Elements of Solar Energy Conversion
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Lecture - 30

Hello everybody, welcome back to the series of lecture on Elements of Solar Energy Conversion. We started looking at the photovoltaic conversion method; we have started with the basics of how the semiconductor, the carrier, or the charge carriers concentrations can be tweaked; what are the major factors in quantifying those concentrations these we have looked at. And at the end of the last class, what we have started is the carrier motion.

So, today we are going to continue on that carrier motion, and then we will introduce few new concepts to understand this carrier motion in more detail. Today, we are here at lecture number 30.

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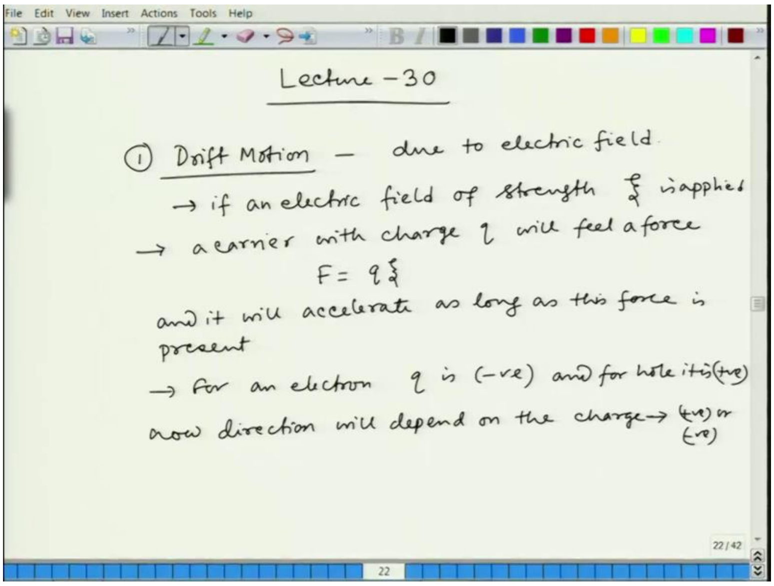

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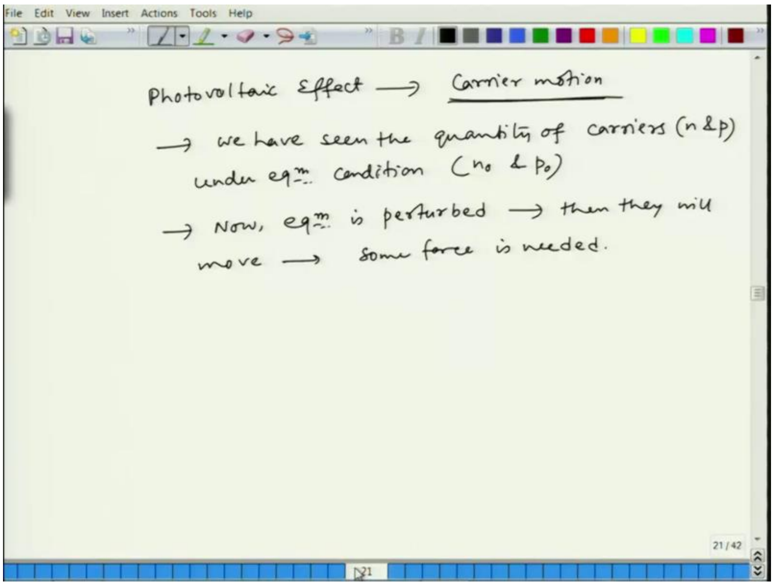

So, in the last class, we have looked at this carrier motion and this n_o and p_o; these are the equilibrium concentrations. Now, if this equilibrium is perturbed either by applying a voltage or by putting solar light on these photovoltaic materials, then the equilibrium is perturbed, and then the carrier motion becomes real.

So, here we now try to understand what kind of motions are there and what are the factors on which those motions depend? So, the first kind of motion we called is drift motion. What is drift? We; whenever we have some electric field, then these charge carriers are charged particles, either electron or hole; due to the electric field, it will go move in directions, and that is called the drift motion.

So, if an electric field of strength ξ is applied, a carrier, it can be anything with charge q will feel a force of magnitude F equal to q multiplied by ξ. So, this is just like Newton's first law; here, the acceleration and mass are replaced by charge and field. And it will accelerate as long as this force is present; that is Newton's first law, as long as the force is applied or force is present; it will increase its velocity.

So, for electron, this q is negative, and for a hole, it is positive, is not it? Now, the direction, of course, will depend on the charge sign right. So, whether it is positive or negative, that will tell what would be the direction of motion.

