Muffler Acoustics-Application to Automotive Exhaust Noise Control
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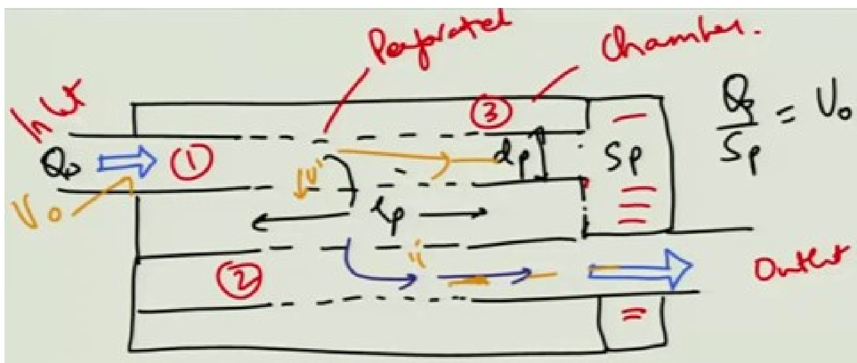
Lecture - 42 and 43
Cross Flow Elements: MATLAB Demonstration for Simple Configuration

Welcome to lectures 2 and 3 of week 9. These lectures 2 and 3 are combined. So, in the last lecture, as you as you noticed we were we were discussing about the cross flow elements for the first time. So, you know when you whenever you have a cross flow element that necessarily means that you must have something like a this kind of a system where you have a 3 interacting ducts.

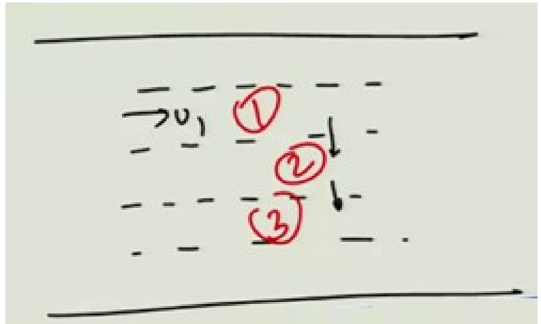


$$Q_0 = (\sigma.\pi d_p\ lp)U' = U_0 Sp$$

$$U' = \frac{U_0 Sp}{(\pi d_p\ lp)\sigma}$$

So, there are basically two perforated pipes, for simplicity only I have assumed that the perforated section is a common section between the two pipes through which the or the air ways through which flow happens. And surrounding that is a jacket or annular region which basically contains or houses these pipes. So, the idea is that the flow has to go through the pipes and come to the other pipe. So, as a result we must have at least 3 interacting ducts.

$$\frac{D_1 = D_3}{\zeta_2 = \zeta_1}$$



$$C_1 \ \rho_0 \frac{\partial \widetilde{U}_1}{\partial z} + U_1 \frac{\partial \tilde{\rho}_1}{\partial z} + \frac{4}{d_1} \rho_0 \widetilde{U}_{1,2} = -\frac{\partial \tilde{\rho}_1}{\partial t} \qquad (1)$$

$$U_{1,2} = \frac{p_1 - p_2}{\rho_0 \ C_0 \zeta} \qquad (2)$$

Now, with this understanding you know we were we are beginning to derive the equation. So, this was the continuity equation. I just wrote it down the different equations, the namely the continuity equations, momentum equations in duct 1, 2, duct 3. So, that is the nomenclature we follow.

$$\rho_0 \left\{ \frac{\partial \widetilde{U}_1}{\partial t} + U_1 \frac{\partial \widetilde{U}_1}{\partial x} \right\} = -\frac{\partial \tilde{p}_1}{\partial z} \qquad (3)$$

$$\rho_0 \frac{\partial \widetilde{U}_2}{\partial z} + U_2 \frac{\partial \widetilde{U}_2}{\partial x} - \frac{4 d_1}{d_2^2 - d_1^2 - d_3^2} \ \rho_0 \ \widetilde{U}_{1,2}$$

$$+ \frac{4 d_3}{d_2^2 - d_1^2 - d_3^2} \ \rho_0 \ \widetilde{U}_{1,3} = -\frac{\partial \tilde{p}_1}{\partial t} \qquad (4)$$

$$\widetilde{U}_{2,3} = \frac{\tilde{p}_2 - \tilde{p}_3}{\rho_0 C_0 \zeta} \qquad (5)$$

$$\rho_0 \left\{ \frac{\partial \widetilde{U}_2}{\partial t} + U_2 \frac{\partial \widetilde{U}_2}{\partial z} \right\} = -\frac{\partial \tilde{p}_2}{\partial z} \qquad (6)$$

$$\rho_0 \frac{\partial \widetilde{U}_3}{\partial z} + U_3 \frac{\partial \tilde{\rho}_3}{\partial z} - \frac{4}{d_3} \rho_0 \widetilde{U}_{1,3} = -\frac{\partial \tilde{\rho}_3}{\partial t} \qquad (7)$$

$$\rho_0 \left\{ \frac{\partial \widetilde{U}_3}{\partial t} + U_3 \frac{\partial \widetilde{U}_3}{\partial z} \right\} = -\frac{\partial \tilde{p}_3}{\partial z} \qquad (8)$$

$$\tilde{\rho}_1 = \frac{\tilde{p}_1}{C_0^2}, \qquad \tilde{\rho}_2 = \frac{\tilde{p}_2}{C_0^2}, \quad \tilde{\rho}_3 = \frac{\tilde{p}_3}{C_0^2} \qquad (9)$$

So, we get all these things as a result, we get all the equations. Now, it becomes actually quite messy to kind of simplify that. And note that we here we are assuming time harmonicity. So, we whatever time variables are there

$$\frac{\partial(\,)}{\partial t} = \frac{j\omega}{C_0}$$

we assume the variable to have a $e^{j\omega t}$ dependence.

So, it is called; so, it is basically it gives you jω times that particular variable. So, support is it is p, so we get

$$p = \tilde{p}\, e^{j\omega t}$$

So, we get $e^{j\omega t}$ and you know basically we get $e^{j\omega t}$ is cancelled on both sides of the equation. So, eventually we get this quantity.

$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \\ \tilde{p}_3 \\ \rho_0 C_0 \tilde{U}_1 \\ \rho_0 C_0 \tilde{U}_2 \\ \rho_0 C_0 \tilde{U}_3 \end{Bmatrix}' = \begin{bmatrix} A_{11} & A_{12} & \cdots & \cdots & \cdots & A_{16} \\ A_{21} & \cdots & \cdots & \cdots & \cdots & A_{26} \\ A_{31} & \cdots & \cdots & \cdots & \cdots & A_{36} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ A_{61} & \cdots & \cdots & \cdots & \cdots & A_{66} \end{bmatrix}_{6\times 6} \begin{Bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \\ \tilde{p}_3 \\ \rho_0 C_0 \tilde{U}_1 \\ \rho_0 C_0 \tilde{U}_2 \\ \rho_0 C_0 \tilde{U}_3 \end{Bmatrix}$$

And this is often divided by the sound speed, so we replace this thing by k_0. So, you know, so that is the standard simplification process you would have kind of followed it by now. So, we hope to put the matrix in this sort of a form is equal to a X, where you know where the X vector is basically nothing, but this particular vector this is the X vector, ok. So, and this is your A matrix the entries of which we will have to populate.

Now, in the last class I just stopped here, and what we could do as a good way to start is basically write down the different entries on the A matrix. So, basically you know written in this form where all the pressure things are in one side and velocities are the in the other side, we would get the following form of the A matrix.

So, A matrix then is sort of given by; and here note that is one more thing that I sort of forgot to tell and that is that we have incorporate the convective effects of mean flow by the underlying terms. So, here we have these underlying terms you know U_2, U_3 and so on and U_1 also in this duct, but for simplicity we can sort of ignore the convective effects of mean flow altogether in the ducted system. And we can just consider the dissipative effects of mean flow.

And how do we do that? So, we incorporate the dissipative effect. So, the perforate impedance expression. So, you know it is been shown in the previous papers like there is one famous paper by Munjal and others published in about 87 in the Journal of Sound and Vibration, where they are shown that they have also ignored the mean flow convective effects. So, mean flow because it makes life easy. You know it kind of simplifies the algebra and lot of terms drop out.

They are just, they just evaluate to 0, so we need not consider them, but they can still account for the most important dissipative effects of mean flow which we are going to see now by using appropriate perforate impedance expressions. Now, as a result all these equations that you see here, you know the equations in this line this I mean this kind of a term here, this term, and your this term, this one all these terms will one by one start dropping out. And you know the A matrix will be much more simplified.

So, with this assumption write down the entries of the of the A matrix when convective effects are ignored. So, here it is written in the following form. A matrix let us do this thing and just for argument sake we will put this where the subscript two denotes to denotes the middle or the annular cavity or the chamber itself.

So, let us make appropriate things here. So, the A1 4, we are now writing the momentum equation first and let us also divide this into 6 parts, 2, 3, 4, 5, 6. So, here we what we will do? You know we will write down the momentum equation in the first duct at the very first. So, A1 4 this term will become

$$
\begin{Bmatrix} \frac{\partial p_1}{\partial z} \\ \frac{\partial p_2}{\partial z} \\ \frac{\partial p_3}{\partial z} \\ \rho_0 C_0 \frac{\partial U_1}{\partial z} \\ \rho_0 C_0 \frac{\partial U_2}{\partial z} \\ \rho_0 C_0 \frac{\partial U_3}{\partial z} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -jk_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -jk_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -jk_0 \\ -jk_0 \frac{4}{d_1\zeta_1} & \frac{4}{d_1\zeta_1} & 0 & 0 & 0 & 0 \\ * & ** & *** & 0 & 0 & 0 \\ 0 & \frac{-4}{d_1\zeta_2} & -jk_0 \frac{-4}{d_2\zeta_2} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ \rho_0 C_0 U_1 \\ \rho_0 C_0 U_2 \\ \rho_0 C_0 U_3 \end{Bmatrix}
$$

You see because you know here you have a state variable X dash. So, what it really means is that $\frac{\partial p_1}{\partial z}$ is equal to $-jk_0$ times $\rho_0 C_0 U_1$.

So, here we write the momentum equation first. Now, you know this is with the understanding this is the A matrix. So, I am just sort of rubbing it off other equation of course, is your momentum equation, is not it. So, what will be this entry? So, this is $-jk_0$. So, here you will get your 3, it is A3 6, sorry this will be something like this. So, $-jk_0$. And there will be, there will be a term here also corresponding to del z of p_2, that will be $-jk_0$here.

So this, this thing this goes with this particular guy, is not it. So, like this we have written down the momentum equation, ok. Now, comes the continuity equation which is; obviously, more challenging because here it is an exchange like I have been mentioning at the very beginning of the lecture on the perforated mufflers. It is basically the exchange of mass between through the perforates and that is why the continuity equation is kind of modified it has to be modified, ok.

So, we get this sort of a thing, and we get this. Let us write down the entries corresponding to the continuity equation. So, here we have $-jk_0$+ 4 / d_1 ζ_1, so this goes here, this guy goes here, $-jk_0$ -4 /d_1 ζ_1, ok. And then you have your p_2 term. So, this will, this guy will go in here ζ_1 and this will be 0, 0, 0. So, now, there is one thing that I want to introduce here. By default by default this a matrix is initialized at 0s.

In this case, 0 cross 0 something like this, ok. So, this is what it is. Now, once we have your the second momentum equation, I mean the momentum equation in the annular duct. What do we get? We will get.

$$\frac{4d_1}{(d_2^2 - d_1^2 - d_3^2\ )\zeta_1}$$

So, apologies for the lack of space, but this will be the term here. So, what I am going to do actually? Let me sort of, let me sort of rub this guy completely, and what are we doing is that I will put a star mark here and a double star mark here, ok. And as well, ok.

So, I will put this and then I will also put another triple star mark here and I tell you what these expressions, what these expressions are. And in the meanwhile wherever there is nothing, its 0. So, like I said it is all initialized with 0, so 0, 0, 0.

Now, the entry is pertaining to the continuity equation in the third duct or the second perforated pipe that is your outlet pipe, ok. So, here you will get, $\frac{4}{d_1\zeta_2}$, ok. And here it will be $-jk_0$ and $-jk_0$ and $\frac{-4}{d_1\zeta_2}$. So, and everywhere else its will be 0, 0, and 0. So, what are these stars? So, these stars are they are pretty much expressions looking of the same form just that they have few more terms.

$$* \quad = \frac{4d_1}{(d_2^2 - d_1^2 - d_3^2)\zeta_1}$$

So, you should avoid the typo. In some of the books you know people what they have done they have put full thing here and then they have put this kind of a thing and then there are some sort of typographical error which would really completely corrupt the results. So, be mindful of that.

