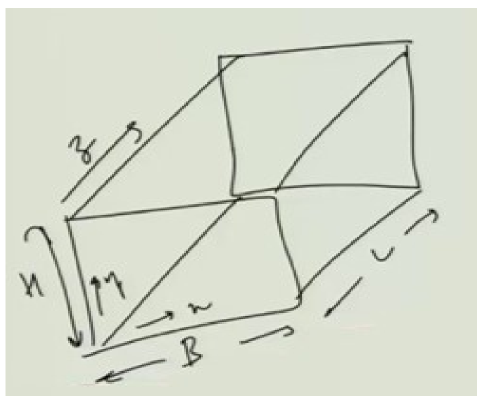
Muffler Acoustics - Application to Automotive Exhaust Noise Control
Prof. Akhilesh Mimani
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Week-11
Lecture - 54 and 55
Circular Chambers: Characterization and TL Analysis using 3-D Piston-Driven Model

Welcome back to our NPTEL course on Muffler Acoustics. So, this is again the combined lectures 4 and 5 and in this extended lecture what we will do is that we will consider a three dimensional analysis of circular cylindrical muffler subject where the ports are located arbitrarily on the inlet or out arbitrary on the either on the end faces or on the side surfaces. So, single inlet and single outlet system. So, and hopefully we will be also able to see some MATLAB simulations.



$$at\ x = x, B$$

$$\frac{\partial \tilde{p}}{\partial x} = 0, \qquad U = 0$$

$$\frac{\partial \tilde{p}}{\partial y} = 0, \qquad V = \ \ y = 0, 4$$

$$\frac{\partial \tilde{p}}{\partial z} = 0, \qquad W = \ \ z = 0, L$$

$$k_{lmn}^2 = \left(\frac{l\pi}{L}\right)^2 + \left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{B}\right)^2$$

$$\tilde{p}(x,y,z) = \sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{lmn}\cos\left(\frac{l\pi z}{L}\right)\cos\left(\frac{m\pi y}{H}z\right)\cos\left(\frac{n\pi x}{B}z\right)e^{j\omega t}$$

- Finite Sized or Cross Section Area is Represented by Mathematical Points
- Green's Function Definition Response to a Point, Point Source in Space or Impulse Time
- Uniform Piston Driven Model Planar Wave Fronts in The Port, Right at the Chamber Interface

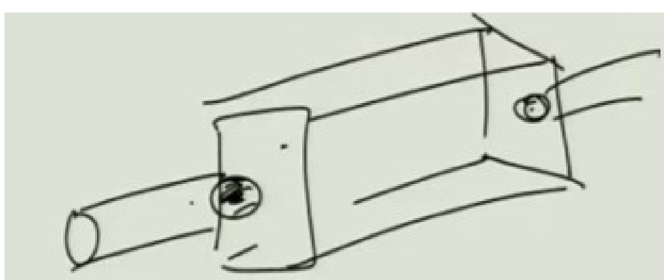
And so, what we will do is that we will we need to probably revisit the contents of the last presentation you know. So, basically what we had? We had the Green's function coming in somewhere here you know this was the Green's function in the Cartesian system and subject to I am sorry the point source represent representation of the port and this was for the uniform piston model in both in Cartesian system.

$$\left(\Delta^2\tilde{p} + \frac{\omega^2}{C_0^2}\tilde{p}\right) = -j\omega\rho_0 Q_0 \delta(x-x_0)\delta(y-y_0)\delta(z-z_0) \qquad (3)$$

$$\frac{\omega}{C_0} = k_0 = \text{ excitation wave number } \frac{1}{\text{m}}\ \ k_0^2 = \frac{1}{m^2}$$

$$= -j\omega\rho_0 V_0\ \delta(x-x_0)\ \delta(z-z_0)$$

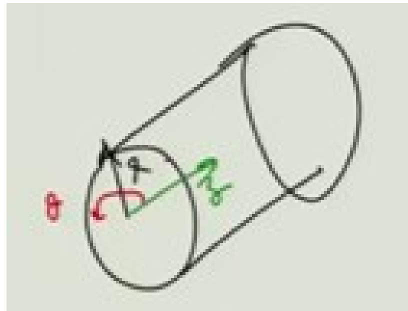
$$(\Delta^2 + k_0^2)\ \tilde{p} = -j\omega\rho_0 Q_0 \delta(x-x_0)\delta(y-y_0)\delta(z-z_0) \qquad (4a,b)$$



Inhomogeneous Helmholtz equation

Now the key thing for this lecture for a circular cylindrical chamber circular cylindrical chamber is that we need to make some subtle changes we need to make certain some important changes to the governing equation.

Circular Cylindrical Chamber



So, basically what we have again your excitation wave number. And t is of course, is varying harmonically. So, this is

$$(\nabla^2 + k_0^2)\tilde{p}\,(r, \theta, z) =$$

$$-j\omega\rho_0 Q_0\, \frac{\delta(r - r_0)}{h_r}\, \frac{\delta(\theta - \theta_0)}{h_\theta}\, \frac{\delta(z - z_0)}{h_z}$$

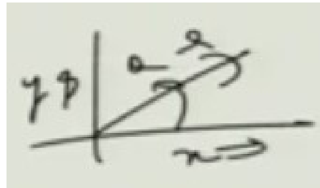
Now direct data function definition will change that is the key point. So, let us say the source is located at r naught. So, we have a you know a circular chamber and r this is your distance theta and let us say this is z ok. So, for theta you have the h factor. So, you here you have h r h θ and h z. So, what are h r h θ and h z? They are basically your scaled factors.

$$h_r, \sqrt{\left(\frac{\partial x}{\partial r}\right)^2 + \left(\frac{\partial y}{\partial r}\right)^2}$$

$$x = r\, cos\theta$$

$$Y = rsinQ$$

So, you know let us you know I think we did something regarding the scale factors a long time back in week 2. So, you know so, this is given by the following thing for the 2D system z will obviously, not be there because mind you this is x = r cos θ y = r sin θ.



So, here you have r theta x and y ok. So, once we substitute this guy we will see that this is sin square theta plus cos square theta root over this is 1 and h theta is.

So, this will be your r square, r square you can take this thing common and this will become

$$= \sqrt{sin^2 r + cos^2 r} \ = 1$$

$$h_\theta = \sqrt{\left(\frac{\partial x}{\partial \theta}\right)^2 + \left(\frac{\partial y}{\partial \theta}\right)^2}$$

That is a Laplacian in the circular polar coordinate circular cylindrical coordinates. And that is basically

$$= r\sqrt{S^2 + C^2} \ = r$$

$$h_\sigma = r \quad h_r = 1 \quad h_z = 1$$

$$\Delta^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

So, you know you get this Laplacian form ok. So, with this with these substitutions all this thing, if we substitute back in this equation, what will it look like? So, it will look like, we basically substitute in place of h r θ z the equations I mean the expressions that we have derived are here and that is

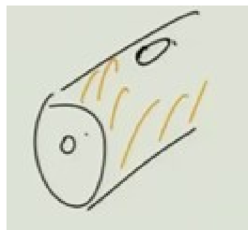
$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z}\right)p$$

$$= -j\omega\rho_0 Q_0\,\delta(r - r_0)\frac{\delta(\theta - Q_0)\delta(z - z_0)}{r}$$

So, these are direct delta functions in cylindrical polar coordinates. Remember one thing there is a small digression that I like to make here is that you know here r minus r naught into theta minus theta naught in the direct delta system.

$$\oint_V \delta(r - r_0) \frac{\delta(\theta - Q_0)}{r} \, \delta(z - z_0) r \, d\theta \, dr \, dz$$

$$= 1$$



You know over when you do dv over this cylindrical volume you know what does it mean? So, dv is r so, instead of dv you write r dr dθ that is your d that is a small r here and the d. So, if you consider basically an elemental volume in this in the cylindrical polar coordinates, basically your you know dθ dr small this thing and something a top kind of a thing here.

So, we get r dr dθ dz ok. So, r r gets cancelled and eventually what you will get is this is 1. So, basically you know this the idea is that this representation is consistent with the definitions of direct δ function and of course, I guess I forgot the term here that is k naught square I am sorry for that.

So, k naught square is; obviously, there the wave number. So, basically what we do is that we need to have if you recall our discussions in the week 2 lecture, we need to find out for simplicity let us say you know we set Q_0 to 0. And actually incidentally you know if you have a piston driven excitation.

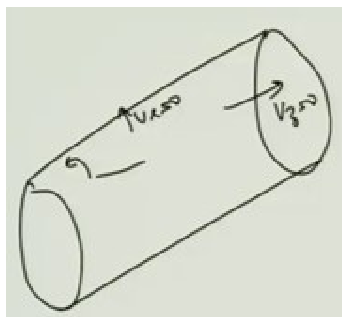
So, what we will do is that instead of this we will have u naught into f of you know f of r theta into delta z minus z naught ok that will be there if the port is located on the r theta on the you know on the end phase somewhere here. If the port is located on the side surface somewhere here, then your this will become minus then you know instead of this thing you will have your this thing say

$$= -j\omega\rho_0 U_0\, \delta(r - r_0)\, f(L, \theta)$$

where f of L comma theta is unity its basically unity over the port area and it is 0 over all the side surface area where the port is not there ok. So, that is one thing.

So, we get basically this sort of a we get this sort of a thing. Now the key idea behind underlying this derivation now is to substitute the solution of homogeneous equation which is which I am going to write down.

Now, is basically p r, θ, z t I am not writing is understood *ejωt*. So, this is a model summation solution which is nothing but this subject to if you consider a long cylindrical circular cylindrical thing with rigid wall condition U or W is 0 here; U or I would say U z = 0, U r = 0 and there is no and Uθ mind you U θ is there is no condition on U theta ok.



$$\tilde{p}\,(r, \theta, z) = \sum_{l=0,1,2}^{\infty} \sum_{M=0,1,23}^{\infty} \sum_{N=0,1,2,}^{\infty} \cos\left(\frac{l\pi z}{L}\right)$$

$$\left(C_{1,l,m}\, cosm\theta\ + C_{2,lm\, sinm\theta}\right) J_m \left(\frac{\alpha_{mnr}}{k_0} r\right) cosm\ \theta\ sinm\ \theta$$

Only thing is that $\tilde{p}\,(\theta) = \tilde{p}\,(\theta + 2\pi)$ this will always be there ok.

So, this is the periodicity condition as a result of that periodicity condition is very important. So, you will get the solution you know eventually when we do a variable separate from this I think we discussed also sometime back. You know we have m as an integer value. So, m is the so, this you know the modes like this

$$J_m\left(\frac{\alpha_{mnr}}{k_0}\right)$$

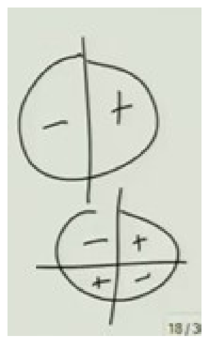
So, we can also write this as $e^{jm\theta}$ $e^{-jm\theta}$ and all that. So, basically these are called you know when for m is equal to 0, you know we get something like a radial mode.

$$M = 0 \quad J_0\left(\frac{\alpha_{0n}}{k_0} r\right)$$

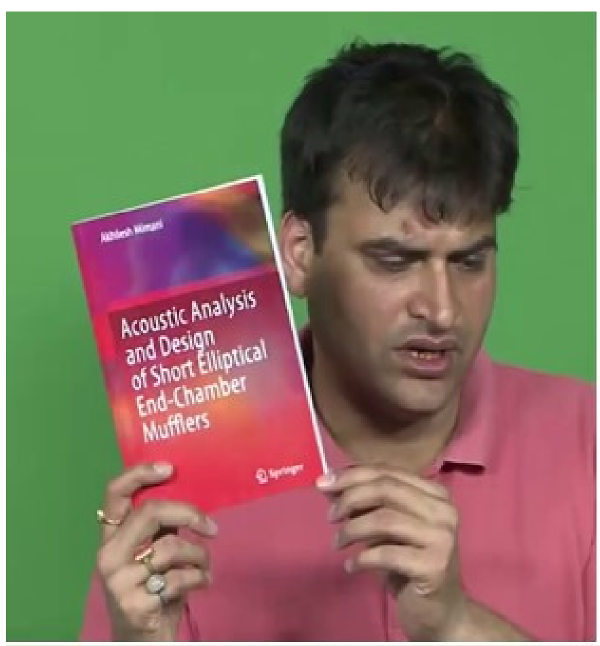
So, you get basically radial mode and when

$$M \neq 0$$

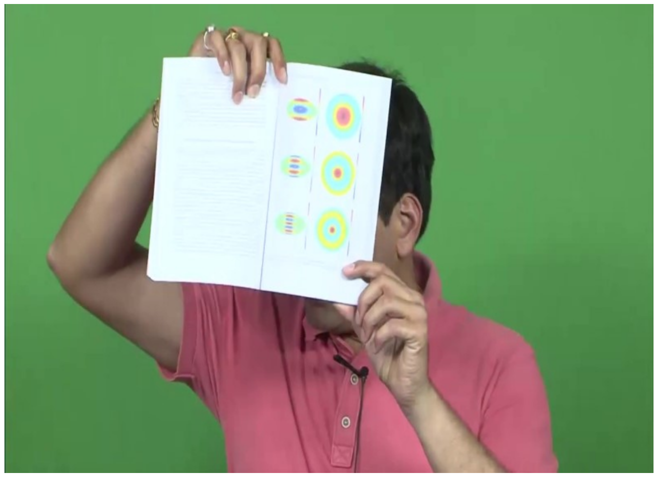
Whatever it is then you get your you know circum and circumferential mode, but then in that case your n should be 0. So, $n = 0 \quad (1,0), (2,0), (3,0)$ mode and so on; these are all your circumferential mode as we saw it will be something like



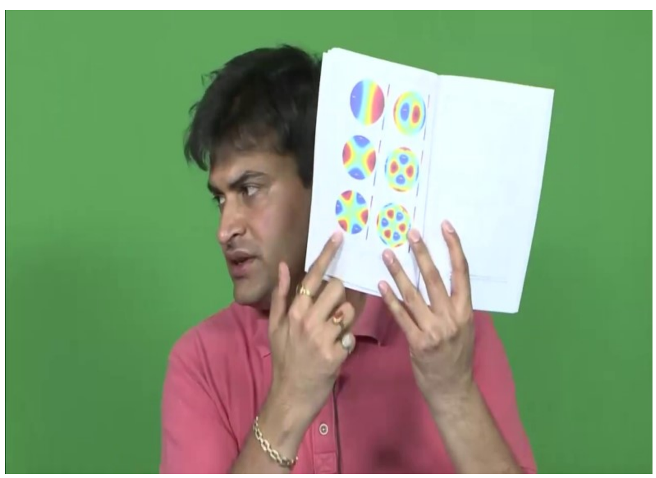
So, it is given in nicely in my book here.