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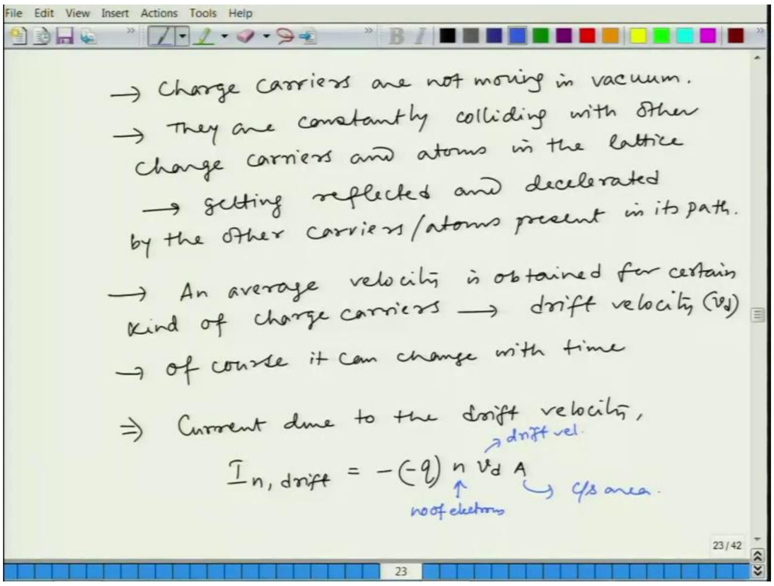

Now, it will accelerate, but it will be opposed as well because the charge carriers are not moving in a vacuum. They are interacting with all the other charge carriers and the atoms in the lattice.

So, they are constantly colliding with other charge carriers and atoms in the lattice, is not it? So, by the process, what is happening? They are getting reflected and decelerated by the other carriers, atoms, etcetera present in its path. So, it is not only just the application of field that is introducing a force; it is also allowing the deceleration mechanism. So, ultimately, what will happen? That you can find an average velocity is obtained for a certain kind of charge carrier.

So, for all the electrons, if you take the average of all these collisions and the field applied field, then what will happen? That you will get an average velocity which is called, let me write these charge carriers that average velocity is called drift velocity or v subscripted with d. And of course, it can change with time, so there might be some acceleration; net acceleration on average for all the electrons, then you will have an increasing function increasing drift velocity with time.

Now, if you have that drift velocity, the effect of that on current, so current due to the drift velocity would be; let us say its I_n; I stands for current, n stands for electron, and let us put it drift because it drifts only current.

Now, it will be

$$I_{n.drift} = -(-q_n)nV_dA$$

And the charge of each electron is minus q, and the current is always measured in the opposite direction of an electron, so there is another minus sign in front. And what are other things? So, this is the number of electrons; this is already drifting velocity, and this is the cross-sectional area because we are not talking about current density but current. So, that will depend on the cross-sectional area, so this is straightforward.

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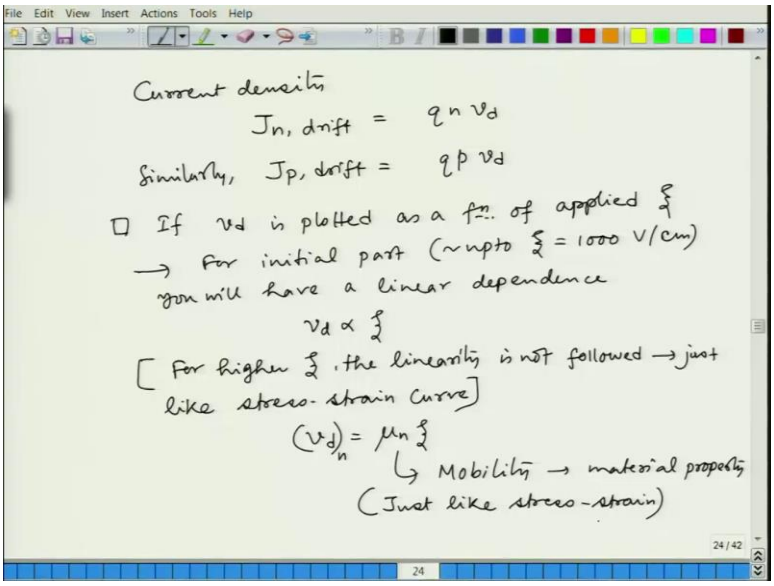

Now, similarly, if we put current density, that we represent through the symbol J. So, $J_{n,drift} = qnv_d$ two positives make it negative; two negative make it positive, and then you have this drift velocity v_d. And similarly, for the holes, you can find J p drift that will again be $J_{p,drift} = qpv_d$. So, p is the number of holes available.

So, now, if v_d is plotted; so, now we want to know some material property which will tell you the v_d. So, acceleration is the force is known that is due to the field q into ξ. Now, the drift velocity will depend on the lattice structure, other carrier concentration, and everything, so it will depend on some material property.

So, we want to quantify the drift velocity, or we can, or we want to relate that drift velocity with the particular material property. So, to do that, if we plot this v_d as a function of an applied field, then for the initial part, so up to about 1000 Volt per centimeter; if that is the field strength, then you will have for that initial part you will have a linear dependence.

