Manufacturing Processes - Casting and Joining
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Lecture - 08
Stress, Strain and Strain Rate and Shear Plane Angle

Hello and welcome back to the course on Manufacturing Processes - Casting and Joining. Let me remind you that in the casting, we have already discussed the total solidification time - how to find it out, we have discussed how the solidification goes in a pure metal and in a in alloys.

So, those things should be now clear to you. Then, we started discussing about the application of the theory part whatever we have learnt including the Chvorinov's rule which says that the TST, total solidification time is, $TST = C_m \left(\frac{V}{A}\right)^2$.

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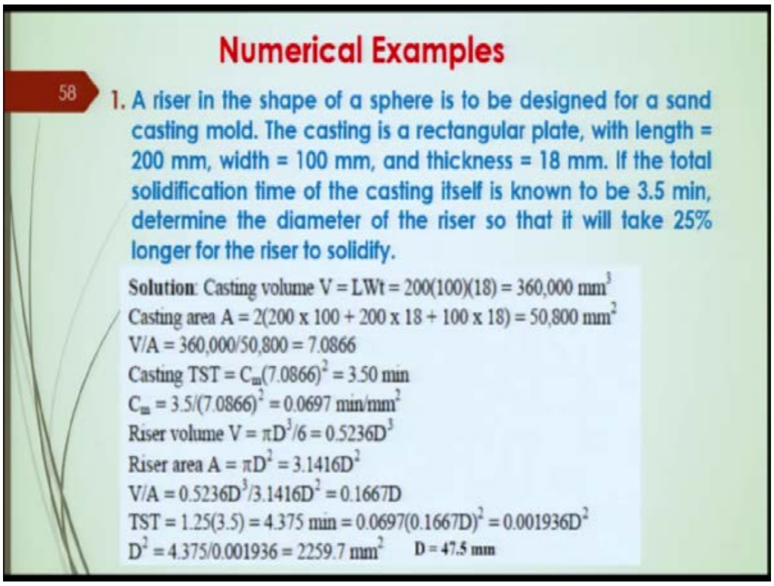

This is a direct application of the Chvorinov's rule in practice through the numerical example. It says that a riser had to be designed in the shape of a sphere, that is for the sand mold casting and the casting has to be a rectangular plate with 200 millimeter length, 100 millimeter width and the 18 millimeter thickness.

The total solidification time is given which is 3.5 minute, we have to determine the diameter of the riser so that it will take 25 percent longer for the riser to solidify. Before we go into details of this numerical example, I would like to clarify this Chvorinov's rule - what it actually identifies is that the riser design we can make according to Chvorinov's rule.

It is already said that it depends on the $\left(\frac{V}{A}\right)$ratio and we can design in such a way that the the liquefied material in the molten metal inside the riser solidifies later than the molten metal inside the mold cavity. Then only we can ensure the feeding of the molten metal to the shrinkages that happens during the solidification.

Now, coming back to the numerical example, casting rectangular plate sizes are given; length, width and thickness, we can multiply them and find out the casting volume. So, the casting volume can be found out which is equal to $360{,}000\ mm^3$. Similarly, the area can be also found out because it is a rectangular plate with a length, width and thickness given. this is becoming 50800 mm^2. So, the volume by area ratio of the rectangular plate is 7.0866.

Now, according to the Chvorinov's rule, the total solidification time, therefore, since we know the $\left(\frac{V}{A}\right)$, this will be equal to the C_m which is a mold constant into the $\left(\frac{V}{A}\right)^2$ and this is given which is 3.5 minute. Let us say it is the total solidification time of such kind of a rectangular plate already known in practice and it is given as 3.5 minute.

From this equation, we can find out the value of the C_m which is 3.5 minute divided by $(7.0866)^2$. This is in $\min/mm^2$. This is 7.0866, it is in mm^2. Now, the riser volume; riser is given as the sphere.

So, the riser volume is $\frac{\pi D^3}{6}$. This is 0.5236 D^3, π is constant. The D, we can actually put it in here $\frac{\pi}{6}$is 0.5236. So, $6.5236D^3$ is actually the riser volume. The riser area is πD^2 and this is a sphere.