So, this is the book that has been published recently on the elliptical muffler design. We are not going to discuss elliptical cylindrical mufflers in this NPTEL course is an advanced course, but what I did was that I presented the solution of a circular cylindrical case as a degenerate case of elliptical case elliptical mufflers. And I can what I can do is that I can show you the mode shapes of how these things would look like. So, just bear with me for a minute and we can have a look at the PDF file as well.



So, you see these are the these are the radial mode shapes of a circular chamber we can also have a look at the PDF file sometime later



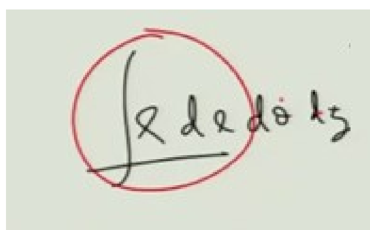
and these are your circumferential modes in the first in the first column ok. So, plus minus minus plus and alternating signs and then you have a cross modes when m and n are both non zero ok. So, it is all documented in my on my in the recently published book on elliptical muffler analysis circular cylindrical case being a part of that.

$$\rightarrow \quad \left.\frac{dJ_m\left(\frac{\alpha_{mn}}{k_0}r\right)}{dl}\right|_{r=k_0} = 0 \quad e^{-jm\theta}$$

So, what we do is that this is of course, your non dimensional resonance frequencies value also tabulated in this book and this subject to this thing. So, you basically have to solve this numerically this is all **Neumann functions**. So, that is what we that is what we do, we basically do that.

And now once we once we get the modal solution which is now I think it is getting a bit untidy. So, just box it something like this ok. So, we get this sort of a thing. Now we need to substitute this guy back in this equation ok. Once we do that you know once we substitute this particular thing in homogeneous Helmholtz equation where the inhomogeneity can be a point source or piston source and then invoke more orthogonality.

So, unlike you know unlike circular function there are certain things with Bessel functions. So, it is orthogonal with respect to the weighing function r.



$$\int_0^{k_0} J_m\left(\frac{\alpha_{mn}}{k_0}r\right)J_{m1}\left(\frac{\alpha_{m1n1}}{k_0}r\right)rds \quad \neq 0$$

$$only \quad m = m_1$$

$$n = n_1$$

$$= 0 \;\; m \neq m_1$$

$$n \neq n_1$$

if either of this condition is not met or is met I am sorry then this is 0.

And cos θ the trigonometric function or orthogonal in the regular sense essentially what you do is that you multiply by r dr dθ and integrate over the volume and then just take these this particular term to be along with the Bessel function, this is with trigonometric function, this is with the also trigonometric function along the z direction and then you know invoke mode orthogonality get the modal coefficients which are given here $C_{1,l,m}$ $C_{2,l,m}$ ok.

Once you get that then back substitute to get the Green's function. So, what we will do is that we look at the Green's function solution and also the piston driven solution just in a while.

So, this is the book I was referring to. So, this is the book that was published a while back by me it is a spring of paper. So, what I am going to do is that I am going to go to the second chapter where you know the solution of this the solution modal solution is given for the circular cylindrical case.

$$p(r,\theta,z,t) = \left\{ \sum_{m=0,1,2\ldots}^{\infty} \sum_{n=0,1,2\ldots}^{\infty} J_m\left(\alpha_{mn}\frac{r}{R_0}\right)\left(A^1_{m,n}\cos m\theta + A^2_{m,n}\sin m\theta\right)\left(C^1_{m,n}\, e^{-jk^+_{z,m,n}z} + C^2_{m,n}\, e^{-jk^-_{z,m,n}z}\right)\right\} e^{j\omega t} \quad (2.58)$$

So, these are all elliptical things which are more complicated you know. So, you see this equation (2.58) is the case where you have this modal summation as I was talking about of course, here you have you are not and really enforce the you are not really enforce the rigid wall condition along the z direction.

$$p(\xi,\eta,z,t) = \left\{ \begin{array}{c} \displaystyle\sum_{P=0,1,2\ldots}^{\infty} \sum_{m=0,1,2\ldots}^{\infty} \sum_{n=1,2\ldots}^{\infty} C_{P,m,n} Ce_m(\xi,\eta,z,t) \times \\ Ce_m(\eta, q_{m,n}) \cos\left(\frac{P\pi z}{L}\right) \\ \displaystyle + \sum_{P=0,1,2\ldots}^{\infty} \sum_{m=0,1,2\ldots}^{\infty} \sum_{n=1,2\ldots}^{\infty} S_{P,m,n} Se_m(\xi, \bar{q}_{m,n}) \times \\ Se_m(\eta, \bar{q}_{m,n}) \cos\left(\frac{P\pi z}{L}\right) \end{array} \right\} e^{j\omega t} \quad (3.1)$$

But once you do that what we can do that we can go to the third chapter of this book where the Green's function solution is given and this is what you get; this is the modal summation solution for the elliptical duct, but let us focus on the circular cylindrical duct. And once we put the modal summation back we are going to get this as the Green's function solution ok.

$$\frac{p(r_R,\theta_R,z_R|r_S,\theta_S,z_S)}{\rho_0 Q_0} = \frac{G(r_R,\theta_R,z_R|r_S,\theta_S,z_S)}{\rho_0 Q_0} = \mathrm{j}k_0 c_0 \left\{ \sum_{P=0,1,2\ldots}^{\infty} \sum_{m=0,1,2\ldots}^{\infty} \sum_{n=0,1,2\ldots}^{\infty} \right.$$

$$\left. \frac{J_m\left(\alpha_{mn}\frac{r_R}{R_0}\right)\cos\left(\frac{P\pi z_R}{L}\right) J_m\left(\alpha_{mn}\frac{r_S}{R_0}\right)\cos\left(\frac{P\pi z_S}{L}\right)\cos\left(m(\theta_R-\theta_S)\right)}{\left\{\left(\frac{P\pi}{L}\right)^2+\left(\frac{\alpha_{mn}}{R_0}\right)^2-k_0^2\right\} N_{l,m,n}} \right\} \quad 3.13$$

So, I guess. So, you know you said you see this J_m α_{mn} R / R_0 and these kind of a thing. So, these are J_m is like I said these are Bessel functions ordinary Bessel function the first kind and you know Neumann functions do not feature in the circular cylindrical case because the origin is obviously present and the Neumann functions would blow up causing similarity at the origin. So, they are not included in the solution.

However, in case of a circular cylindrical pipe with a circular cylindrical chamber with a concentric rigid pipe then the origin would not have been present and in which case it is necessary to include the Neumann functions ok. But here in this case the Neumann functions are not quite there ok. So, this is one of that now basically what we get is this is the Green's function response port source port and

$$N_{l,m,n,P} = \left\{\int_{r=0}^{r=R_0} r\left(J_m\left(\frac{\alpha_{mn}}{R_0}r\right)\right)^2 dr\right\}\left\{\int_{\theta=0}^{\theta=2\pi}\left(\cos(m\theta)\right)^2 d\theta\right\}\left\{\int_{z=0}^{z=L}\left(\cos\left(\frac{P\pi z}{L}\right)\right)^2 dz\right\} \quad (3.14)$$

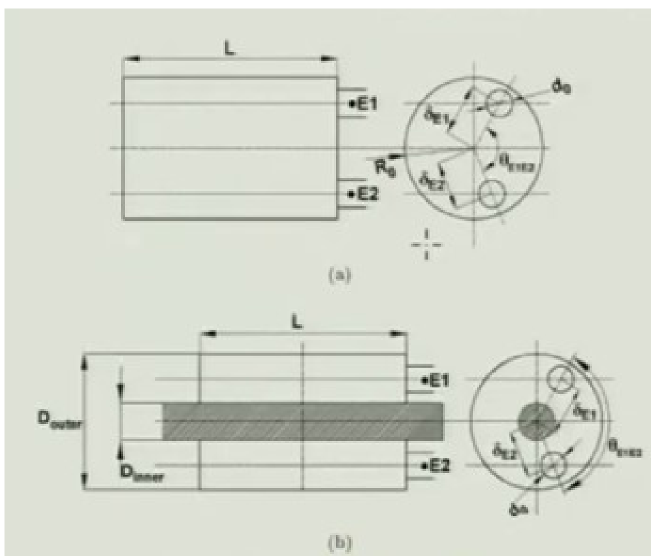
you know cos eventually you will get things like cos m θ R into cos of m θS and your plus plus sin of m theta R into sin m theta S, which can be combined into one trigonometric form which basically tells you that because of axisymmetric of the nature axisymmetric of the problem only the relative location relative angular difference between the inlet and outlet ports are of importance not their absolute thing unlike a elliptical thing.

Now these are like I was saying your you know integral of the square of the product of particular set of mode shape functions.

$$\alpha_{mn} = k_{m,n}\,R_0 = \left.\frac{dJ_m(k_{m,n}r)}{d(k_{m,n}\,r)}\right|_{r=R_0} = \frac{1}{2}\{J_{m-1}(\alpha_{mn}) - J_{m+1}(\alpha_{mn})\} = 0$$

So, these are found by orthogonality they are basically you know they are basically closed from the expressions for that for example, a closed form expression for the first integral that is this one in equation 3.4 is given by 0.5 are not this thing which can be found out from different mathematical handbooks and other functions are sort of trivially you can find it out.

And it is also given such a function in you know other paper that was published also sometime back in journal of computational acoustics for a circular cylindrical chamber you know you have you know this kind of a thing. So, I guess we can have a look at the muffler configuration.



So, this is a end inlet and end outlet flow reversal chain, but forget about the pipe we are not interested in the pipe just yet ok.

So, we are interested only in the circular cylindrical without a concentric pipe. So, this is the more well a little more larger font size of you know this Green's function.

They are essentially the same, they are the same and using that you can find out the impedance matrix parameter characterize that and this is the roots non dimensional roots which have been tabulated also in my monograph you know for the first as much as, as much as first 10 roots first 9 orders and 10 roots each or that has been tabulated.

$\alpha_{m,n}$	*m = 0*	*m = 1*	*m = 2*	*m = 3*	*m = 4*	*m = 5*	*m = 6*	*m = 7*	*m = 8*
n = 0	0	1.8412	3.0542	4.2012	5.3176	6.4156	7.5013	8.5778	9.6474
n = 1	3.8317	5.3314	6.7061	8.0152	9.2824	10.5199	11.7349	12.9324	14.1155
n = 2	7.0156	8.5363	9.9695	11.3459	12.6819	13.9872	15.2682	16.5294	17.7740
n = 3	10.1735	11.7060	13.1704	14.5858	15.9641	17.3128	18.6374	19.9419	21.2291
n = 4	13.3237	14.8636	16.3475	17.7887	19.1960	20.5755	21.9317	23.2681	24.5872
n = 5	16.4706	18.0155	19.5129	20.9725	22.4010	23.8036	25.1839	26.5450	27.8893
n =6	19.6159	21.1644	22.6716	24.1449	25.5898	27.0103	28.4098	29.7907	31.1553
n =7	22.7601	24.3113	25.8260	27.3101	28.7678	30.2028	31.6179	33.0152	34.3966
n =8	25.9037	27.4571	28.9777	30.4703	31.9385	33.3854	34.8134	36.2244	37.6201

Table 26. The Roots of the derivative of Bessel function of the first kind: non-dimensional cut-on frequency in a circular cylindrical waveguide $D_2 = D_1 = D_0$ (e = 0, $\xi \rightarrow \infty$)

Let me just show you that quickly. So, first 9 orders including the m is equal to 0 order and first 9 roots. So, such things are also seen in engineering books. So, engineering maths text books or things like that or maybe acoustics book you can have a look at these tables, but for rigid wall condition that is the duct satisfying the Neumann condition these tables are there ok.

Now, you get this Green's function solution and mode orthogonality. And now basically you know again you have to integrate the Green's function over the port area. So, if the ports are located both at the at the end faces as we saw in the configuration just a while back; you just have to integrate the Green's function

$$Z_{EkEi} = \frac{jk_0c_0}{S_{Ek}S_{Ei}} \left\{ \begin{array}{c} \displaystyle\sum_{l=0,1,2\dots}^{\infty} \sum_{m=0,1,2\dots}^{\infty} \sum_{n=0,1,2\dots}^{\infty} \\ \left(\iint_{S_{Ek}} \cos(m\theta) J_m\left(\alpha_{mn}\frac{r}{R_0}\right) \mathrm{rdrd\theta}\right) \\ \times \dfrac{\times \left(\iint_{S_{Ek}} \cos(m\theta) J_m\left(\alpha_{mn}\frac{r}{R_0}\right) \mathrm{rdrd\theta}\right)}{\left\{\left(\frac{l\pi}{L}\right)^2 + \left(\frac{\alpha_{mn}}{R_0}\right)^2 - k_0^2\right\} N_{l,m,n}} \end{array} \right\}$$

And you know the other function you know if this n ports located on the end surfaces then z value this particular thing z are that can be simply be 1 or minus 1 depending upon whether the ports are on the same surface on the opposite surface. If ports if there is a end inlet and end outlet configuration we said simply you know z is equal to on the same phase slow flow reversal chamber.

Then z is equal to 0 you can substitute and you get 1 and if it is 1 port is on 1 of the end face the other port is on the other end face you said 0 here and l here. So, it can be minus 1 you know you get your cos l π l depends on the value of l it can be 1 or minus 1 for l is equal to 0 you get you know 1 l is equal to 1 you get cos π l is equal to 2 you get cos 2 π and so on. So, you get 1 or - 1 depending on the value ok.

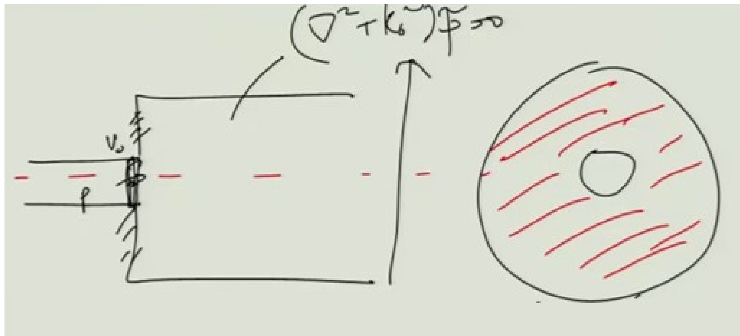
In this case we get just 1 you know that is why this the z term is taken care of and you have to integrate the Green's function over both the receiver port and the source port and divide by the e the port area of the of the inlet and outlet ports to get the average response you get the impedance matrix parameters.

