Muffler Acoustics - Application to Automotive Exhaust Noise Control
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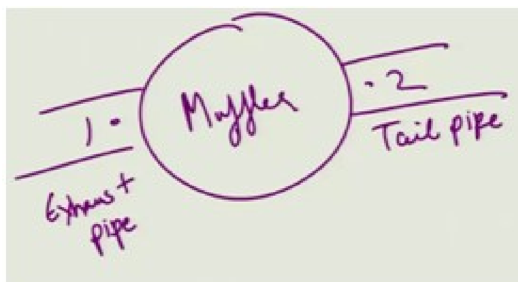
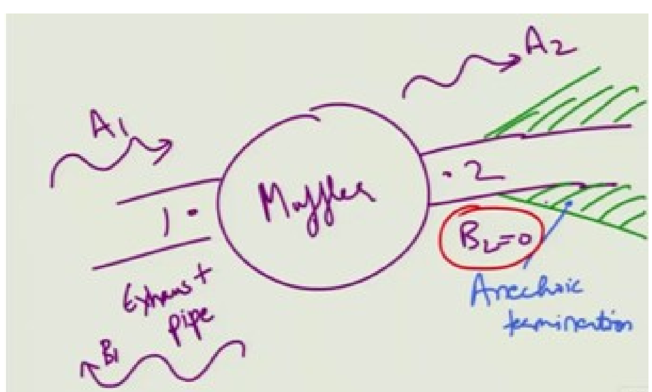
Lecture - 24
TL analysis of Extended-Inlet and Extended-Outlet Muffler (MATLAB)

Welcome to week 5, lecture 4 of the NPTEL course on Muffler Acoustics. So, if you recall the in the last week, we just about to derive the transmission loss of a generalized muffler elements in terms of the four pole parameters or the T matrix parameters. So, the idea was that once we have a general expression for transmission loss in terms of the T parameters T_{11}, T_{12}, T_{21}, T_{22} and so on.



Then, we can probably consider the special case of extended inlet and outlet mufflers and go about doing the transmission loss analysis. So, to this end what we will do is that basically put. This is what we are going to get.

$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{V}_2 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} \tilde{p}_2 \\ \tilde{V}_2 \end{Bmatrix}$$



Now, recall that actually when we consider such an element, we have your, we have the implication or the I am sorry the implementation of anechoic boundary conditions to for the transmission loss setup. So, you have this kind of a thing. So, we have anechoic termination. So, the waves let us say the waves A_1 wave is incident here and d_1 goes here and A_2 goes here; but B_2 is not allowed to come back because it is an anode termination. So, you get $B_2 = 0$. This is what we are going to get.

$$\tilde{p}_1 = A_1 + B_1, \quad \tilde{V}_1 = \frac{A_1 - B_1}{Y_1}$$

And obviously, we will do some special cases, where the exhaust pipe and tail pipe have the same diameter that is to say they have the same characteristic impedance and

$$\tilde{p}_2 = A_2, \qquad \tilde{V}_2 = \frac{A_2}{Y_2}$$

So, what we are going to do is basically substitute these expressions here and then, worry about what happens next.

$$\begin{Bmatrix} A_1 + B_1 \\ \dfrac{A_1 - B_1}{Y_1} \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} A_2 \\ \dfrac{A_2}{Y_2} \end{Bmatrix}$$

So, you get 1 here. Let us call this equation 1 and what about the other one?

$$A_1 + B_1 = \left(T_{11} + \frac{T_{12}}{Y_1}\right) A_2 \qquad (1)$$

So, what we probably could do is simplify this further and to that end, we multiply both the sides by Y_1. So, remember there was a Y_1 here.

$$A_1 - B_1 = \left(Y_1 T_{21} + Y_1 \frac{T_{22}}{Y_2}\right) A_2$$

$$A_1 - B_1 = \left(Y_1 T_{21} + \frac{Y_1}{Y_2} T_{22}\right) A_2 \qquad (2)$$

$$A_2, B_1 = f(A_1)$$

So, we multiply both sides by Y_1 to get the following, which can be nicely simplified or written in a more cleaner manner by the following expression. This is 2. So, obviously, we need to find out, what do we need to do? We need to find out A_2 and B_1 as a function of some function of A_1.

