0^0 or negative rake angle and so on. Higher values are used for softer work material as I said like aluminium. In general, harder is the work material lower is the rake angle that is the thumb rule; that harder is the work material obviously the tool has to be stronger and the rake angle is smaller.

For example, the recommendation for the α_s, α_0 or α_n these values lie between 8 and 12^0 but for Cast Iron these values are 5 to 10^0 as you can see that these values are less than the values in case of steel because the cast iron is the brittle material, it is a very strong material as well, I mean hard material as well. For hard material like cast iron, the tool has to be stronger and the rake angle therefore to be chosen is smaller.

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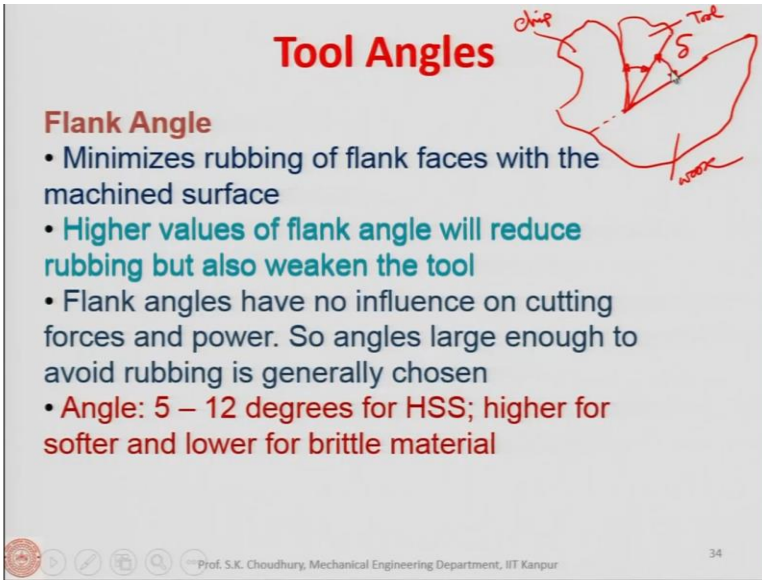

Now as far as the flank angles are concerned, a flank angle minimizes the rubbing of the flank faces with the machine surface. I will again draw this picture. Let us say I am not drawing fully the workpiece, this is the tool. This is the flank angle; this is the workpiece, this is the chip flowing, this being the tool.

This is already a worked material, work surface is already made. When the tool removes chip material then the flank face rubs against the already machined surface. Therefore, we have to have the flank angle to clear the flank face from being rubbed over the already machined surface. Therefore, the flank angle is also called the clearance angle because we need to clear the flank face from the already machined surface.

Now higher values of flank angle will reduce rubbing but also weaken the tool because if the flank angle increases in that case the cutting angle decreases and the tool becomes weaker, less material will be available to conduct the heat. There will be thermal deformation. Therefore, we cannot infinitely increase the flank angle like in case of the rake angle that we have discussed.

Because it will weaken the tool and it will have less material for the conduct of the heat. Flank angles fortunately have no influence on the cutting forces and power like in case of the rake angle. So, angles large enough to avoid rubbing is generally chosen because we are safe if it if it is bigger. I mean the tool with larger value of flank angle will never rub off the work surface, so we are safer.

But it should not affect the tool strength. We have to see that we should increase the value of the flank angle only to that extent, so that the tool strength is sufficient for that particular process. And here also the angle range is about 5 to 12^0 degrees for HSS tools; higher values for softer and lower for brittle material meaning that if we have the softer workpiece material in that case the flank angle can be higher.

Because in that case tool strength is not required to be very high like I already discussed in case of the rake angle. And vice versa, that is, the flank angle should be lower if the workpiece material is hard or the brittle material like the Cast Iron for example.