And you know other thing I just wanted to talk about is that you know the all the very briefly solving you know getting the impedance matrix parameters by solving this inhomogeneous system of equation subject to homogeneous boundary condition; that is everywhere it is the everywhere it is the same or the boundary conditions.

That is the same as solving a homogeneous Helmholtz equation subject to inhomogeneous boundary condition. So, what do I mean? You know this would require a little bit of elaboration more of a conceptual thing.

Now, let us consider you know a circular cylindrical we are looking at the let us say the side view we have a concentric chamber something like this and we have a port like this. So, what it means that over this domain you still have this thing p is equal to 0 ok and then you can express this as you know modal summation.

$$\tilde{p} = \sum_{m=0,1,2}^{\infty} \sum_{n=0,1,2}^{\infty} \left\{ C_{mn}^{1}\ J_m \left(\frac{\alpha_{mn}}{R_0} r \right) (\cos\ m\theta)\ e^{-jkzm}\ z \right.$$
$$\left. + C_{mn}^{2}\ J_m \left(\frac{\alpha_{mn}}{R_0} r \right) (\sin\ m\theta)\ e^{jkzmn} \right\}$$



And cos of m theta plus some e to the power minus j kz mn into z this entire thing plus you can call this C_{mn}^2 ok you get this sort of a thing ok. The solution of this is a solution of this equation there is no source term here, but what happens really you know what the annular surface where I have shown with the red hatched surface velocity $U_{z=0}$ and U_0 you are as let us assume that uniform piston driven model here. So, it is u naught.

So, $U_0 = U_z$ which is non zero ok. So, and in this port immediately from the port chamber interface you can assume planar wave. So, $U_p U_0 = U_z$port and in the port area you are assuming U naught velocity U naught into e j omega t that is a given and that is equal to the velocity particle velocity along the axial direction in the chamber and the axial velocity in the chamber is 0 at the annular surface that is at these points ok

And pressure other condition of course, is the pressure at the port is equal to the pressure in the chamber over the annular area. So, you know what we do is basically you know use these conditions to get this thing. So, how do we do that? You know for the axial particle velocity you have this condition U_z t is equal to minus dou dou x of dou z. So, from this expression we can find out this term and you know this is nothing but

$$j\omega\rho_0 U_z = -\frac{\alpha_0}{\partial z}$$

you can find out U_z in terms of these things. So, U_z is this entire modal summation is set to 0 here and it is U_p U_0 here and then again multiply throughout by multiplied throughout by

$$J_m\left(\alpha_{mn}\frac{r}{R_0}\right)\, cosm_1\theta\; rdrd\theta\theta$$

And you know integrate over the r dr dθ and due to orthogonality of the modes only that mode will survive where $m_1 = m_1$ and $n_1 = n_1$ and only the cos term will survive similarly we need to multiply by sin and then we will get the C_{mn}^2 coefficient C_{mn}^2 and C_{mn}^1, like this we can evaluate back substitute and get the final modal summation solution in term in the cylindrical lattice assuming a planar piston excitation that is to say there is no coupling between the this chamber and here.

So, you know this is a homogeneous equation subjected to inhomogeneous boundary condition in homogeneous B c's why? Because you know it is you know is used at a 0 or the annular part and this is equal to this over this part and.

So, basically you know we get this sort of a thing and once we get that we will basically get the solution impedance matrix parameters can also be obtained like this. So, now basically after that we can evaluate the transmission loss of such a system.

$$TL \approx 10log_{10}\left(\frac{1}{4Y_{E1}Y_{E2}}\left|\frac{Z_{E1E1}Y_{E2}+Z_{E2E2}Y_{E1}}{Z_{E2E1}}\right|^1\right) \approx 10log_{10}\left(\frac{(Y_{E1}+Y_{E2})^2}{4Y_{E1}Y_{E2}}\right)$$

So, what we are going to do now is that I can just show you some probably run some codes in MATLAB and talk to you about how the transmission loss is evaluated and all the integrals over these ports are done. So, actually before I do that let me also just show you the how does it how does it look.

$$p(\xi_r = \xi_{0,}\eta_{S2}, z_{S2}|\xi_{0,}\eta_{S1}, l_{S1}) = \frac{1}{S_{side\ 2}}\left\{\iint_{side2}\frac{1}{S_{side\ 1}}\right.$$

$$\left.\iint_{side1}\{G(\xi_r = \xi_{0,}\eta_{S2}, z_{S2}|\xi_S = \xi_{0,}\eta_{S1}, l_{S1})h_\eta d\eta_{S1}dl_{S1}\}h_\eta d\eta_{S2}dl_{S2}\right\} \quad (3.34)$$

So, for the end port for the so, these are the formulation integral formulations for the end port, but I guess we can have a look at such formulations for this port just in a while. So, this is my thesis in which you know the integral expressions represented.

So, if the port is located on the end surface you know and it is you know the distance between the center of the port and the center of the circular chamber is less than the radius then this expression would apply cos mθ or sin mθ these integral values are which derived carefully.

$$\delta < r_0 \int_{\theta=\theta_0}^{\theta=\theta_0+2\pi} \begin{matrix}\cos(m\theta) \\ \sin(m\theta\end{matrix} \left\{ \int_{r=0}^{r=\delta\cos(\theta-\theta_0)+\sqrt{r_S^2-\delta^2 sin^2(\theta-\theta_0)}} rJ_m\left(\frac{\alpha_{mn}}{R_0}r\right)dr \right\} d\theta$$

$$= \int_{\theta=\theta_0-z}^{\theta=\theta_0+z} \begin{matrix}\cos(m\theta) \\ \sin(m\theta\end{matrix} \left\{ \int_{r=0}^{r=\delta\cos(\theta-\theta_0)+\sqrt{r_S^2-\delta^2 sin^2(\theta-\theta_0)}} rJ_m\left(\frac{\alpha_{mn}}{R_0}r\right)dr \right\} d\theta \qquad (4.62)$$

$$\delta = r_0, \int_{\theta=\theta_0-\pi/2}^{\theta=\theta_0+\pi/2} \begin{matrix}\cos(m\theta) \\ \sin(m\theta\end{matrix} \left\{ \int_{r=0}^{r=2m\cos(\theta-\theta_0)} rJ_m\left(\frac{\alpha_{mn}}{R_0}r\right)dr \right\} d\theta \qquad (4.63)$$

$$\delta < r_0 \int_{\theta=\theta_0-sin^{-1}}^{\theta=\theta_0+sin^{-1}} \begin{matrix}\cos(m\theta) \\ \sin(m\theta\end{matrix} \left\{ \int_{-\sqrt{r_0^2-\delta^2 sin^2(\theta-\theta_0)}}^{r=\delta\,co\ (\theta-\theta_0)+\sqrt{r_0^2-\delta^2 sin^2(\theta-\theta_0)}} rJ_m\left(\frac{\alpha_{mn}}{R_0}r\right)dr \right\}$$

$$d\theta \qquad (4.64)$$

Or you know you can also write this as theta naught you know you can redefine the limits and all that. If it is exactly equal to the radius then you then these limits apply and if it is more than r_0 then the other limits sort of apply. And for the it is easy to see when you have m is equal to 0 and n is equal to 0 you get back the plane wave mode and in which case you know you know this will just become 1 and r dr dθ you know in such a case this will also be 1.

And you see it is easy to see that you know you will eventually get back $\pi d_0^2 / 4$ and you know theta naught is describes the angular location of the center of the end port with reference to the X-X axis as shown in the previous schematic. So, eventually you know for a plane wave case. So, that is for the lowest order mode plane wave mode you will get back $\pi d^2 / 4$ which is important for the check for the sake of self consistency.

$$-J_m\left(\frac{\alpha_{mn}}{R_0}r_S\right) = 0, r_S = 0, \qquad m \neq 0 \ \ and$$

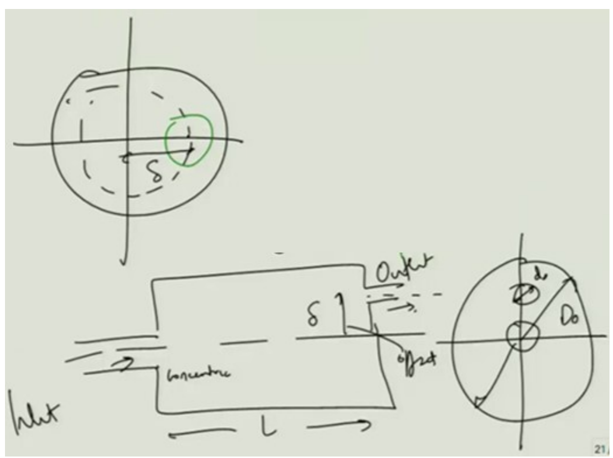
$$J_0\left(\frac{\alpha_{0n}}{R_0}r_S\right) = 0,\ \ r_S = 0\ m = 0\ \ m \neq 0 \quad (4.66a, b)$$

$$\delta = 0, \quad \left\{ \int_{\theta=\theta_0}^{\theta=\theta_0+2\pi} \begin{matrix} \cos(m\theta) \\ \sin(m\theta \end{matrix} \; d\theta \right\} \left\{ \int_{r=0}^{r=r_0} rJ_m\left(\frac{\alpha_{mn}}{R_0} r\right) dr \right\}$$

$$= 2\pi \left\{ \int_{r=0}^{r=r_0} rJ_m\left(\frac{\alpha_{mn}}{R_0} r\right) dr \right\}, \qquad m = 0 \qquad (4.65)$$

$$= 0, \qquad m \neq 0$$

And for you know delta is equal to 0 you know these things the theta and r directions will decouple and you will get that is for basically for the concentric case and you get back your r this thing. So, that would basically mean that only axisymmetric modes will propagate and non axisymmetric modes will not propagate. So, it brings a lot of important thing a lot of important conclusions that you know if you center the port center the inlet port.



So, let us get back to the presentation you know if you have circular chamber and you center your port somewhere here

$$\begin{gathered} m = 0 \\ (0,0) \\ (0,1) \\ (0,2) \\ 0.6276 R_0 \end{gathered}$$

or the radial modes will propagate 0, 1, 0, 0, 0, 2 and so on. If the port is not suppose it is offset you know at certain distance here then of course, all the other modes all will propagate.

$$m = 0, 1, 2$$
$$(0, 0)$$
$$0,6276ro = \delta$$

And based on the location of offset location of this port here you know delta you can actually suppress you know the propagation of the 0, 1 mode if it is located at exactly 0.6276 times r naught if delta is equal to this thing you know then the 0, 1 radial mode will not propagate because you see a its a nodal circle is there and if you center it the you know the integral will evaluate to 0.

Like this a lot of interesting you know rules that a lot of interesting observations that has come out you know all this is possible by analytical formulations. And finite elements we can obviously validate. So, that is what I am saying is that

$$-J_m\left[\frac{\alpha_{mn}}{R_0} r_S\right] = 0, r_S = 0,$$

$$m \neq 0 \ \ and \quad J_0\left[\frac{\alpha_{0n}}{R_0} r_S\right] = 0, \ r_S = 0 \ m = 0 \ \ m \neq 0 \quad (4.66)$$

and this is and this is not equal to 0 for r s is equal to and for the 0th order Bessel function that is the purely radial mode this is not 0.

So, basically the number of conclusions corollaries that we made that for z matrix parameter due to a concentric end port as well as between two concentric end ports located on the opposite faces is independent of the azimuthal modes and the z matrix parameter or impedance matrix parameters between a concentric end port and an offset code is also independent of the azimuthal modes.

So, you know and similarly they are expressions for deriving integral expressions when the port is located on the ends on the curvilinear surface that is the curved surface.

$$S_{sideport} = \pi r_0^2 \left\{1 + \frac{1}{8}\left(\frac{r_0}{R_0}\right)^2 + \frac{3}{64}\left(\frac{r_0}{R_0}\right)^4 + \frac{35}{384}\left(\frac{r_0}{R_0}\right)^6\right\} \quad (4.68)$$

So, you know the different cases the general expression is this for m is equal to 0 mode it is like this and m naught is equal to not equal to 0 you get this thing. So, what we can was possibly you know do is that what we can possibly do is basically go to the go to the MATLAB code now.

And we can run a couple of simulations you know just to show the expansion chamber or kind of a behavior or you know short chamber behavior side end inlet side outlet or side line inside out side port. So, those special cases we can see we just see that in a while.

```
function [] =transmission_loss_plot_two_port()
tic

c0=343.14;

frange1=5;
frange2=2500;

f=frange1:5:frange2;
n1=size(f); n=n1(1,2);

L=300/1000;
z21=L;
```

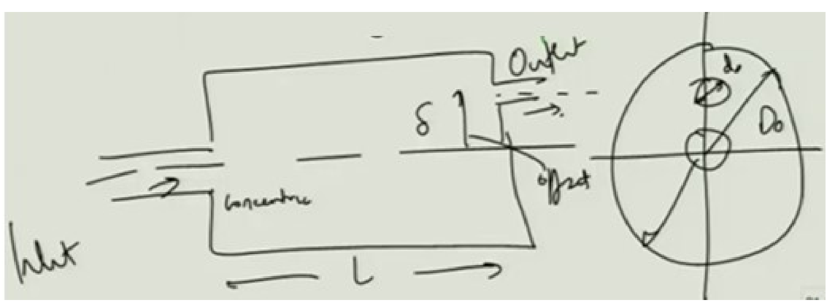
What we do now is basically you know for the first time in this course we are doing the three dimensional analysis of although of only reactive mufflers and circular cylindrical geometry you know rectangular I kind of avoided showing you the code because of time management thing. But for this thing I like to spend some time and show you how the how these codes were written some of you can try on your own system.

```
sp1=load('Port1.dat');
sp2=load('Port2.dat');


sp21=load('Port2.dat');

Tl=zeros(n,1);
  for i=1:n

Tl(i)=transmission_loss_two_port(Mode,sp1,sp21,s
i
  end
  ch='k';
```

```
z21=L;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
k0=(2*pi*f)/c0;

Mode=load('Mode.dat');

sp1=load('Port1.dat');
sp2=load('Port2.dat');


sp21=load('Port2.dat');

T1=zeros(n,1);
```

So, as usual we have transmission loss port which is the main file which cause the which invokes the transmission loss port where the actual with the final computation of transmission loss from the overall impedance matrix is computed using in the terms of scattering matrix parameters. So, number of parameters have to be computed first.