So, it is really something like $-jk_0$ is a separate entity, otherwise dimensionality also dimensionality wise also it will not be correct. So, $-jk_0$ is separate. You know let me write it a bit more separate. So, I would write it here $-jk_0$ and in the, in the numerator of this term.

$$** \quad = \frac{4\left(\frac{d_3}{\zeta_2}+\frac{d_1}{\zeta_1}\right)}{(d_2^2-d_1^2-d_3^2)} - jk_0$$

And this term

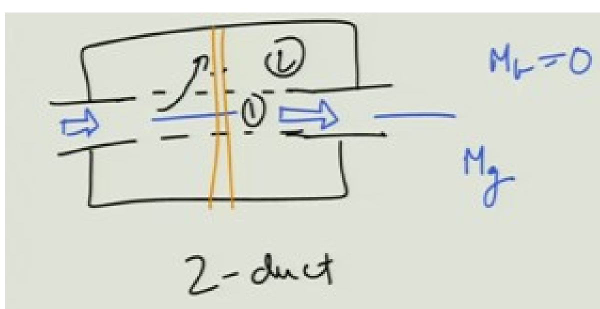
$$*** \quad = \frac{4d_3}{(d_2^2-d_1^2-d_3^2)\zeta_2}$$

Now, all these terms have to be put inside the appropriate entries here and this of course, will also be 0. So, this is the matrix. Again, we are getting back the form you now again as like I said X' is equal to a Ax, where X' is, dash means d /d z, that is the spatial derivative.

$$\begin{Bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix}_{z=0}^{-X} = \frac{expm\ (-[A]L)}{[T]} \begin{Bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix}_{z=1}$$

$$A = zeros\ (6,6)$$

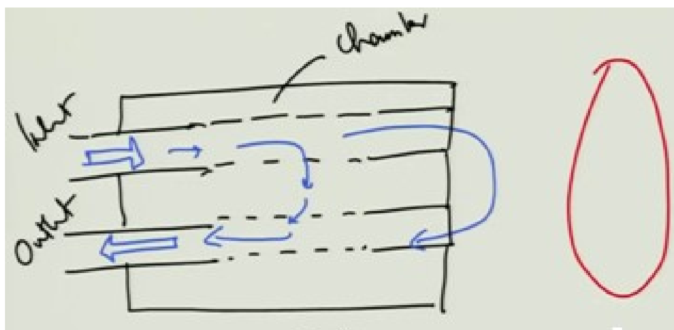
So, again you know this is the command. Now, just one small digression that I would like to sort of make, prior to this work you know ages back when the analysis of perforates first started the work by Sullivan and Croker what they did was they use the segmentation approach.



So, they used to divide the duct into number of small parts and analyze you know get transfer matrix matrices across each part. So, for example, you know let us go back to our original the configuration let us say this one. So, you know does not matter whether 2 or 3 ducts. The point is that they used to divide the ducts into small regions and get transfer matrices across this one and this one. And then what their approach used to really do is that they could account for the exact number of holes in each of the things.

So, they could account for non-uniform porosity also. So, this is like a you know it is like a discretize system. And another thing that I want to tell you is that you know flow we are assuming the grazing flow in a straight through duct to be constant throughout, but it will never be constant.

It will be gradually be you know sort of as the flow passes the grazing flow will tend to decrease and more of a, little bit of nonzero flow will be there across the a perforated element even in a straight through muffler that is a two duct muffler, two interacting duct muffler.



Cross-Flow Reversal Chamber

And then it will be minimum sum in the middle, and then we will gradually pick up and be a kind of a steady value when it reaches. CFD analysis would do a proper justice, and specially, when you come to a configuration like this one or perhaps this one, the flow would actually gives us something that I have to discuss in a greater detail. The flow usually comes the grazing flow and it tends to have become go to a almost 0 value, suppose this guy is not there it is a closed cavity, ok.

So, this will be 0 here, from maximum value it will go to 0, and in the process your grazing flow I am sorry your bias flow will be nonzero here, and that will be entering this value this perforate here. And you will be having some nonzero convective value in the duct in the central duct as well which you sort of conveniently ignored.

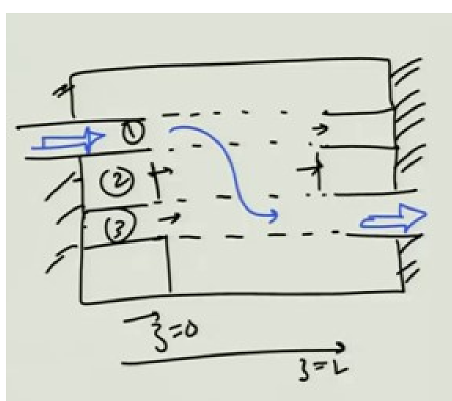
And then we will be picking up from 0 value here to a maximum value here in the outlet duct. So, the idea is that the flow is gradually varying and segmentation approach by

Sullivan and Croker and in basically in Sullivans paper in 19, I guess 60, late 60s I believe or probably 70s those papers basically talked about considering the mean flow effects also, gradually varying mean flow effects also. And but in this particular thing in the analysis that we are doing we are, we are considering the perforate impedance to be at least uniform throughout the duct.

And it is like a distributed parameter approach as opposed to the discrete segmentation approach by Sullivan. And so, this basically everything has its own benefits as well as drawbacks. So, in our approach we probably cannot take the porosity well, porosity is one parameter or impedance to vary along the length that is this thing. So, we are assuming really in all these expressions that we that you are seeing wherever you are considering well, perforate impedance this guy.

We are considering this to be uniform throughout the cross section because porosity is assumed to be uniform and whole distribution pattern is assumed to be uniform and all that, but it need not be. If it is varying that is if we have a non-uniformly perforated pipe it will have its own sort of effect.

Now, for such a thing we can consider a segmentation approach which will be slightly different. So, we still assume over small segments things do not vary much and we can probably analyze the special muffler configuration called a CCTR conical concentrate to resonator that is outside cavity is of conical shape. And then we can do all this matrix and approach in a you know in its full form, means you know it completely harnesses the power of this nice mathematical approach.



So, this is something here now coming back to the problem were left with integrating this particular matrix and to this end we will have to go to MATLAB and integrate in an with the understanding that we also have to apply appropriate boundary conditions. So, before we actually go to MATLAB let us also write down the different boundary conditions that we have you know for basically for ducts of this sort is called a across flow expansion duct, ok as you know.

It is an expansion kind of a thing. So, here you are, ok. So, the flow leaves from here, goes on here, ok. Now, what are the boundary conditions? If we fix our z is equal to 0 here and you know z is equal to l is really here, ok. So, what do we get? At X is equal to 0; you what you basically you know get is your the boundary conditions are

$$Z_2(0) = \frac{\tilde{p}_2(0)}{-U_2(0)} = -j\rho_0 C_0 cot k_0 la$$

So, basically 2 is this region, 1 is this guy 3 is here. So, basically at this point we are assuming a rigid world end plate, ok.

They need not always be rigid, but you know like I have mentioned before let me write it down on a sort of different page or at least let me sort of reduce the length of this and utilize the space properly. So, z_2 (0) at this point that is somewhere here and this interface.

And the same thing applies for the duct here. So, cot cannot, you know cot cannot a level or cot cannot always keeps on occurring you know as we have seen from the very beginning of this lecture where for a rigid and cavity. So, apply all these things here, ok. That is why it is important to you know kind of pay attention at the expressions, right from the beginning because these will be useful now. So, again why is a minus sign? Because velocities considered positive along this direction, ok.

But because you are looking into the cavities of - U_3 times that thing will give you will basically change the direction of velocity and then you have this expression readily applicable. So, here also,

$$Z_3(0) = \frac{\tilde{p}_3(0)}{-U_3(0)} = -j\rho_0 C_0 cot k_0 la$$

Now, at this point you know this cavity is close.

$$Z_1(L) = \frac{\tilde{p}_1(L)}{-U_1(L)} = -j\rho_0 C_0 cot k_0 lb$$

Now, we have really one more boundary condition and that is at this point. So, let me write it down here only. We get this kind of a thing.

$$Z_2(l) = \frac{\tilde{p}_2(l)}{-U_2(l)} = -j\rho_0 C_0 cot k_0 lb$$

You know implementation of all this boundary condition in a MATLAB code is certainly not trivial. You know it was already messy you know in the last for this particular case, you know I guess these were the things and then we had all these mathematical or algebraic manipulations. And all these boundary conditions eventually led to you know quite tedious expressions. They are it is not like they are very complicated, but they are definitely tedious.

$$\{X\} = expm\left([C]x\right) \begin{Bmatrix} \tilde{p}_1 \\ \rho_0 C_0 \widetilde{U}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \widetilde{U}_2 \end{Bmatrix}_{x=0} \quad inv\,(.)$$

$$\begin{Bmatrix} \tilde{p}_1 \\ \rho_0 C_0 \widetilde{U}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \widetilde{U}_2 \end{Bmatrix}_{x\,=\,L} = expm\,\{-[C]L\} \begin{Bmatrix} \tilde{p}_1 \\ \rho_0 C_0 \widetilde{U}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \widetilde{U}_2 \end{Bmatrix}_{x=0}$$

$$expm\,\{-[C]L\} \begin{Bmatrix} \tilde{p}_1 \\ \rho_0 C_0 \widetilde{U}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \widetilde{U}_2 \end{Bmatrix}_{x\,=\,L}$$

$$= \begin{Bmatrix} \tilde{p}_1 \\ \rho_0 C_0 \widetilde{U}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \widetilde{U}_2 \end{Bmatrix}_{x=0}$$

$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \tilde{U}_1 \\ \rho_0 C_0 \tilde{U}_2 \end{Bmatrix}_{x=0} = \begin{bmatrix} T'_{11} & T'_{12} & T'_{13} & T'_{14} \\ T'_{31} & \cdots & \cdots & \cdots\ T'_{34} \\ T'_{21} & \cdots & \cdots & \cdots\ T'_{24} \\ T'_{41} & T'_{12} & T'_{13} & T'_{44} \end{bmatrix} \begin{Bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \tilde{U}_1 \\ \rho_0 C_0 \tilde{U}_2 \end{Bmatrix}_{x=L}$$

upstream variable upstream variable

$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \tilde{U}_1 \\ \rho_0 C_0 \tilde{U}_2 \end{Bmatrix}_{x=0} = \underbrace{\begin{bmatrix} T'_{11} & T'_{12} & T'_{13} & T'_{14} \\ T'_{31} & T'_{32} & T'_{33} & T'_{34} \\ T'_{21} & T'_{22} & T'_{23} & T'_{24} \\ T'_{41} & T'_{42} & T'_{43} & T'_{44} \end{bmatrix}}_{[T]} \begin{Bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \tilde{U}_1 \\ \rho_0 C_0 \tilde{U}_2 \end{Bmatrix}_{x=L}$$

$$\left.\begin{aligned} \frac{\tilde{p}_2(0)}{-\tilde{U}_2(0)} &= -j\rho_0 C_0\, cotk_0 la \\ \frac{\tilde{p}_2(L)}{\tilde{U}_2(L)} &= -j\rho_0 C_0\, cotk_0 lb \end{aligned}\right\}$$

$$\begin{Bmatrix} \tilde{p}_1(0) \\ \\ \rho_0 C_0 \tilde{U}_1(0) \end{Bmatrix} = \begin{bmatrix} T_a & T_b \\ \\ T_c & T_d \end{bmatrix} \begin{Bmatrix} \tilde{p}_1(L) \\ \\ \rho_0 C_0 \tilde{U}_1(L) \end{Bmatrix}$$

$$T_a = T_{11} + A_1\, A_2, \quad T_b = T_{13} + A_2\, B_1$$

$$T_C = T_{31} + A_1\, B_2, \quad T_d = T_{33} + B_1\, B_2$$

$$A_1 = \frac{(X_1 T_{21} - T_{41})}{F_1} \qquad B_1 = \frac{(X_1 T_{23} - T_{43})}{F_1}$$

$$A_2 = T_{12} + X_2 T_{14} \quad B_2 = T_{32} + X_2 T_{34}$$

$$F_1 = T_{42} + X_2 T_{44} - \quad X_1 = T_{22} - X_1 X_2 T_{24}$$

So, imagine the thing for a 4 cross 4 system. So, to eliminate appropriate variables and you know eventually what we want, we must ask ourselves what exactly are we looking for. So, we are looking for a relation between the state variables. So, basically what we are sort of seeking for is really this kind of a thing, is not it.

$$\begin{Bmatrix} p(0) \\ \\ \rho_0 C_0 U_1(0) \end{Bmatrix} = \begin{bmatrix} T_a & T_b \\ \\ T_c & T_d \end{bmatrix} \begin{Bmatrix} p_3(l) \\ \\ \rho_0 C_0 U_3(l) \end{Bmatrix}$$

We really are seeking this kind of a thing. And then to eliminate all these things and to finally, arrive at this form it is not trivial. It is definitely not trivial. You know it involves a lot of thing expressions. So, we can do it in a number of ways starting beginning of course, from this guy. You know eventually we need to integrate this and get it in this form. And once we have this kind of a thing we can call this as a T matrix, ok T matrix.

```
function [] =transmission_loss_plot(ch)
tic

frange1=5;
frange2=3500;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
sigma=3.9;   %%% Porosity...
sigma=sigma/100;

th=2/1000;
dh=3/1000;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

f=frange1:1:frange2;
```

So, T is equal to x e to the power minus A into l, where A is a matrix. So, what we will do now is that we will directly go to MATLAB and have some fun with our scripts. Well, it is fun for me, so I am saying so. Now, this is the overarching function as we know from the programming logic that I have adopted. An overarching function causes another subroutine and which calls in another in turn its own the heart of the program or the main thing that models the system.

So, this is your frequency range which is fairly simple have set. Deliberately set some very low sigma values, we can modify that porosity. So, this really is your porosity, ok. And these are the thickness of the pipe and the hole diameter respectively, frequency range and all these things are known.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

mg=0;

D2=148.1/1000;  %%% Diameter of the chamber...
D1 =49.3/1000;  %%% Perforated duct diameter (eq
L =141.4/1000;  %%% Chamber length
la=6.4/1000;      %%% Extended-inlet length
lb=6.4/1000;    %%% Extended-oulet length...
l_perf = L -(la+lb);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for i=1:n
T1(i)=transmission_loss(D1,D2,l_perf,la,lb,k0(i)
```

Now, D_2, let us come to the business end of the things. So, D_2 is your diameter of the chamber. So, going back to the presentation, so D_2, by D_2 I really mean this thing. And you know here I am assuming now one thing that I want to make clear here I am assuming D_1 is equal to D_3, ok.

The diameters of the perforated duct or the airway are equal and so, the distribution of perforates. In other words, the porosity is same, but perforate impedance they are basically same,

$$\begin{aligned} D_1 &= D_3 \\ \zeta_2 &= \zeta_1 \end{aligned}$$

So, now we have this kind of a thing. So, D_1 and D_1 and D_3 are same and ζ_1 and ζ_2 are same, ok.

Now, with this sort of understanding let us hope on back to the MATLAB. So, L is your overall chamber length and la is the extension neck extension of the inlet and lb is the neck extension of the outlet. So, what is the perforate distance? It is L-(la + l b).