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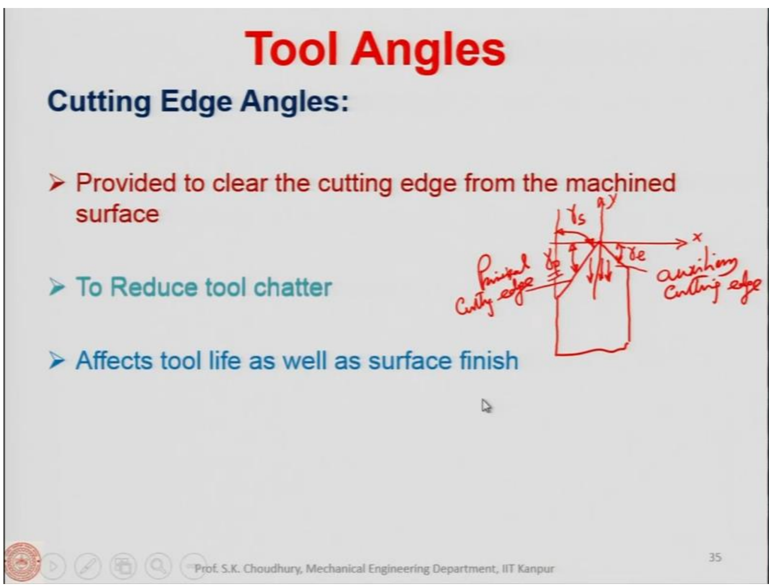

The cutting edge angles are provided to clear the cutting edges from the machined surface. I would like to remind you that the cutting edge angles are like this, if this is the tool in that case this is the principal cutting edge and this is the auxiliary cutting edge. Now with respect to the X-axis, if this is the system of axes, let us say this is X and this is Y, this angle in the ORS and NRS this is given as the cutting edge angle, this is one of the cutting edge angles and this is another cutting edge angle.

This is the principal cutting edge angle and this is the end cutting edge angle, in one of the systems. Principle cutting edge and auxiliary cutting edge of the tool make cutting edge angles with respect to the x axis. In case of the ASA system this was the angle given and that was the side cutting edge angle.

Therefore, we said that the side cutting edge angle is 90 minus this principal cutting edge angle. These angles are provided to clear the cutting edges, principle cutting edge and the auxiliary cutting edge from the already machined surface. This is to reduce the tool chatter which affects the tool life as well as the surface finish.

In case the angles γ_p and γ_e are incorrectly selected and if they are smaller than required and the tool is not able to clear the cutting edges sufficiently from the machined, in that case the components of the forces which try to detract the tool from the workpiece start growing Let us say this is the turning process going on and this is how the tool is located; this is the workpiece.

This is the component of the radial force which will try to retract the tool from the workpiece. This force fluctuates and will try to retract the tool from the workpiece because the cutting edge angles are not appropriate. Therefore, what happens is that there will be a vibration and that vibration is the self excited vibration. It is called the chatter.

Chatter is the self excited vibration where the source of energy comes from within the system not from outside. When this source is from the outside, it is a force vibration. To reduce the tool chatter we have to have the appropriate value of the principal cutting edge and the γ_p. Because this also will affect the surface finish.

So, when the source of vibration is from within the system it is the tool chatter. To reduce the tool chatter, we have to have the appropriate values of this. Now as I was explaining that these components, when they increase, the tool starts chattering. This will also affect the tool life as well as the surface finish because as the tool chatters the tool life reduces. Moreover, because of the tool chattering there will be feed marks on the workpiece, so the workpiece surface finish will be spoiled.