Basically the you know the n l mn value the mode shapes integral of the square of the particular mode shapes of the chamber and then the integrals for the particular port. So, you know let us consider let us consider what we are trying to study here is the let us say we have a expansion chamber ok this port can be located somewhere here and circular thing. So, inlet and outlet ok.

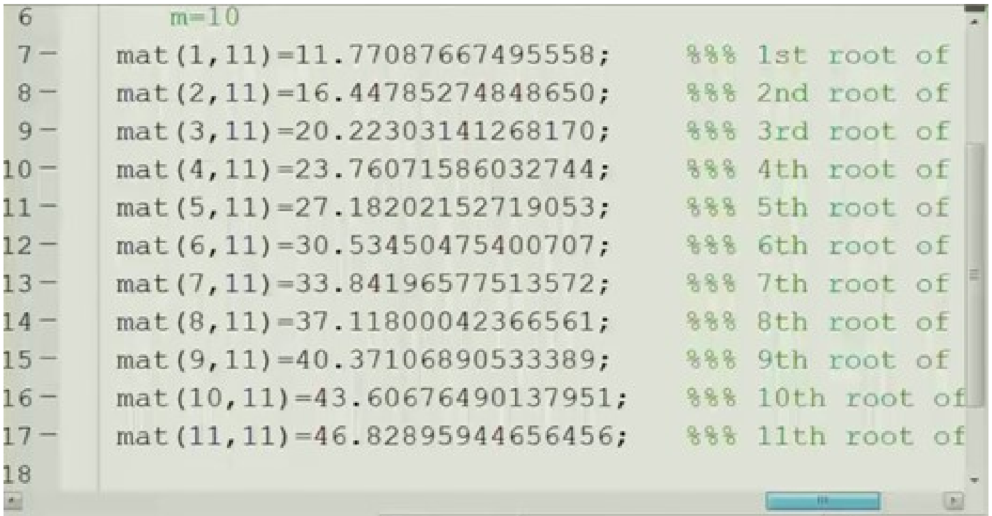


So, we have this sort of a thing. Now this is delta distance then this is concentric port this is offset port. Now based on the value of the offset distance this is the length l diameter D and D_0 and this is all port diameters are assumed to be equal.

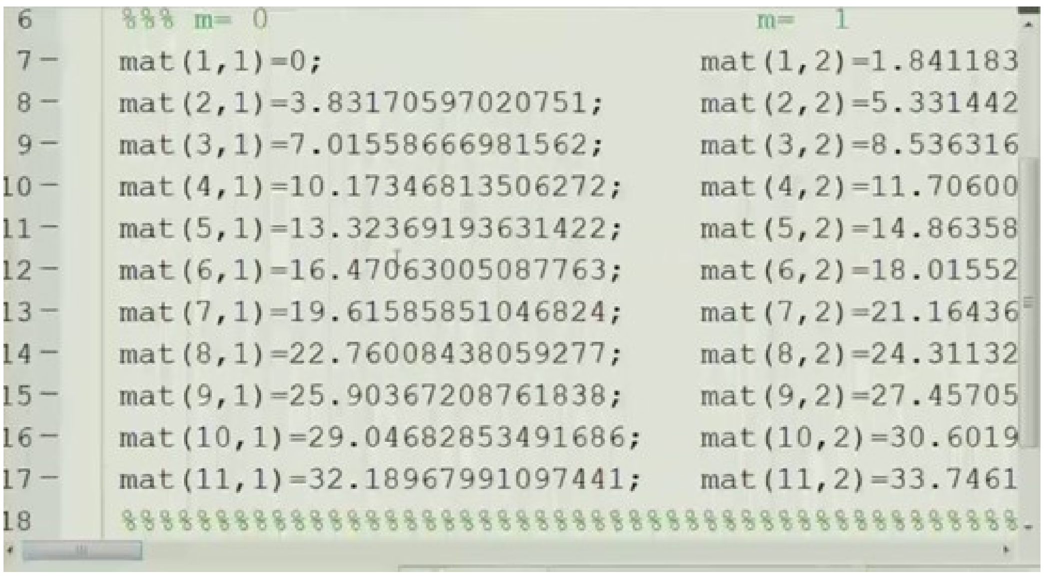
So, let us say D is equal to 250 mm L is L we can vary, but we can also vary D and port diameters, but we just have to compute some integral values before, this is 50 mm and so on. So, now, we get all these things. Now what is happening here is that delta we just get back to the code we are going to analyze this thing for different values of delta.

```
function [int_val] = modeshape_precompute(i1,i2,
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Sc=(pi/4)*(D^2);
i3=i1+1; i4=i2+1;
%%% i1 is the order of the Bessel function, whil
beta=non_dim_cut_on_freq(i3,i4); %%% root number
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R=D/2; %%% Radius of the circular expansion cham

if i1==0 && i2==0  %%% plane wave mode (0,0)...
int_val=Sc;

elseif i1==0 && i2~=0 %%(m=0,n) mode
```

```
%%% m= 0                            m=  1
mat(1,1)=0;                         mat(1,2)=1.841183
mat(2,1)=3.83170597020751;          mat(2,2)=5.331442
mat(3,1)=7.01558666981562;          mat(3,2)=8.536316
mat(4,1)=10.17346813506272;         mat(4,2)=11.70600
mat(5,1)=13.32369193631422;         mat(5,2)=14.86358
mat(6,1)=16.47063005087763;         mat(6,2)=18.01552
mat(7,1)=19.61585851046824;         mat(7,2)=21.16436
mat(8,1)=22.76008438059277;         mat(8,2)=24.31132
mat(9,1)=25.90367208761838;         mat(9,2)=27.45705
mat(10,1)=29.04682853491686;        mat(10,2)=30.6019
mat(11,1)=32.18967991097441;        mat(11,2)=33.7461
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```



```
    m=10
mat(1,11)=11.77087667495558;     %%% 1st root of
mat(2,11)=16.44785274848650;     %%% 2nd root of
mat(3,11)=20.22303141268170;     %%% 3rd root of
mat(4,11)=23.76071586032744;     %%% 4th root of
mat(5,11)=27.18202152719053;     %%% 5th root of
mat(6,11)=30.53450475400707;     %%% 6th root of
mat(7,11)=33.84196577513572;     %%% 7th root of
mat(8,11)=37.11800042366561;     %%% 8th root of
mat(9,11)=40.37106890533389;     %%% 9th root of
mat(10,11)=43.60676490137951;    %%% 10th root of
mat(11,11)=46.82895944656456;    %%% 11th root of
```



```
%%% m= 0                                m=   1
mat(1,1)=0;                             mat(1,2)=1.841183
mat(2,1)=3.83170597020751;              mat(2,2)=5.331442
mat(3,1)=7.01558666981562;              mat(3,2)=8.536316
mat(4,1)=10.17346813506272;             mat(4,2)=11.70600
mat(5,1)=13.32369193631422;             mat(5,2)=14.86358
mat(6,1)=16.47063005087763;             mat(6,2)=18.01552
mat(7,1)=19.61585851046824;             mat(7,2)=21.16436
mat(8,1)=22.76008438059277;             mat(8,2)=24.31132
mat(9,1)=25.90367208761838;             mat(9,2)=27.45705
mat(10,1)=29.04682853491686;            mat(10,2)=30.6019
mat(11,1)=32.18967991097441;            mat(11,2)=33.7461
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

So, let us get back to the code where you know we compute the mode shapes pre compute the mode shapes first and this is the cut on frequencies beta non dimension values I was referring to in my book. You know these are for large. Now large orders are considered as many as first 10 11 orders I guess including the including m is equal to 0 order and first 11 roots of such things.

So, such square matrix where the thing is done and these values are computed by numerically solving the derivative of the Bessel function going to 0 or evaluated using some Newton quadrature sorry Newton-Raphson's method or bisection method I do not remember exactly, but it some numerical method was used.

```
function [int_val] =simpson_three_eight_disc_val
m1=size(F); m=m1(1,2);
sum=0;
for i=1:m
    val=F(i);
    if i==1 || i==m
    sum=sum+val;
    elseif  rem(i,3)==1
            sum=sum + (2*val);
    else
        sum=sum + (3*val);
    end
end
```

And this Simpson's three-eighth rule is used to you know do general numerical integration numerical quadrature and what we what I have done you know is basically. You know these are two separate files where I have written you know first is to compute the integrals over the port surface area depending upon the location of the port.

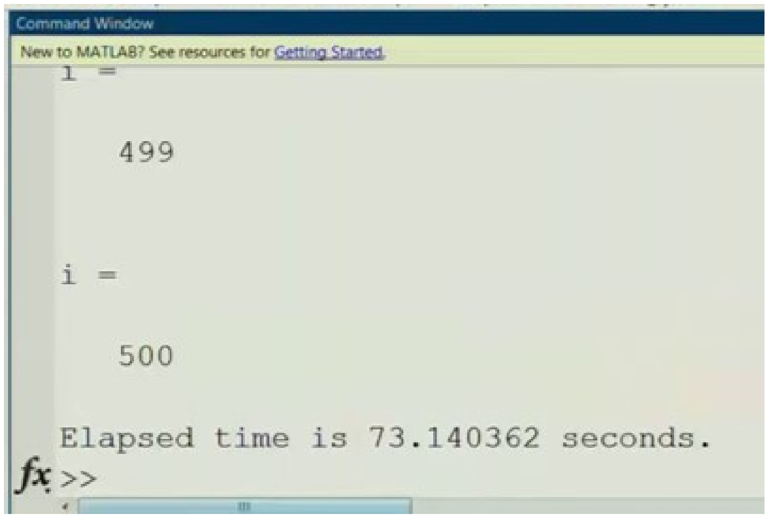
```
function [mode_val]=mode_shape_integration_pipe
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
tol=10^-6; iter=30;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
r1=d1/2; R=D/2;
%%% theta1 is the reference angle (in radians)
%%% center of the excitation port....
i3=i1+1; i4=i2+1;
beta=non_dim_cut_on_freq(i3,i4); %%% root numbe
kr=beta/R;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if od < r1
```

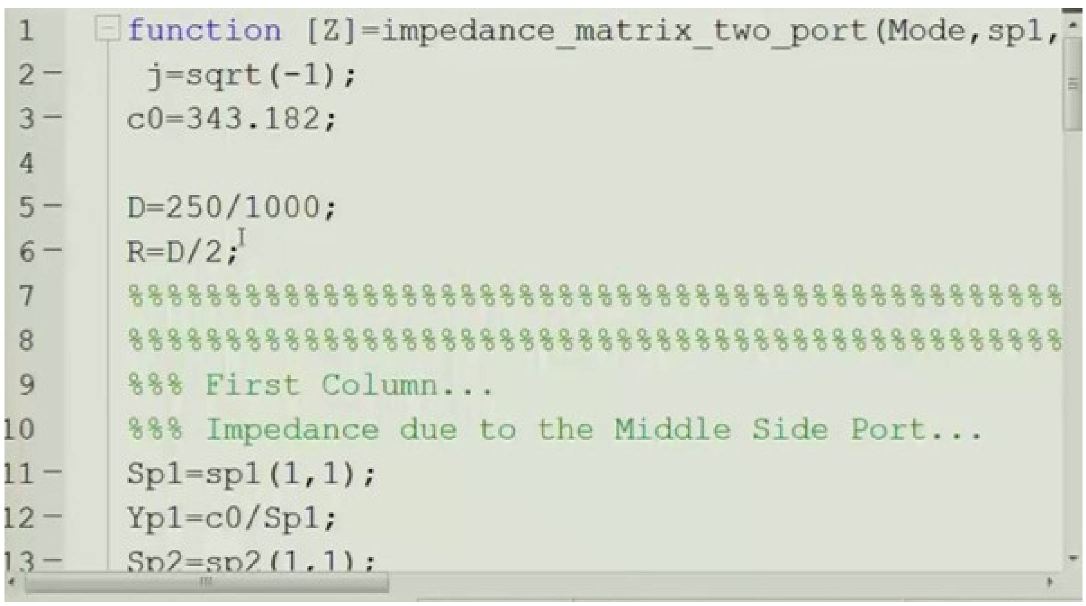
So, these are the radius of the port and the chamber tolerance limit where how many iterations is required. This was found to be good enough and different cases based on the plane wave mode we have this and you know I will not bother going through each and every line of the code.

But basically they are based on what is given in the you know PDF files that I showed you about my papers and thesis and all that and then you have your mode shape pre computed and they are kind of they are nothing but integrals of you know Jr Jm square dr and all that.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R=D/2; %%% Radius of the circular expansion cham

if i1==0 && i2==0  %%% plane wave mode (0,0)...
int_val=Sc;

elseif i1==0 && i2~=0 %%(m=0,n) mode

 int_val=-((R^2)/8)*besselj(i1,beta)*( besselj(i
 int_val=int_val*(2*pi);

elseif i1~=0

```

```

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% To store the precomputed modeshape values co
%%% azimuthal dependence, run the following code

% for i1=0:1:10
%     for i2=0:1:10
%  M(i2+1,i1+1) = modeshape_precompute(i1,i2,250
%     end
% end
%
```





So, this for the plane wave mode this is for the you know. This for the 0 0 n mode or generally mn mode and this is the these were pre computed using things that were run and then what we need to do is that let us fix up some values. So, we that is why we first compute this diameter is fixed port diameter and chamber diameter fixed length *l* can you can vary.

But if you want to change d capital D_0 and small d_0 you just have to re compute these values again. So, basically what is happening here is that we compute these things and then enter the same value diameter in the impedance matrix which computes the which once you and then you load in these functions you know sp1 sp12 mode. All these things are loaded here and then they are passed on to this function impedance matrix function where the actual response functions are computed I guess here.