```
function [Tl]=transmission_loss(D1,D2,L,la,lb,k0,
j=sqrt(-1);
c0=343.1382; %%% sound speed...

S1=(pi/4)*(D1^2);    Y1=c0/S1; %%% upstream pipe
S2=(pi/4)*(D1^2);    Y2=c0/S2; %%% downstream pipe
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[Tf] =transfer_matrix(D1,D2,L,la,lb,k0,th,dh,sigm

v=abs(      ((Y2/Y1)^0.5)*(Tf(1,1)+(Tf(1,2)/Y2)+(T
Tl=20*log10(v/2);
```

```
n [Tl]=transmission_loss(D1,D2,L,la,lb,k0,th,dh,s
-1);
1382; %%% sound speed...

4)*(D1^2);    Y1=c0/S1; %%% upstream pipe
4)*(D1^2);    Y2=c0/S2; %%% downstream pipe
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

ransfer_matrix(D1,D2,L,la,lb,k0,th,dh,sigma,mg);

    ((Y2/Y1)^0.5)*(Tf(1,1)+(Tf(1,2)/Y2)+(Tf(2,1)*
og10(v/2);
```

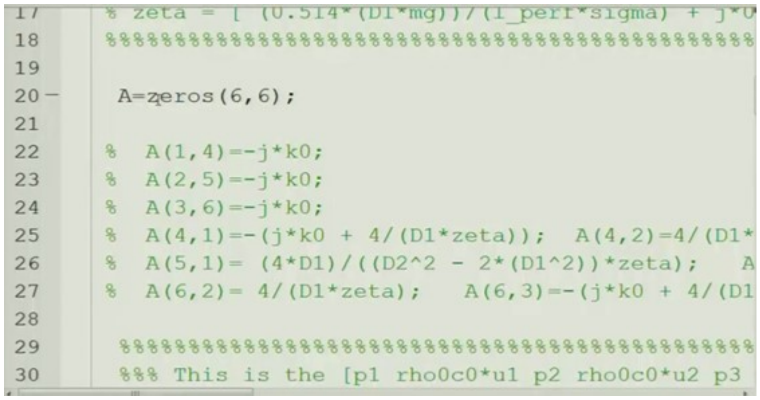
So, here we have that that is the perforate perforated distance, ok. So, we get this sort of a thing. Now, this function in turn call invokes the function in which transmission loss computation actually happens, but then in turn this invokes the heart of the system. That is it passes all the functional parameters, and what it does it calls the main subroutine.

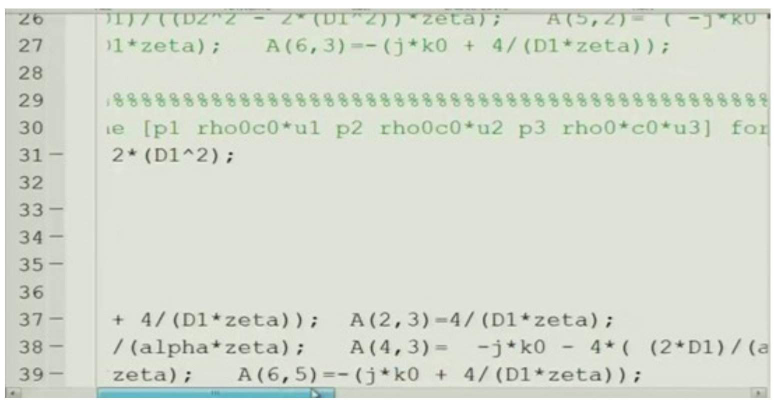
```
n1=size(f); n=n1(1,2);
c0=343.1382;
%%%%
k0=(2*pi*f)/c0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

mg=0;

D2=148.1/1000;  %%% Diameter of the chamber...
D1 =49.3/1000;  %%% Perforated duct diameter (eq
L =141.4/1000;  %%% Chamber length
la=6.4/1000;      %%% Extended-inlet length
lb=6.4/1000;     %%% Extended-oulet length...
l_perf = L -(la+lb);
```

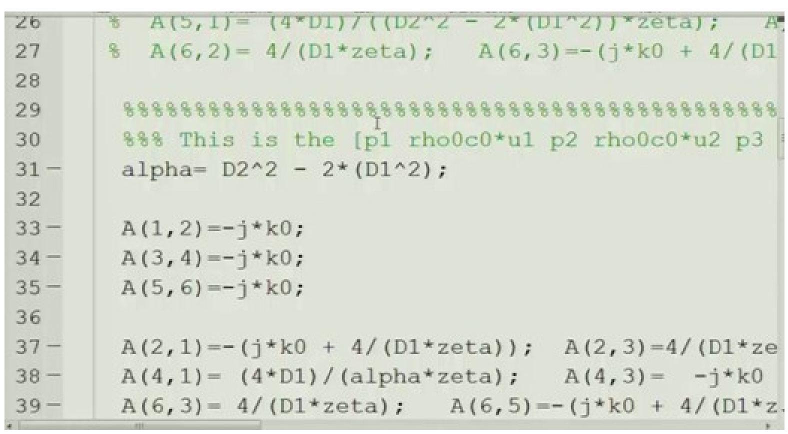
```
%%% D1 is the diameter of the perforated pipe..
Spipe= (pi/4)*D1^2;

% zeta = ( 6*(10^-3) + j*k0*(th + 0.75*dh) )/si
% zeta = ( 6*(10^-3) + j*(4.8*10^-5)*((k0*c0)/(

%   zeta=(7.337*10^-3)*(1+72.23*mg) + j*(2.2245
%   zeta=zeta/sigma;
  mb = mg*(D1/(4*l_perf));
% %
   zeta = perforate_impedance_singlepipe(k0, c0
%
% zeta = [ (0.514*(D1*mg))/(l_perf*sigma) + j*0
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```




)))/sigma;

-5)*(1+51*th)*(1+204*dh)*((k0*c0)/(2*pi));



gma, th, dh, 18.3*(10^-6), 1.21, 0.8, mg, mb);

k0*(th + 0.75*dh)]/sigma;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

So, let us assume what exactly how I assume for the mean flow. I think I have a I have set me grazing flow have taken it to be 0. So, we can continue with this thing. And let me show you how we go about. So, we can you know sort of assume we can still use the expression given by Elnady, expression given by Elnady that is your this thing.

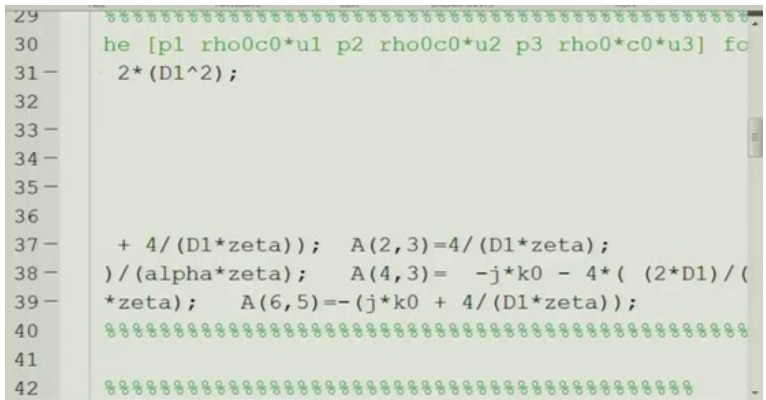


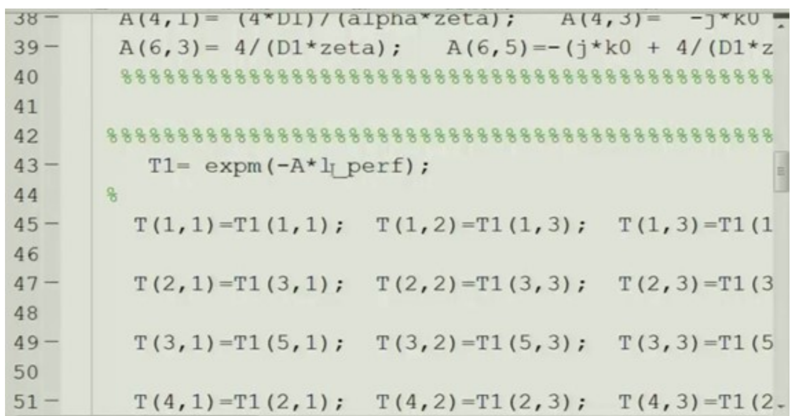




So, bias flow is D_1 by 4 times l perforate the expression that we derive which is approximately sort of valid and these are your you know density values and kinematic viscosity and all that. So, this is a same routine that we call. Now, here is the thing here is a deal A 6 x 6 matrix.

So, we slightly rearrange it in this form p1 $\rho_0 C_0 U_1$ because this is shown to be a bit more stable than the other form. This is again obtained, this experience is obtained only through number of numerical experiments or these things. So, A1 to A3 these are all something that you know, the thing is that this is all what I have written in the slide which I presented.



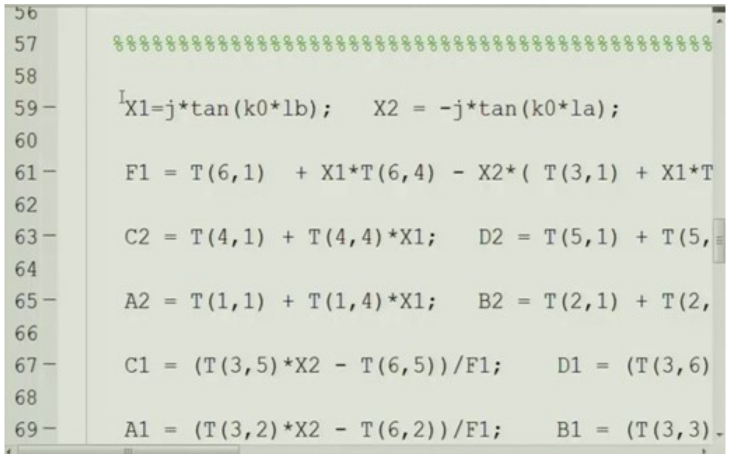


```
%
T(1,1)=T1(1,1);  T(1,2)=T1(1,3);  T(1,3)=T1(1

T(2,1)=T1(3,1);  T(2,2)=T1(3,3);  T(2,3)=T1(3

T(3,1)=T1(5,1);  T(3,2)=T1(5,3);  T(3,3)=T1(5

T(4,1)=T1(2,1);  T(4,2)=T1(2,3);  T(4,3)=T1(2

T(5,1)=T1(4,1);  T(5,2)=T1(4,3);  T(5,3)=T1(4

T(6,1)=T1(6,1);  T(6,2)=T1(6,3);  T(6,3)=T1(6

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
);  T(1,5)=T1(1,4);  T(1,6)=T1(1,6);

);  T(2,5)=T1(3,4);  T(2,6)=T1(3,6);

);  T(3,5)=T1(5,4);  T(3,6)=T1(5,6);

);  T(4,5)=T1(2,4);  T(4,6)=T1(2,6);

);  T(5,5)=T1(4,4);  T(5,6)=T1(4,6);

);  T(6,5)=T1(6,4);  T(6,6)=T1(6,6);

%%%%%%%%%%%
```

Now, this when, but however, if you put recast the equations in this particular form p_1 $\rho_0 C_0 U_1$ p_2 $\rho_0 C_0 U_2$ and so on, you we will get this sort of a thing. And T1 is exponential times A into l perforates, ok. And then, we again rearrange back by doing this operation we are is a bit.

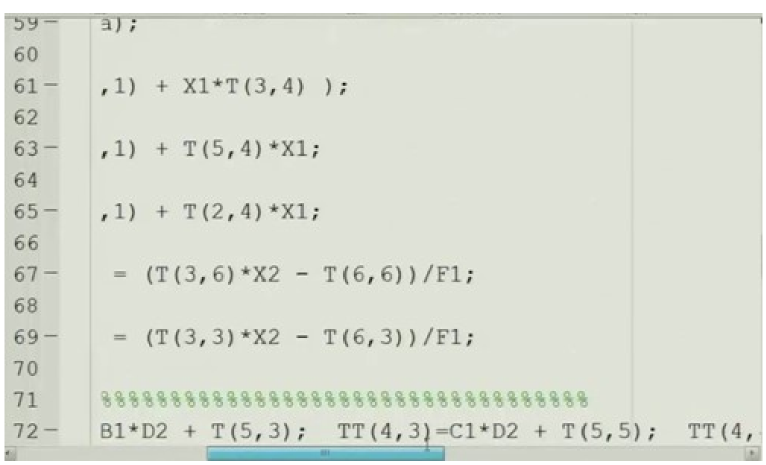
It requires a lot of coding, it requires a lot of patience. So, you know coding in my opinion is a very humbling experience, you will make a lot of mistakes, but do not get disheartened by that. You might have to spend hours to debug your code. I would encourage all of you to write your own codes by a simple expansion chamber.



And gradually moving onto you know step by step move on to more complicated muffler configurations. And then, when you do that you get you have to you know your T matrix is integrated from 0 to l. Now is the time when you apply your boundary conditions. So, here we have we have defined you know like I said I was saying these boundary conditions are quite messy. They are not that trivial. Just that they are very tedious. So, we have to define a few variables with a view to finally, get a transfer matrix.

So, you know the detailed derivation you can find in the third chapter of the book on ducts and mufflers by Professor Munjal, but again I am presenting it here. So, here we have X_1, this these there are something that you can do it on your own also. The expressions are given in the book you can derive on your own also.

Just follow the road map and you will get it. So, here we are defining X_1 and X_2 these are your cavity at the because of, if you said *l*a and *l*b 0 you get back your fully perforated duct, ok. So, F1, C2, A2 all these expressions are kind of interrelated.