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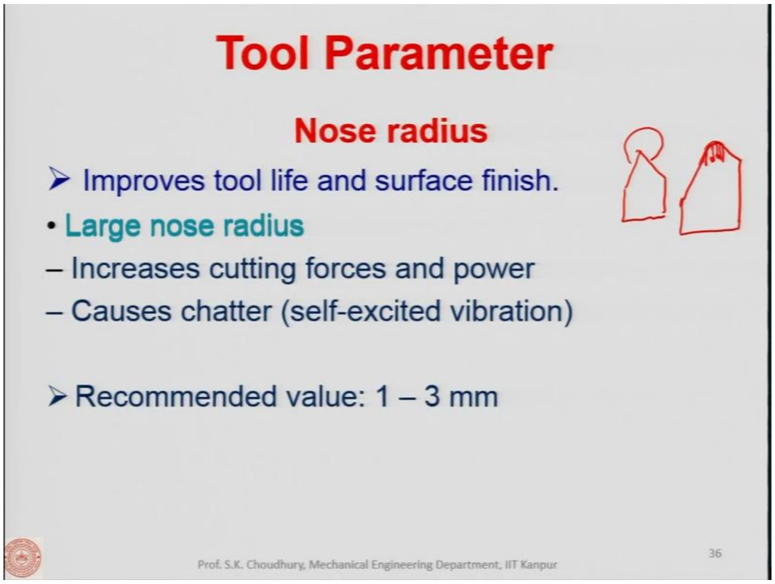

Now the nose radius. About the nose radius I told you that nose radius is given because sharp tool will always have a very high stress concentration around this point. Therefore, the tool is given a tool nose radius and that tool nose radius decreases the stress concentration around it and also improves the surface finish.

I told you during our last discussion session that when we will discuss the chapter on surface finish, then I will tell you how the surface finish improves when the tool nose radius is given. Large nose radius however, increases cutting forces and power because it will have more area for cutting.

Therefore, the cutting forces and the power will be more and this causes chatter. The chattering occurs due to the radial components of the radial forces which will detract the tool from the workpiece, will increase because of the higher nose radius. So that is the self excited vibration and the tool life will be less, surface finish of the workpiece will be bad and so on.

Normal recommended values of the nose radius are between 1 to 3 mm. Below that there will have more stress concentration and with the value higher than 3 mm, chattering will be more and the tool life will be less.

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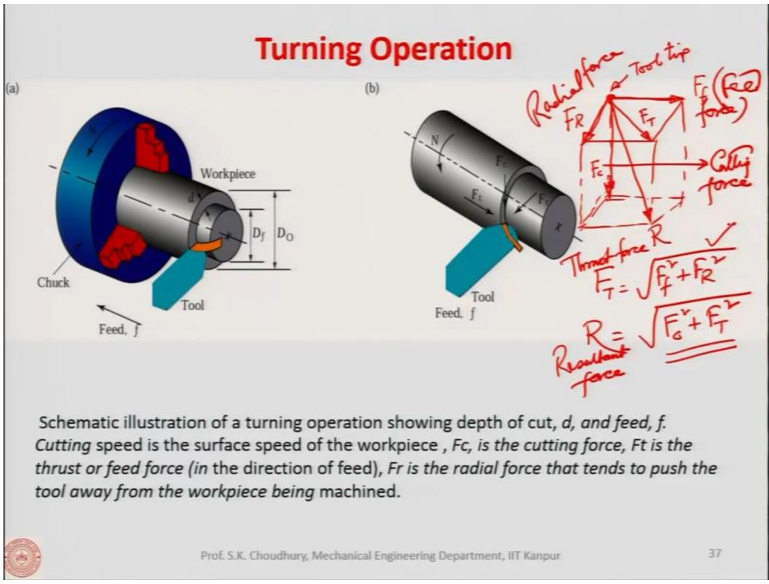

Here the turning operation is shown. This is the 3-jaw chuck, this is the workpiece mounted on the 3-jaw chuck. I will demonstrate this principle of the working in the laboratory when we will go to the lab. As you see here that the workpiece initial diameter is D_0 and the diameter that we are finally getting is the D_f. Therefore, this value is the depth of cut, that is, $\left(\frac{D_f - D_0}{2}\right)$.

This is the feed direction, this is the tool, this is how the chip is being formed and the chip is flowing along the rake face of the tool. This is the 3-jaw chuck, here the rotation is given to the workpiece. The forces which act on the tool during the cutting process are shown here. This is the force which acts in the feed direction, this is the force acting in the radial direction and along this direction is a cutting force which is in the vertical direction.