So, it is 3.1416 which is πD^2. The volume by area of the riser will be 0.5236 D^3divided by 3.1416 D^2, which gives you 0.1167 D. Now, the total solidification time in the riser , this will be 1.25 times more because it is 25 percent longer; longer than the casting TST which is 3.5 minute.

So, we are finding out the total solidification time of the riser and here it is given that we have to design the riser in such a way that it takes 25 percent longer time for the riser to solidify. Therefore, the total solidification time of the riser will be 1.25 into 3.5 we have found out for the casting total solidification time, which gives you 4.375 minute,

Because we found out C_m is equal to 0.0697. This is the C_m and this is the $\left(\frac{V}{A}\right)$of the riser because we are finding out the total solidification time of the riser. Therefore, we will take this value $\left(\frac{V}{A}\right)$of the riser square and C_m we already know; this is 0.0697. Why we are taking this C_m? You understand this, I already said that C_m of the riser and the casting will be the same.

We found it out for the casting. So, this can be taken as 0.0697. This will give you 0.001936 D^2 and from here D is equal to we can find out root over 2259.7. This will be 47.5 mm. This as you can see is quite simple; based on the basic principle that the total solidification time is equal to mold constant into $\left(\frac{V}{A}\right)^2$.

This is the Chvorinov's rule and accordingly, we can find out the $\left(\frac{V}{A}\right)$of the casting $\left(\frac{V}{A}\right)$of the riser and then, we can solve this because in one initially we can find out the C_m and that C_m will be the same for the riser. So, put that value and find out what is the total solidification time. From there, you can find out the dimension. This is how it is solved.

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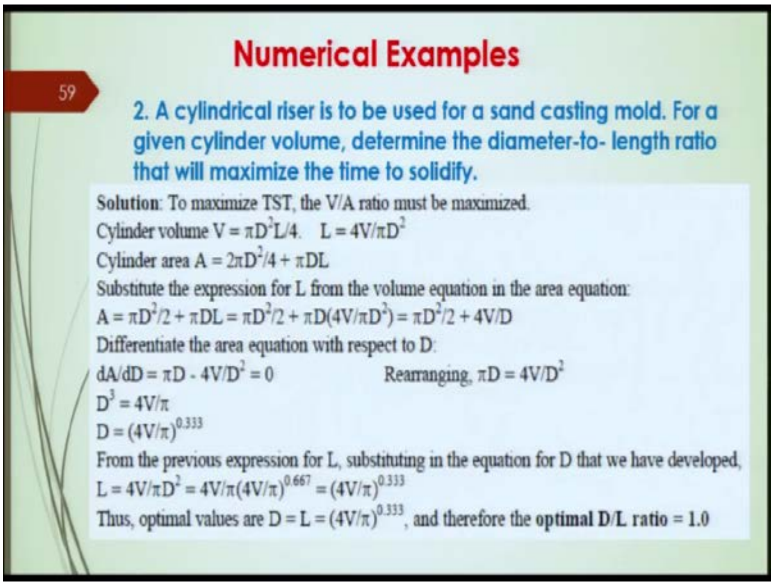

Now, the second example based on the same principle, that is the Chvorinov's rule. It says, a cylindrical riser is to be used for a sand-casting mold. For a given cylinder volume, determine the diameter to length ratio that will maximize the time to solidify. This question is little tricky, little different than the earlier one.

Because here, it is only the conditions are given; that you have to design a cylindrical riser for the sand casting and for that, you have to determine the diameter to length ratio that will maximize the time to solidify. Let us see how to solve this.

Now to maximize total solidification time, the $\left(\frac{V}{A}\right)$ ratio must be maximum because the total solidification time is directly proportional to the $\left(\frac{V}{A}\right)$; $\left(\frac{V}{A}\right)^2$. Now, we have a cylinder. The cylinder volume is $\left(\frac{\pi D^2}{4}L\right)$. From here, we can find out L as $\frac{4V}{\pi D^2}$.

Similarly, cylinder area is $2.\frac{\pi D^2}{4}+\pi DL$. Substitute the expression for L that we found out from cylinder volume, which is $\frac{4V}{\pi D^2}$. Put this in here, from the volume equation in the area equation; this is the area equation. A is equal to $2.\frac{\pi D^2}{4}+\pi DL$.