```
pc1=0; pc21=0;  pc2=0;

for p=0:1:20  %%% axial modes

    for i1=0:1:10  %%% first 10 order of the Bes

        for i2=0:1:10 %%% root number i3 corresp

         alpha=non_dim_cut_on_freq(i1+1,i2+1);

         k_disc= (alpha/R)^2 + ( (p*pi)/L )^2 -

         if p==0
```

```
         if p==0
             int_vol=Mode(i2+1,i1+1)*L;
         elseif p~=0
             int_vol=Mode(i2+1,i1+1)*L*0.5;
         end

         den=(int_vol*k_disc);


     pc1  = pc1  +    (sp1(i2+1,i1+1)^2)/den;
     pc21 = pc21 +  ( cos((p*pi*z21)/L)*(sp1(i2

     pc2  = pc2  +    (sp2(i2+1,i1+1)^2)/den;
```

```
         den=(int_vol*k_disc);


     pc1  = pc1  +    (sp1(i2+1,i1+1)^2)/den;
     pc21 = pc21 +  ( cos((p*pi*z21)/L)*(sp1(i2

     pc2  = pc2  +    (sp2(i2+1,i1+1)^2)/den;

     clear den

        end
    end
end
```

```
+1)^2)/den;
1)/L)*(sp1(i2+1,i1+1)*sp21(i2+1,i1+1)) )/den;

+1)^2)/den;
```

So, this is the characteristic impedance and you know for the first 20 action modes and first 11roots 11 modes were computed and those many modes were considered and for convergence analysis.

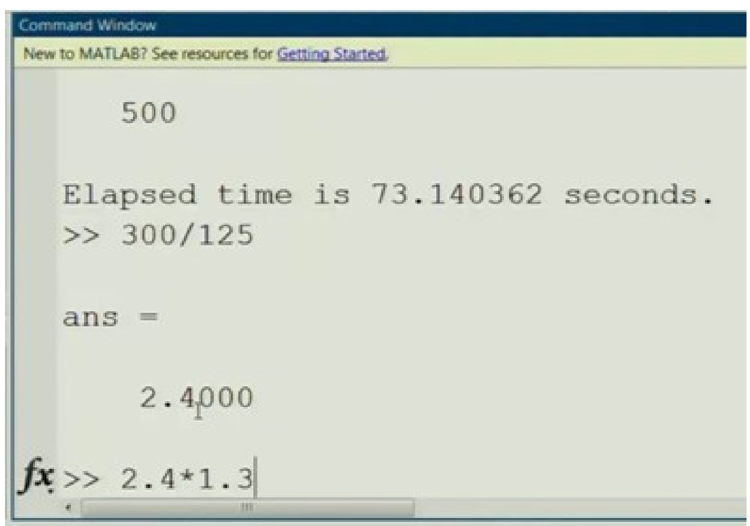
And these are the mode shift value L into L 0.5 and all that depending upon whether p is 0 or not and these are the modal summation approach and the response functions were computed and z21 value was given based on whether you have a flow reversal chamber or a expansion chamber. So, we can do both z21 is given here is specified in the this parameter.

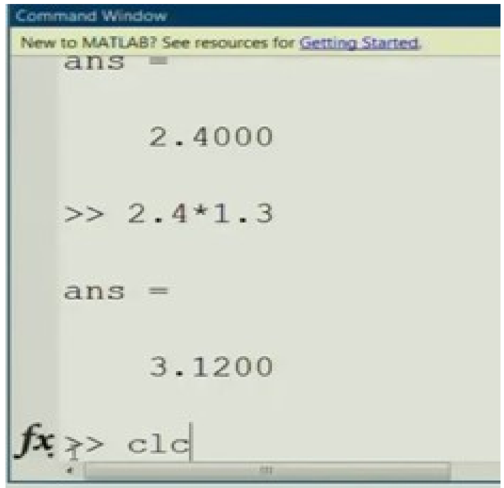
```
frange2=2500;

f=frange1:5:frange2;
n1=size(f); n=n1(1,2);

L=300/1000;
z21=L;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
k0=(2*pi*f)/c0;

Mode=load('Mode.dat');

sp1=load('Port1.dat');
```

So, z21 L means location of the port 2 with respect to 1. So, if you change L you get expansion chamber. So, let us run the case when the port is you know located at the offset and this offset I am deliberately choosing at 0.6276 times R_0 and when I actually when I run the code you know when I run the code I have already done it.

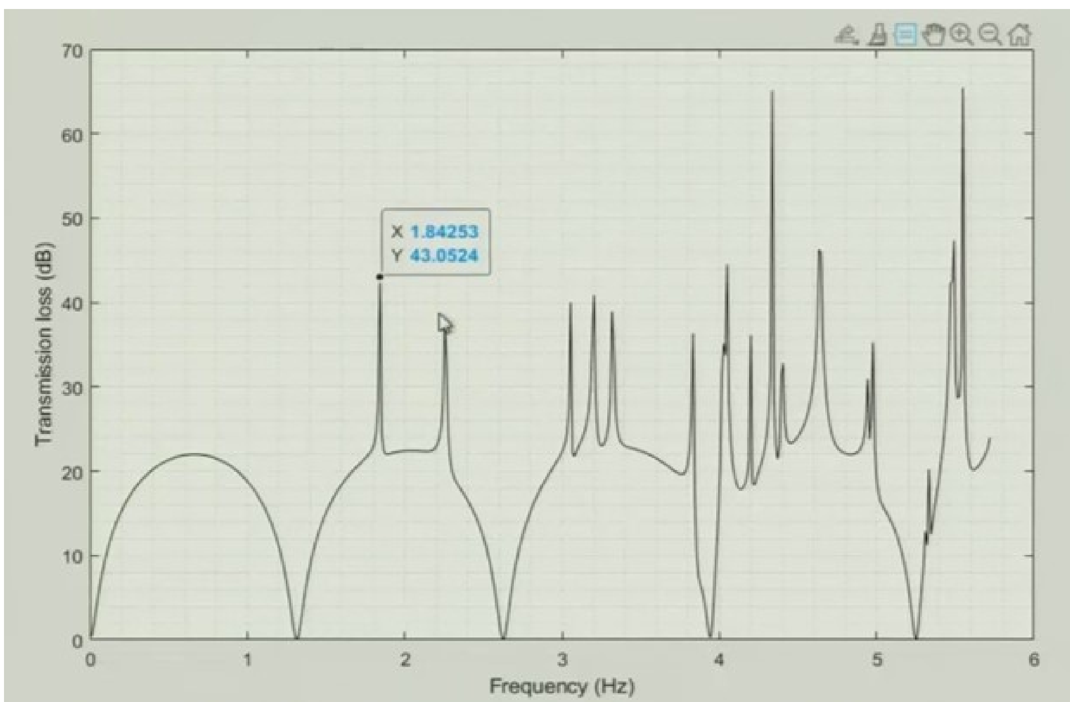
So, we get basically your this kind of a graph ok. So, you get you know this is k naught R naught the non dimensional frequency this is not Hertz by mistake I have written Hertz; so, k_0 R_0. So, you know do you see this expansion chamber kind of a behavior you know this is an expansion chamber dome and trough.

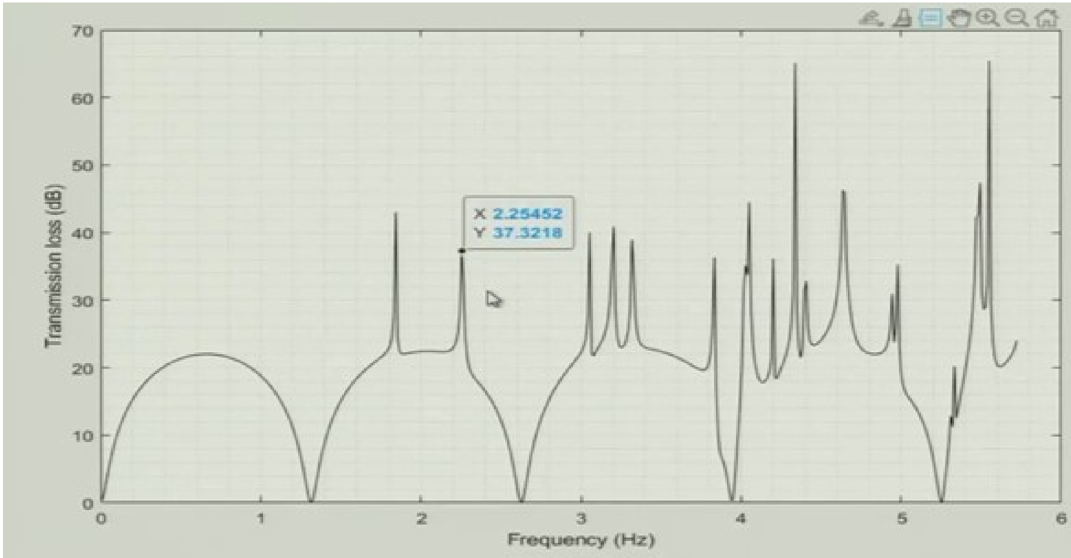
So, this occurs that you know we can figure out certain things you know this is this occurs at one point 1.3. So, you can work out how much based on the l whatever this is this will in terms of k_0 L it will work out to be π ok 3.14 and this is just k_0 R_0.



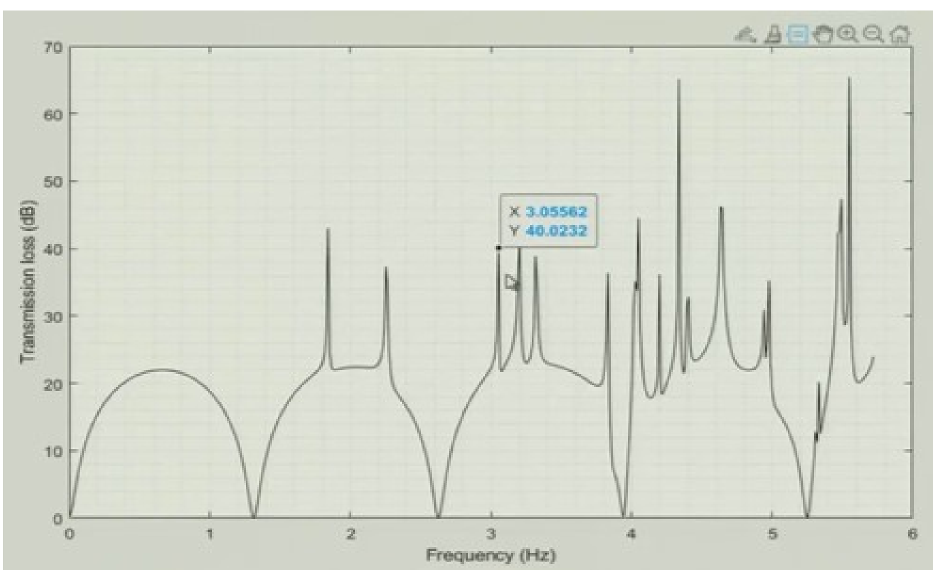


So, because we have basically you know what is L? L is 300 and 145. So, 300 by 125 so, 2.4 if you multiply by 1.3 roughly you are getting close to it ok getting quite close it always occurs at 2 π 2 π 3 π, but this in terms of k naught L in terms of k naught r these are the values, but what is important is that you are you know at look at this value.





You know it is because of the offset nature you are getting a peak you know the azimuthal modes are you know if 1 of the port is centered other port is at offset that is why you are getting a peak otherwise this would have failed right here and you are getting another peak at this thing these are the cross modes and k naught R naught and then 3.05 this is the peak because of the radial offset.



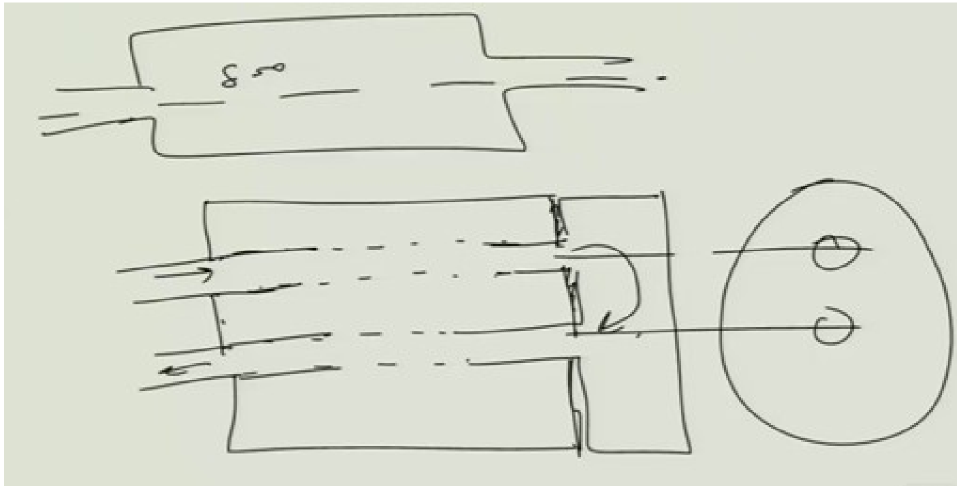
Now, we can have some fun here we can sort of you know close this guy and make some simple changes in the code here. So, this is something probably once you write your code you can do that you who knows you might be asked this in the final exam to run some MATLAB codes based on that.