Just like the stress-strain curve, you know for the initial part, you have a linear dependence, and we call that is the elastic part. And beyond, when the stress value or the strain value is higher than a certain value, then you have this plastic region where the relationship is not linear anymore; here also v_d; if it is plotted against ξ or the field, then you will get the initial linear relationship.

So, for that part, what we can write is v_d is proportional to ξ, and for higher ξ, the linearity is not followed. So, it will help you understand, or you make one-to-one correspondence if you think of the stress-strain curve.

So, for reasonable field strength; what we can write that this v_d is some kind of constant into ξ. So, v_d for an electron is some kind of constant into ξ, and this constant is called mobility and its and material property. It is just like elasticity as we encounter in the stress-strain curve. So this mobility is electron mobility.

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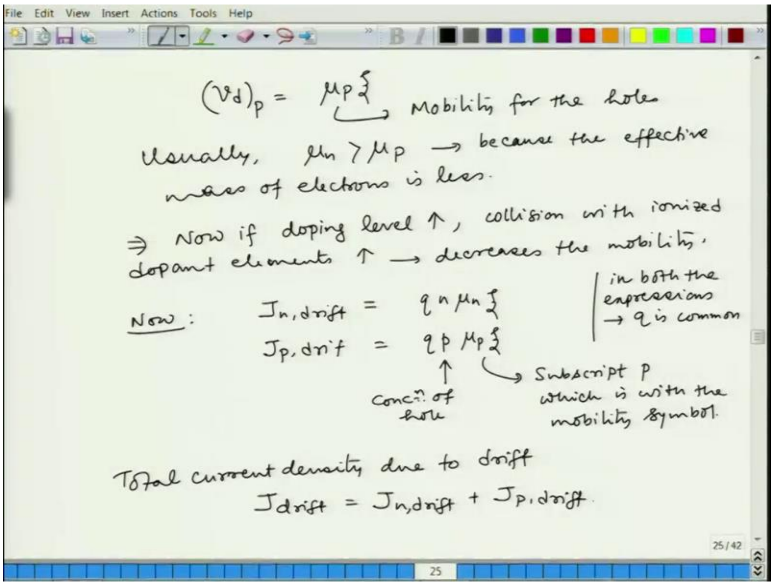

So, similarly, for holes, you can write mobility for the holes multiplied by the ξ or the field strength. So, again this is mobility for, and of course, the μ_n and μ_p will not be equal. For any material, it will be different because they have different mass, they have a different motion, the collision, and everything. So, μ_n and μ_p do not have to be equal.

Usually, μ_n is greater than μ_p; the mobility for an electron is more because the effective mass of electron electrons is less that is why the mobility is more for electrons. Now, if the doping level increases, that means collision with ionized dopant element that also increases and it decreases the mobility.

So, for a certain doping level, you can define mobility which is a material property. Now, if the doping level changes, then mobility can also change. So, now, with the introduction of this mobility, what we can write this $J_{n,drift}$ which is the electron-dense; the drift velocity induced current density due to electron only, we can write $qn\mu_n$. So, v_d for electron, we have replaced that with μ_n and ξ.

So, here I should for whenever we are using v_d; we should use whether it is for electron or hole. So, here let me write it is for electron and for hole ok. So that this mobility dependence is ensured. So, now, v_n we have replaced that with μ_n and ξ. So, $J_{n,drift}$ will be like this, and similarly, $J_{p,drift} = qp\mu_p\xi$.

So, here one thing you note that in both the expressions, q is common because electron and hole have an equal amount of charge; they carry an equal amount of charge, but the sign is opposite electron is the negative hole is positive. And this p, so this p or n just above it is the concentration of hole, and this p is just the subscript p which is with the mobility symbol.

So, please pay attention $qp\mu_p\xi$ that does not mean q, the first p is not a subscript, but the second one is. Now, the total current due to drift will be the summation of these two currents. So, total current density due to drift will be $J_{drift} = J_{n,drift} + J_{p,drift}$. Is not it? Because both of them will carry a charge, and that is why the total current density will be a summation of them.

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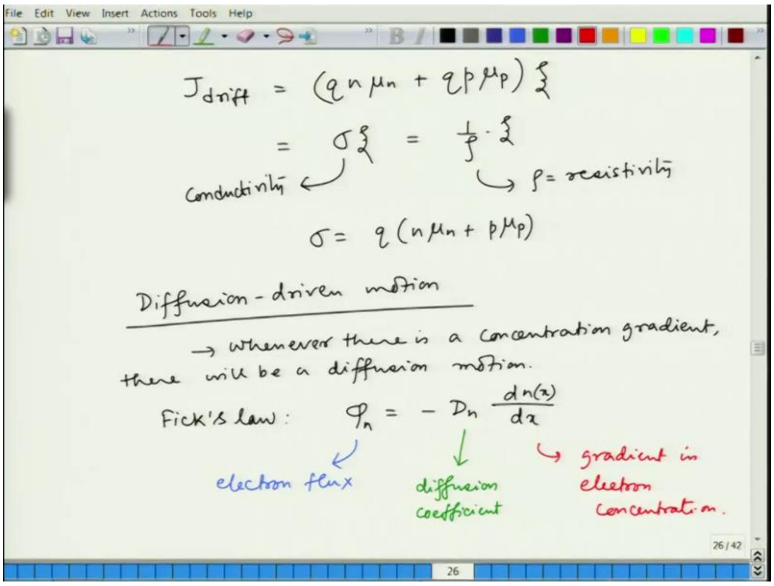

