Muffler Acoustics-Application to Automotive Exhaust Noise Control
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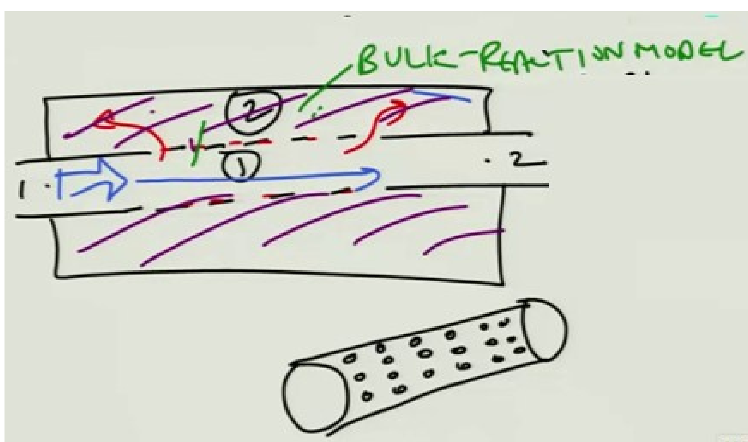
Lecture - 38 and 39
Review of Perforate Impedance Expressions

Welcome to lectures 3 and 4 of week 8. In the last; in the last lecture which was really consolidated lectures 1 and 2 of week 8, what we studied in detail was the derivation of the governing equations of concentrative resonator, a two duct interacting system and we basically, we developed or setup the equations in which we can sort of solve the extend inlet and outlet concentric tube resonator with extensions at the inlet and outlet.

And we were just about to solve it ok, you know we basically got it into the form let me write it week 8 and lectures 3 and 4 of this. So, basically what we studied or where we just about to solve was this guy, this muffler configuration which is quite famous, quite popular because of its low pressure, low back pressure or minimal pressure drop across it is, across the length across the point 1 and 2, the inlet and outlet points and here, this part is your perforated section.



You know what, so, what does it really look like? You know what I have always been insisting that it guides the flow, does not allow the flow to freely expand, it allows the ways to interact with the annular cavity and we know by here in the neck extensions ok.

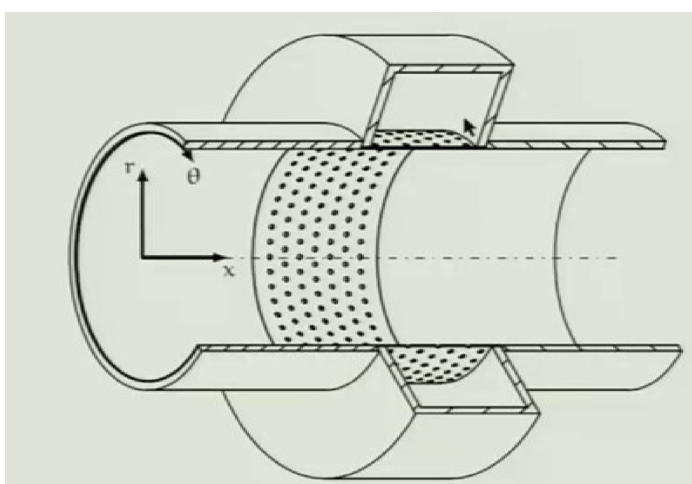
So, basically what it is doing is that here we have a; we have a sort of a pipe so, this is perforates. So, I will what I am going to do now is that show you some nice photographs or the perforates and before we even attempt to solve this, we have to overcome an important hurdle and that is what the perforate impedance what we called we call this as region 1 and we call this as region 2.

$$\frac{p_1 - p_2}{\rho_0 C_0 U^*} = \zeta$$

So, zeta you know this has been a subject matter of several investigations and it is almost you know 40-50 years. In this course to be very honest, it is almost impossible to cover all of them, but in fact, it is also difficult to you know give you a very proper or chronological historical account of the development of such equations; of such expressions for this guy.

But what I am going to do is that may be talk about 3 or 4 of the important expressions used by different group of researchers worldwide. So, let us have a look at before actually we begin to analyze a transmission loss in a MATLAB code, do some kind of parametric study of this you know CTR with extensions CTR's with extensions what we could do is that very briefly review the different perforated impedance that has been developed and used.

So, for that, you know all though I can write it down here on the screen, a better way would also be to have a look at the published literature. So, I have fortunately, I have got some you know thesis and PDF files and different research articles, which talk about different perforate impedance. So, let me show you a few of them.



So, this is the thesis by Claus Lahiri who did some work on the **Acoustic Performance Of Bias Flow Liners In Gas Turbine Combustors**. So, what so, it was very good thesis on review of different perforate impedance is used and in specially in context of liners for gas turbine engine. So, I need to you know show you some photographs or the exaggerated view of the nice, perforated liner that is used in such things.

So, here we see in cylindrical pipe and a part of that has you know array of small, small holes so, these are basically what constitutes the perforates. So, you know the waves can go through them. This region where my mouse is pointing that is probably the area through which this perforated section allows the waves to go and interact with the annular cavity where I am pointing.

Figure 2.4: Simplified geometry of a combustor liner for acoustic studies: a cylindrical perforated liner.

2.2.1 *Orifice Geometry*

The orifice geometry is composed of three main features: the orifice cross-section shape, the orifice edge, and the orifice profile.

The *orifice cross-section shape* is given by a cut through a plane normal to the direction of the orifice. Figure 2.5 gives an overview of the orifice shapes that have been studied in the literature regarding their acoustic properties: circle, ellipse [336, 385], square [7, 89, 169, 181], rectangle [89, 129, 225, 336, 389], oblong [410], triangle [7, 169, 181], cross [89, 282], star [7, 169], crown [181], eye[2] [7, 169], and trapezoid [213].

A single rectangular orifice with a high aspect ratio, i. e. a long and thin slit, is often used for its two-dimensional characteristics

Now, this is these shapes can be they are also known as orifices. You know they are basically small holes in kind of thin bend pipes or something like that. So, they need not be necessarily of circular shape, they can be of a number of shapes.

So, they are called orifices and as he mentions, orifice cross section shape is given by a cut through a plane normal to the direction of the orifice. So, a lot of them has been studied like some of them are circular shape, elliptical, square and all that.