```
z21=L;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
k0=(2*pi*f)/c0;

Mode=load('Mode.dat');

sp1=load('Port1.dat');
sp2=load('Port1.dat');


sp21=load('Port1.dat');

Tl=zeros(n,1);
```

```
Tl(i)=transmission_loss_two_port(Mode,sp1,sp21,s
i
  end
  ch='r';
  figure(1)
  plot(k0*(125/1000),Tl,ch);
   hold on

grid minor
xlabel('Frequency (Hz)')
ylabel('Transmission loss (dB)')
% legend('3-D Analytical: Uniform Piston Approxi
```

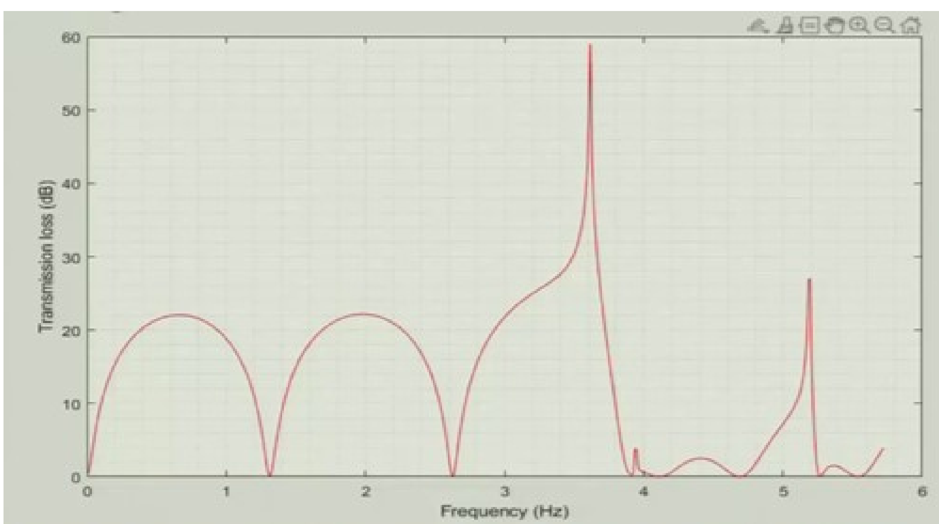
```
Command Window
New to MATLAB? See resources for Getting Started.
i =

    499


i =

    500

Elapsed time is 69.522909 seconds.
fx
```

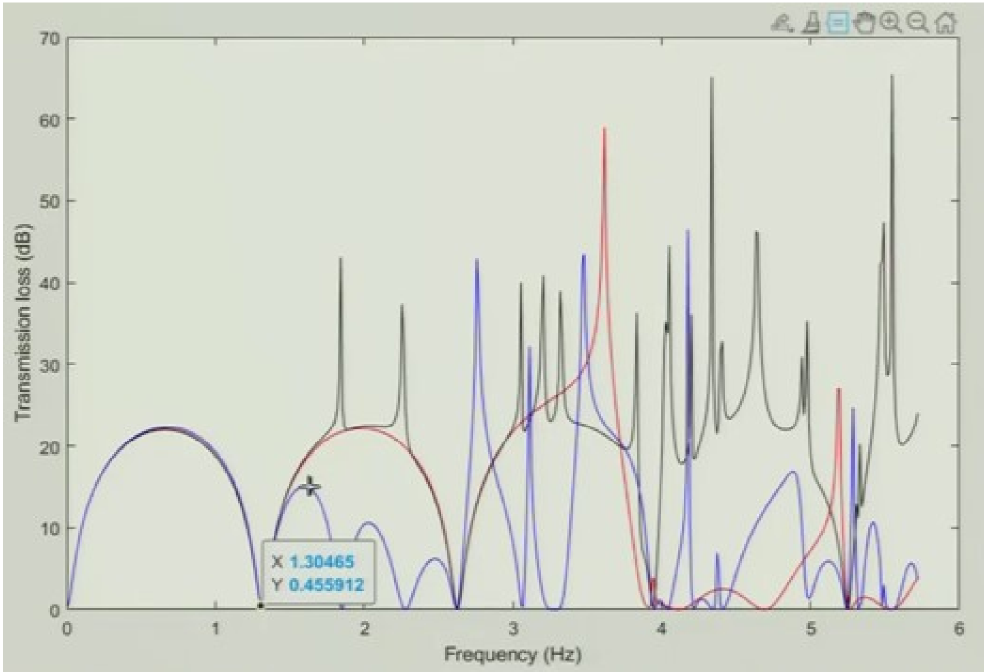
So, instead of port 1 is a concentric 1 and if you have choose this sort of a thing let us see what do we get. So, it will take about a minute or less than that or probably little over a minute to get you the curve. So, this is what we are doing while the code is running let me just go to the presentation.



And in this we are doing a concentric case. So, we are doing something like this sort of a thing. So, δ=0 for both things ok in other words we have bought this delta to 0 here. So, let us get to the presentation I am sorry to MATLAB and see what the progress.

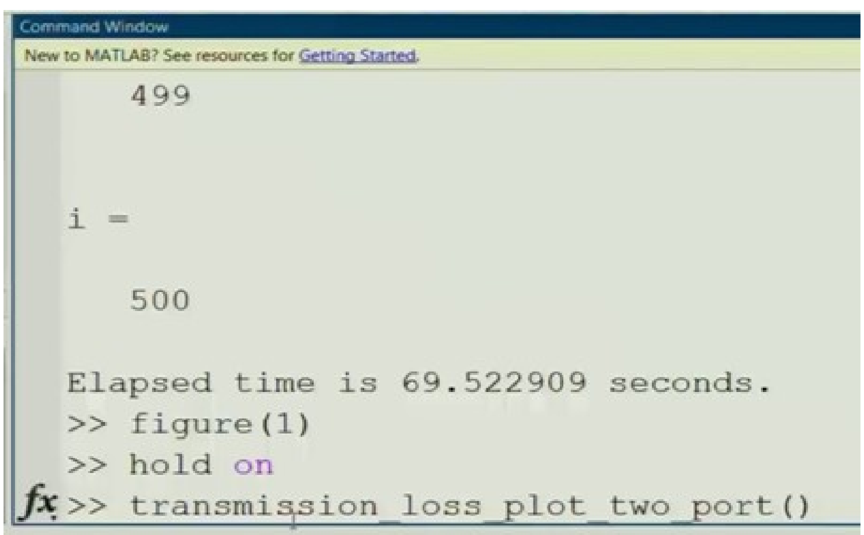
So, it is about 500 time steps of frequency steps that it has to do so, will be there in just a while. So, we will see you know we will still see this expansion chamber kind of a we have a dome and trough, but then the peaks the sharp peaky nature would at an ancient peaks that will happen at a much later frequency you know possibly at from k_0 R_0 = 3.05 onwards; so, constant.





So, you know you can have certain things you can offset one of the port and you can offset one of the ports and this thing. So, basically compare this with the graph here. So, you know this concentric chamber thing kind of breaks down at k_0 R_0 = 3.83 which is basically a first radial mode frequency.

You know and, but at, but this particular thing continues to do well even beyond this dome and troughs continues to do well for ends end centered a concentric inlet and a offset port which is located at the radial mode.

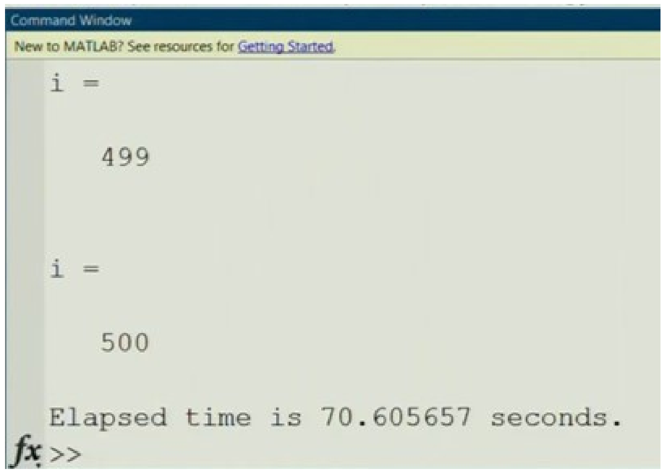


Now, let us do something even more interesting you know we will close this figure and go to MATLAB and say figure 1 hold on and do something interesting, what is that?

```
Tl(i)=transmission_loss_two_port(Mode,sp1,sp21,s
i
  end
  ch='b';
  figure(1)
  plot(k0*(125/1000),Tl,ch);
   hold on

grid minor
xlabel('Frequency (Hz)')
ylabel('Transmission loss (dB)')
% legend('3-D Analytical: Uniform Piston Approxi
```

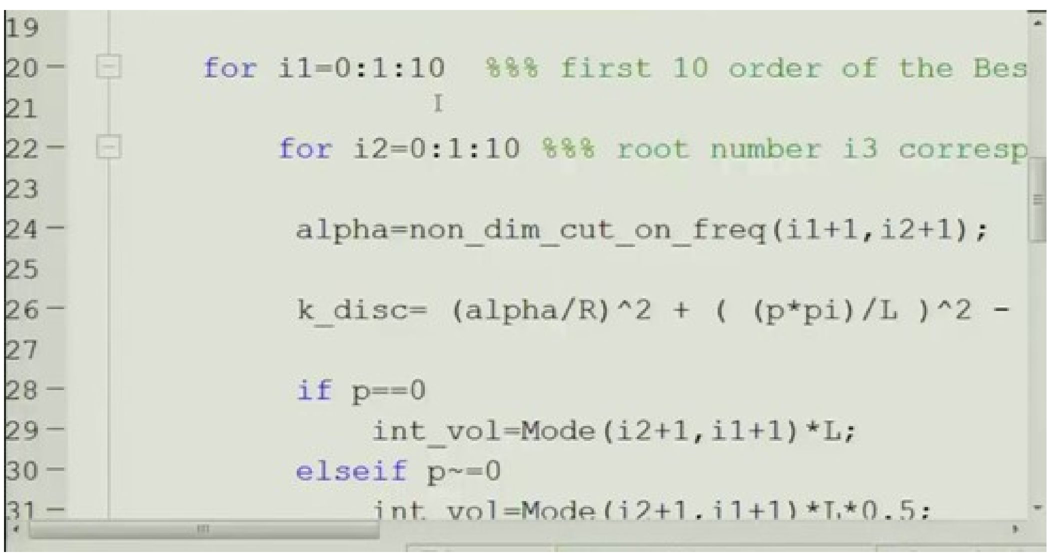
```

Mode=load('Mode.dat');

sp1=load('Port2.dat');
sp2=load('Port2.dat');


sp21=load('Port2.dat');

Tl=zeros(n,1);
  for i=1:n

Tl(i)=transmission_loss_two_port(Mode,sp1,sp21,s
```

So, we will change it to blue color to show different plot and now what we could do is that you know we can put this as a port instead of concentric we could put this as port 2. And this is also port 2 ok that is there and if you put this also as port 2. So, what exactly are we looking at? We are looking at configuration something like this ok. We are looking at this thing ok.



```
Command Window
New to MATLAB? See resources for Getting Started.
i =

   499


i =

   500

Elapsed time is 70.605657 seconds.
fx >>
```

Let us run it and see what does it give; will basically the dome and trough behavior will come down even faster, then that transmission loss behave will come down for even faster. So, you know note that you know in all these things we have considered as many as let us go to the impedance matrix file well.



```
    for i1=0:1:10  %%% first 10 order of the Bes

        for i2=0:1:10 %%% root number i3 corresp

         alpha=non_dim_cut_on_freq(i1+1,i2+1);

         k_disc= (alpha/R)^2 + ( (p*pi)/L )^2 -

         if p==0
             int_vol=Mode(i2+1,i1+1)*L;
         elseif p~=0
             int_vol=Mode(i2+1,i1+1)*L*0.5;
```

You know we have considered as many as first 10 orders in actually first 11 orders of Bessel function and first 11 rules if you put this as 0 0 0 0 0 will just get the plane wave mode. So, we can see what sort of a behavior does the plane wave mode also give us just as a corollary, but we will probably do it for a reversal chamber.

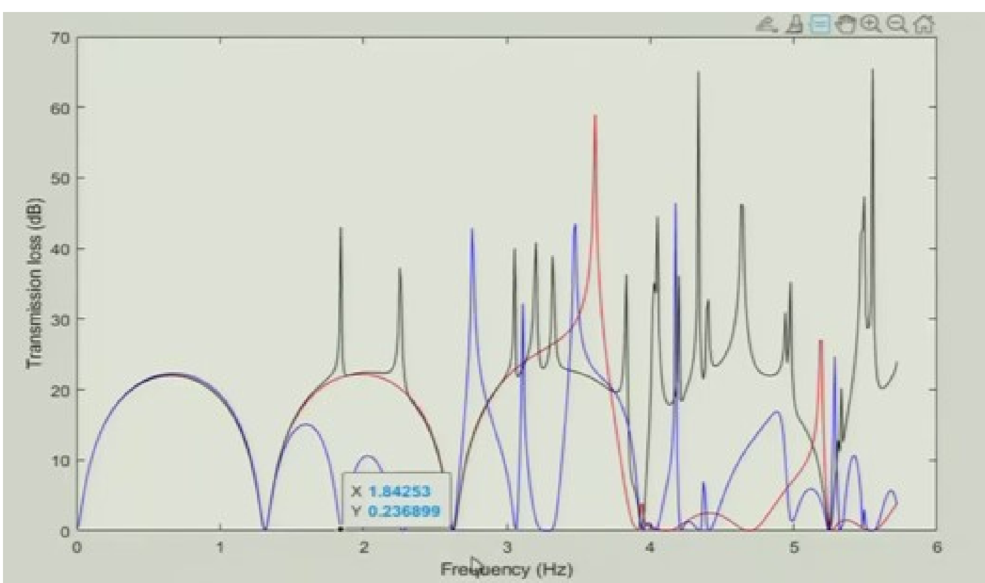
So, we are running short of time. So, what we will do is that we need to skip all not all cases can be considered can be covered. So, we will we let the simulation run perhaps next time we will do it for a shorter frequency range ok. In the in well [FL] well it fails down even faster.

So, it I mean at a much earlier frequency you know at basically at what frequency does it fail? Or the dome trough pattern you know there is a collapse of the dome trough pattern at least dome trough pattern you can get only at isolated frequencies you are getting problematic thing the muffler was transparent.

So, the good thing about dome trough pattern is that you know at least you know where you have the problematic frequency range. But when this fails down you not only get a collapse you get lot of you know troughs and you know attenuation is also completely annihilated.

So, as low as one point well k naught l k naught l is equal to π or the first actual resonance it kind of after that the plane wave would fail. And you know you getting why because as for such a configuration that is shown here you know all the modes will start propagating. So, higher down modes you know 10 mode will also propagate and that. So, you basically get a you know when you had basically this configuration you know ends.

So, concentric inlet an offset port at least the frequency range over the which the dome trough pattern occurred that was enhanced and because this 1 0 mode the suppression of that was proposed that suppressed because of the concentric nature. Similarly 2 0 mode 3 0 everything was suppressed.



Now, similarly when you have the port here one one of the ports at the center this are the offset. So, 0 1 was also suppressed, but you have both the ports at the 0 1 mode node so, but in such a case all the circumferential mode will start propagating. So, we see that you know this kind of fails at k_0 r = 1.84 yes.

So, my guess was right. So, it breaks down at the onset of the first circumferential mode and then the story is over you know it is no longer valid beyond k naught R naught equal to 1.84. So, you know the outcomes of this thing there are lots and lots of things parametric studies can reveal to you.

So, a strong analytical background is very much desirable then you can you know do this 3D effects in a very nice elegant manner. Although this is a piston driven approach is somewhat an approximation, but 1.84 is the frequency at which in which it fails, but in the other case it feels at much larger range. The, what is the point am trying to make? I am trying to make a point that you know the location of the ports has a lot to do with the transmission loss behavior.

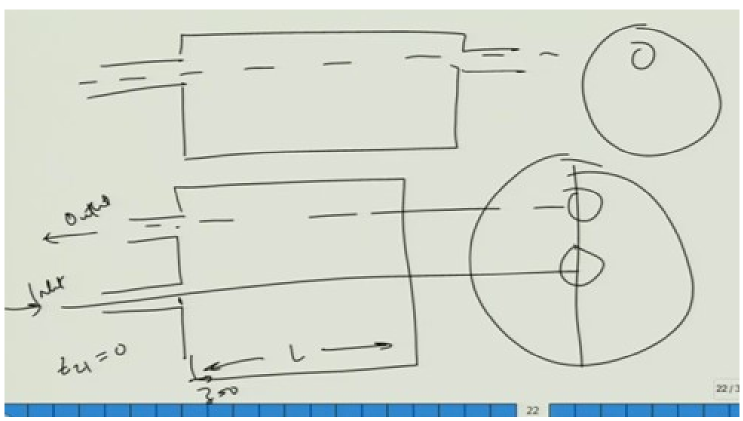
So, that is very important ok. So, if you can at least you can have concentric inlet and outlet ports is still good if you can have one port offset at a certain thing it is even better, but do not have both the ports offset although they are both at the radial nodes of the radial mode, but that is not sort of recommended. So, I will close off this figure and do you know show you something interesting even more interesting for a short chamber.