So, rather we can write the $J_{drift} = (qn\mu_n + qp\mu_p)\xi$, we can write to be some conductivity sigma multiplied by ξ. So, this is the definition of conductivity. So, the amount of, I mean field you are applying, whatever proportionality constant it has with the drift current is called the conductivity or the same thing we can write, as one over resistivity. So, though, so this rho is resistivity.

So, now, you know what the source of this resistivity or conductivity is? So conductivity you can write $\sigma = (qn\mu_n + qp\mu_p)$. So, it is not a black box anymore; you know from where all these things are coming. Now, other than the drift current, what we have is the diffusion-driven motion or diffusion-driven current. What do you mean by diffusion? Diffusion is a thing that is driven by the concentration gradient. Whenever you have a concentration gradient, it will diffuse to a portion where the concentration is less.

So, whenever there is a concentration gradient, there will be a diffusion motion, is not it? It is not only for charged particles; it is for any particles. For charged particles also, if you have some electron concentration more somewhere, it will try to move or diffuse to the portion where electron concentration is less. So, those diffusion phenomena are driven by the Ficks law, and the flux of electron is obtained with the electron; sorry; the diffusion coefficient for the electron multiplied by the gradient in concentration. So, this is what; this is the electron flux, this is the diffusion coefficient, and this is the gradient in electron concentration right.

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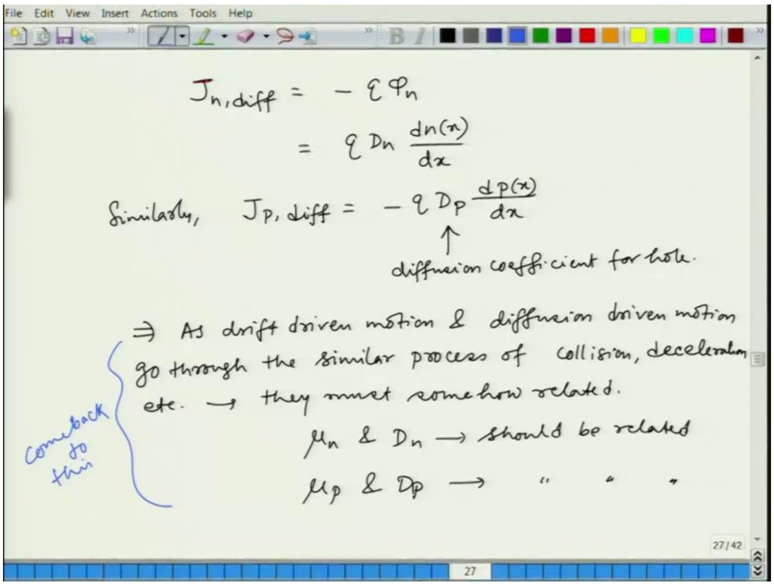

So, now, if we have this diffusion-driven current; what we can write, what we can write is the electron or electron diffusion-driven current density will be $-q\varphi_n$. Because it will be carrying that much charge, and the current density will be negative of that charge. So, if we put that ϕ value, we get q into D_n, then the gradient of the electron concentration.

Similarly, the current density for hole diffusion will be $-qD_p\frac{dp(x)}{dx}$. So, here also, you will see that this is the diffusion coefficient for the hole. So, you can intuitively think of this; the diffusion process will also be driven by similar things. So, electrons will go try to go to a lower concentration region, and in the path, it will collide with the similar carriers and atoms as it does for a drift-driven motion, is not it?

So, there must be some relation between these two; diffusion-driven motion and drift-driven motion or field-driven motion. So, here let me write this observation or this intuitive question which we will come back, after discussing a few other things. So, as the drift-driven motion and diffusion-driven motion go through a similar process of collision and deceleration, etcetera. They must be somehow related, and so; that means this μ_n and D_n should have some relationship, should be related and similarly μ_p and D_p; is not it?

So, we will come back; so come back to this, after we discuss; after we take a little detour because we will need that detour to make this relationship possible.