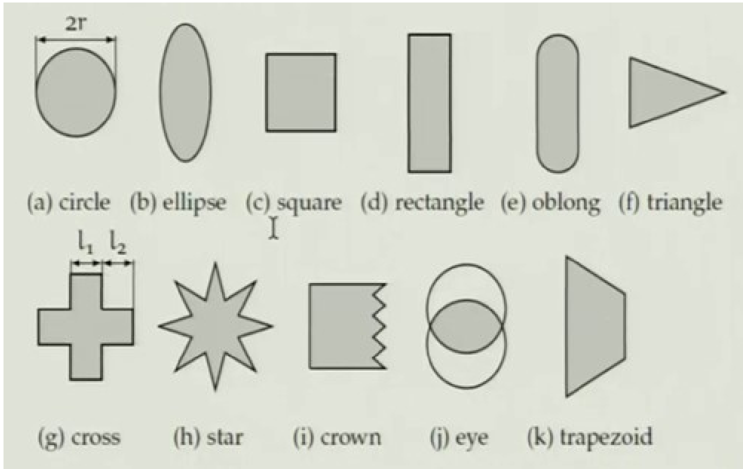
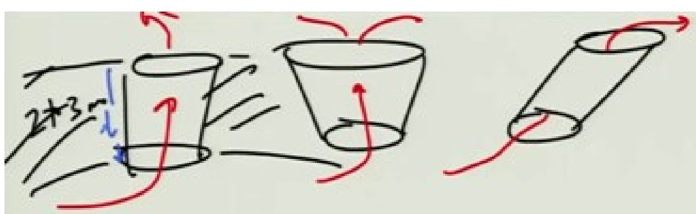


Figure 2.5: Overview of orifice cross-section shapes, that have been studied in the literature regarding their acoustic properties.

So, he has drawn some nice schematic views of different orifices shape or the shape of hole essentially drilled through pipes you know.

And each of them have different acoustic properties that is you know the shape of that definitely effects the perforated impedance. So, an another interesting thing is that it is not just the shape of the holes, the manner in which they are cut for that I probably have to go back to the presentation for just a bit to the slides I mean.



And you know what I am trying to say is that you know if we draw a an exaggerated view of this, they could be something like this, something like this or you know really parallelepiped kind of a thing, they are all arrays so, wave and this is. So, this is like a very small thickness may be about 2 mm or may be 2 to 3 mm, this is the solid section and the waves really go through them and interact here similarly, here ok.

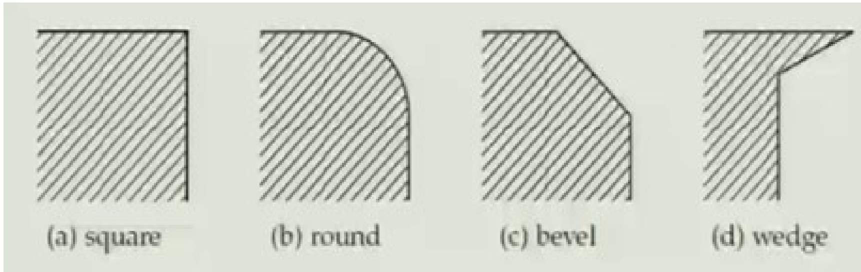


Figure 2.6: Overview of orifice edge shapes, that have been studied in the literature regarding their acoustic properties

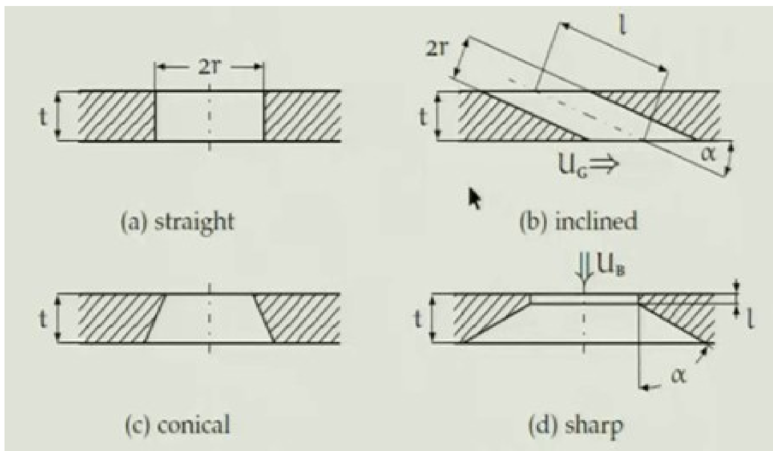


Figure 2.7: Overview of orifice profile shapes,that have been studied in the literature regarding their acoustic properties

So, let us get to the presentation of Clause who shown some nice cool figures. So, these are the different orifices edge shapes, but what is also what I was talking about now is that an overview of the orifice profile shapes that is basically, you know what is happened is that you can see this is really an extended section like this and their holes drilled through conical holes or may be straight holes.

And this is really cylindrical in shape or this conical frustum of a cone or of a sharp edge bevel or something like that or inclined, it is like a cylinder kind of tilted in one direction. So, there are different ways in which the orifices can be cut through you know pipes of kind of small thickness may be 3 mm or 2 mm, 4 mm something like that.

So, now all these will greatly affect the perforated impedance that is you know p 1. So, suppose this is your section p 1, this is p 2 so, p 1 minus p 2 divide by the rho naught c naught into U star which is the radial velocity that is velocity in the direction that I am pointing. So, all these geometries or profiles have been sort of studied in the literature.

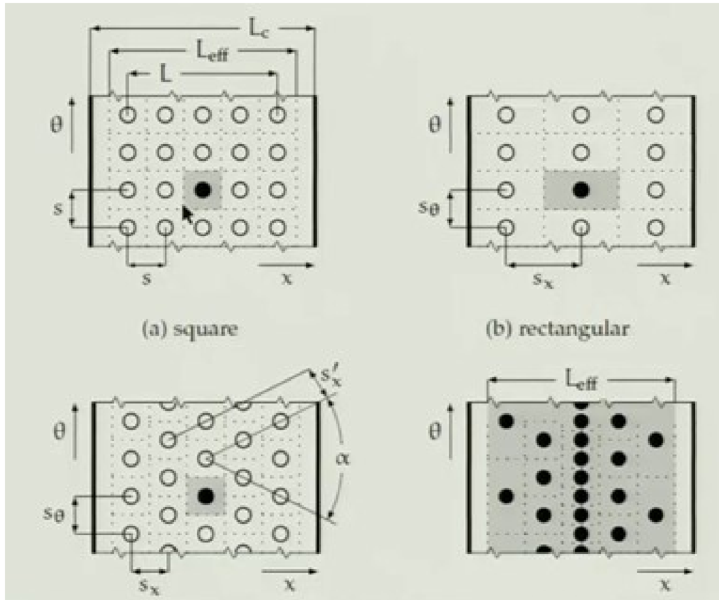


Figure 2.8: Overview of perforation patterns that have been studied in the literature regarding their acoustic properties

And another important thing that one needs to sort of know here before we actually I mean these things one must know before we present the expressions for perforated impedance because there will be a number of symbols so, it is good to know beforehand what these numbers or symbols would mean.

So, these holes can be aligned in different manners. They can be arrive, they can be arranged in a square kind of a pattern like something like this so, basically what is happened is that suppose you unbend the pipe or this is the development view and you have holes of say circular shape, drilled at regular intervals 1, 2, 3, 4 like this at uniform center to center distance along the circumference as well as along the this thing because you can bend the pipe like here.

So, this will become your circumference, and this will be the radial sorry reaction section. Similarly, you can have a rectangular thing that is basically have lesser holes in the circumferential direction if you bend it like this and more number of holes along the axis or vice versa.