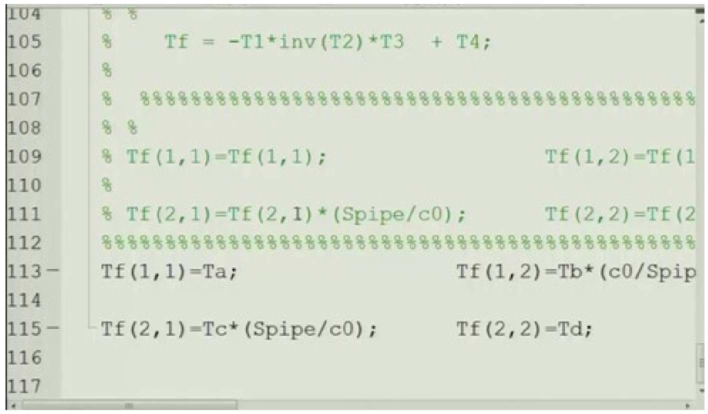


```
A1 = (T(3,2)*X2 - T(6,2))/F1;    B1 = (T(3,3)*

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
TT(4,1) = A1*D2 + T(5,2);  TT(4,2)=B1*D2 + T(5

TT(3,1) = A1*C2 + T(4,2);  TT(3,2)=B1*C2 + T(4

TT(2,1) = A1*B2 + T(2,2);  TT(2,2)=B1*B2 + T(2

TT(1,1) = A1*A2 + T(1,2);  TT(1,2)=B1*A2 + T(1

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

They have to be, you cannot make a mistake in bookkeeping. Finally, your TT matrix which basically relates the p_1, the state variable at z = 0 section in the duct 1 and the annular duct 2 with those at z = l section in the annular duct 2 and the chamber at z =2 and the outlet duct at z = l.

```
TT(1,1) = A1*A2 + T(1,2);  TT(1,2)=B1*A2 + T(

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

F2 = TT(4,1) + X1*TT(4,3) - X2*TT(2,1) - X1*X

D3 = TT(3,1) + TT(3,3)*X1;   C3 = TT(1,1) + (

Ta = TT(1,2) + (A3*C3);  Tb = TT(1,4) + (B3*C

Tc = TT(3,2) + (A3*D3);  Td = TT(3,4) + (B3*D
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

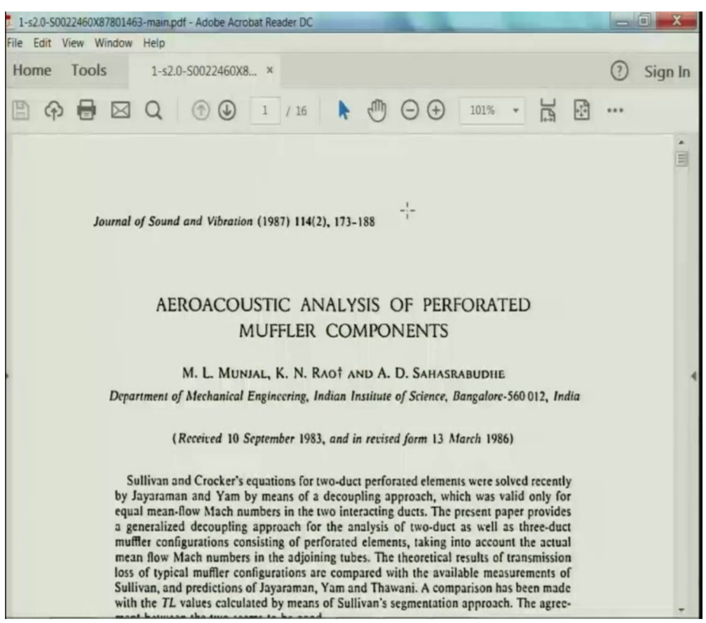
So, with all these boundary conditions will give you these TT matrix terms. And then you will have your parameters defined in terms of the TT matrix and number of other parameters. So, I have purposely committed out this part and finally, you know this Ta, main thing is your Ta, Tb, Tc, Td.



So, these are your full pool parameters and then you just put it back in p form rather than p $\rho_0 C_0 U_0$ for. It is appearing very simple, but when you code it you will realize that each term has its own importance and has to be done quite carefully.

So, with this I think we are good to go. And let us generate some curves for you know some configurations, and particularly demonstrate the effect of you know mean flow or a fully perforated muffler. So, just by just by setting this thing you know we can do a lot of things. So, let me run a few example.

So, this is one example that I thought of taking from the paper by published long time back by Munjal and Rao and Sahasrabudhe. So, very famous paper on air acoustic analysis of perforated components published in Journal of Sound and Vibration. So, these are the parameters that I have taken from there.



Journal of Sound and Vibration (1987) 114(2), 173-188

AEROACOUSTIC ANALYSIS OF PERFORATED MUFFLER COMPONENTS

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(*Received* 10 *September* 1983, *and in revised form* 13 *March* 1986)

Sullivan and Crocker's equations for two-duct perforated elements were solved recently by Jayaraman and Yam by means of a decoupling approach, which was valid only for equal mean-flow Mach numbers in the two interacting ducts. The present paper provides a generalized decoupling approach for the analysis of two-duct as well as three-duct muffler configurations consisting of perforated elements, taking into account the actual mean flow Mach numbers in the adjoining tubes. The theoretical results of transmission loss of typical muffler configurations are compared with the available measurements of Sullivan, and predictions of Jayaraman, Yam and Thawani. A comparison has been made with the *TL* values calculated by means of Sullivan's segmentation approach. The agree-

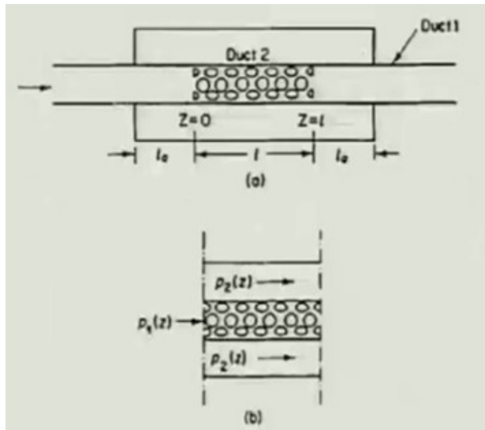


Figure: Partially perforated resonator configuration. (a) Partially perforated resonator (b) concentric perforated section.

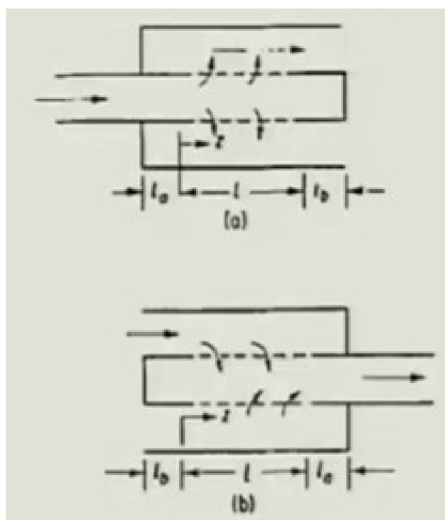


Figure: Cross-flow element (a) Cross-flow expansion (b) Cross-flow construction element

This is the paper that I was talking about. So, what they have and this is a well cited paper, very nice paper. So, they have analyzed different 2 duct configuration, and 3 duct configuration you know with cross flow elements like this one of the grazing flow element like this one and you know plug muffler.

We will we will do all this in today's a class. Let us let us first analyze this one. So, this is called a cross flow configuration because flow really comes here and it has to go through the pipe and go through this pipe and otherwise the flow is coming here it crosses over the pipe and goes through this pipe, ok.

So, then, so when it does so, it definitely there is a, it experiences much more back pressure than the straight through one, but still it is not that the back pressure problem is not that much as compared to the case where you have you know and if you completely leave it open and did or something like that.

Because of perforates and you know and specially, because the flow has multiple has this these paths, so this would allow you different this will basically guide the flow. So, the back pressure although more than the bit more than the straight through element. It will not be, it can still be tolerable in the context of automotive exhaust systems.

So, let us consider the dimensions given here. Basically, 49.3 mm, 148.1 mm or roughly 149 and the other diameters are same as the d 1. Length is about perforate sections about 129 mm and extension is about only very small extension 6.4 mm and 3.9 percent is the small porosity. So, they have considered a very less porous or you know nearly a solid section, but still it can do wonders. It of course, this is a completely a section with almost 0 porosity; obviously, I will not have any transmission loss.

So, the flow cannot go through, this the waves cannot go. So, you have to have need to have certain nonzero value. So, they are chosen a duct with a low very low porosity, that is ok.

So, and the whole diameter is something that I have chosen. What we need to do is we have already put in the parameters for you guys to for demonstration. And I just need to run the code. But before I do, so let us have a look at the transfer matrix here. So, I am using the Elnady's expression these guys this expression is rather modern one and the one presented in the paper this paper is in present was published in 1987, so ages back. They used an different, an altogether different perforate impedance expression. It is not unique like I said by the brief review an attempt to a review that I try to do a few classes back. So, the idea was to demonstrate a few popular expressions. So, these are the two ones,

$$\zeta = (6 \times 10^{-3} + i4.8 \times 10^{-3}\, f)/\sigma$$

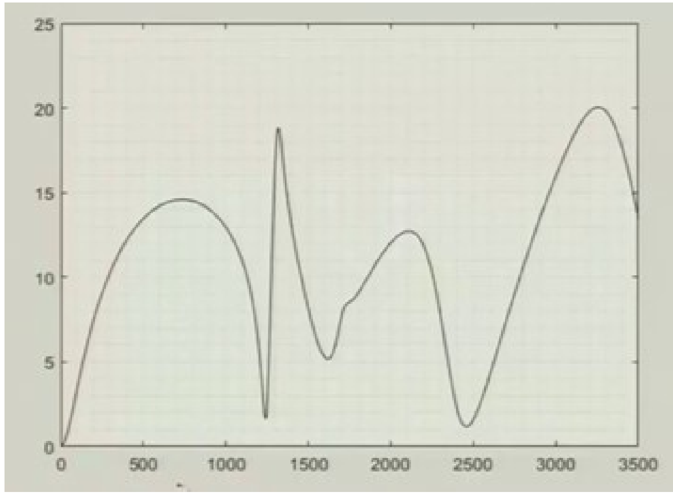
$$\zeta = (0.514 dM/l\sigma + i4.8 \times 10^{-3}\, f)/\sigma$$

So, this is the sort of thing. These are the ones now ah. So, what we need to do now is that instead of using this we will use some let us set the grazing flow to 0 only and let us generate some curves using Elnady's expression where you have this sort of a thing. So, we just said mb bias logarithm by this one and you just plug out this guy.

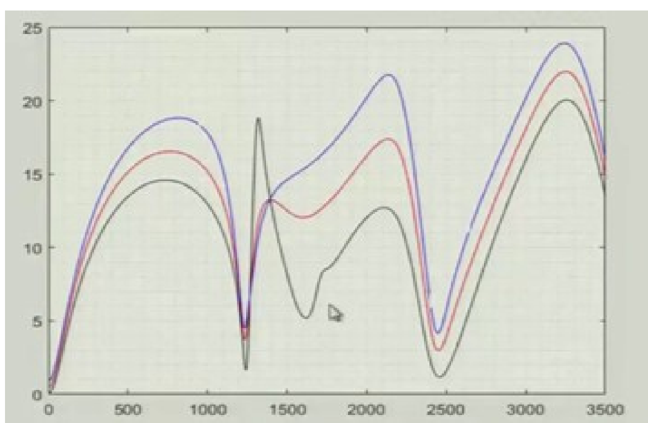
```
%   zeta=(7.337*10^-3)*(1+72.23*mg) + j*(2.2245
%   zeta=zeta/sigma;
  mb = mg*(D1/(4*l_perf));
% %
   zeta = perforate_impedance_singlepipe(k0, c0
%
% zeta = [ (0.514*(D1*mg))/(l_perf*sigma) + j*0
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

 A=zeros(6,6);

%  A(1,4)=-j*k0;
%  A(2,5)=-j*k0;
```

```
*dh)*((k0*c0)/(2*pi));


0^-6), 1.21, 0.8, mg, mb);

igma;
%%%%%%%%%
```

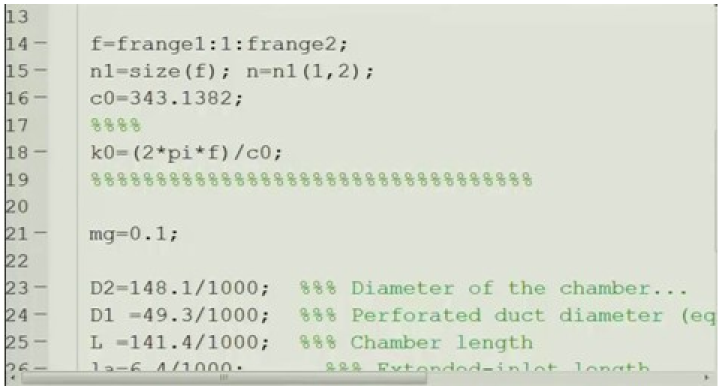
```
Command Window
New to MATLAB? See resources for Getting Started.
>> transmission_loss_plot('k')
Elapsed time is 8.838694 seconds.
>> hold on
>> transmission_loss_plot('r')
Elapsed time is 1.232194 seconds.
>> transmission_loss_plot('k')
Elapsed time is 2.585939 seconds.
>> hold on
>> transmission_loss_plot('r')
Elapsed time is 1.842453 seconds.
>> hold on
>> transmission_loss_plot('g')
```



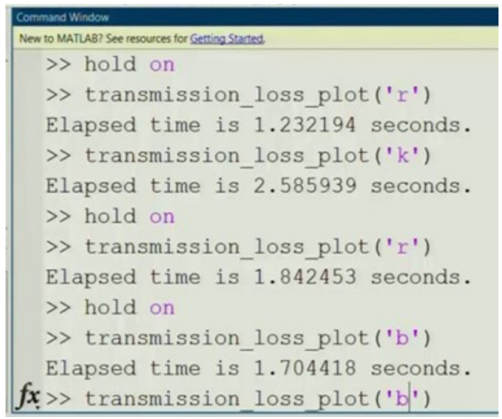

```
f=frange1:1:frange2;
n1=size(f); n=n1(1,2);
c0=343.1382;
%%%%
k0=(2*pi*f)/c0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

mg=0.05;

D2=148.1/1000;  %%% Diameter of the chamber...
D1 =49.3/1000;  %%% Perforated duct diameter (eq
L =141.4/1000;  %%% Chamber length
```



So, we get this kind of a curve. And we will see Munjal; and others are presented it for nonzero mean flow as much as 0.2. So, we will soon get there. And let us see even a what a small difference in the flow field can sort of do. So, definitely does lift the things. So, I do hold on and use another colour, g, green.