```
Mode=load('Mode.dat');

sp1=load('Port1.dat');
sp2=load('Port2.dat');


sp21=load('Port2.dat');

Tl=zeros(n,1);
  for i=1:n

Tl(i)=transmission_loss_two_port(Mode,sp1,sp21,s
```

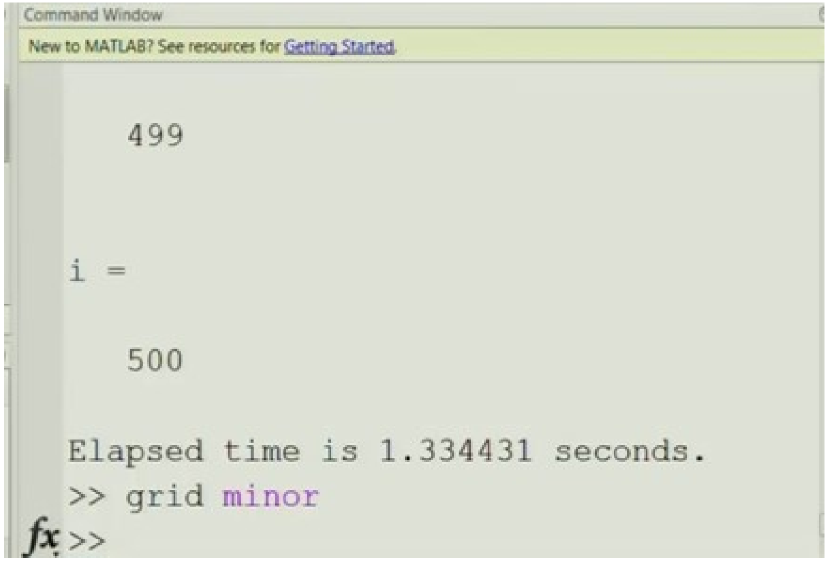
So, I will put this as the concentric port remember port 1 was a concentric 1 port 2 is this thing you know with respect to this thing and. So, basically what is happening is that you know you have this situation. So, this is the port 1. So, z 11 is this thing z 2 is this thing and the and with regards to the z21 parameter that that you see.

So, before we actually run the code you know we can use the same code to analyze a reversal kind of a muffler configuration. So, let us say you know this port is concentric here and this is something like this. So, inlet, outlet ok. So, inlet and outlet and the length is L.

So, what we need to do in the code is that just put z 12 is equal to 0 basically this port if you know consider the coordinate system z is equal to 0 here with respect to the port 1 this is also located at the same value. You know so, basically let us go to the MATLAB code. And just put z 21 is equal to 0 which is which I have done.

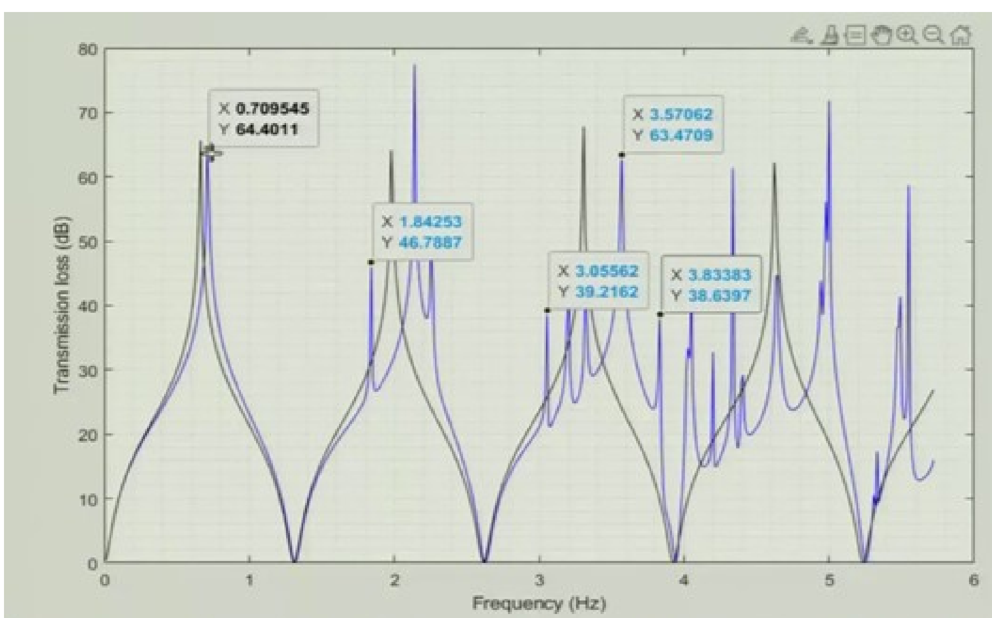


So, it is a flow reversal chamber and port 1 is the concentric one this is the offset one at a certain location and z 21 is also the same because this is the cross impedance matrix kind of a parameter which is evaluated somewhere here this is the cross matrix parameter pc21. So, this will give you the z21 parameter and z12 is the same as z21. So, this is the critical part is p sp21 with reference to the port 1 what is the location of the port 21. So, this is what it is and then let us hit run.



So, you see you know a flow reversal kind of a behavior lot of parametric studies that you can do on your own. So, let us demonstrate the behavior for a long chamber muffler configuration. So, what we are going to do? It is going to take a minute or so. And you and we will also analyze the case where we have plane wave mode.

So, just to put m = 0, n = 0 and we can see the deviation what the end correction does what are what we were discussing all this while. So, you just have to bear for a minute and next what we can do is that we can analyze a short chamber expansion chamber or a reversal kind of a muffler and we see even for short actual chambers things can be good. So, bear for a minute and you will get the results soon.



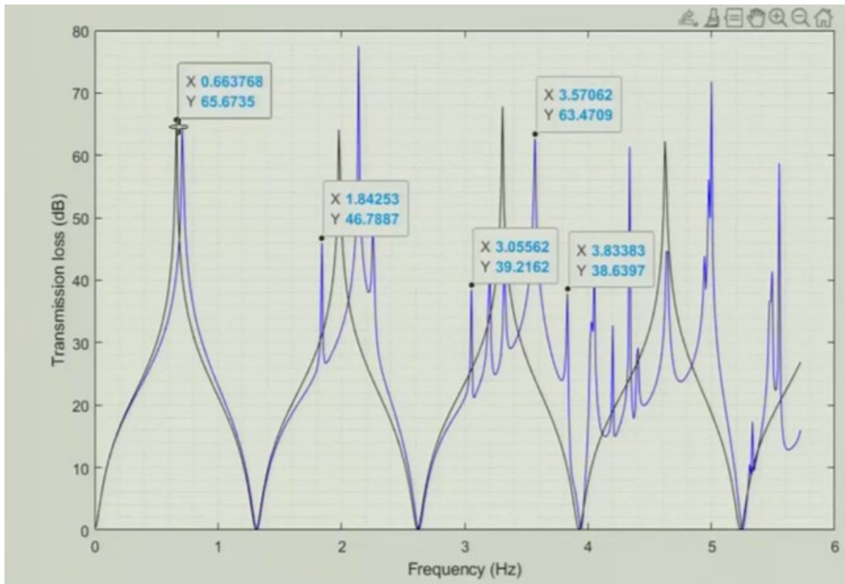
So, well we get the transmission loss characteristics of a flow reversal chamber. So, you see this is a peaky behavior you instead of a dome trough thing you know you are seeing a attenuation peak and a trough thing and there are multiple spikes this always occurs at 1.84 peak and this is occurring at the cross modes of course, and then you are getting a peak at 3.05 and 3.84 I guess. Now 3.57 3.84 somewhere here I guess.

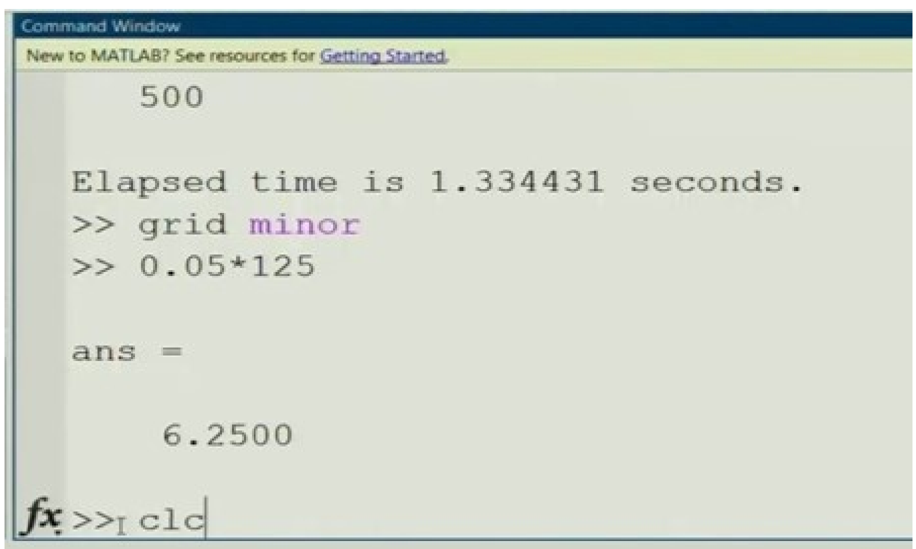
```
1
    end
    ch='k';
    figure(1)
    plot(k0*(125/1000),Tl,ch);
     hold on

grid minor
xlabel('Frequency (Hz)')
ylabel('Transmission loss (dB)')
% legend('3-D Analytical: Uniform Piston Approxi

toc
```

```
pc1=0; pc21=0;  pc2=0;

for p=0:1:20  %%% axial modes

    for i1=0:1:0  %%% first 10 order of the Bess

        for i2=0:1:0 %%% root number i3 correspo

         alpha=non_dim_cut_on_freq(i1+1,i2+1);