Then, you have a staggered thing. Basically, staggered is like a may be something like having one hole in the center of two rows on the diagonal joining the two holes something like this where I am pointing with different parameters or non-uniform completely, random holes that kind of a thing.

> pattern that is s^2). This is indicated by the dashed grid lines in Figure 2.8. For a uniform perforation pattern the *porosity* σ is defined as the ratio of the open-area of one orifice to its unit area[4] $\sigma = A/(s_x s_\theta)$. For a circular orifice in a square perforation pattern that yields $\sigma = \pi r^2/s^2$. This is illustrated by the filled black orifice and the gray area around it. For a specified surface area the same

So, basically the reason that it affects the perforate impedance because it affects the, it decides the porosity. So, that is basically your ratio how much permeable this surface is. Because remember we are modeling this by an impedance, it is like the, it is like saying that the surface of the or the perforated part is a permeable surface, it allows waves to pass, but with the certain resistance or impedance you know some oppositions. So, that is what we trying to model.

Now, so, one parameter that obviously, comes becomes very important here is the porosity that is basically the ratio of the total cross circumferential area and what is the fraction of the circumferential area that is open so, that is obviously, given by the number of holes. So, basically the total open area divided by your overall circumferential area that is your open area ratio which is highly related to the porosity.

So, what I would suggest now is that let us briefly review the expressions that have been developed for the this thing. So, there is lot of literature and at times, it can be sort of really over whelming. So, to make things very simple because this is really is a first course or a fundamental course on muffler acoustics so, I would you know stick to a very simple kind of a presentation.

$$\zeta = \frac{p}{\rho_0 C_0 u} = \left[0.514\frac{d_1 M}{l\sigma} + j0.95k_0(t + 0.75d_h)\right]/\sigma$$

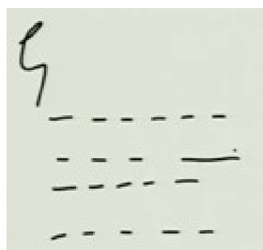
where

d_1 is diameter of the perforated tube,
M is the mean-flow Mach number in the tube,
l is the length of perforate,
σ is porosity,
f is frequency,
t is the thickness of the perforated tube, and
d_h is the hole diameter.

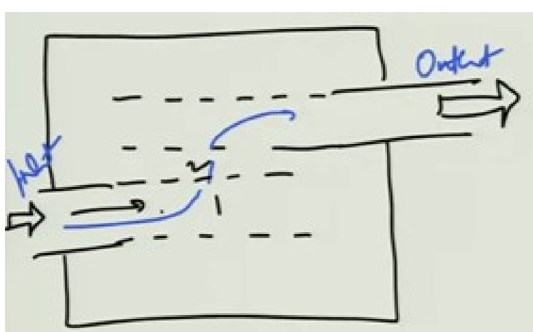
Perforates in stationary medium [25,26] (linear case)

$$\zeta = [6 \times 10^{-3} + jk_0(t + 0.75d_h)]/\sigma$$

So, first things first, they are so, this is an extract from the textbook by Munjal in which this basically perforate impedance expression for a cross flow is given. Then, there is also expression for the case when there is no mean flow that is a stationary medium. So, this is what it is, let me sort of write it down on the slide.



So, what we really have here is zeta for a cross flow. So, remember cross flow was something like this. So, you know you have your overlapping section say.



BIAS-FLOW CROSS-FLOW Sllivan, JASA a, 1979

And the flow was coming, and it was forced to go like this, enter the cavity here and then may be sort of it is going here. So, it was something like this kind of a muffler ok. So, we will worry about that somewhat later you know something like this, the flow has to necessarily cross from one perforate; perforated pipe through another one perhaps in a overlapping section and that is why this zeta for this cross flow configuration becomes important cross-flow.

Cross flow basically they let me tell you I will also obviously, discuss this at a later thing when I discuss about the relevant maths behind this. But one thing about cross flow elements is that they have slightly better attenuation performance than simple grazing flow a muffler configurations, but at the expense of higher back pressure. So, that is one thing you need to keep in mind. It is not like it is, it can be used everywhere just because the attenuation performance is good, the other constraints we also need to be mindful of.

So, let us say zeta is your impedance here. So, p1 minus p2 assuming this is duct 1, duct 2 and so on is your rho naught. So, flow has to necessarily traverse to this, it is not like the flow range is grace the surface what we saw for this thing. In this, the flow can simply you know go like this straight with and only the waves interact, the flow does not have to you know go through this.

Although, there will be some non-zero flow, but we can sort of ignore it for most practical purposes, the flow mean flow just goes like this, there will be no flow through the perforates. But in this case, the flow has to go because otherwise, how it will go outside go through the outlet. This is your outlet pipe, this is your inlet pipe ok.

Now, for this guy, we

$$\zeta = \frac{p_1 - p_2}{\rho_0} = \frac{0.514 d_1 M_0}{l\sigma} + \left[\frac{j0.95 k_0 (t + 0.75 dh)}{\sigma}\right]$$

So, remember t thickness means this; this guy is your thickness. So, t plus 0.75 diameter d h; and this entire thing you know is divided by sigma, this entire thing is sort of divided by sigma. So, this was for the cross-flow configuration and from where did we get this expression?

Which research article proposed this thing? So, it was given in Sullivan’s paper. J W Sullivan a long time back in JASA, a Journal of Acoustical Society of America. J Sullivan published this formula or this proposed this thing in JASA acoustic society of America in 1979 ok.

So, then for the stationary medium, you know in the same researcher proposed another expression for

$$\zeta = \frac{6 \times 10^{-3}\, jk_0(t + 0.75dh)}{\sigma}$$

One thing that is important in perforates and we will soon see is that, this perforate impedance expression is complex, it is of the form a plus ib. Resistive part or dissipative part and reactive part or something that does not dissipate energy.

So, now, in case of what this expression really tells you, this is the expression for a stationary medium. So, when you have a stationary medium at you do not really have a concept of a cross flow so, basically it is just like a simple perforated pipe without any mean flow.

So, 6×10^{-3} quantity is very small compared to at least this thing and this thing particularly increases with frequency because k naught will increase, reactive part will increase. The point I tend to make is that in the absence of mean flow that is for the stationary medium considered, we do not really have the resistive part is basically very small or there is no dissipation.

So, one; one reason perforates do well is because they introduce additional damping into the system, they are like you know they are small viscous loses. So, all these holes that you have just seen the array of holes that you have seen you know may be in like this; this guy Lahiri gives you very good this researcher gives you very good schematics.

So, you know there are small, small holes and this is the orifice cross section what he has really given. So, basically what I am saying is that when the waves are forced to go through this small, small holes, viscous loses become very important here. So, one hole will not do anything, it is only a cumulative action of small, small affects of each holes that really adds up and give you the overall effect in terms of some non-zero resistive part.

So, but for a stationary medium, when it is not very significant, it is only the mean flow. When there is a non-zero mean flow, even for a grazing flow. So, all this thing that you saw, this expression was for a cross flow also known as bias flow. So, flow really has to go through the pipes. In this stationary medium, there is no concept of bias flow, but then there is thing like perforates with grazing flow.