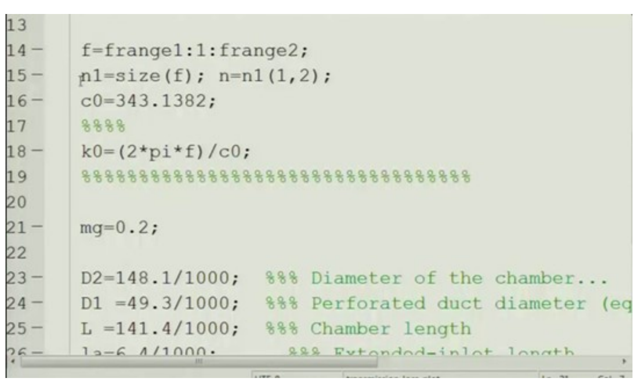
```
f=frange1:1:frange2;
n1=size(f); n=n1(1,2);
c0=343.1382;
%%%%
k0=(2*pi*f)/c0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

mg=0.1;

D2=148.1/1000;  %%% Diameter of the chamber...
D1 =49.3/1000;  %%% Perforated duct diameter (eq
L =141.4/1000;  %%% Chamber length
la=6.4/1000;     %%% Extended-inlet length
```



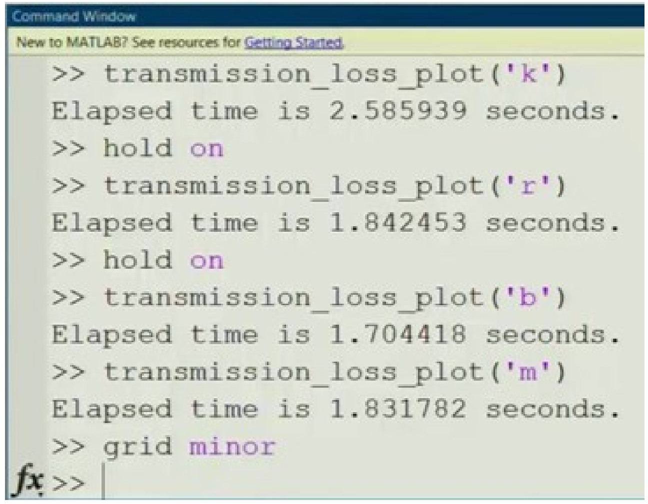
```
Command Window
New to MATLAB? See resources for Getting Started.
>> hold on
>> transmission_loss_plot('r')
Elapsed time is 1.232194 seconds.
>> transmission_loss_plot('k')
Elapsed time is 2.585939 seconds.
>> hold on
>> transmission_loss_plot('r')
Elapsed time is 1.842453 seconds.
>> hold on
>> transmission_loss_plot('b')
Elapsed time is 1.704418 seconds.
fx >> transmission_loss_plot('b')
```

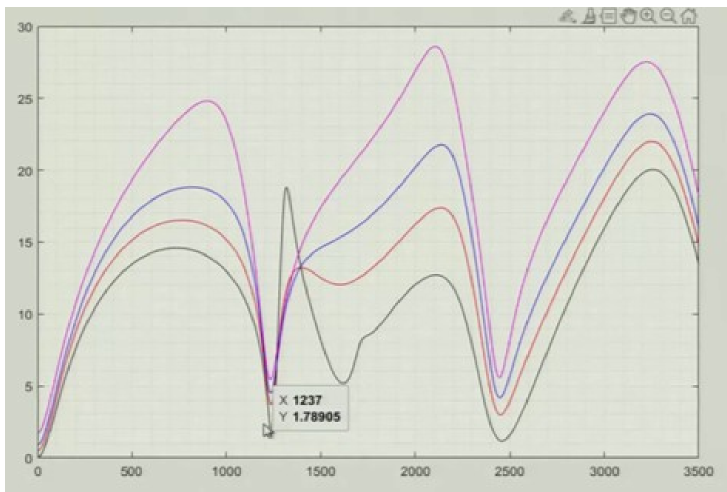
And put in another value for the flow field like about 0.1 or so. Let us see what it does. Green might be difficult to see, so let us see use blue. So, it gives you this kind of a thing, ok ah. Well, I must say that plane wave may not be valid for such a large frequency, rigid might just break down here.

But for sake of completeness we will do that. So, there are obviously, much more advanced topics in muffler acoustics. It is a huge field in itself, like you know 3-dimensional numerical mode matching of perforate elements ah. It is very interesting topic. But I do not think so we can cover it in course in this course.



```
f=frange1:1:frange2;
n1=size(f); n=n1(1,2);
c0=343.1382;
%%%%
k0=(2*pi*f)/c0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

mg=0.2;

D2=148.1/1000;  %%% Diameter of the chamber...
D1 =49.3/1000;  %%% Perforated duct diameter (eq
L =141.4/1000;  %%% Chamber length
la=6.4/1000;     %%% Extended-inlet length
```





May be in more advanced courses perhaps in future. But for now we have to be contained with plane waves, that is the world. And of course, a glimpse of some 3D analysis using analytical things later on. So, then we do mean magenta colour 4.2 Mach number. So, you know see the dramatic effect the flow has.

So, we will do grid minor and have a look at the transmission loss graphs, ok. Let us see how they look. Well, using Elnady's expression this is this is what we sort of get. Transmission loss always starts from 0. The fact that it is still at 5 hertz, at 5 hertz frequency it is still about 1.84. It is you know Elnady's expression is tends to, you know slightly well the modern expressions tend to slightly over the transmission loss even at very low frequencies, but of course, we need to do a more thorough you know maybe experimental corroboration of all these things.

But nevertheless one thing is clear using the modern expressions where you know mean flow and mean bias flow and mean grazing flow occur simultaneously and these expressions can readily take care of that. We will also use the previous the expressions known before and see how the shape of the curve changes.

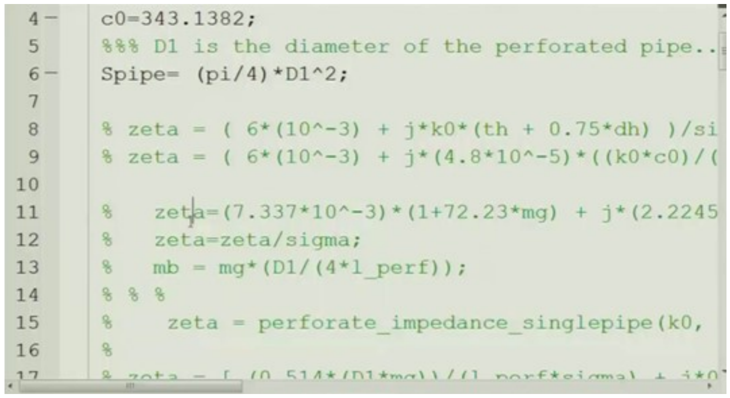
What we however notice that mean flow definitely tends to increase these dooms at least for the cross flow expansion chambers. And you know black one was for 0 flow. So, with the with nonzero flow these peaks tend to come down, but then these troughs which are more notorious more problematic, they are significantly lifted.

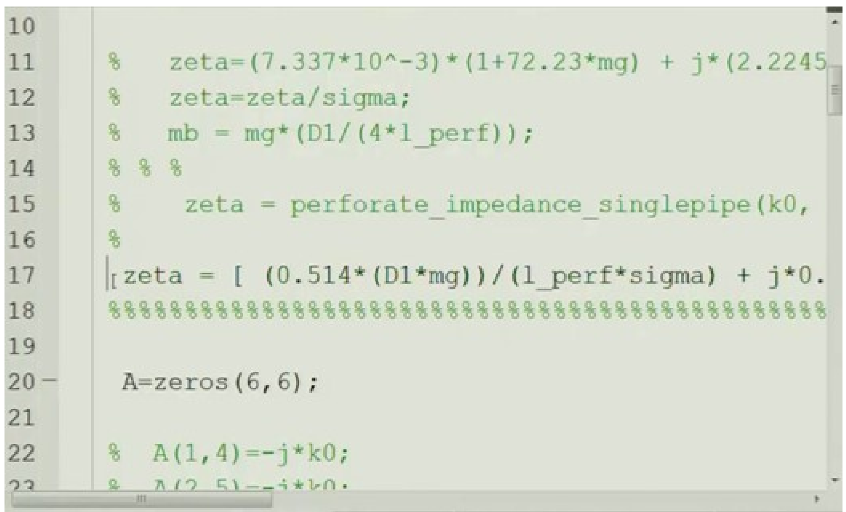
For 0.2, it is almost about you know 5.63 d v, 5.6 dv and this one was only about one and half dv, 1.7 dv or so. So, it is basically lifted the troughs which is more important than you know the peaks coming down and these peaks are obviously, going up. So, you know in in effect it is basically you know the mean flow always tends to lift the trough and the same time it tends to lower the peaks.

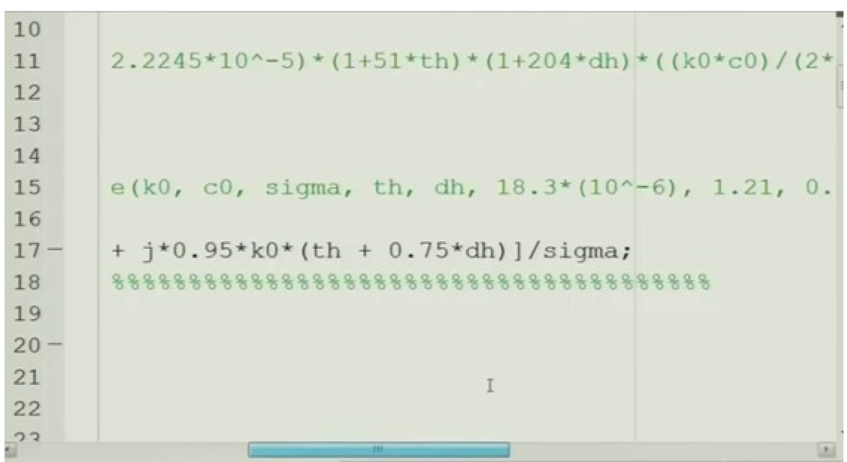
So, this effect is known. So, notice one thing that we have completely ignored the convective effects of mean flow that will be sort of quite tedious to incorporate if you have to need to have B inverse A matrix and do all those sort of things and get approximate expressions for m2 within the cavity.

And it is going to be very tedious. So, that would have just changed the dv value by a few db here and there. So, we will be contented with using ignoring the convective effects you know and just sort of focusing on these values, ok. Now, let us see what perforate impedance expression for we change it for a mean flow, what it does really.

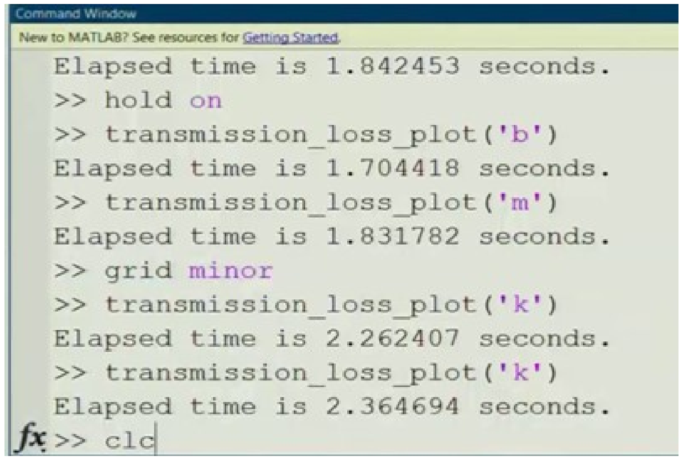
```
D1 =49.3/1000;   %%% Perforated duct diameter (eq
L =141.4/1000;   %%% Chamber length
la=6.4/1000;       %%% Extended-inlet length
lb=6.4/1000;      %%% Extended-oulet length...
l_perf = L -(la+lb);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for i=1:n
Tl(i)=transmission_loss(D1,D2,l_perf,la,lb,k0(i)
end
figure(2)
plot(f,Tl,ch)
grid minor
toc
```

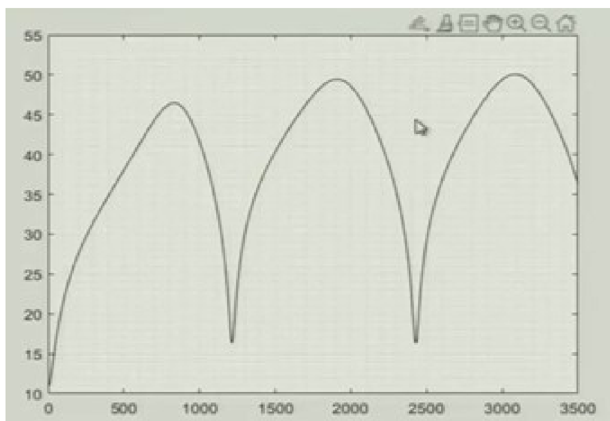


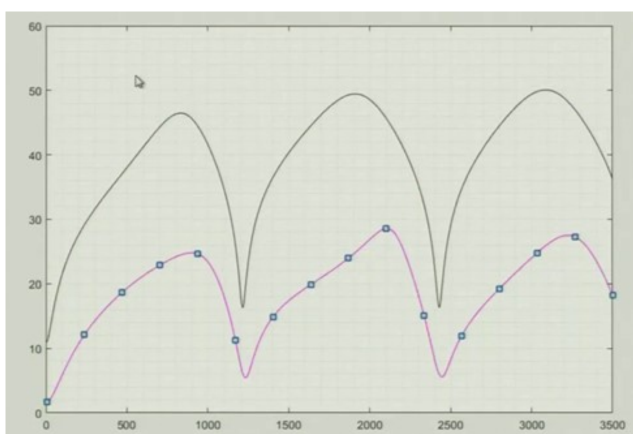




If we just take 0.2 and let us say you plot in a different terms sort of figure and use the expressions that available you know its 2021 now. It is about you know 30 35 years back or so, or maybe more perhaps 35 years, 40 years back whatever it is ah. You know those expressions those classical expressions, if we just change these values let us comment out these things. So, D1 is the diameter of the duct, mg is the grazing flow, Mach number and these are the things ah. So, let us see what do we get.