This gives you that A is equal to $\frac{\pi D^2}{2} + \pi DL$ and instead of *L*, we are putting the value of the $\frac{4V}{\pi D^2}$. Ultimately, it will give you $\frac{4V}{D}$. So, all together it will be $\frac{\pi D^2}{2} + \frac{4V}{D}$ which is equal to the area.

Now differentiate the area equation with respect to *D*. Now, this will be *dA* first derivative of the *A* which is this with respect to capital *D*; this will be, $\frac{dA}{dD} = \pi D - \frac{4V}{D^2}$ if you take the first derivative of this equation, and that will be equal to 0. That is, the first derivative is equal to 0.

If we rearrange that, it will be πD is equal to we will take it to the right side, $\frac{4V}{D^2}$. From here, we can find out the *D* value. *D* will be $\left(\frac{4V}{\pi}\right)^{0.333}$; Now, from the previous expression for *L*, substitute in the equation for *D* that we have right now developed. *L* is equal to $\frac{4V}{\pi D^2}$. Initially, from the cylinder volume, we found out that.

In here, we can put the value of the *D*, that *D* we just now found out from the first derivative of the area with respect to the capital *D* ; putting the value of *D* and after rearranging as shown in the slide, we can find out the value of *L* as $\left(\frac{4V}{\pi}\right)^{0.333}$, which is the same as the value of *L*.

Thus, the optimum values are *D* is equal to *L*, you can see that the *D* is equal to $\left(\frac{4V}{\pi}\right)^{0.333}$ and we got the value of the L the same. So, the *D* and the *L* are the same which is $\left(\frac{4V}{\pi}\right)^{0.333}$. Therefore, this is the optimum $\left(\frac{D}{L}\right)$ ratio because we have taken the first derivative is equal to 0 and this will be equal to 1 because *D* is equal to *L*.

That is how we can find out the optimum design of the $\left(\frac{D}{L}\right)$. As you can see that this also looks like a tricky example; but as it is, not so and it is simple, you are using again, the Chvorinov's rule and then, you are finding out the volume and the area of the casting and the riser subsequently.

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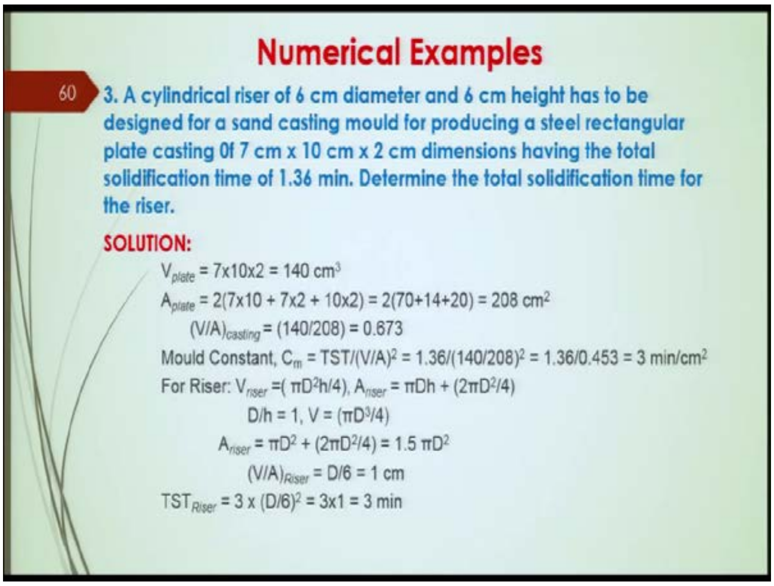

Probably to clarify the entire theory little more, we will discuss some more examples and the third example says that a cylindrical riser, again the riser is cylindrical of 6 centimeter diameter and 6 centimeter height has to be designed. Now, let me tell you in between that why we are concentrating more on the riser design. I will once again repeat that this is the most important factor that we have to get the casting with proper dimensions, with proper accuracy as far as possible.

Now, you cannot avoid the shrinkage process because this is the metallurgical property of the material; Molten metal solidifies in three different stages, it will have the contraction and therefore, it will have the shrinkage. So, it is very important for us that during the production of the casting, during the fabrication of the casting, we have to compensate for the shrinkages.