So, another group of researchers sometime back they proposed a formula for perforates with grazing flow and that was let me write down the expression and we can sort of verify also from the book it

$$\zeta = \frac{7.337 \times 10^{-3}(1 + 72.23M) + j22245 \times 10^{-5}\,(1 + 51t)(t + 2045dh)f}{6},$$

this expression was published by Munjal and M N Rao and Munjal. So, this I am sort of writing it down we might use one of them for our MATLAB demonstration for evaluating the transmission laws.

So, this is this becomes a bit messy, let me just go directly to the to the book by Munjal where these expressions are given. For the cross flow, d 1 is obviously, the diameter of the perforated pipe and M is the mean flow Mach number M or M naught whatever you want to say. The σ is a porosity.

Now, porosity varies between you know anywhere between 5 percent or you know typically, 7 percent to about 28 to 30 percent. So, 30 percent porosity means it is highly porous. So, it acoustically transparent as though waves can go through one pipe, the inner pipe to the annular section without you know; without much obstruction.

So, 30 percent porosity means the it is nearly acoustically transparent and as though acoustically speaking, the pipe, the perforated way does not exist. So, one clever way what engineering trick what people do is that if they want to do some good maintain, some good aerodynamic performance.

And you know just get rid of the flow dynamic flow induced noise due to the flow expansion, they use the highly perforated section, 30 percent porosity and so that the waves are allowed to go through allowed to pass through very easily. But at the same time, acoustically the performance will be very same, very similar to that of a extend inlet and outlet muffler.

So, it would not sort of effect much acoustically speaking, but it definitely has a big, very good influence or on the on reducing or kind of eliminating the separation noise that would otherwise occur. So, f is the frequency like I was saying, t thickness of the perforated pipe and d_h is the hole diameter. So, this is the, this t is the thickness and 2r or dh is the hole diameter.

So, what about this expression? This was the expression given in Munjal, which I just sort of copied down on the presentation. So, M here is I mean basically symbols are the same.

an experimental setup as shown in Figure 3.15, Rao and Munjal [35] evaluated impedance of a perforated plate by measuring the impedance without the plate and subtracting this value from all the measured values with the perforated plate in position (see Figure 3.13). The acoustic impedance of a perforate is a function of many geometrical and operating parameters, namely, grazing-flow Mach number M, porosity σ, plate thickness t, and hole (or orifice) diameter d_h.

These parameters were varied one at a time, in steps, as follows:

$M = 0.05, \underline{0.1}, 0.15, 0.2$
$\sigma = 0.0309, \underline{0.0412}, 0.072, 0103$
$t = 1/24, \underline{1/16}, 1/8$ in.
$d_h = 1.75, 2.5, \underline{3.5}, 5.0, 7.0$ mm.

The underlined values indicate the default option values when some other parameter is being varied. Least squares fits were obtained for the dependence of normalized impedance of

And these were obtained through a large number of experiments and this paper was published by these researchers perforates with grazing flow, a long time back again, these are all historical expressions which have been used.

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So, I will talk about that, but this was published in **1986, Experimental Evaluation of impedance of perforates with grazing flow published in Journal of Sound and Vibration.** So, ever since that a large number of papers have been published in its separate investigations were reported on the perforate impedance things.

So, the one paper is by the group in Egypt and in Sweden by Elnady and Abom. So, that paper I can just show a glimpse of that things already in this book I believe the expressions there so, just bear with me for a minute.

So, another impedance expression which has been used quite popularly since the last several years is the one by Elnady and others. So, the that expression is given you know is been used in several investigations and takes into account both the mean grazing flow and the mean bias flow and so, this is the expression that is here. So, this is zeta is given by

4.5 Meanflow Lumped Resistance Network Theory

Mean flow affects the perforate impedance as shown in Equations 3.128 and 3.129. In muffler configurations, particularly of the type of Figure 4.3, grazing flow and bias flow occur simultaneously, and then the acoustic impedance of a perforate is given by the expression [9]

$$\zeta = \theta + j\chi$$

Where,

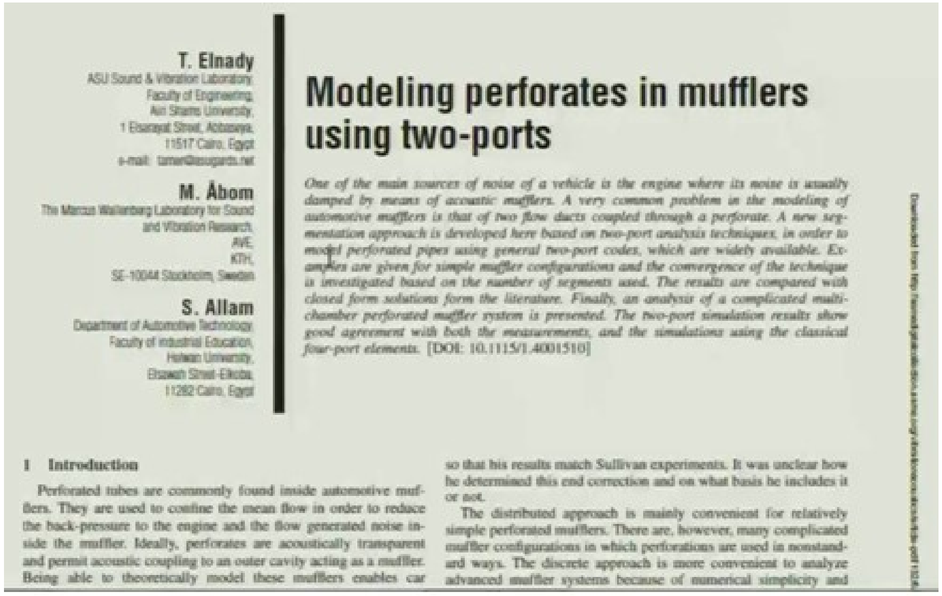
$$\theta = Re\left\{j\frac{k}{\sigma C_D}\left[\frac{t}{F(\mu)} + \frac{\delta_{re} f_{int}}{F(\mu)}\right]\right\} + \frac{1}{\sigma}\left[1 - \frac{2f_l(kd)}{kd}\right] + 0.3\,\frac{M_8}{\sigma} + 1.15\frac{M_b}{\sigma C_D}$$

$$\chi = Im\left\{j\frac{k}{\sigma C_D}\left[\frac{t}{F(\mu)} + 0.5\,\frac{d}{F(\mu)}\right]\right\}$$

$$K = \left(-\frac{j\omega}{v}\right)^{0.5} \quad K = \left(-\frac{j\omega}{v}\right)^{0.5} \quad F(Kd) = I - \frac{4j_1(Kd/2)}{Kdj_0\,(kd/2)}$$

$$\delta_{re} = 0.2d + 200d^2 + 16000d^3, f_{int} = 1 - 1.47\sigma^{0.5} + 0.47\sigma^{1.5}$$

So, this nu is basically what they have done is the sum total of four different parts.