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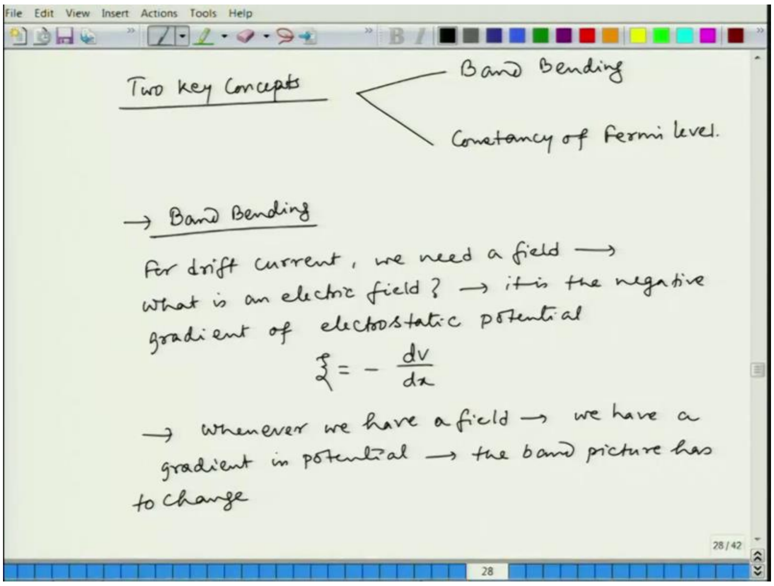

So, what is that detour? Now, we will come to two key concepts, which are key to the photovoltaic effect. So, what are those two key concepts? One is called band bending; band means the electronic band structure it bends, under what circumstances? We will see.

So, that is one concept that you will require, and the third, the second one, is the constancy of the Fermi level. Earlier, we have seen that to quantify the concentration of electrons and holes, we needed two key concepts, one was the density of state, and the other one was the Fermi function. Here also, for going to the next level, to input the carrier motion, and that is how the photovoltaic effect is generated, we need to have these two key concepts; one is band bending, and the other one is constancy of Fermi level.

So, now, you can think of; first, let us look at this band bending. Now, intuitively you can think of for drift current, we need a field right that we already talked about. It is a field-driven current which is called drift current. Now, what is an electric field? It is the negative gradient of electrostatic potential, so that is the definition of an electric field. So, you can write the ξ as a negative gradient of electrostatic potential; here, v is the voltage or potential, and x is the distance.

So, ξ initially we told that in the initial part up to 1000 Volt per centimeter. So, that is the unit of the field; Volt per centimeter here also we can see that it is $\frac{dv}{dx}$ with a negative sign. So that means, physically, that whenever we have or have a field, or we say that a field is applied, that means we have a gradient in potential. If the potential is flat, then we do not have a field; when the potential has a gradient, then only we can have a field.

So that means the band picture has to change. Because band picture is what? It is the potential energy description of the electronic states that is the band picture.

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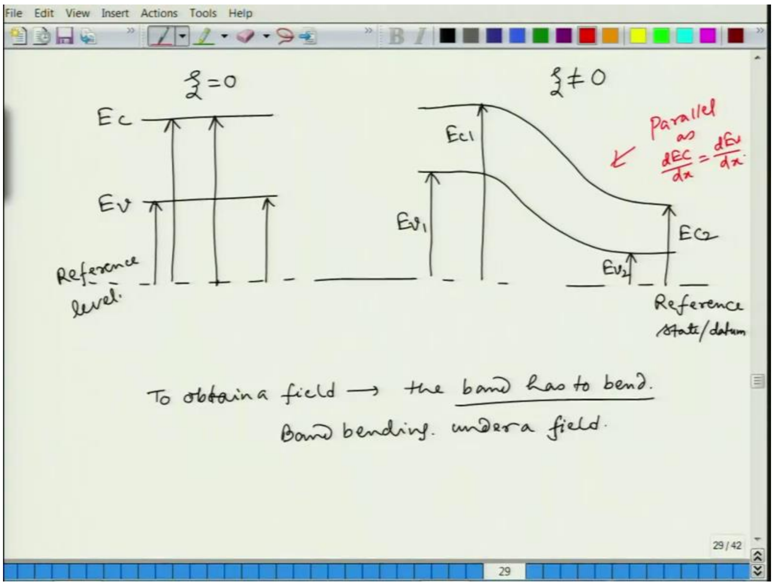

So, whenever you have a potential gradient, it has to change. So, initially, when you do not have any potential, or you do not have any potential gradient or a field, then we can write this to be conduction band and this to be valence band with flat potential ok; so, this is the case for ξ equal to 0.

So, if you have this datum, this reference level ok, this everywhere has the same height for the conduction band or valence band ok, and that is why you do not have any gradient of it. Now, whenever we call ξ is not 0, then that means it has to have some gradient. Now, everything is bending ok, because now, with respect to the same datum, if you say reference state or datum; now, you have here E_v is larger; E_{v1} is larger than another location where it is E_{v2}.

Similarly, here E_{c1} is greater than E_{c2}; is not it? So, this has to be the case when ξ is non-zero. So, to get a field, to obtain a field, the band has to bend right. That is the concept of band bending, which is called band bending under a field ok.