Here is a schematic illustration of turning operation showing depth of cut, feed in this direction, cutting speed is the surface speed of the workpiece given by the rotation of the workpiece. F_c is the cutting force acting vertically this F_t is the thrust force. I will explain it to you a little later what is thrust force.

Fr is the radial force. I told you earlier that this component of force is directed in this direction that tends to push the tool away from the workpiece being machined. I told you that if the radial forces increasing chatter occurs and this chattering comes particularly when we have the larger than required nose radius for example.

Let us say this is the tool tip. first I will draw a parallelopiped like this, and let us say here we have the tooltip. Here this is the feed force F_f, this is in the direction of the feed motion, the tool is moving like this, this is in the radial direction. So, this is the F_R in the radial direction. The summing of that will be the F_t ,the thrust force.

And the cutting force, F_c will be in this direction and the resultant force will be from this point to this point as shown in the slide. If you connect these 2 points, this will be the resultant force. This is the tool tip and F_t therefore becomes $F_t = \sqrt{F_f^2 + F_R^2}$. This is in the horizontal plane. In the vertical plane we have the F_c and the resultant force R.

Therefore, $R = \sqrt{F_c^2 + F_t^2}$ .This is the resultant force, R. F_R is the radial force, this is the radial force; this is the feed force and this is the cutting force F_c component. This is what exactly is shown in the diagram. F_t is the resultant force of the F_f and the F_R , i.e. feed force and the radial force. Therefore, 2 basic forces are F_t and F_c.

The resultant force can be found out from $\sqrt{F_c^2 + F_t^2}$. We measure the cutting forces on the machine tool, which I will discuss a little later, with the help of the dynamometer. It can measure the forces in turning forces in drilling, forces in milling and the forces in grinding.

In case of the turning that we are discussing, the dynamometer will measure basically F_c and F_t. So, you have to determine the F_f and the F_R in case it is required analytically from the formula $F_t = \sqrt{F_f^2 + F_R^2}$

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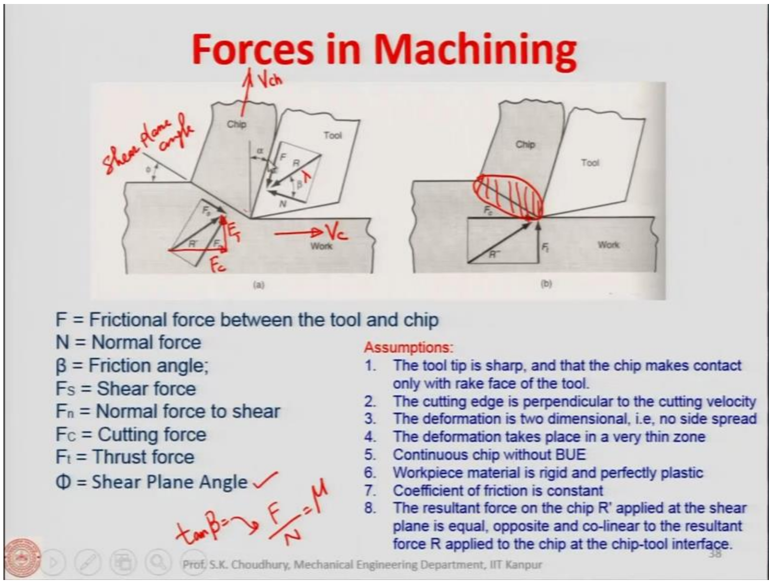

These are the forces in machining that are shown here. If we assume that the chip is a rigid body and it is moving at a constant velocity over the rake face of the tool with the cutting velocity V_c then the resultant of the forces acting on the chip from the tool through the rake face of the tool and the force acting from the workpiece through the shear plane of the process should be equal to zero. .

Once again I will explain it to you. Looking at this figure, let us assume that the chip is a rigid body, homogeneous body and this is moving along the rake face of the tool with a constant chip velocity V_{ch} .Chip velocity can be considered to be constant if the resultant forces acting on the chip from the tool and from the workpiece will be the same meaning that the sum of these resultant forces is zero.