We get for this one much more. Now, if you go to simply pick up the curve here for this guy and place it here, heaps of difference. There is a lot of difference although the shape remains intact, but difference in perforate impedance expression can completely give you different you know attenuation values.

So, which one is correct? Well, I am not quite sure here to be honest . I mean, one thing is for sure that we need to use the more modern expressions because they have been sort of they have been obtained by much more accurate mathematical models and they have been corroborated quite well experimentally. So, I would say that you know, I would sort of trust the predictions based on modern expressions for perforate impedance more at least for a cross low configurations when mean flow is present.

You know we show if you recall your if you play back your last lectures at your leisure time you will see that you know we used a variety of different perforate impedance expressions and we did not quite see much of a difference, even when the flow was about 0.15, 0.15 or something like that. We did not quite see much of a difference. So, but for cross flow configurations it is its quite important. Because here now the you have a nonzero bias flow that is very important.

And you know that will definitely add a lot of damping, you know lot of attenuation. So, you know we are seeing a number of things there. So, it definitely does impact, but the black colored one curve is what Sahasrabudhe, Munjal and Rao all these guys reported in their paper and that sort of a thing. But these are the more modern predictions.

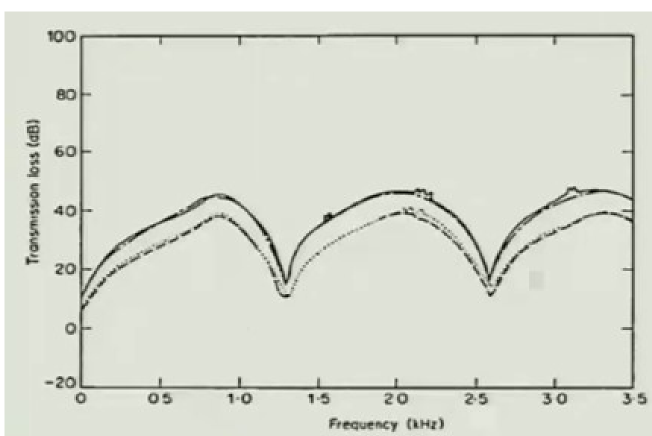
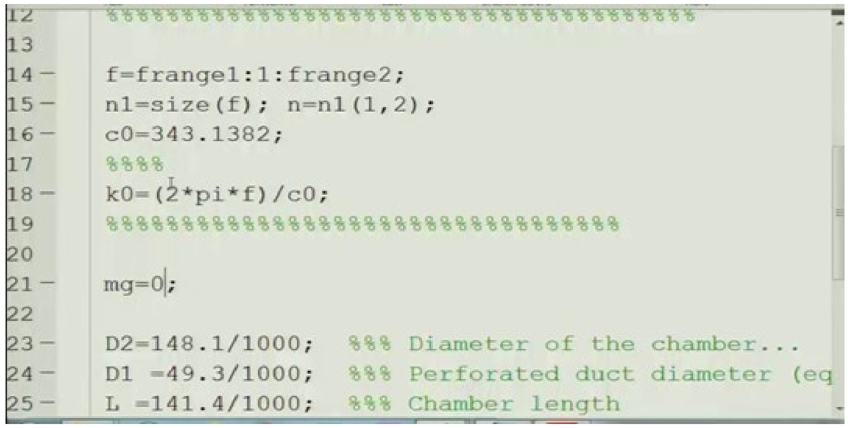
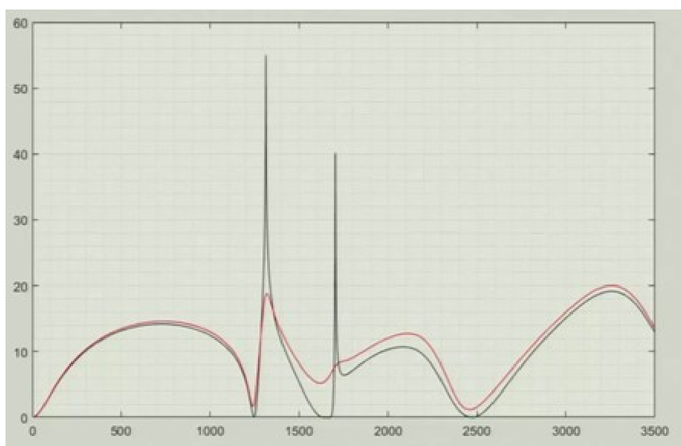


Figure: Transmission loss for the tree-duct cross-flow element of figure 4a --- Prediction by segmentation approach (16 segments). M = 0.1; …, prediction by distributed parameter approach, M = 0.2; -



```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

f=frange1:1:frange2;
n1=size(f); n=n1(1,2);
c0=343.1382;
%%%%
k0=(2*pi*f)/c0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

mg=0;

D2=148.1/1000;  %%% Diameter of the chamber...
D1 =49.3/1000;  %%% Perforated duct diameter (eq
L =141.4/1000;  %%% Chamber length
```

So, I would, I would probably go with the modern one and would urge you also to use the more, I mean more accurate expressions for impedance. So, this I mean, and there is one more thing that I could sort of point out here even for a cross slow configuration and that is your if we said the grazing flow to 0 and put and you know sort of compute it for the using the classical expressions I written. So, this is what we get really.

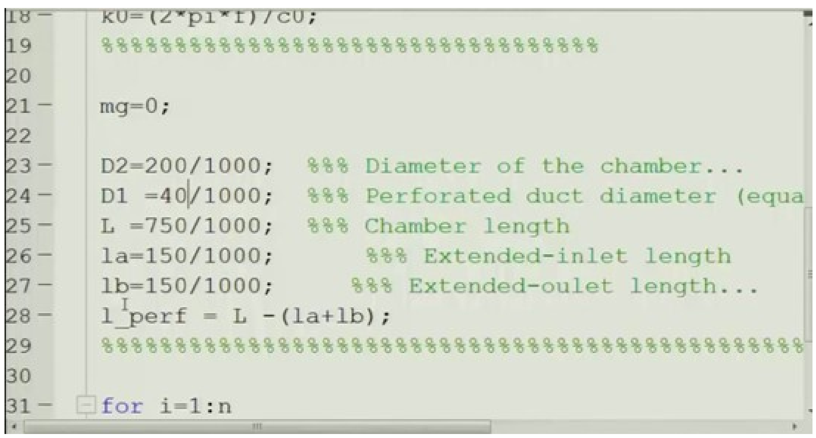


But and if we take this curve maybe you like to use sort of another colour, this was obtained using Elnady's expression. And let us change the colour. So, you get this kind of a curve a red coloured curve.

But you know the point I am trying to make is that these black and red colour curves are nearly the same, in the low frequency and there is qualitatively they remain the same you know as we go in the higher frequency range. And of course, there is a limit up to which plane waves are there.

So, and then deviations will happen as the mean flow becomes approaches a nonzero value. So, there is a lot of interesting things that one can do. You know there is lot of room for, lot of research show, lot of people can take up research projects in this in this area of muffler acoustics because perforates, you know analysis of perforated components are really the a very important topic. And there is still a lot of things that are not quite analyzed.

So, this is an interesting opportunity you know to take up new projects maybe masters project or doctorate investigations in such fields ah. And it is practically very important field you know of one of the important areas with a noise control engineering. So, basically with this thing I will close of this figure and probably try to analyze just one more cross flow expansion chamber configuration where you will find some totally different sort of a transmission loss curve.

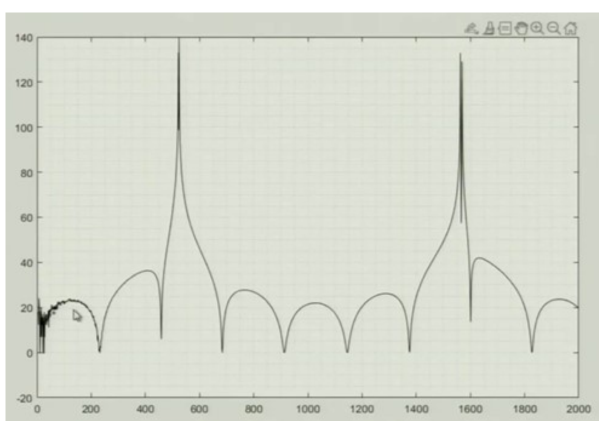


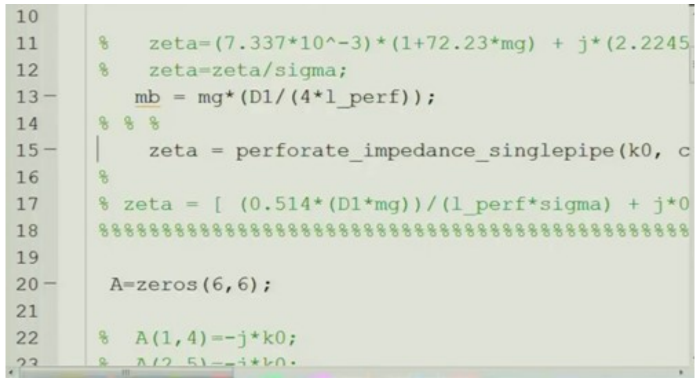
So, basically, we need to change a few parameters. And what we are considering now is cross flow expansion chamber with slightly different dimensions and with more extension. This is just to bring out some qualitative kind of a kind of a difference. So, let us say this is 200, 200 mm, ok. And your length is about, well length of the chamber is about 750 mm, ok. This is about 150 mm, 150 mm and this diameter is about 40 mm or the perforated duct. So, this is effectively a perforate section is about 450 mm, ok.

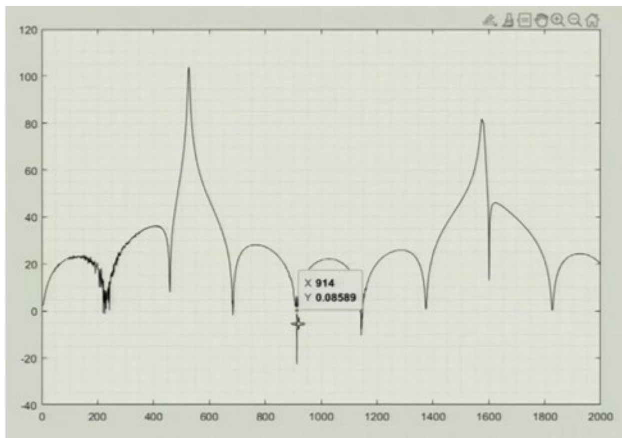
```
function [] =transmission_loss_plot(ch)
tic

frange1=5;
frange2=2000;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
sigma=20;   %%% Porosity...
sigma=sigma/100;

th=2/1000;
dh=3/1000;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

f=frange1:1:frange2;
```

No mean flow is considered. The idea was to bring some substantial la neck extension values compared to the perforated length and see what sort of a curve you are getting. And let us take some realistic value. Well, highly porous duct I would say. Let us say 20 percent, ok and we may not go to such a large frequency because plane wave is really would not be valid anyways for this thing.

```
Command Window
New to MATLAB? See resources for Getting Started.
>> transmission_loss_plot('k')
Elapsed time is 1.609604 seconds.
>> transmission_loss_plot('k')
Elapsed time is 1.665169 seconds.
>> transmission_loss_plot('k')
Elapsed time is 1.507316 seconds.
>> transmission_loss_plot('k')
Elapsed time is 1.318123 seconds.
>> transmission_loss_plot('k')
Elapsed time is 1.221527 seconds.
fx >> clc
```





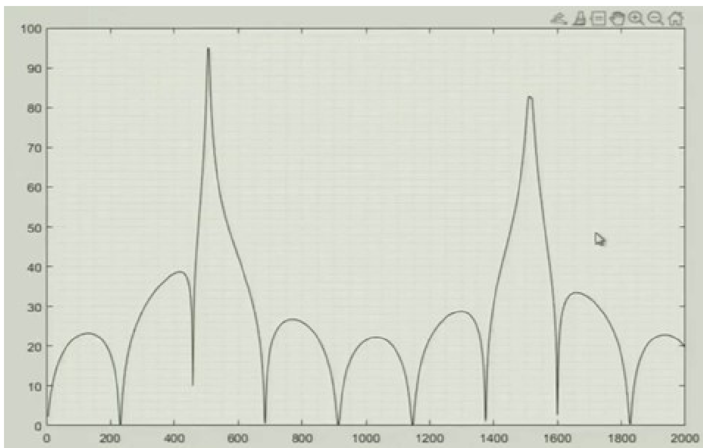


So, I will clear out this guy and plot it. So, there is some instabilities that are happening here. So, let us see, I hope we do not get such a problem for this thing. We have changed the expression. Let us see are we getting the same thing. Well, it seems we are. Well, slightly shifted. So, you know again these are all artifacts of numerical error or instabilities. Transmission loss can never be negative. At least for a passive system which does not have energy generation inside the system transmission loss is always positive.