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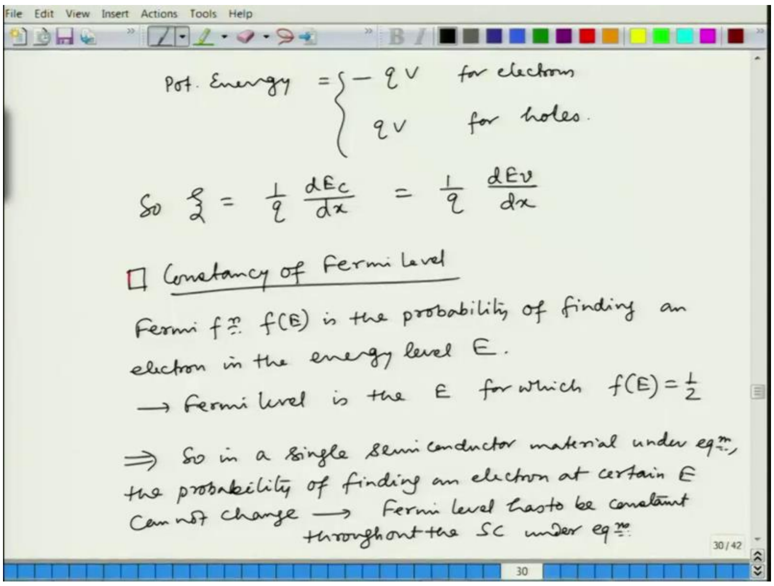

So, now, if we want to quantify that; so the potential energy is equal to $-qv$ that is the voltage for electrons ok, and it is equal to qv for holes.

So, what can we write? $\xi = \frac{1}{q}\frac{dE_c}{dx}$.$E_c$ is the conduction band energy; that means, for the electrons, the conduction band is associated with electrons, and the valence band is associated with holes.

So, we can write $\xi = \frac{1}{q}\frac{dE_v}{dx}$. And everywhere, you will see that both the conduction and band and the valence band will be affected equally, and that is why they stay parallel. Here, they are parallel as this $\frac{dE_c}{dx}$ is equal to $\frac{dE_v}{dx}$ everywhere ok, and that is why they stay parallel, but the overall band bends.

Now, the second concept we are going to look at is the constancy of the Fermi level. Now, what is the Fermi function? Fermi function is the probability of finding an electron at a certain energy state E right. So, Fermi function f€ is the probability of finding an electron in the or yeah, in the energy level E ok; or an electronic state at energy level E, whether it will be occupied or not that is that probability is represented by the Fermi function right.

So, and Fermi level is what? The level is the energy E, for which this probability is half right. So, it is one point on that Fermi function; a particular value of that Fermi function which we call Fermi level. So, it is important to think of this physical meaning because otherwise, you can mathematically juggle many things, but the fundamental idea or concept will never be clear.

So, please pay attention to why the Fermi level has to be constant; the constancy of the Fermi level principle is based on this physical concept that it is probability. So, what you can write; so in a single semiconductor material under equilibrium ok; the probability of finding an electron at certain E cannot change right.

So, that means the Fermi level has to be constant throughout the semiconductor material under equilibrium. So, if the probability changes, then there will be some motion, and under equilibrium, that motion is not allowed right.

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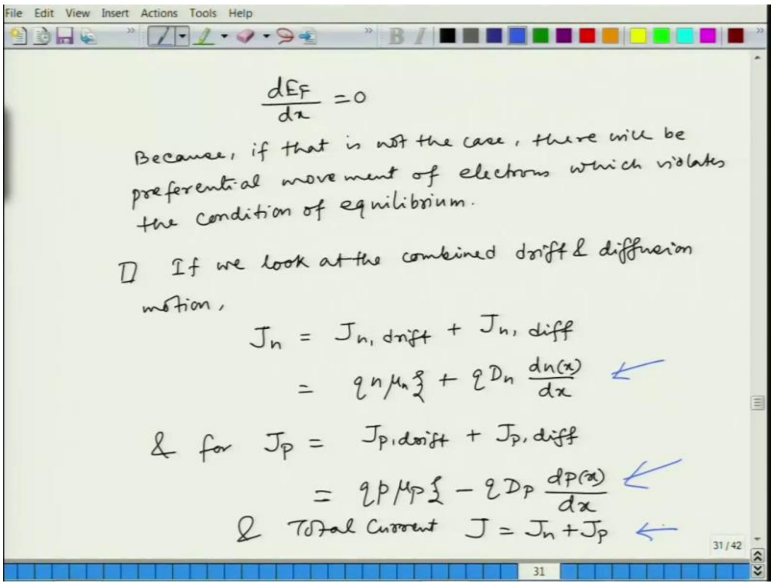

So, that means, under equilibrium, the Fermi level has to be constant everywhere. So, what it means? That $\frac{dE_F}{dx}$ will be 0, because if that is not the case. So, this is called the principle of negation ok. If it is not the case, there will be preferential movement of electrons which violates the condition of equilibrium right.