Let us say the resultant force acting on the chip from the tool through the rake face is R and the resultant force acting from the workpiece side through the shear plane is R'. So, R and R' have to be equal opposite and collinear. Their sum is zero because they are equal opposite and collinear. Then only the chip being a rigid body can move along the rake face of the tool with a constant velocity V_{ch}.

Now let us say this R that is the resultant force is resolved into 2 components, one component will be along the rake face of the tool and another component will be perpendicular to the face of the tool. This is the F this is the N. Now the N is the normal force since it is acting through the tool. Because of the normal force, there will be a friction force F.

Once again, this resultant force, R acting on the chip from the tool through the rake face is resolved into 2 components, namely friction force and the normal force. The normal force is the force which is acting through the tool to the workpiece for cutting process. And because of that normal force the friction force is created. Therefore, between the N and the resultant force, R the angle will be called the friction angle, λ.

In many places it is shown as λ. In the diagram here it is shown as β. Similarly, the resultant force acting on the chip from the workpiece through the shear plane R' can also be resolved into 2 components, one component is along the shear plane and another component along the perpendicular to the shear plane. Here there are two more components, one is the angle shown here.

And this is the angle between the shear plane and the direction of the cutting velocity vector. This line is parallel to the cutting velocity vector. This angle is the shear plane angle, ϕ. Once again, his is the friction force, F between the tool and the chip, N is the normal force. This is a friction angle we are taking as β, F_s is the shear force.

This R' we are resolving into 2 components, F_s which is parallel to the shear plane which is the shear force, since it is parallel to the shear plane and perpendicular to the shear plane is the F_N normal force to shear. Cutting forces F_c and thrust force $F_t$are shown here. Let me explain it to you what happens because of the change in the shear plane angle, ϕ and the rake angle, α.

Let us say when the shear plane angle, ϕ is changing or the α is changing, in that case the resultant forces will change. Let me show it to you that if the α is different that means if the inclination of the tool is different, then the direction of the F will be different and the direction of the N, R will be different. Similarly, the ϕ can also change that means the inclination of the shear plane can also change.

In that case the inclination of the F_s will also change since it is parallel to the shear plane, so it will change the direction and so on. So, the R' direction will change meaning that with the change in the α or ϕ the directions of R or R' are not unique, they do not remain the same.

This resultant force which is acting on the chip again can be further resolved into two more components, one component will be parallel to the V_c and another component will be perpendicular to the V_c. So, this is called the cutting force because this is parallel to the V_c.

Therefore, this force will be responsible for the power consumption because F_c and V_c product is the power and F_t, perpendicular to $F_c$will be always the thrust force. Therefore, now we have 6 components, 2 components by resolving the R and 4 components by resolving the R' that is the F and the N in the rake face of the tool, that is along the rake face and perpendicular to the rake face.

F_s and F_N will be along the shear plane and perpendicular to shear plane and F_c, F_t are the cutting force and the thrust force components respectively. So, once again, $F_c$and $F_t$we have taken because as you change the ϕ or the α, the direction of the F_c and F_t will not be changed that is another reason that we have to take that F_c and F_t components.

Because the direction of the F_c, F_t components will not be changed with the change in the ϕ and α like in case of R or R' meaning that F_s, F_N, F and N these directions will be changed as we change the α and ϕ but F_c and F_t directions will remain constant, it will not change with the change in the ϕ or the α. Now these angles that is the α, ϕ and the friction angle, λ or β can be correlated with the components of the forces F, N, F_s, F_N, F_c and F_t .

This relationship has been established for the first time by the Merchant and Ernst in 1944. This is still being used as a valuable analysis or a valuable model showing the insight of the mechanics of metal cutting. This is called the model for the mechanics of metal cutting which will give you the insight of the mechanics.