So, it is simple. Add 1 and 2 to get,

$$2A_1 = \left(T_{11} + \frac{T_{12}}{Y_2} + T_{21}Y_1 + T_{22}\frac{Y_1}{Y_2}\right)A_2$$

Because when you add this one, equation (2) and this one plus B_1 and minus B_1 gets cancelled and you are left with only A_1 and what we get is basically T_{11}.

So, we get a nice clean expression T 11 and the contribution from this side is T 12 divide by

$$\frac{A_1}{A_2} = \frac{\left(T_{11} + \frac{T_{12}}{Y_2} + T_{21}Y_1 + \frac{Y_1}{Y_2}T_{22}\right)}{2} \qquad (3)$$

$$= 20log_{10}\left|\frac{1}{2}\ \left(T_{11} + \frac{T_{12}}{Y_2} + T_{21}Y_1 + T_{22}\frac{Y_1}{Y_2}\right)\sqrt{\frac{Y^2}{Y_1}}\right|$$

So, remember for getting the transmission loss, all we need to do is basically evaluate the following;

So, what does it tell you basically? It tells you that the ratio of the incident wave amplitude to the amplitude of the wave that propagates downstream into the anechoic termination is nothing but sum of the T matrix parameters and obviously, some of the parameters are multiplied by the characteristic impedance or the ratios and half of and multiplied by a factor half.

So, all we really need to know for getting the transmission loss is really this guy A_1 / A_2. Why?

$$TL = 10log_{10}\left|\frac{W_{inc}}{W_{trans}}\right|$$

Because if you recall the discussions in the last several lectures, last week I guess, this was transmission loss was nothing but ratio of the acoustic power that is incident to that of the acoustic power transmitted downstream. So, this can be very nicely

$$= 10\ log_{10} \frac{\frac{|A_1|^2}{2\rho_0 Y_1}}{\frac{|A_2|^2}{2\rho_0 Y_2}}$$

So, we get this sort of a thing. So, what it means

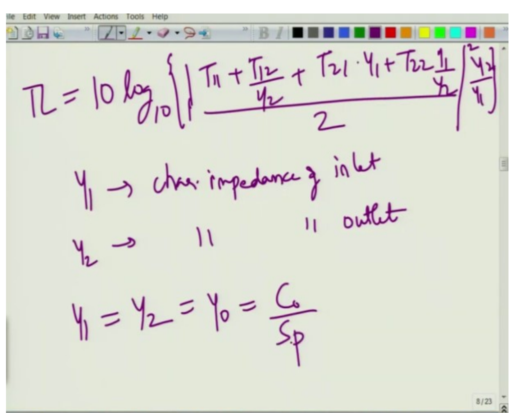
$$TL = 10\ log_1 \left|\frac{A_1}{A_2}\right|^2 \frac{Y^2}{Y_1}$$

$$= 20\ log_{10} \left|\frac{A_1}{A_2}\right|^2 \sqrt{\frac{Y^2}{Y_1}}$$

Now, basically everything reduces to the substitution of well let us say equation 3 in the above expression. So, what do we get then? We get

$$= 20 log_{10} \left| \frac{1}{2} \left(T_{11} + \frac{T_{12}}{Y_2} + T_{21} Y_1 + T_{22} \frac{Y_1}{Y_2} \right) \sqrt{\frac{Y^2}{Y_1}} \right|$$

But remember, you also have root over Y_2/ Y_1 sitting here. So, this would be somewhere here and the entire thing mod. Let us quickly see what are the special cases that these expressions can take if you go about simplifying this, what are the special cases that we can get.



So, one way of writing the entire thing would be

$$TL\ 10\ log_{10}\left\{\left|\frac{T_{11}+\frac{T_{12}}{Y_2}+T_{21}Y_1+T_{22}\frac{Y_1}{Y_2}}{2}\right|^2\frac{Y_2}{Y_1}\right\}$$

and the idea is to eliminate that square root factor. What we probably could do is get this thing in the following form.

Now,

$$Y_1 \rightarrow \text{ characteristic impedance of inlet}$$

$$Y_1 \rightarrow \text{ characteristic impedance of outlet}$$

So, what it probably means is that if we have

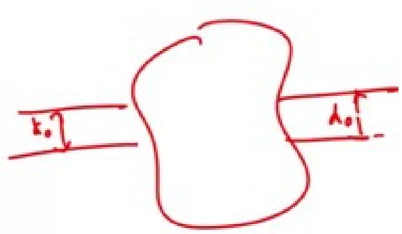
$$Y_1 = Y_2 = Y_0\ \frac{C_0}{S_p},$$

then things would simplify and let us call this as Y_0 as we were doing it for the extend inlet and extend outlet element.