So, you start with some hypothesis, ok; if this is true and then you end up with something which is unphysical or not possible. Then you say ok; the first thing with that we have assumed that cannot be true. This is how the principle of negation or the argument of negation works and that what tells us that constancy of Fermi level has to be maintained.

Now, so we talked about that we will come back to this motion; we have to connect this diffusion coefficient and the mobility of that material or mobility of electron in that material, we will connect them after taking the detour. So, now, we have talked about the band bending, as well as the constancy of Fermi level. Now, we are coming back to the motion again.

So, if we look at the combined drift and diffusion motion ok, then what we have? This total current density due to electron will be current density due to drift plus the current density of electron due to diffusion right. And we have also quantified them at least in terms of this mobility and diffusion coefficient

$$J_n = qn\mu_n\xi + qD_n\frac{dn(x)}{dx}$$

These two terms we obtained earlier.

And for hole J_p, the current density due to the hole will be again due to drift plus diffusion of holes. And here what we have

$$J_p = qp\mu_p - qD_p\frac{dp(x)}{dx}$$

And the total current or total current density; we can write $J = J_n + J_p$.

Because in the original current density itself, we have taken care of the direction. So, the electrons go in the opposite direction to the current, and holes go in the opposite; in the same direction to the current. So, that is already incorporated in these expressions. So here we can write; it is just simple addition.

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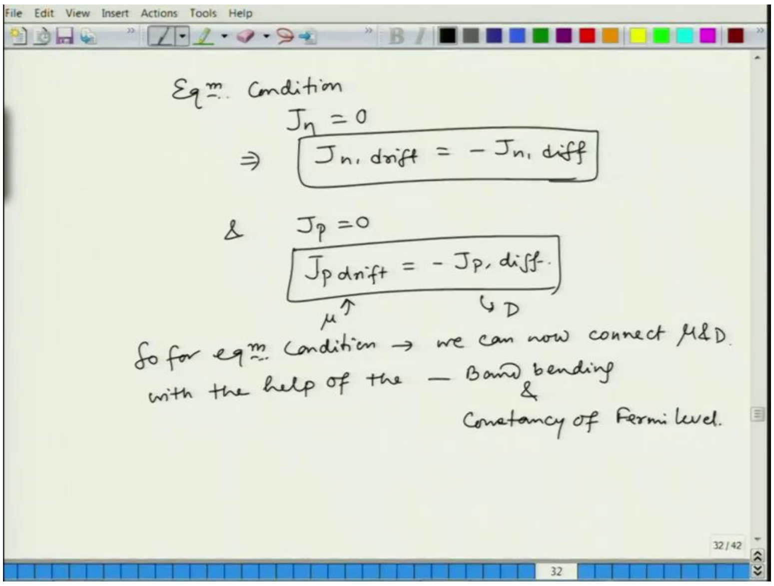

Now, under equilibrium conditions, there should not be any net motion, is not it? So, we can write J_n will be 0 ok; no electron should, in an effective way, move.

So, that means $J_{n,drift}$ has to be equal to minus $J_{n,diff}$ and similarly, holes will also not have any effective motion or net motion, so this will tell that $J_{p,drift}$ will be equal to minus $J_{p,diff}$. Now, we are in a position to connect these two; the diffusion coefficient and mobility. So, for the equilibrium condition, we can now connect because you can see that on the left-hand side, you have this μ, and on the right-hand side, you have capital D or diffusion coefficient ok.

So, under equilibrium conditions, we can now connect this μ and D with the help of the two concepts that we have introduced; one is band bending and the constancy or Fermi level ok, how can we do that? Let us see.

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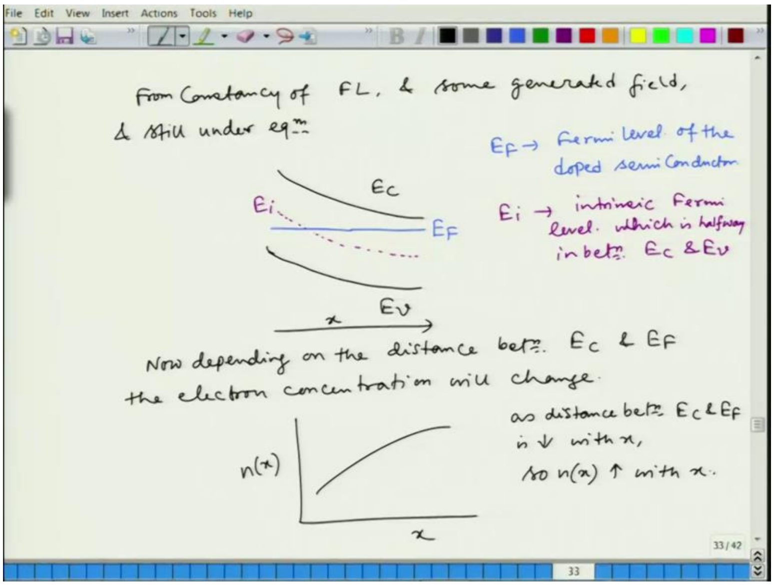

So, for or from the constancy of Fermi level and some generated field, what we can write and it has some generated field, but still under equilibrium. So, the net motion will be 0, but it has some field; if there is no field, the drift motion will be 0. And if drift motion is 0, then you know that the diffusion motion will also be 0 under equilibrium, but here at equilibrium, you have some field, and the equilibrium is also maintained, so the two currents have to cancel each other.