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Modeling perforates in mufflers using two-ports

One of the main sources of noise of a vehicle is the engine where its noise is usually damped by means of acoustic mufflers. A very common problem in the modeling of automotive mufflers is that of two flow ducts coupled through a perforate. A new segmentation approach is developed here based on two-port analysis techniques, in order to model perforated pipes using general two-port codes, which are widely available. Examples are given for simple muffler configurations and the convergence of the technique is investigated based on the number of segments used. The results are compared with closed form solutions form the literature. Finally, an analysis of a complicated multi-chamber perforated muffler system is presented. The two-port simulation results show good agreement with both the measurements, and the simulations using the classical four-port elements. [DOI: 10.1115/1.4001510]

1 Introduction

Perforated tubes are commonly found inside automotive mufflers. They are used to confine the mean flow in order to reduce the back-pressure to the engine and the flow generated noise inside the muffler. Ideally, perforates are acoustically transparent and permit acoustic coupling to an outer cavity acting as a muffler. Being able to theoretically model these mufflers enables car

so that his results match Sullivan experiments. It was unclear how he determined this end correction and on what basis he includes it or not.

The distributed approach is mainly convenient for relatively simple perforated mufflers. There are, however, many complicated muffler configurations in which perforations are used in nonstandard ways. The discrete approach is more convenient to analyze advanced muffler systems because of numerical simplicity and

And let me get to the paper which was published in the you know this work appeared in the **Journal of Vibration and Acoustics ASME journal, American Society of Mechanical Engineers in I guess sometime in well 2010.**

$$K = \sqrt{-\frac{j\omega}{v'}} \;\; K' = \sqrt{-\frac{j\omega}{v'}}$$

So, what they have done, this expression what is given in the book on Ducts and Mufflers by Munjal is adopted from the paper here where they basically.

What they did was that basically, it was given by modeling perforates as a system of two port elements and what they said was that the grazing and bias flow impedance were calculated according to previous work and the resistance part or the resistive part of the impedance consist of four terms like I was just about to mention.

First is the viscous losses inside the hole so, what is that? This is this part viscous losses inside the hole. They are very small holes about 2 or 3-mm in dia and radiation resistance. You know in typical courses in fundamental of acoustics and all that, you consider piston oscillating in a infinite baffle so, then you find out the radiation impedance or acoustic impedance right at the piston surface.

So, all these formulas can now be, there might be a reason why we study all these things so, their number of applications related to the simple models which are now useful. You know you see the utility of those models now. One is clearly the radiation resistance to the vibrating piston.

So, you basically model the air column inside the orifice as vibrating piston and then, you know the radiation impedance is given by

$$\theta = Re\left\{\frac{jk}{\sigma C_D}\left[\frac{t}{F(\mu')} + \frac{\delta_{re}}{F(\mu)} f_{int}\right]\right\} + \frac{1}{\sigma}\left[1 - \frac{2f_1(kd)}{kd}\right] + \frac{0.3}{\sigma} M_g + \frac{1.15}{\sigma C_D} M_b$$

$$\chi = lm\left\{\frac{jk}{\sigma C_D}\left[\frac{t}{F(\mu')} + \frac{0.5d}{F(\mu)} f_{int}\right]\right\}$$

the Bessel function of the first order and the first kind, ordinary Bessel function of the first order first kind.

And you know again, sigma is the porosity, Mg is the grazing flow so, basically the flow that just touches the perforates does not really cross it and Mb is the flow that kind of goes through the duct. So, basically you can have kind of different you know this formula really combines the mean and mean grazing and bias flow, they can be perforated mufflers very complicated ones like the one, the photographs I had shown in the beginning of this lecture.

Where you have you know a four duct interacting system, three perforated pipes and the big annular cavity, which contains a big cylindrical, elliptical cylindrical or circular cylindrical shell that contains the three perforated pipes. So, it is a multiple interacting duct problem and multi-path propagation.

So, in such a situation, grazing and bias flow can occur simultaneously, and determination was these things are also not trivial, it is a research problem in itself. So, in such a situation, this formula is applicable, but it is also applicable in other simplified situations like you know CTR's where you just have grazing flow, or you can also have a cross flow in the other configuration that I just had briefly talked about.

And now chi is your imaginary part or the inductive part or it is called also called reactance. So, the basically this is the reactance is assumed to be caused only by the mass plug and any flow affects on the reactance are neglected.

So, F mu dash is basically these are all sort of functions where derived through number of theoretical models and these expressions, these constants are basically fitted to some experimental data by these researchers. So, that is this is one sort of one popular expression.

Bessel function, $\nu = \mu/\rho_0$ is the kinematic viscosity, ρ_0 is the fluid density, μ is the adiabatic dynamic viscosity, $\mu' = 2.179\,\mu$, M_g is the grazing flow Mach number, and M_b is the bias flow Mach number inside the holes of the perforate. Equation 4.72 is a correction factor for the orifice interaction effects.

Evaluation of the bias flow Mach number, M_b and the grazing flow Mach number, M_g in each of the perforated pipes in a multiply-connected perforated-element muffler may be carried out readily by means of a lumped flow resistance network making use of the electrical circuit analogy with Kirchhoff's first law for the nodes or junctions and second law for the closed loops.

Flow resistance R of the lumped element is defined here as

$$R = \frac{\Delta\rho}{Q[Q]}, or \;\; R = \frac{\Delta\rho}{Q^2}$$

if directionality is immaterial or understood. Here, Δp is the stagnation pressure drop across the element and Q is the volume flow rate passing through the element.

This definition is substantially different from the definition of electrical resistance which is defined as voltage drop across the element divided by the current passing through the element.

If Δp is written in terms of the dynamic head, $H = \frac{1}{2}\rho_0 U^2$, and loss coefficient ζ, as is the practice in fluid mechanics of incompressible flows, then

$$\Delta P = g\left(\frac{1}{2}\rho_0 U^2\right), U = \frac{Q}{S},$$

And your sigma which is your porosity so, what is sigma? How do you how do you determine porosity and actually, what is also there is another thing that what do you let us also talk about different symbols here, C D is the coefficient of discharge, k is as usual the wave number k or k naught forcing frequency you know and c is the sound speed and nu is mu by rho naught that is the kinematic viscosity and nu dash is 2.17 times mu. So, there are number of things here. We just talked about this sigma which is the porosity.

So, what is, how do you define porosity in fact, how do you like with let us say you know if you are given the job of fabricating this you know perforated tube which was fabricated by us sometime back so, how do you define like it is clearly seen that this guy you know where my mouse is pointing, this is much more porous than this one.

So, porosity is sigma; porosity is sigma is defined as the total area of the hole. So, let us say in all like for example, if you go back to this figure, the area; the total area of hole is really the area of one hole let us say diameter is dh.

$$\frac{6 = nh\ \frac{\pi dh^2}{4}}{\pi D_0 L_p}$$

the number of, total number of holes ok that you can have equal number of holes per ring. And they can be something like you know m number of rings and n number of holes in one ring or you can have different configurations like this is clearly come something like a square kind of a configuration, you can have rectangular, you can have staggered. In the most generalized sense, if n h it is the total number of holes and $\frac{\pi dh^2}{4}$ it is the total it is the area of each hole ok.