If you see here there are about 8 assumptions and the assumptions are that tool tip is sharp and the chip makes contact only with the rake face of the tool, it does not have the round shape, I mean there is no tool nose radius, the cutting edge is perpendicular to the cutting velocity, we have said that in case of the orthogonal cutting, the cutting edge is perpendicular to the cutting velocity.

The deformation is 2 dimensional that is, no side spread. Two-dimensional means the cutting edge is absolutely straight and it is perpendicular to the cutting velocity. So, if the cutting edge is perpendicular to the cutting velocity, deformation has to be 2-dimensional, there is no side spread. No side spread of the chip, the chip will be flowing exactly perpendicular to the cutting edge and it will not be inclined to any side of the normal to the cutting edge.

The deformation takes place in a very thin zone. This we are indicating by a line. And we are saying that this is the shear plane angle and this is the shear plane along which the plastic deformation occurs. But as a matter of fact, when actual cutting process takes place the shear deformation takes place not along a plane but in a zone that means this is a zone and not really a plane.

We are assuming that this is plane for the simplicity of the analysis. All these assumptions are made by Merchant and Ernst to simplify the analysis . Because none of these assumptions can be valid in the actual machining process. The tool does not remain sharp, as soon as it comes in contact with the workpiece it gets rounded off.

Cutting edge is perpendicular to the cutting velocity - does not remain true in case of oblique cutting. Although this is simplified and this model is given for orthogonal cutting where tool is exactly perpendicular to the cutting velocity vector, nevertheless with this analysis we first started understanding the mechanics of metal cutting. Therefore, even these days this model is used in practice.

Now the next assumption is the continuous chip without the built-up edge, it cannot have the built-up edge because in that case this analysis will not be valid, it cannot be analyzed.

Workpiece material is rigid and perfectly plastic. If the workpiece material is not rigid then we cannot assume that the R and R' are the equal, opposite and collinear. If it is homogeneous and if it is rigid and perfectly plastic then only it can move along a constant V_{ch} when the R and R' are equal, opposite and collinear.

Next assumption is the coefficient of friction is constant along the chip tool contact length. This is the chip tool contact length and along this length the coefficient of friction, F / N i.e. the ratio of friction force to normal force, this is μ and $\tan\lambda = \left(\frac{F}{N}\right)$. So, this coefficient of friction is assumed to be constant along the chip-tool contact length.

Next assumption is the resultant force on the chip R' applied at the shear plane is equal opposite and collinear to the resultant force R applied to the chip at the chip tool interface; this is the chip tool interface. So, this R and this R' are assumed to be equal, opposite and collinear.

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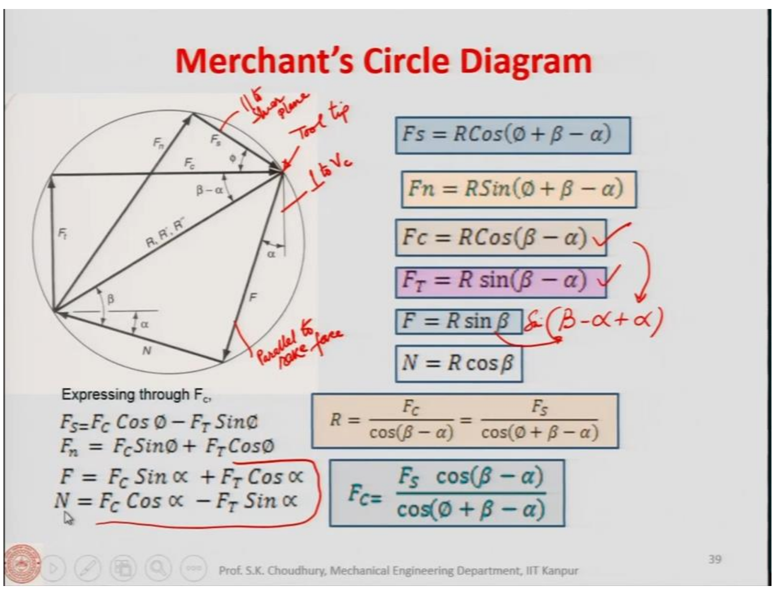

Now if we draw the circle with a diameter of R or R', they are the same, in that case the tips of all these components will be lying on the periphery of this circle because all these components F, N, F_s, F_N, F_c and F_t are the components of the one in the same resultant force R or R'.