So, in this case, what we have that due to generated field, we have this band bending. So, let us say this is the change in the potential for the conduction band, and this is the change in potential in the valence band. And we have seen that they will be parallel because they are measured under the same datum, and the constancy of Fermi level says that throughout the material, the Fermi level should stay the same ok. It cannot be different.

Now, we can also draw the intrinsic Fermi level here; intrinsic Fermi level is where? It is exactly in between the conduction band and the valence band right. So, this is the intrinsic Fermi level E_i, and this particular thing is our actual Fermi level under the equilibrium and under a certain field. So, let me write that E_F is the Fermi level of the doped semiconductor, ok. And E_i; intrinsic Fermi level which is in between E_c and E_v or rather which is halfway in between; it is exactly bisecting the E_c and E_v ok.

But E_F; can have a different distance from the E_c or E_v, depending on the doping level right. So, that is why we have kept E_F to be the doped semiconductor or extrinsic semiconductor Fermi level, but it has to be constant throughout. Now, depending on the distance between the conduction bandage and the Fermi level, the electron concentration will change right.

This we have seen earlier, ok. How the distance between the Fermi level and the conduction bandage I mean it is you can quantitatively relate it with the electron density. And similarly, the distance between the Fermi level and the valence band edge; you can relate it to the concentration of hole ok.

So, in this case, in the above figure, you can write the concentration of electron with the distance will be somewhat like this. It will increase as you go up in x. So, let us say this is x ok;

so as the distance between E_c and E_F is decreasing with x; so the electron concentration will increase with x ok. So, this we have seen earlier; we are just making the connection between these two.

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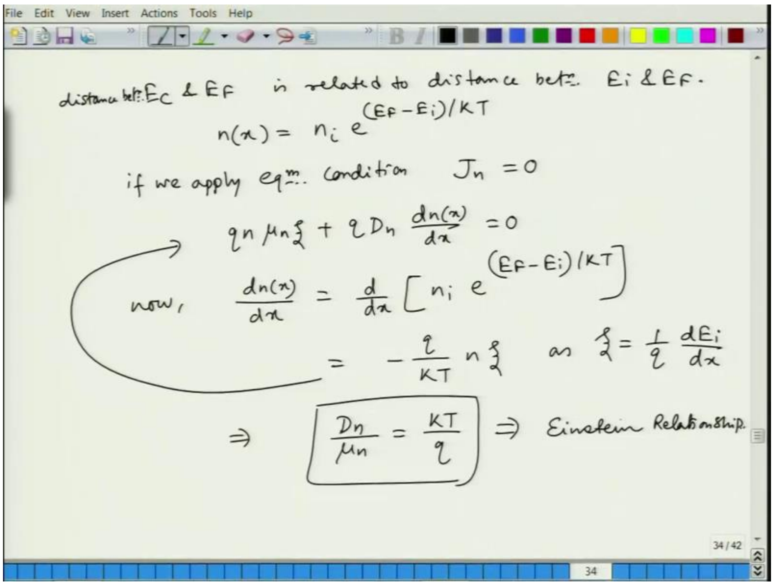

Now, this E_c and E_F, or rather the distance between E_c and E_F, is related to distance between E_i and E_F right. You can see here that you can also write in terms of E_c minus or (E_F- E_i). So, what you can write; this n(x) will be

$$n(x) = n_i e^{(E_F - E_i)/KT}$$

So, this is nothing we have done; we have just used the formula that we earlier derived in the last class that the concentration of electron will depend on the distance between the intrinsic Fermi level and extrinsic Fermi level. And the n_i is the pre-exponential factor which is the intrinsic concentration of electron ok.

So, if this is the case, now, if we apply the equilibrium condition, that means the effective flow of electron will be 0. We can write

$$qn\mu_n\xi + qD_n\frac{dn(x)}{dx} = 0$$

So; that means, $\frac{dn(x)}{dx}$ is now, I mean it is not related. Now, this one is we can write

$$\frac{dn(x)}{dx} = \frac{d}{dx}[n_i e^{\frac{E_F - E_i}{KT}}]$$

So, from here, if you put this information here in this expression, what you get is

$$\frac{D_n}{\mu_n} = \frac{KT}{q}$$

That is what we were after, right. So, we had to relate the diffusion coefficient and the mobility D_n and $\mu_{n,}$ and they are related this way ok, and this is the famous Einstein relationship ok. So, we will elaborate on this in the next class, and we will go further in the motion of these electrons and holes.

Thank you for your attention, we will see you in the next class.