So, this is the total area due to the all the perforated holes ok and what is the cross sectional, what is the cylindrical or curved surface area of the perforated pipe? It is not from this pipe, this point to this point, it is from just from the extreme like for example, just where the hole the array ends like this is the array where it ends, from this section until this section where this pipe is there, that is the perforated length.

In this case, you know it is probably from you know the ring, the length let us say from here from this point all the way until this length. So, basically the idea is if capital D is your diameter of the perforated duct so, π D into Lp you know π D into Lp will give you the curved surface area, it is like really like a rectangular when you open the when you do the development.

So, π D is the circumference or 2π r_0 or 2 π D_0 divide by Lp So, basically, this is the porosity and clearly if numerator is more that is the total area covered by the holes is much more significantly large or you know substantial compared to the this curved surface area, then porosity is more.

$$0.5 \leq 6 \leq 0.3 \quad \rightarrow \quad \text{Acoustically transparent}$$

So, typically, we do not have porosity greater than 0.3, it is always about less than 30 percent and like I said it is almost acoustically transparent; like as if the perforate did not exist, but mechanically, it does and it you know it does not make much sense to go less than 0.5 even 0.05 is less, it is quite small.

So, typically, we vary it from you know 0.08, 0.1 all the way until 0.28, 0.3 something like that. So, these are the typical values. There is also something like open area ratio. So, open area ratio is related to sigma that is the total area of the covered by the perforated hole divide by the cross-sectional area of the pipe.

So, you know in place of this guy, you would write $\frac{\pi D_0^2}{4}$ you know. So, you can then simplify matters and you know get the things, but you know so, open area ratio is important for getting the back pressure due to these things, we are not probably going to talk about that in this course, but just for knowledge sake, I thought of mentioning OAR, Open Area Ratio also.

Anyhow, let us get back to the review, a very brief review of the perforated expressions that are available in the literature. So, this is this one and then Lahiri like I said gives a excellent review of the different perforated impedance that is there in the literature. So, what we have to do is that look at the different expressions. So, impedance $Z = \frac{p}{q}$ or pressure by volume velocity. Typically, like is mention, Lahiri also says impedance is resistive part

$$Z = R + i\chi, \;\; z = r + i\chi \; and \; \zeta = 0 + i\chi$$

the reactive part. Real part is resistance, imaginary part is a reactance. So, they are different model.

$$Z = R + i\,[\omega M - 1/(\omega C)]$$

The acoustic impedance can be written as sum of three quantities, you know R resistance for damping and M is the mass inductance and this C is the compliance. So, we have the lumped parameters.

So, we had number of mathematical or theoretical models based on different you know researchers over different years. So, noticeably what is important is like there are few parameters like internal impedance of the orifice as well as the mass end correction that is due to radiation reactance I was talking about, resistance end correction and number of things.

$$\zeta = \frac{ik}{\sigma}\left(\frac{1}{F(k'_s r)} + \frac{16r/(3\pi)}{F(k_s r)}\right)$$

So, this is one; formula where ζ is ik times this thing. So, here of course, the mean flow effects are not quite considered.

$$\psi(\sigma) = \sum_{n=0}^{8} an\ \left(\sqrt{\sigma}\right)^n,$$

This is the orifice interaction. There are number of formulas by

$$\zeta\ Melling = \frac{ik}{\sigma}\left(\frac{1}{F(Sh')} + \frac{16r/(3\pi)}{F(Sh)}\psi\right)$$

$$\psi(\sigma) = \sum_{n=0}^{8} a_n\left(\sqrt{\sigma}\right)^n, \qquad with$$

$$a_0 = 1, \qquad a_1 = -1.4092, \qquad a_2 = 0,$$

$$a_3 = 0.33818, \qquad a_4 = 0, \qquad a_5 = 0.06793$$

$$a_6 = -0.2287, \qquad a_7 = 0.03015, \qquad a_8 = 0.01614,$$

Where you know these are the different things given in the work by Fok.

$$v'_{rms} = \frac{1}{2\sigma C_D}\left(\frac{p_{ref}10^{\frac{SPL}{20}}}{\rho c}\right)$$

$$\zeta Melling, nl = \frac{ik}{\sigma}\left(\frac{1}{F(Sh')} + \frac{16/(3\pi)}{F(Sh)}\psi\right) + \frac{1.21 - \sigma^2}{2C\ (\sigma C_d)^2} V'_{rms}$$

So, it is pretty hard to review all of them right now, but just for the sake of knowledge, it is good to understand this like, this is the more comprehensive formula involving resistance sum as well.

$$\theta_c = \frac{0.3 M_C}{\sigma}$$

Then, you; then, you have a grazing flow impedance models and large number of such models are there. So, the entire thesis is devoted to that.

$$\zeta_{Betts} = \frac{4\nu l}{C\sigma C_D r^2} + \frac{\sqrt{2\omega\nu l}}{C\sigma C_D r} + \frac{1 - \sigma^2}{\sigma C_D}\left|M_B^{eff}\right|$$

$$+i\left(\frac{Kl}{\sigma C_D} + \frac{\sqrt{2\omega\nu l}}{C\sigma C_D r} + \frac{16}{3\pi}\frac{r}{\psi\psi}\right)$$

Now, I will just; I will just go through that may be to the people who are interested, they can read it. For example, Betts model is this thing where there are some expressions one needs to understand that.

$$|\tilde{v}| = \frac{1}{2\sigma C_D}\left(\frac{p'_{ref} 10^{SPL/20}\sqrt{2}}{\rho c}\right)$$

So, basically, I will kind of stop the review of this thing here at least with the understanding that at least when there is no, when the annular area is empty that is there is no dissipative material in the lining, one would, one needs to really consider an appropriate expression. For our cases, we will use most likely we are going to use the expression given by Elnady and others and that is quite like I said they has been quite popularly used.

And just for the sake of completeness, I must also mention that there are also other models you know available in the literature where the annular area so, you know let us go back to the presentation, you know if this area is filled with an absorbent material like a rock wool, glass wool, then of course, quite complicated expressions for the impedance of the perforate exists.

So, here, we consider bulk reaction may be if we get time, we will consider this in the course bulk reaction models here; bulk reaction models, bulk reaction wave propagation and here there is a now there is a regular wave propagation in the airway and, but before the wave interacts or goes in this thing, it has to cross through the impedance through the perforated bridge ok. So, for that, we need to develop suitable expressions, people have already done it.