And only way to compensate for that is to feed extra molten material to the shrinkages, to the solidified metal inside the mold cavity so that those shrinkages can be to some extent, to the larger extent compensated for. That is why the design of the riser is very important that the total solidification time of the riser should be more than the total solidification time of the casting.

Because casting should solidify fast, faster than the riser and therefore, the total solidification time of the riser should be moreso that it solidifies at a slower rate than the casting. That is why in all examples we are giving importance to the riser design; in practice also we will see the riser design will be one of the most important factors, depending on the kind of casting that you are making.

Because the riser design will be very different if the design of the casting is different. All the time, we have to consider the $\left(\frac{V}{A}\right)$ratio of the casting and depending on that ratio of the casting, we have to design the volume and area ratio of the riser .

A cylindrical riser of 6 centimeter diameter and 6 centimeter height has to be designed for a sand casting mould. So, producing a steel rectangular plate, again the plate is rectangular casting. This is of 7 centimeter into 10 centimeter into 2 centimeter height. This is the length, this is the width, this is the height; having the total solidification time of 1.36 minute.

Now, here also you can see the total solidification time is given like in one of the earlier examples because this is taken from the earlier practice. Like if we have made a similar casting somewhere else, so you know what is the solidification time of that if the material is the same and if the dimensions are the same.

This is known, let us say, therefore, determine the total solidification time for the riser. Now, here the plate is given. This is the rectangular plate. 7 x 10 x 2 will be the volume of the plate in cm^3. Volume of the plate is (7 x 10 x 2) which is 140 cm^3. Now, the area of the plate will be, we have done it earlier, 2(7x10 + 7x2 + 10x2). This will be 208 cm^2.

Now, the $\left(\frac{V}{A}\right)$of the casting, this will be 140 divided by this 208. this is coming out to be 0.673, unit less , it is a ratio. Now, the mould constant. By the way the mould constant always comes out as minute by centimeter square. This is the mould constant which is total solidification time divided by $\left(\frac{V}{A}\right)^2$.

Total solidification time is known which is 1.36 minute divided by $\left(\frac{V}{A}\right)$,already we found out, which is $(0.673)^2$. This is coming out to be 0.453. $\left(\frac{1.36}{0.453}\right)$,which is 3 min/$cm^2$. This is the mould constant. Now, for the cylindrical riser , the volume of the riser is $\frac{\pi D^2}{4}h$,h is the height. Area of the riser, A is $\left[\pi Dh + 2.\frac{\pi D^2}{4}\right]$, this also we have done it earlier that how to find out the volume and the area.

So, the $\left(\frac{D}{h}\right)$is equal to 1 . Therefore, the V is equal to $\frac{\pi D^3}{4}$. Now, the area of the riser we have found out to be $\left[\pi D^2 + 2.\frac{\pi D^2}{4}\right]$. From here, you put these values which will be $1.5\pi D^2$, if we simplify this. Therefore, the $\left(\frac{V}{A}\right)$of the riser because we have the V riser and we have the A riser, we take the ratio, it will be $\left(\frac{D}{6}\right)$ and $\left(\frac{D}{6}\right)$is 1 cm, since D = *6 cm* given. .

Now, the total solidification time is $3.\left(\frac{D}{6}\right)^2$because C_m is equal to 3. We took the C_m here and $\left(\frac{D}{6}\right)$is the $\left(\frac{V}{A}\right)$ of riser square. This is equal to 1 and therefore, it will be 3 minutes. The total solidification time, that is what we had to determine, what is the total solidification time for the riser, this has come out to be 3 minutes.