Therefore, all the tips will be lying on the periphery of the circle and that this circle is called the Merchant's circle diagram because this was invented, as I said, way back in 1944 to explain the mechanics of metal cutting. In this diagram this is the tool tip. Therefore, if we draw the line which represents the perpendicular to the cutting velocity then between this line and the *F* will be the rake angle because *F* acts parallel to rake face.

This line is normal to cutting velocity vector. So, the line between the rake face and a line perpendicular to the cutting velocity vector is the rake angle. Therefore, this angle is the rake angle. Then we said that the angle between the normal force and the resultant force would be the friction angle. Here it is shown as the β.

Therefore, perpendicular to this line and perpendicular to this line will make the angle which is the rake angle. Now the angle between the line which is parallel to the V_c, and the F_s which is parallel to the shear plane is the shear plane angle.

And if this is the shear plane angle then you can find out, this being α this will be the friction angle minus the α a, because this angle is 90^0 ,this angle is 90 minus α and so on. This angle you is 180 minus 90 minus β and so on. So, you will find out that this angle is equal to the friction angle minus the rake angle. These angles we have already determined. Now let us see how we can correlate them.

Let us say the F_s from this triangle is equal to $R'\cos(\phi+\beta-\alpha)$. This is what it is written here. Similarly, the F_N which is perpendicular to F_s will be $R'\sin(\phi+\beta-\alpha)$. If this is cos this will be sin because F_s and F_N are perpendicular to each other . Now the F_c similarly, will be $R'\cos(\beta-\alpha)$. Now this angle we have seen that this is the $(\beta-\alpha)$ and therefore F_c will be $R'\cos(\beta-\alpha)$.

F_t perpendicular to F_c will be $R'\sin(\beta-\alpha)$ then the *F* and *N* are remaining. *F* will be R' and the *cos* of this angle. Since it is $(90-\beta)$ so this will be $F=R'\sin\beta$ or $F=R'\cos(90-\beta)$. Similarly, perpendicular to that is *N* that is the normal force. This will be $N=R'\cos\beta$ or $R'\sin(90-\beta)$.

These are the 6 equations that we can find out which will correlate the force components, namely shear force, normal to shear force, cutting force, thrust force, friction force and the normal force with the 3 angles, the shear plane angle, the friction angle and the rake angle. So, from here you can find out that $R = \frac{F_s}{\cos(\phi + \beta - \alpha)} = \frac{F_c}{\cos(\beta - \alpha)}$.

Therefore, $F_c = \frac{F_s \cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)}$. I mentioned earlier that when you measure forces using the dynamometer, you normally measure 2 components, namely F_c and the F_t. You cannot really measure other forces like shear force, friction force etc. These forces can be calculated analytically from the measurement that you have for the F_c and the F_t.

Therefore, in this slide whatever way we have expressed F_s, F_N, F and N , this can be translated through the F_c. So, F_s can be expressed as $F_s = F_c \cos\phi - F_T \sin\phi$. Similarly, F_N, F and N can also be expressed in terms of F_c and F_T as shown in the slide.

This is simple. You can find out that as you open up the equations of F_N, F and N and then put the value of the F_c from the equation $F_c = R\cos(\beta - \alpha)$.

So, F_s and $F_N$we can convert through the F_c and F_T. similarly F and N which is $R'\sin\beta$ and $R'\cos\beta$ can also be expressed through the F_c and F_T . This is because $\sin\beta$ you can write as $\sin(\beta - \alpha + \alpha)$. So, finally you can get $F = F_c \sin\alpha + F_T \cos\alpha$.

Similarly, the normal component will be opposite to that, $N = F_c \cos\alpha - F_T \sin\alpha$. The rest of the material related to this I will continue discussing in the next session of discussion. Thank you for your attention.