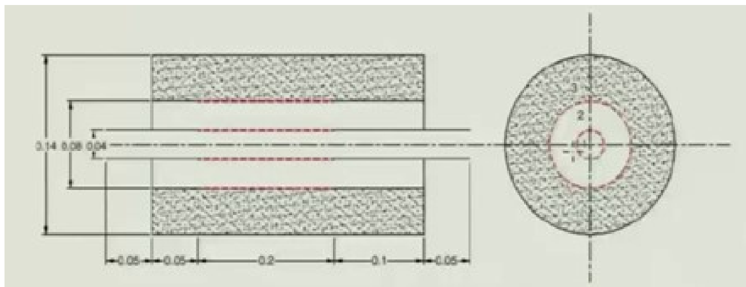


Figure: 4.18 Schematic of lined duct with annular air-gap between the airway and the absorptive material (all dimension are in meters) (Adapted from [11])

So, another like I was mentioning, here this is an important configuration schemating of a line duct with annular air gap between the central airway that is your, you know this one, central airway and the absorptive material. This is like a hybrid kind of a muffler; you have a resonator or neck extensions at the inlet and outlet plus the waves can also go through the interact with the annular cavity.

So, this you know it is like the best of both the world because this dissipative things you see hopefully, we will see in the later parts of the course. This is affective only at high frequencies. So, at low frequencies, you know when the reactive parts can do their job by providing suitable attenuation and at high frequencies the dissipative parts probably can take over.

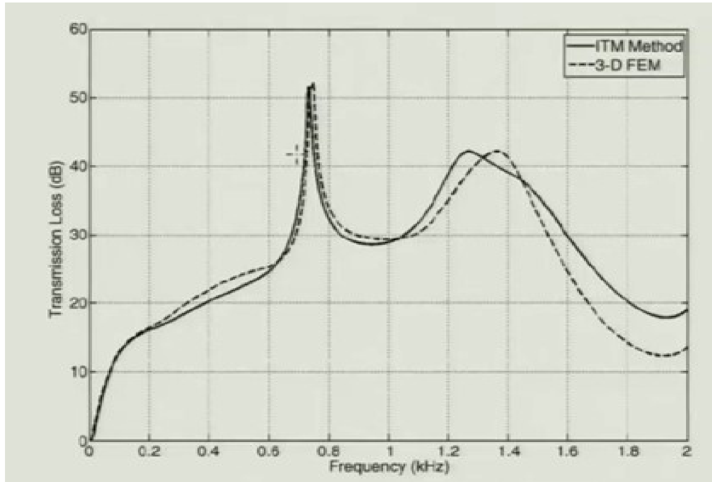


Figure 4.19 Comparison of TL spectrum from the 3D FEM analysis with that of the ITM method for the configuration of Figure 4.18 (Reproduced with permission from [11])

So, this is one result also only from the plane wave analysis, the ITM method which is called the Integrated Transfer Matrix method derived by Munjal and his group. So, it matches quite well for the configuration dimensions considered with a three-dimensional results, but for larger mufflers, we still have to use a 3D thing.

But what I was trying to show you is this expression. Now, what does it do? It is the modified impedance expression. Now, notice the fact that here you have a very small real part 0.06 and here

$$Z_{p=}\left[0.06 + jk_0\left\{t_w + 0.35d_h\ \left(1 + \frac{Y_w k_0}{Y_0 k_w}\right)\right\}\right]/\sigma$$

are the characteristics impedance and the wave propagation constant wave number in the dissipative material and k naught Y naught are the counter parts in air.

So, when this is 1 so, this becomes when that is basically when you have Y w and k w are equal to Y naught and k naught respectively, when the medium is air so, this becomes 1. So, this is 2 and 0.375 will become 0.75 d h. So, you basically get back the medium expression for the case of a stationary medium in case of zero mean flow and there is no dissipative material on the other side, there is on both sides of the edge perforate there is only air that is one thing.

$$\Gamma = a_1\xi^{-a_2} + i[1 + a_3\xi^{-a_4}]$$

and

$$\tilde{\rho} = -\Gamma[a_5 \xi^{-a_6} + i(1 + a_7\, \xi^{-a_8})]$$

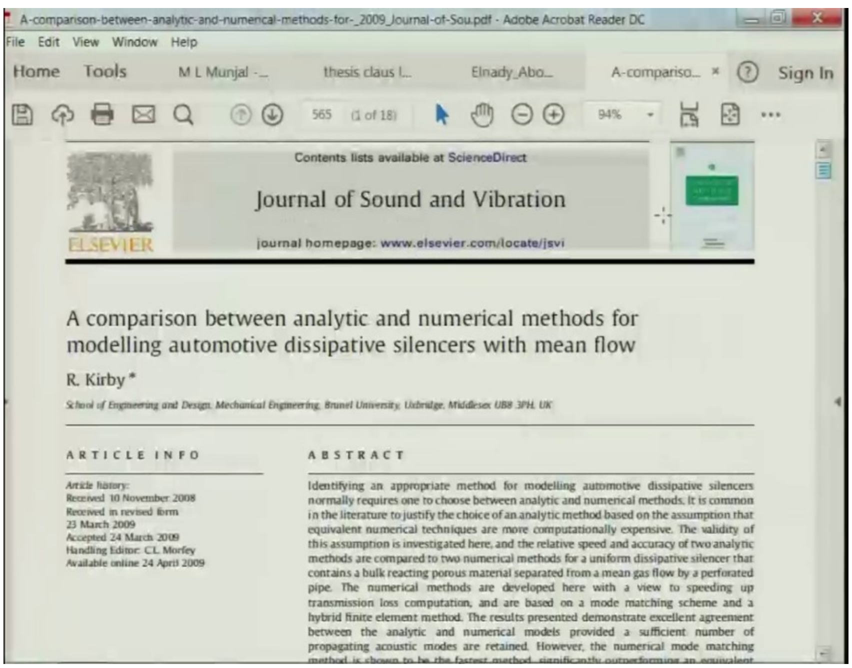
And then, there are other expressions also, you know they are lots of them; you know it is almost impossible to cover the entire thing.

$$\zeta = \frac{[\zeta + i0425 k_0 d\ (\tilde{\rho} - 1) F(\sigma)]}{\sigma}$$

where,

$$F(\sigma) = 1 - 1.06\sigma^{0.5} + 0.17\sigma^{1.5}$$

But and they have been obtained either through some theoretical models and validated by some impedance tube test or by you know basically fitting, directly fitting some empirical formula to the experimental values.