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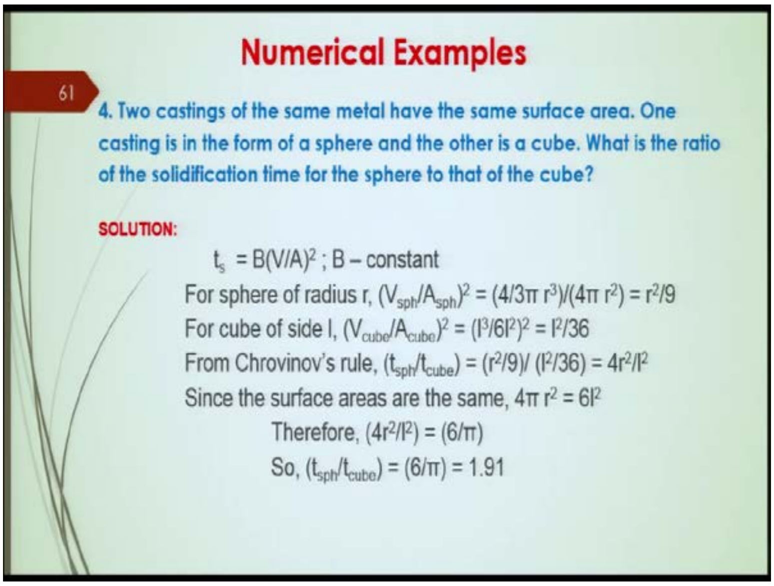

The next numerical example that we will discuss is the following. The two castings of the same metal have the same surface area; one casting is in the form of a sphere and the other is a cube. What is the ratio of the solidification time for the sphere to that of the cube? See if we look at this numerical example, no data is given except that it is sphere and cube. Knowing very well that we can find out the volume of the sphere, volume of the cube, area of the cube, area of the square and so on. Let us see. It is an interesting question, interesting example.

Now, the solution is the total solidification time t_s, this is the mould constant multiplied by $\left(\frac{V}{A}\right)^2$ which is Chvorinov's rule. *B* is the mould constant. Well, earlier if we designated that as the C_m; does not matter, here we are saying suppose it is *B*, this is the mould constant again equivalent to that C_m. Now, for sphere, one casting is in the form of a sphere.

So, for the sphere of radius r, $\left(\frac{V}{A}\right)^2$; *V* sphere is $\frac{4}{3}\pi r^3$ is the formula for the volume of a sphere; when the *r* is the radius of the sphere. Now, that is divided by the *A*, area of the sphere and area of the sphere is $4\pi r^2$; *r* is again the radius of the sphere.

So, $\left(\frac{V}{A}\right)$,that is volume by area ratio will be $\frac{r^2}{9}$. Now, for cube, we have in the form of a sphere one casting and another is in the cube; but both of them have the same surface area. You mind it, this is the condition for this example. Now, let us find out for the cube. Let us say the cube side is *l*.

Volume of the cube V is l^3 and the area is the $6l^2$. the ratio is $\frac{l^3}{6l^2}$which is. All together, $\left(\frac{V}{A}\right)^2$ will give you l square divided by 36 because it will be l by 6 inside and square of that will be $\frac{l^2}{36}$.

Once again, we have found out for the sphere what is the $\left(\frac{V}{A}\right)$ratio, this is clear. Because the volume of a sphere is $\frac{4}{3}\pi r^3$ and the area of the sphere is $4\pi r^2$. So, the ratio will be $\frac{r^2}{9}$. Now, for cube, the volume of the cube is l^3. Let us say side is l; small l and the area of cube is $6\,l^2$.

So, their ratio will be $\frac{l^3}{6l^2}$and the $\left(\frac{V}{A}\right)^2$ as it is used in the Chvorinov's rule will be $\frac{l^2}{36}$, this is what has been said. Now, from Chvorinov's rule, the ratio of t_{sphere} and t_{cube} that is the total solidification time of the sphere and the total solidification time of the cube will be equal to these two ratios because the constant is getting cancelled, it is the same.

So, it will be $\frac{r^2}{9}$which is for the sphere volume by area ratio square divided by the volume by area ratio square of the cube. This is $\frac{l^2}{36}$ and this will give you the $\frac{4r^2}{l^2}$ this you are getting from the Chvorinov's rule, that what is the ratio of the total solidification time of the sphere and the total solidification time of the cube.

Because we found out separately; we can find out the total solidification time here, as you can see the *B* constant, that is the mould constant is not coming into picture because it is getting cancelled. Therefore, this value is not given and this value is not required actually because this is getting cancelled and we already said that the mould constant for the casting and mould constant of the riser will be the same.