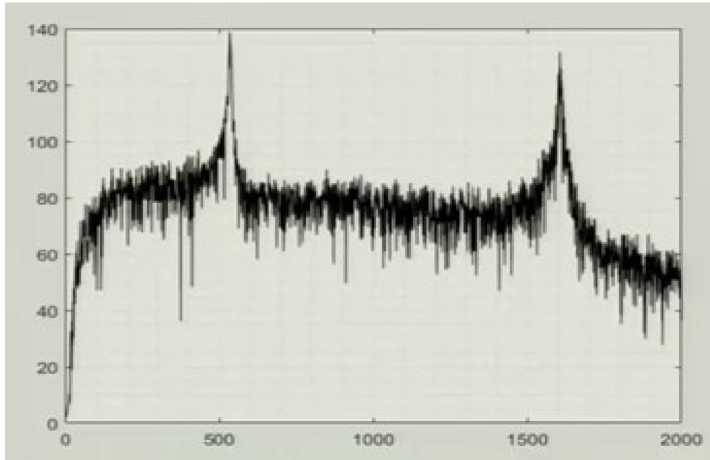
Why? Because it is a ratio, logarithm of the ratio or the energy or the acoustic pardon sorry incident to that transpired downstream of an termination. So, incident thing is always incident acoustic power is always greater than the power that is transpired and that can happen only when you when you have a passive system, not an active system.

When active system you might have some additional sources. So, specially, if there is a flow separation going on and of course, we are not considering that in our mathematical modeling flow generated noise, and anyways there hopefully that would not be there because of perforate guides the flow as I have been repeatedly telling. So, this basically the idea that these guys cannot be negative, ok. They cannot really be negative. They have to be positive these are all artifacts of numerical instabilities.

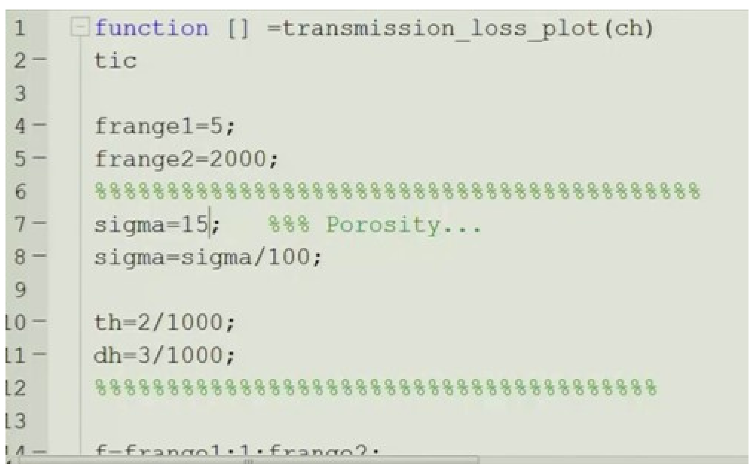
```
function [] =transmission_loss_plot(ch)
tic

frange1=5;
frange2=2000;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
sigma=10;    %%% Porosity...
sigma=sigma/100;

th=2/1000;
dh=3/1000;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

f=frange1:1:frange2;
```



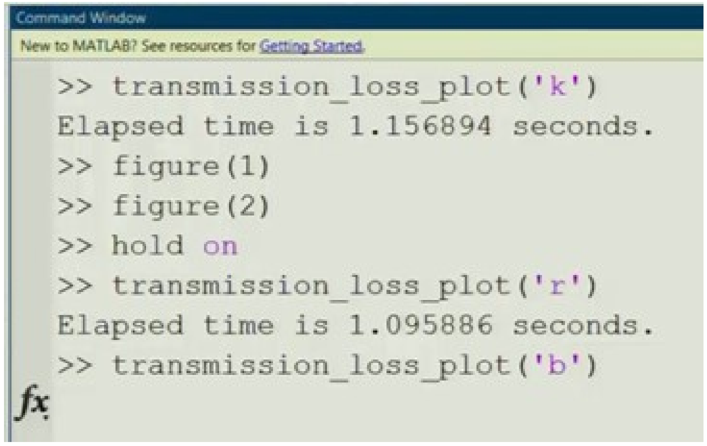

```
function [] =transmission_loss_plot(ch)
tic

frange1=5;
frange2=2000;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
sigma=30;    %%% Porosity...
sigma=sigma/100;

th=2/1000;
dh=3/1000;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

f=frange1:1:frange2;
```

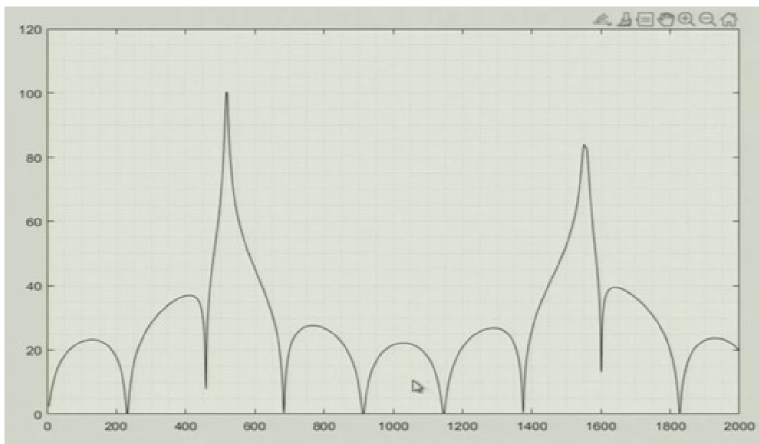


And you see you know if you change the porosity value, you might end up with slightly better thing. Maybe if we consider only 10 percent porosity, let us see what happens. For high enough porosity always also tends to, yeah look it is much better. So, if we use like a 30 percent porosity thing in here something like this thing you will see a lot. So, this is an issue although MATLAB is very advanced, expm it is like you see it is its garbage, it is pretty garbage.

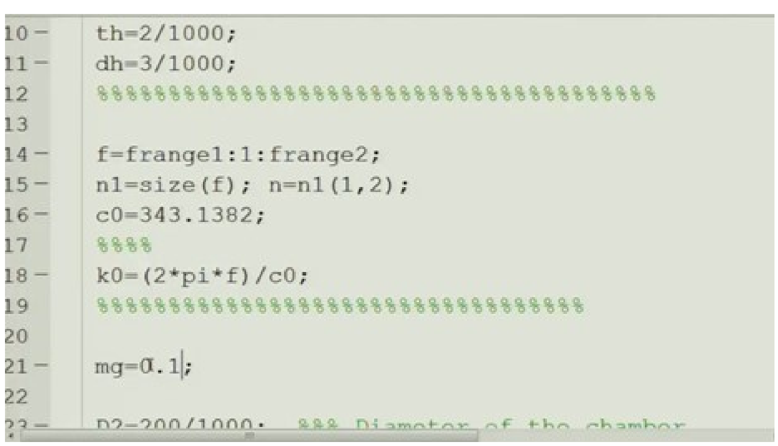


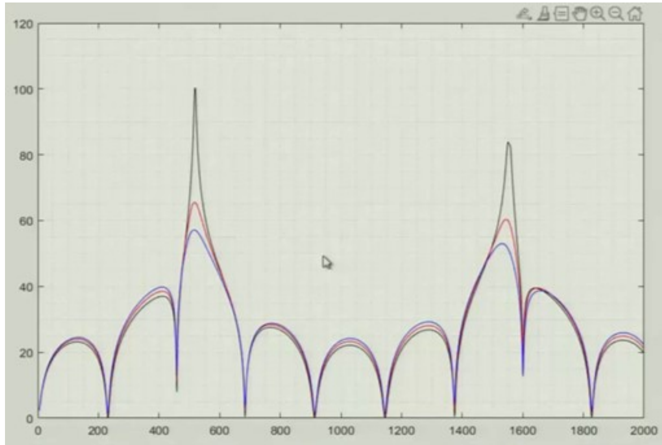
It is completely kind of unstable. So, you know these are all numerical instabilities at works ah. So, this is another challenge. So, these are some of the things I wanted to purposely demonstrate in front of you guys. So, I would say let us go with the nominal values.



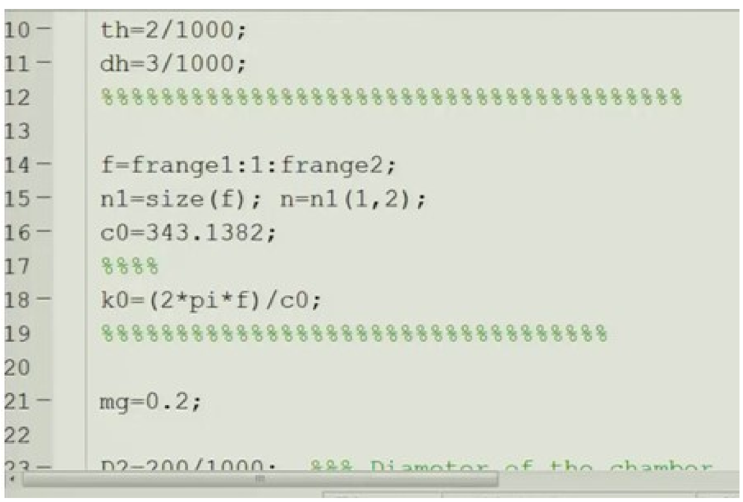


Let us say you know 15 percent porosity. Hopefully you should be able to demonstrate suitable things. It still 15 percent porosity still means you know quite porous duct you know. So, it is about something like this, ok something like this. So, we get peaks here and these are really it happening due to your neck extensions, ok.





Now, if I were to sort of increase the grazing floating, let us say we take 0.1 and let us say we do figure 1, which popes up I am sorry figure 2 is what I called. If they hold on. So, it plots next time it plots, it plots on this graph, ok. So, the peaks are started to come down, troughs are slightly lifted.



The same story being repeated again and again. So, let us do with the maximum value that you typically encounter and automotive exhaust. Well, the effect is more sort of prominent, is not it. So, we get these wonderful nice results and apart from the occasional mishap that happened because of instability.

So, you know 15 percent porosity, 20 percent porosity still very highly porous section only, but typically suppose if we have 30 percent or more porosity it is almost like acoustically transparent. But in the process what happens you know your perforated impedance expression tends to become because you have your zeta.

So, zeta has its dependent upon porosity you know in the denominator. So, typically you have you might end up with the exponentially large numbers when you know you use the function expm. So, it goes beyond the machine precision and it is kind of failing there. So, those are definitely problematic things beyond the machine precision. But the point I am trying to make here is that even 15, 20 percent porosity value if you are obtaining a decant smooth stable curve.

So, you should be happy with that and with the understanding that if it, if it vary like 30 percent, 40 percent porosity although you are not able to calculate this due to instabilities numerical round of errors and all that. The curve would eventually become you know would be very similar to these curves, because you see highly porous section is nearly equivalent you know the results would converge as the porosity tends to 100 percent because it is like an open ended section there is virtually nothing there.

So, it is only because of machine problems round of errors and all that you are not able to calculate beyond the certain value, but the curves would approach that of a open ended thing. That is the physical explanation that we can give, that if it is the perforate is not at all existing for such high porosity, but in physical reality it is there, ok.

So, you should be happy with until instability happens and then go along those lines of predicting. Now, what we will do is that we will analyze another configuration which is called reverse flow configuration. So, you need to bear with me for a minute before I get on with that configuration.

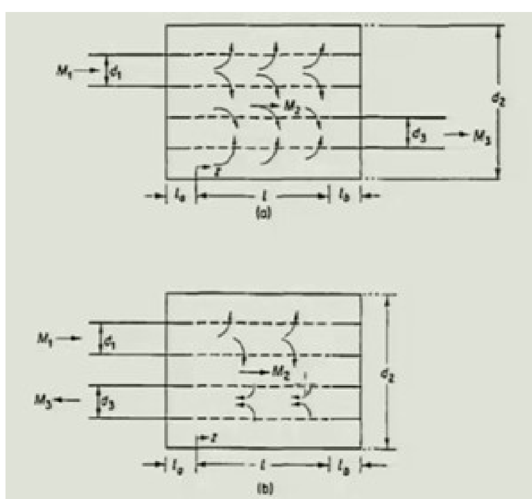
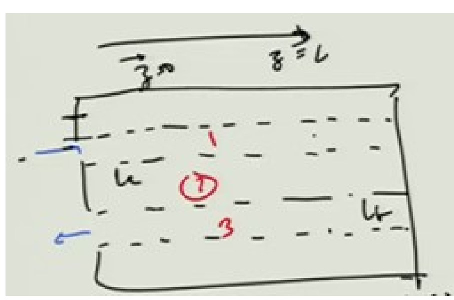


Figure: Three-duct muffler components. (a) Cross-flow expansion chamber; (b) reverse flow chamber. $d_1 = 0.0493m, d_2 = 0.1481m. d_3 = 0.0493m. l = 0.1286m. l_a = l_b = 0.0064m, \sigma = 3.9\%$.

This configuration is shown in this paper also given in the book ducts and mufflers by Munjal. So, this flow goes here and comes here the only thing that we have now is that we have a rigid bond condition, rigid boundary condition here as well as here. So, for simplicity what we will do, we will consider a completely perforated duct that is with no extensions and you know let us make some suitable changes in our codes and you know analyze such a system.

So, far we have considered a cross low expansion chamber. But you know if you recall we also have another configuration where we just slightly change the boundary conditions. So, instead of the flow going somewhere like this we it reverses. So, such a configuration is called a cross low reverse expansion chamber. You know, so the boundary conditions here let me just very quickly draw this configuration here and for simplicity. Let us well we could still take partially perforated duct something like this, ok.