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A comparison between analytic and numerical methods for modelling automotive dissipative silencers with mean flow

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ABSTRACT

Identifying an appropriate method for modelling automotive dissipative silencers normally requires one to choose between analytic and numerical methods. It is common in the literature to justify the choice of an analytic method based on the assumption that equivalent numerical techniques are more computationally expensive. The validity of this assumption is investigated here, and the relative speed and accuracy of two analytic methods are compared to two numerical methods for a uniform dissipative silencer that contains a bulk reacting porous material separated from a mean gas flow by a perforated pipe. The numerical methods are developed here with a view to speeding up transmission loss computation, and are based on a mode matching scheme and a hybrid finite element method. The results presented demonstrate excellent agreement between the analytic and numerical models provided a sufficient number of propagating acoustic modes are retained. However, the numerical mode matching

But and one such expression you know this is the paper by Kirby, published in JSV a long time back.

$$\frac{1}{C_q^2}\frac{D^2 {p'}_a}{Dt^2} - \Delta^2 {\mathrm{p}'}_{\mathrm{q}} = 0$$

Where C_{q} is the speed of sound $\mathrm{p'}_{\mathrm{q}}$ is the acoustic pressure, and r is time. The hard wall boundary condition is given by

$$\Delta {\mathrm{p}'}_{\mathrm{q}}.\mathrm{n}_{\mathrm{q}} = 0$$

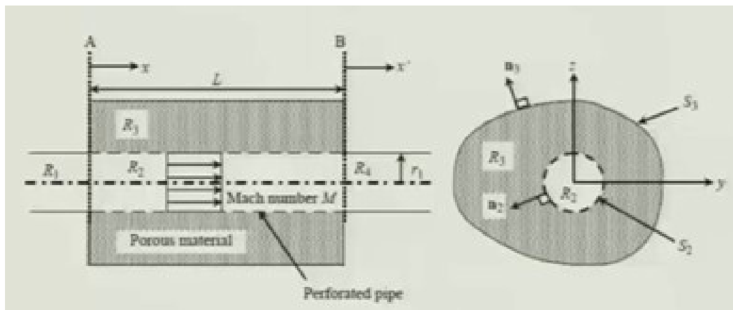


Figure 1: Geometry of silence.

Where he basically considered this kind of a muffler configuration, a straight through muffler filled with dissipative material on the in the annular cavity and again, it is a same problem one needs to find out the perforate impedance with backing on one side with the absorbent material.

So, well, this paper is devoted to something else, but it is quite I mean it is it will be good if we can just have a, it is a finite element development paper for analyzing these such configurations.

But what is, what we can take the take home message from this particular thing is that to have a look at the perforate impedance expression. So, for the perforated pipe the normalized impedance zeta is given by this thing, which is given in reference 5, zeta dash plus this thing where F; F of sigma that is given in the paper by Fok.

And these are different expressions for F sigma and zeta dash is also found out previously, zeta dash is orifice impedance measured in the absence of an absorbent material and values of zeta dash is measured by Kirby and Cummings. And so, there are number of such things where one uses the proper material and all these constants that we see here, they are specific for a particular material ok.

algorithm to find the roots of the silencer eigenequation (necessary in the AMM method), and the number of modes required to obtain an accurate solution is relatively small. Accordingly, the number of degrees of freedom required in the (transverse) finite element mesh is relatively small for the NMM method, even for an asymmetric cross-section. However, what is perhaps surprising is that the new HFE method also provides relatively fast solutions. Here, if one is interested only in generating transmission loss predictions accurate to one decimal place, then the HFE method is as least as fast as the AMM method, and in most cases faster. This is because, in addition to avoiding the need to use iterative root finding techniques, the HFE method delivers a banded symmetric matrix and this facilitates fast matrix inversion despite the increase in the number of degrees of freedom. Accordingly, predictions can be generated quickly when using the HFE method and the normal assumption in the literature that analytic equals fast, and numerical equals slow, are not necessarily true, at least for automotive dissipative silencers. Therefore, for more complex silencer geometries the flexibility of the HFE method becomes attractive and for silencers that include, say, inlet and outlet extensions, a numerical approach can readily be applied in the knowledge that this will not necessarily be slower than the more usual analytic approach.

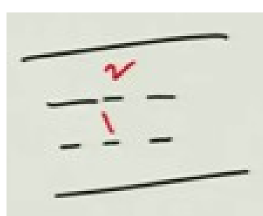
It is shown here that automotive dissipative silencers that contain mean flow and a perforated pipe can be modelled accurately and even when using numerical methods these models can run very quickly on a desktop PC. Accordingly, such models can readily be applied in an iterative design environment and from the results presented here, the most efficient technique for a uniform dissipative silencer is a numerical mode matching technique.

References

The point is that based on the type of muffler configuration that one is analyzing so, suitable thing can be found out, a suitable perforated impedance can be found out and used.

$$Z_{p=}\left[0.06 + jk_0\left\{t_w + 0.35d_h\ \left(1 + \frac{Y_w k_0}{Y_0 k_w}\right)\right\}\right]/\sigma$$

So, for our cases, since in this, we can possibly you know in the next week, we will be doing reactive mufflers without any dissipative thing. I think we will probably use the expression given by Elnady. So, let us now it is a, I guess it is a time now to get back to some muffler analysis using MATLAB and get back to; get back to the presentation.



So, we already have got

$$\frac{d}{dz}[\Delta]\{X\} = [A]\{X\};$$

we saw that, and X was our state variable for two duct interacting system something like this ok, we had such a system ok. What do we need to do now from here? We need to basically, we

$$\{X\} = \begin{Bmatrix} \tilde{p}_1 \\ \rho_0 C_0 \widetilde{U}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \widetilde{U}_2 \end{Bmatrix}$$

So, we need to do B inverse A and get the C matrix wasn't it? We did B inverse A times B inverse A and

$$\frac{d}{dz}\{X\} = [B^{-1}][A]\{X\}$$

So, this guy was your C matrix.

$$[A] = \begin{bmatrix} 0 & -jk_0 & 0 & 0 \\ -jk_0 & 0 & \frac{1}{d_1\zeta} & \\ 0 & 0 & 0 & 0 \\ \frac{4d_1}{(d_2^2 - d_1^2)\zeta} & 0 & -jk_0 - \frac{4d_1}{(d_2^2 - d_1^2)\zeta} & 0 \end{bmatrix} \begin{matrix} M_1 \\ C_1 \\ M_2 \\ C_2 \end{matrix}$$

$$X\{0\} = [T]X[L]$$

$$expm\,(-[C]L)$$

$$[B] = \begin{bmatrix} 1 & M_0 & 0 & 0 \\ M_0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad B = I$$

Now, one thing that we realize that the because of the particular rearrangement of the equation from the right-hand side, we put the continuity equations first and momentum second and so on so, it led to a different form of the B matrix. So, we kind of slightly rearrange that momentum equation is the first row now, continuity equation is second row and so on.

So, with this kind of a small rearrangement, it is easy to see that when M naught is 0 that is stationary medium, we get B as I, identity matrix which is very obvious and now, when M naught is not 0, then we need to invert that matrix and that is what we are doing here ok.

So, the point is now with this understanding that we have got this equation, we will sort of use the exponential matrix to get the finally, the form that we talked about that is your {0} to state variable at X [T] matrix times X [L], we talked about this thing, where your T matrix is really your exponential M times minus C L, it just becomes all the more easy.

Now, once we know your perforate length L that is overlapping of L perforate length L and diameter of the perforate pipe and the annular cavity on just the cavity d_2 and the suitable and the frequency at which we are solving, it is a particular perforate model, we can sort of easily put in the expressions and go about doing that.

So, we will get, we will now go to the MATLAB code where the code is already written and I will explain the stuff to you and then, run some codes and run the codes with different extension lengths LELB and get different curves and see how do we reduce to the case of a concentric resonator and we will talk about that in the next lecture.

So, thanks for attending.