         k_disc= (alpha/R)^2 + ( (p*pi)/L )^2 -

         if p==0
```

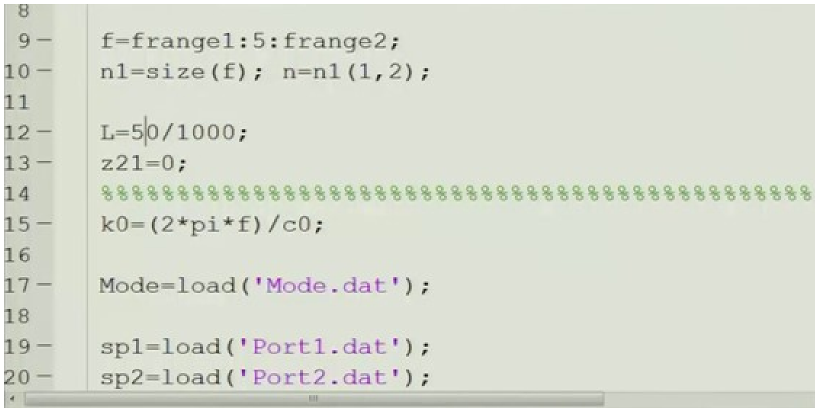
So, anyways what we will do is that we will say figure 1 hold on because all the high order modes at least the first several modes were considered. Now what we can do is that we can change it to black color and in the impedance matrix file how about we change it to 0 and 0 that is we are considering only the plane wave modes. You can put 100 here not a problem and this hopefully should be much faster well you are there.

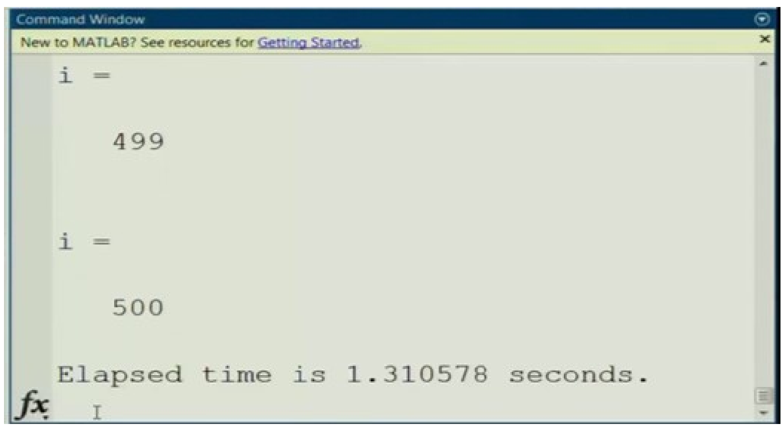
So, plane wave does not take too long to validate because they are lesser number of modal terms. So, we get this. So, you see a small shift; that means so, based on 3D the peak was occurring at 0.709 0.71 $k_0 R_0 =$ 0.71. So, that we can work out the frequency and for just on the plane wave it is occurring at I guess 0.68 or something like that 0.66. So, there is a difference.

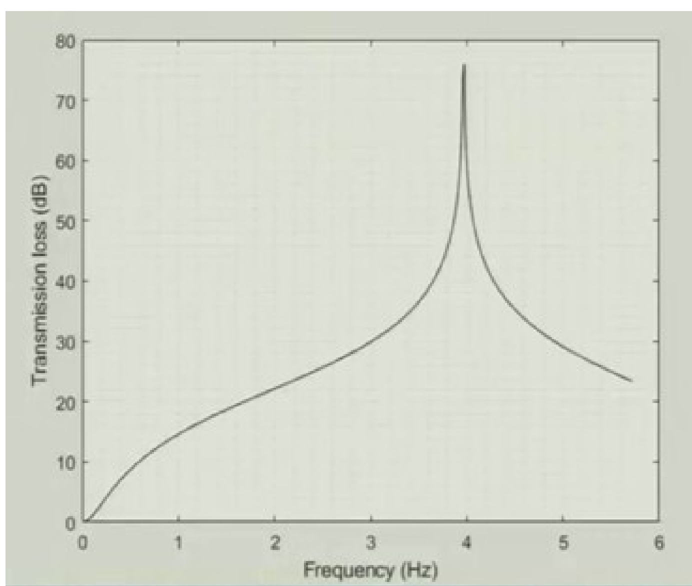


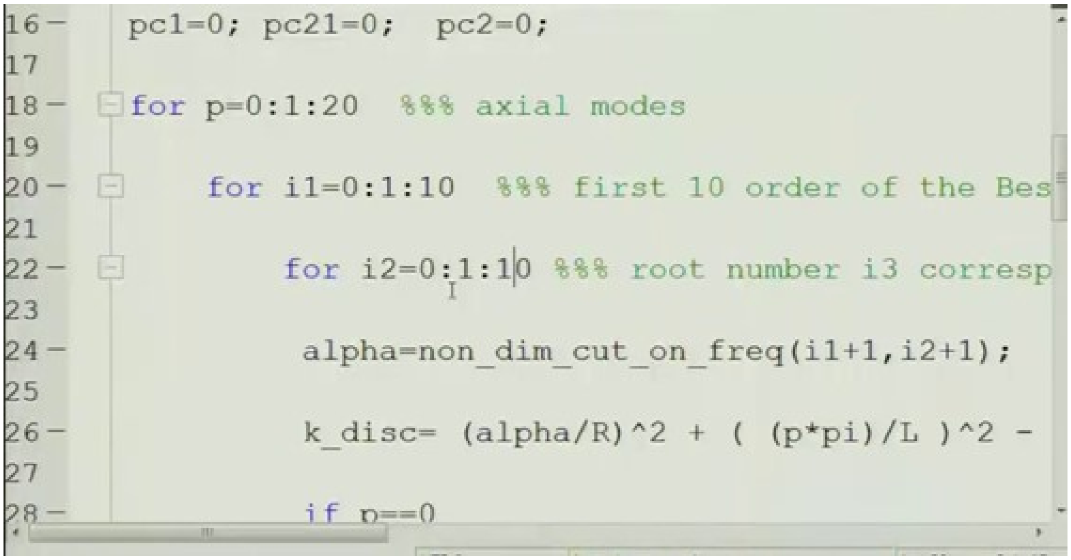


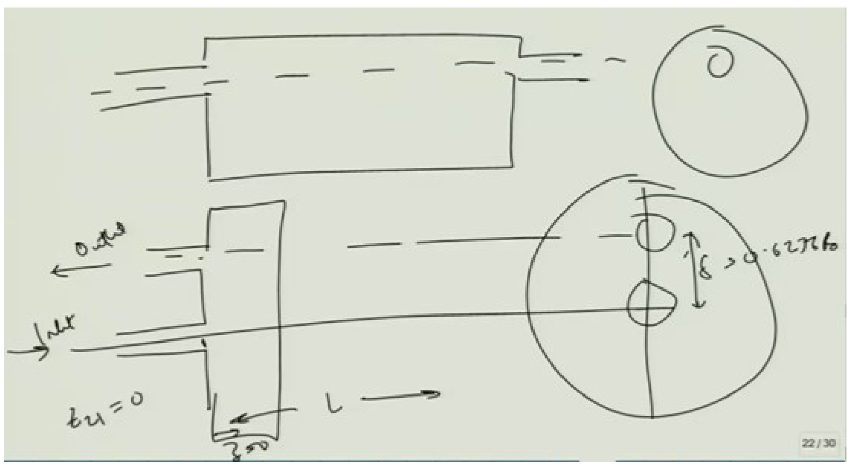
So, point 0.05 R_0 that is a difference. So, 0.05 into 125 if you do well you need to work out how much it would evaluate in terms of Hertz, but there is a good 20-30 hertz difference in the attenuation peak for different configuration you can have different attenuation ranges different shift the variable shift in the peak frequency predicted by the plane wave and the 3D effects. Now basically you know if you put z naught is equal to point instead of this thing if we let us have a shorter chamber.





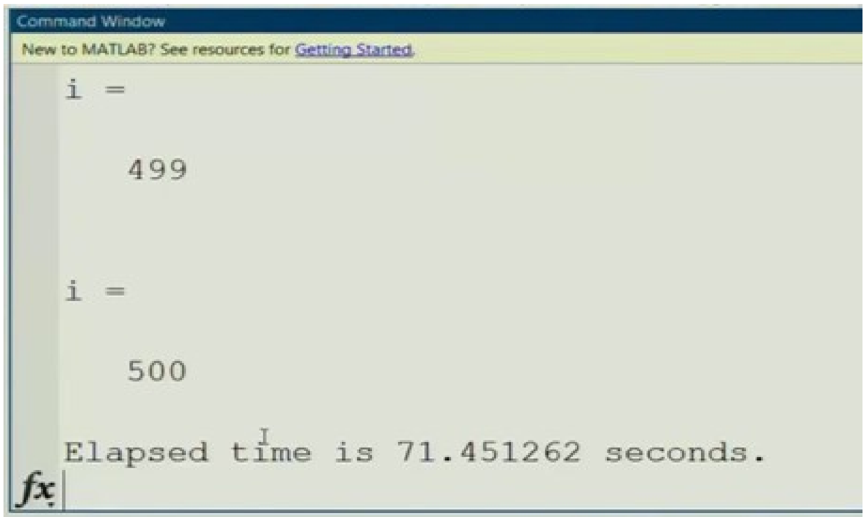


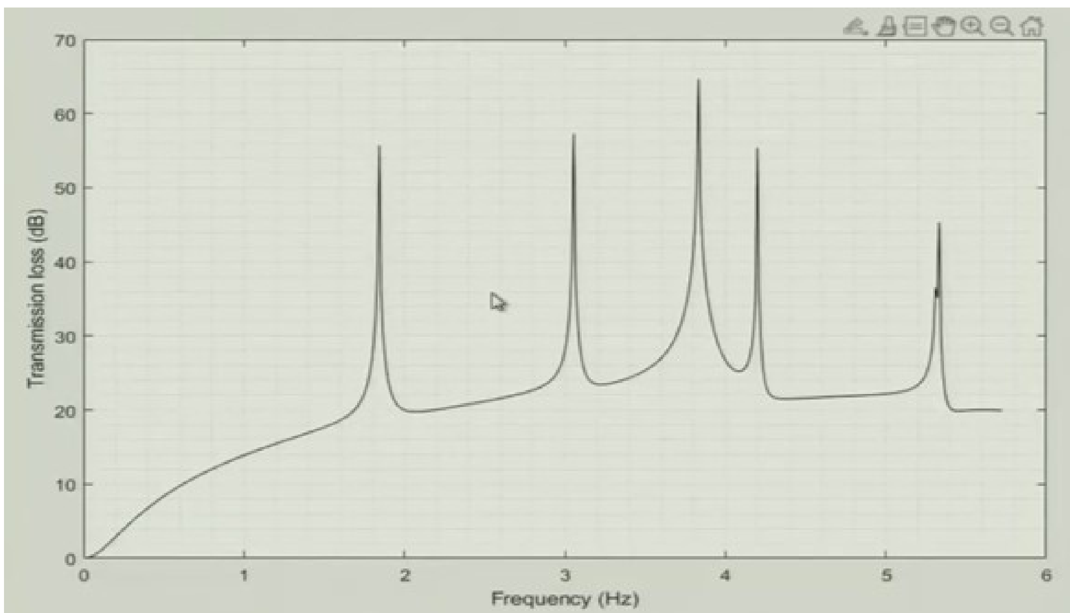




And let us see what the behavior would be like. You know well this is not quite correct because we have not considered the hard or more terms, but what I am what I am kind of let us get back to the original thing and you know what it means is that you know we are considering a much shorter chamber. So, one port is at the center other is it at is at delta and delta is 0.6276 R_0.

So, you see you will see just in a while a fantastic broadband annotation even a short chamber can give you provide that you can offset the inlet and outlet properly. So, this is what I presented in my book. So, I am going to show you the graph of such a configuration while it is doing its job. So, if you go back to the close of this guy in my monograph, I have talked about this configuration let it load we should be nearly there you should be getting a very broadband attenuation well.





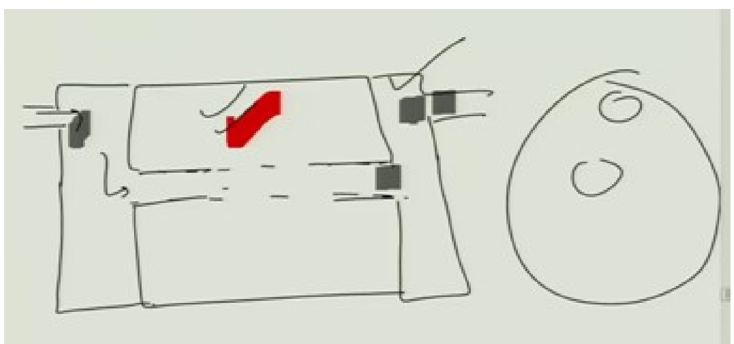
This is what we get and compare it with the other configuration. So, you know although at slow frequency at attenuation amount of attenuation is relatively low, but you are getting a broadband attenuation. And this is what you know I kind of got in my springer monograph which is basically your it is not loading somehow I am not sure why.

But, well, anyhow the point is that you know even a short chamber can get you a broadband attenuation and you are able to get you know as much as if you continue to go beyond it is going to fail at a much higher frequency probably about $k_0 R_0 =$ 6.8 or 6.9 something like that.

But you know. So, these reversal chambers what is presented here they are used a lot in your as an end chamber in a more 3 pass or multi pass perforated muffler system as we discussed in you know week the previous weeks you know 8 to 10 we are discussing all those things.

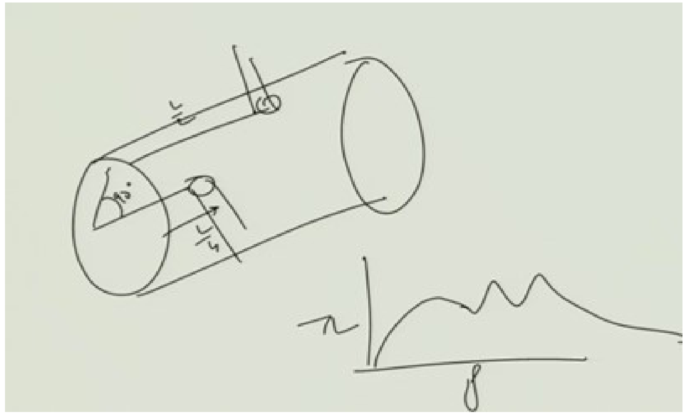
So, the point is that if you have a bigger system something like. So, if you have a system like this the problem one of the research problems that comes in my mind and that is a that is been there for some time you have a perforated tube you can try you know this is the length here, what is the location of this inlet and outlet ports? Or the ports of the end chamber this is perforated fully or partially depending upon what you want this got some thickness this baffle plate.

And this end chamber which facilitates the flow reversal. These are the configurations what I presented in my book and you can alternatively have another configuration you know something like this you know single pass tube like this. The flow comes here has to negotiate this you can have this is a partially perforated tube.



So, what is basically your optimal configuration? Basically this guy we know this guy we know, but we have to design this guy and this guy what are the optimal location of this port here and so on. So, all this needs to be done and so, mat so, 3 D analysis of such configurations can be can prove to be very handy and that is what that is what we did.

So, other thing that we can also look into is that analysis of side inlet in a side outlet configuration, but you know this something that you should probably try on your own. There is something similar what we saw for a rectangular chamber with the end inlet and a side inlet and a side outlet.

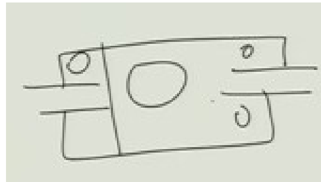


So, let me just summarize the result you know rather than doing the calculations again you know if you have things like if you have a port in here and one of the ports is say located here at L / 4 other port is at L / 2, but this is say on this thing and this is at 90 degrees you know sort of not able to draw it. This is something like this and this is like this. So, the included angle between this is 90 degrees then you would get a broadband attenuation.

You have multiple peaks and you know you will get a broadband annotation first, second, third resonance drops are basically eliminated and with 90 degree you can also get rid of the 1 0 mode, but 2 0 mode you are going to have problems that at that point attenuation is going to fail.

So, anyhow the point is that such codes can also be used for analyzing a side inlet and side outlet or possibly an end inlet in a side outlet kind of chamber reports can be arbitrary placed and these results are you know match pretty well with 3D analysis 3D FEM analysis where there is a proper coupling between the inlet and outlet ports you know that because the entire thing is taken as a continuum.

So, with this thing I will stop here and what we are go. So, this brings us to the end of week 11 lecture, we just had a glimpse of the 3D analysis. But what we could also do is that you know there are lots of geometries there as a final node for this lecture you know piston driven model can be used to analyze a lot of configurations, but for systems such as extended inlet and outlet systems.



You know one necessarily has to consider modal coupling at least within between the annular cavity that is you know that is between you know if you have a chamber like this something like this between this part and this part and this part, there is a fully coupled system one must consider that. So, hopefully we will take up that in the next week the first few lectures.

And then touch upon some just touch upon the dissipative muffler theory because this is huge theory is hard to cover in one week. You guys just have a glimpse of that and then you know, but as a final note in this lecture, I want to say that you know analytical solutions are possible for a lot of geometries in which Helmholtz equation is separable.

There are lot of cordial systems circular cylindrical and rectangular I mean Cartesian system present only the simplest one the other one is elliptical system for which I have written the monograph in terms of Mathieu functions, radial Mathieu functions and angular Mathieu functions. So, they can be used to analyze elliptical chamber mufflers which are used a lot in practical applications.

So, other than that of course, there is a concentric I mean circular chamber with a concentric pass tube and so on then there are hemispherical chambers which can also be analyzed using piston driven model and all that. So, they are a lots of things conical pipes perhaps conical chambers and then there are advanced mode matching techniques. So, all these can be used to analyze reactive mufflers quite well.

But I will probably stop here because the idea was to just to get you a glimpse of the 3D analysis and we will basically meet in the next week where we will discuss the extent inlet and outlet system using analytical approach. Let us see how much we can analyze that using MATLAB code based on the time and the codes how much it is giving us results. So, till that time I will stop here and I will see you in the next week.

Thanks.