And we could do the following. And here we have another partially perforated duct, and it is a solid thing in here, and it goes here like this, ok. So, the flow goes in here and it is like this. So, you know at this end we let us consider this as z = 0 and this as z = L, ok. So, this is what it is. At z = 0 we have you know the following boundary conditions which are relevant to this particular configuration. So, z 2, z 2 that is basically this is your duct 2, 1, and 3. So, this is the rigid wall boundary condition here. And similarly at this is la and actually there should be a minus sign.



$$Z_2(0) = -j\rho_0 C_0 cot k_0 la$$

$$Z_2(L) = -j\rho_0 C_0 cotk_0 lb$$

Similarly, we can apply the appropriate boundary conditions here. Here will be it will be something

$$\left.\frac{p_1}{U_1}\right|_L = -\rho_0 C_0 cotk_0 la$$

$$\left.\frac{p_3}{U_3}\right|_L = -\rho_0 C_0 cotk_0 lb$$

And here we can similarly apply the appropriate boundary conditions. There will be a 4 boundary conditions 1, 2, 3, 4, what I have already sort of mentioned. And here at this point we do not have any boundary conditions this is just the inflow condition, outflow condition, at this these are all rigid plate. So, now, basically what is happened is that you can have a look at the text by Munjal for those boundary conditions is the simplification of that are rather tedious.

So, what we will do, what I suggest is that a simple configuration of these fully perforated duct. So, here you know all these terms will be will be going to infinity here. So, you know there are lot of simplifications that one can sort of do.

$$\begin{Bmatrix} p_1 \\ \cdot \\ p_3 \\ \rho_0 C_0 U_1 \\ \cdot \\ \rho_0 C_0 U_3 \end{Bmatrix}_{z=0} = [T] \begin{Bmatrix} \\ \\ \end{Bmatrix}_{z=L}$$

And eventually you know if you write the state vector. So, you know with appropriate simplifications what we what we could do is that you know basically; let me just write it down. Simplify matters here,

$$\begin{Bmatrix} p_1 \\ \cdot \\ p_3 \\ \rho_0 C_0 U_1 \\ \cdot \\ \rho_0 C_0 U_3 \end{Bmatrix}_{z=0} = \begin{matrix} [T_1] & [T_2] \\ [T_3] & [T_4] \end{matrix} \begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ \cancel{\rho_0 C_0 U_1} \\ \cancel{\rho_0 C_0 U_2} \\ \cancel{\rho_0 C_0 U_3} \end{Bmatrix}_{z=L}$$

So, when we have such a thing, what we do is basically you know since all these things are going to 0, we what all this is what it is and because all these things are going to 0.

And similarly

$$\begin{Bmatrix} p_1 \\ \\ p_3 \end{Bmatrix}_0 = [T_1] \begin{Bmatrix} p_1 \\ \cdot \\ p_3 \end{Bmatrix}_L \quad \& \quad \begin{Bmatrix} \rho_0 C_0 U_1 \\ \\ \rho_0 C_0 U_3 \end{Bmatrix}_{z=0} = [T_3] \begin{Bmatrix} p_1 \\ \cdot \\ p_3 \end{Bmatrix}_L$$

So, this will become

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \end{Bmatrix}_{z=L} = [T_3]^{-1} \begin{Bmatrix} \rho_0 C_0 U_1 \\ \rho_0 C_0 U_2 \\ \rho_0 C_0 U_3 \end{Bmatrix}_{z=0}$$

And once we get that we just need to put this guy here, ok in terms of things here. So, this will become

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \end{Bmatrix}_{z=L} = \underset{[T_4]_{3\times 3}}{[T_1][T_3]^{-1}} \begin{Bmatrix} \rho_0 C_0 U_1 \\ \rho_0 C_0 U_2 \\ \rho_0 C_0 U_3 \end{Bmatrix}_{z=0}$$

If you remember recall this was at z = 0 and so U_1 at z =0. And in place of this we are writing the inverse of this guy. So, when you it is easy to see that when you sort of simplify we will get this. So, you know and then U_2 at z is equal to 0 is also 0. Eventually let us say this is the matrix you know you can call this as matrix T 4 which is really a 3 cross 3 matrix. So, then you know if we take $T^4 11$, $T^4 13$ with a minus sign, I tell you why it is minus, and here we will get our final T matrix representation for such a system, ok.

$$\begin{Bmatrix} p_1 \\ \\ p_3 \end{Bmatrix}_{z=0} = \begin{bmatrix} T_{11}^4 & -T_{13}^4 \\ \\ T_{31}^4 & -T_{33}^4 \end{bmatrix} \begin{Bmatrix} \rho_0 C_0 U_1 \\ \\ \rho_0 C_0 U_3 \end{Bmatrix}_{z=0}$$

Now, this is minus because originally you know if for this configuration the transfer matrix when you derived here the velocity was in this direction, but now we have to take the velocity for the outlet in the direction going out of the duct. So, we have to multiply this with minus. So, because once you multiply with minus and take things inside, so this will become your p1, p3 and you know you will get this kind of a thing relating things at z = 0 also 0, ok.

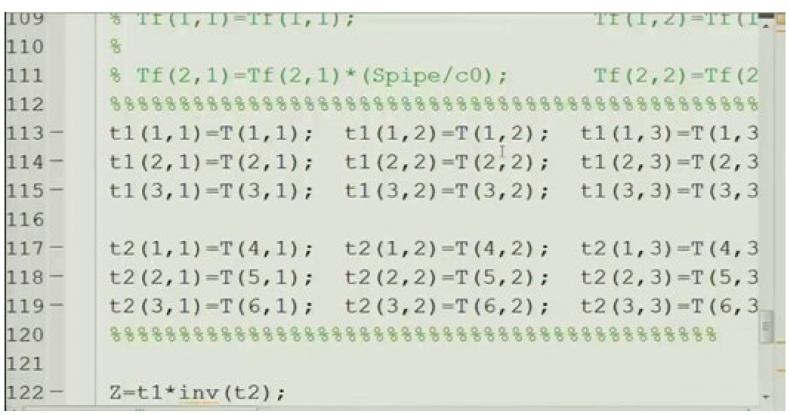
Now, we need to do further things. Eventually, we would want the this is more like an impedance matrix something that we have not really come across pressures are related to velocity. So, now, we need to do something like a transfer matrix kind of a representation.

What we have got now here is the impedance matrix. We need to convert this to a form like this thing. So, I will sort of you know do away with the showing with the simplification, but let us

$$\begin{Bmatrix} p_1 \\ \rho_0 C_0 U_1 \end{Bmatrix} = \begin{bmatrix} T_f(1,1) & T_f(1,2) \\ T_f(2,1) & T_f(2,2) \end{bmatrix} \begin{Bmatrix} p_3 \\ \rho_0 C_0 U_3 \end{Bmatrix}$$

We can have this sort of a form. So, what I am going to do is that just jump onto mat lab and show you the relevant thing.

So, everything else, interestingly everything else in the code remains the same the same because it is really a 3 duct interacting system, you can use the same thing ah. So, you have this kind of a thing, you integrate it and then rearrange it in a form like this to have things in p and $\rho_0 C_0$U form rather than this particular form.



```
% Tf(1,1)=Tf(1,1);                    Tf(1,2)=Tf(1
%
% Tf(2,1)=Tf(2,1)*(Spipe/c0);         Tf(2,2)=Tf(2
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
t1(1,1)=T(1,1);  t1(1,2)=T(1,2);  t1(1,3)=T(1,3
t1(2,1)=T(2,1);  t1(2,2)=T(2,2);  t1(2,3)=T(2,3
t1(3,1)=T(3,1);  t1(3,2)=T(3,2);  t1(3,3)=T(3,3

t2(1,1)=T(4,1);  t2(1,2)=T(4,2);  t2(1,3)=T(4,3
t2(2,1)=T(5,1);  t2(2,2)=T(5,2);  t2(2,3)=T(5,3
t2(3,1)=T(6,1);  t2(3,2)=T(6,2);  t2(3,3)=T(6,3
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Z=t1*inv(t2);
```

And then I just comment out the other codes, and basically this is another, now this is a kind of a restructuring or just extracting out certain information only a partial information from the T matrix what we found out up here you know by reshuffling the different rows and columns of the T1 matrix and putting them in the T matrix. So, once we extract certain rows and columns on the T matrix into T1 and T2 matrices. This is really the inversion thing that I was talking about and this is really your impedance matrix.

```
Z=t1*inv(t2);
%
Za = Z(1,1);    Zb = -Z(1,3);
Zc = Z(3,1);    Zd = -Z(3,3);

t3(1,1)= 1; t3(1,2)=-Za;
t3(2,1)=0;  t3(2,2)=-Zc;

t4(1,1)=0;  t4(1,2)=-Zb;
t4(2,1)=1;  t4(2,2)=-Zd;

t5=inv(t3)*t4;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

So, it relates pressures at different ports namely at port 1 and 3 to the velocities at ports 1 and 3. Now, you get these things. So, these are all sort of you know conversion of the impedance matrix in to transfer matrix. We need not do that when we come at a later stage of the course, in the next maybe next week or so. But for now I am just trying to kind of write down certain piece of code which essentially transforms the impedance matrix parameters into transfer matrix parameters.

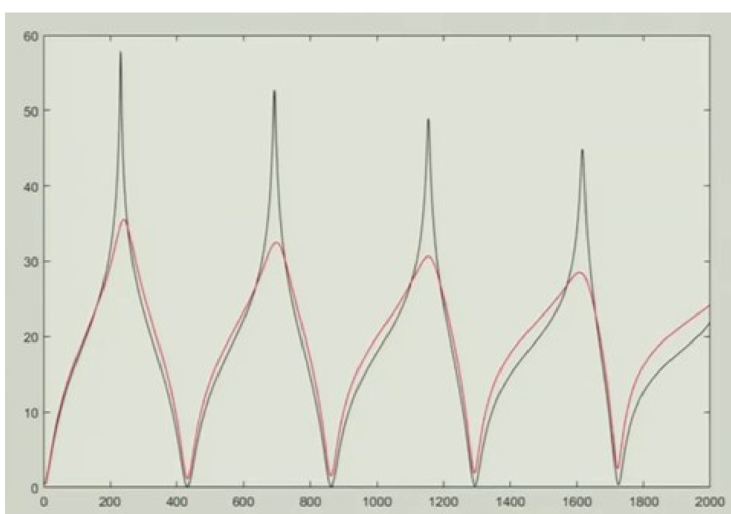
```
t3(1,1)= 1; t3(1,2)=-Za;
t3(2,1)=0;  t3(2,2)=-Zc;

t4(1,1)=0;  t4(1,2)=-Zb;
t4(2,1)=1;  t4(2,2)=-Zd;

t5=inv(t3)*t4;
  INV(A)*b can be slower and less accurate than A\b. Consider using A\b for INV(A)*b or b/A for b*INV(A). Details
%
Tf(1,1)=t5(1,1);                       Tf(1,2)=t5(1,2

Tf(2,1)=t5(2,1)*(Spipe/c0);            Tf(2,2)=t5(2,2
%
```

```
Za;
Zc;

Zb;
Zd;


%%%%%%%%%%%%%%%%%%%%%%

             Tf(1,2)=t5(1,2)*(c0/Spipe);

e/c0);       Tf(2,2)=t5(2,2);
```

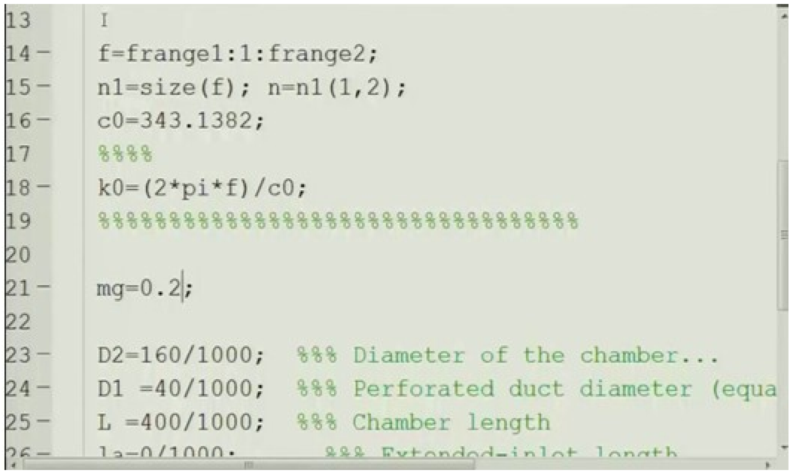
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
sigma=10;   %%% Porosity...
sigma=sigma/100;

th=2/1000;
dh=3/1000;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

f=frange1:1:frange2;
n1=size(f); n=n1(1,2);
c0=343.1382;
%%%%
k0=(2*pi*f)/c0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

You know p_1 and $\rho_0 C_0 U_1$ and then we get your this kind of a thing, ok. So, now I am good to go, I am good to run the code. So, let us let us choose a nominal porosity 10 percent σ and 160 mm is a diameter, 40 is the perforate diameter; there are no extensions. The code, at the moment the code cannot take care of extensions for reverse flow configuration, so one needs to write a lot of things, one need to develop proper routines where you can have *l* a, *l* b values.

```
Command Window
New to MATLAB? See resources for Getting Started.
>> transmission_loss_plot('k')
Elapsed time is 1.341703 seconds.
>> hold on
>> transmission_loss_plot('r')
Elapsed time is 1.293355 seconds.
>>
```



But for now we, just for the sake of demonstration I am just putting in 0 values and some value of this thing. You see it will behave exactly like your flow reversal end chamber, so you get lot of peaks and troughs. So, even as the flow is there let us see what flow kind of does to this configuration.



If you put like a large flow you see and I will say hold on hold on and do this. So, really, that is magic. Flow really lowers the attenuation peaks and the troughs again are lifted. So, you keep getting the same kind of conclusions again and again, but of course, you know these results cannot be completely relied.

Even in the low frequency range for flow reversal configurations because you know for flow reversal configurations we really have you know end connections. So, even the peaks that you are seeing here, in this case still be shifted towards the right side of the spectrum because of produced effective length of the chamber. So, you know these are all end correction businesses, maybe we can discuss all those some of them when we cover come to the stage of 3-dimensional analysis, ok.

So, now, I will wrap up today’s class. It was rather lengthy lecture. I will move on to the next lecture.

Thanks.