Why it is same? Because the same material is going into the riser, same molten metal which is fed to the cavity mould cavity, it goes into the riser. Therefore, the C_m is same for the casting and for the riser. In this example very interesting is that mould constant is not actually taken into consideration at all and while taking the ratio of the total solidification time of the sphere and the cube, we are directly taking the $\left(\frac{V}{A}\right)^2$ of both of them that is the sphere and the cube separately taken.

See how interesting it is and you can find out the surface area of the same. Therefore, $4\pi r^2$ is equal to $6l^2$, from here it will be coming. Now, because the surface area we know; surface area of this $4\pi r^2$ and here for the cube it is $6l^2$. They are the same; $4\pi r^2$, area of the sphere equal to area of the cube.

So, from here, you can simplify this and you can find out the ratio of $\frac{t_{sphere}}{t_{cube}}$, this will be $\frac{6}{\pi}$ which is 1.91 because that $\frac{4r^2}{l^2}$, this will be $\frac{6}{\pi}$; $\frac{6}{\pi}$ is equal to 1.91.

The ratio of the total solidification time of the sphere and the cube will be equal to 1.91. As you can see that the examples are simple, this is how you can understand the exact application of the theory that we have gone through, particularly, as far as the riser design is concerned and riser design is very important to get the proper kind of a casting.

So that the casting could be defect free and by defect, we mean to say not all the other defects which will be discussing at a later stage; but particularly, in the dimensions, in the form of the dimensions, in the form of the shrinkages. To compensate for the shrinkage, the design for the riser is extremely important and this is how we can take care of that.

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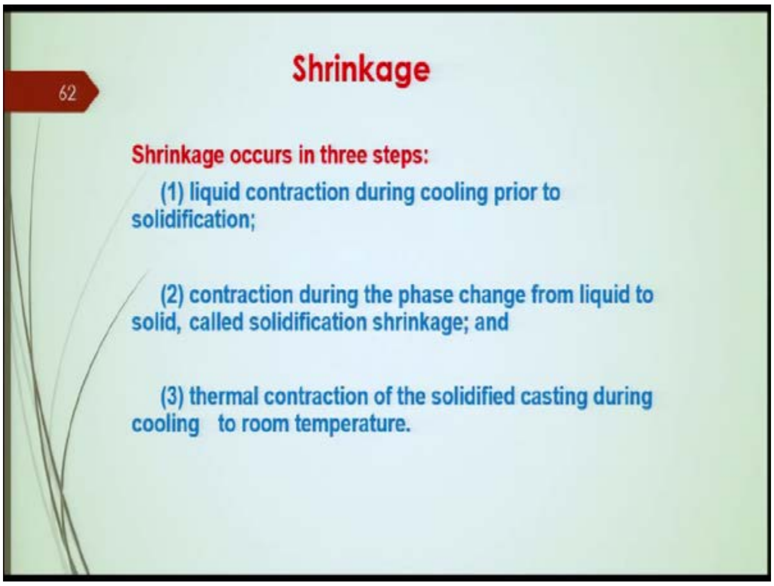

I hope this is understood. Now, we will discuss the shrinkages. What kind of shrinkages we have and why these shrinkages happen and how those shrinkages can be taken care of. Let us see this. Shrinkage occurs in three stages. I will remind you that the first stage will be the liquid contraction during the cooling period to the solidification.

This is the cooling period, when it is starting or it started cooling. Contraction during the phase change from liquid to solid is called the solidification shrinkage and the thermal contraction of the solidified casting happens during the cooling to the room temperature.

So, these are the three stages when the shrinkage happens and we have to take care of the shrinkage through the design of the riser, by the appropriate design of the riser. Once again, it will be liquid contraction during the cooling prior to the solidification. Just solidification starting at that point prior to solidification during the cooling, there will be a contraction.

During the phase change from liquid to solid, it is getting changed from liquid to solid, there will be contraction, there will be shrinkage; and the thermal contraction of the solidified casting happens during the cooling to the room temperature, that is the last stage. If you remember in that curve, there the shrinkage again takes place.

So, in these three stages the total shrinkages will be taking place that have to be taken care of. The rest of the material, we will discuss in our next session of discussion.

Thank you.