So, under this situation, transmission loss then becomes 20 times log to the base 10 and this would be Y_0 Y_0 and this would cancel out and so, would this term or probably this term would be unity, this term would be just be 1.

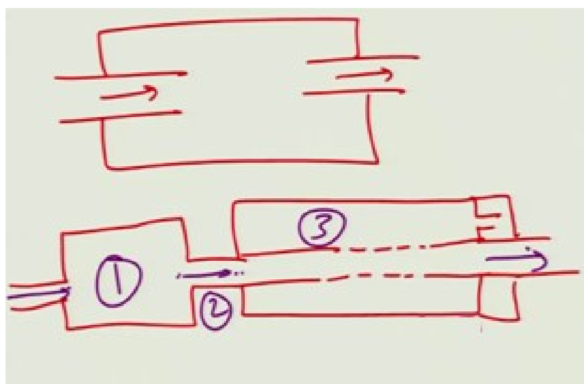
$$TL\ 20\ log_{10}\left\{\frac{T_{11}+T_{22}+\frac{T_{12}}{Y_0}+T_{21}Y_0}{2}\right\} \qquad (4)$$

So, we get this sort of a thing for the transmission loss of a, any general element; but with the condition that inlet, this is d_0 and this is also d_0. Inlet and outlet are equal.



So, we have once we have this condition, then of course your things would simplify a bit and probably as we go deeper into the analysis of such expressions, transmission loss, we can actually show that transmission loss and insertion loss are equal in the limiting case of the radiation impedance or basically, the instead of exposing the outlet pipe or the tail pipe to the atmosphere is that way to be the if that were directly fitted to the anechoic termination, insertion loss will tend to the transmission loss under the limiting conditions of anaerobic termination.

So, we can easily show that and actually, there are large number of things that we could probably study here; one of them being if we have, look this is a generalized expression in terms of you know general T_{11}, T_{22} and all these T parameters also known as the four parameters like I was mentioning. But although, we will revert back to the extended inlet and extended outlet case in a while.



But what I am trying to tell you guys here is that if you have a general element like this a simple expansion chamber or yeah and you certainly have a bigger chamber like this, but with things like extensions or maybe with perforates, we have not really discussed that element now.

But perhaps you could do and actually, you could have a thing like small sorry small slit here and this goes here and there could be a resonator something like this, you could have all sorts of complication different elements.

So, with understanding that there is a uniform unidirectional propagation of acoustic power. So, well, under such a situation, what we really get is that the transfer matrices can be cascaded that is to say let us say this is the transfer matrix for this element is 1, this is element 2. Because now, this length becomes important although it is uniform because its connecting to two different muffler elements.

So, and this is 3 and so on and we have transfer matrix between this point and this point, this point and this point, this point and this point; then, it is

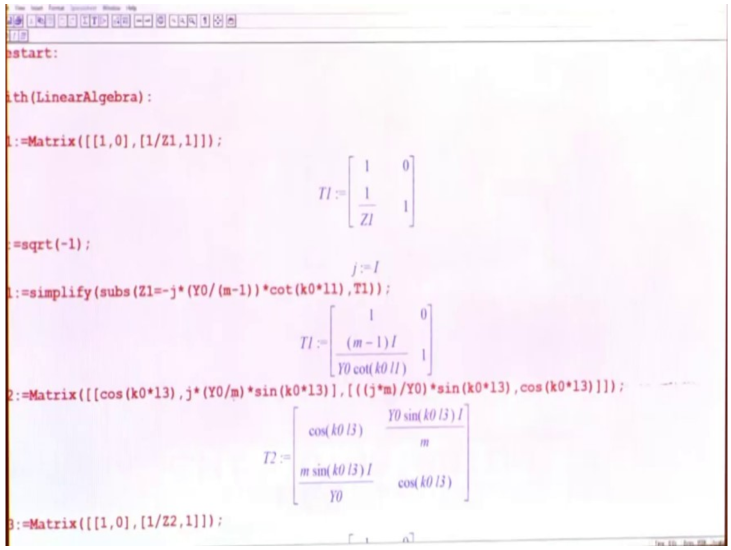
$$T = [T_1]\,[T_2][T_3].$$

So, we have this sort of a thing. So, once we obtain the overall transfer matrix by sequentially multiplying the transfer matrices of individual elements, we can use possibly the generalized expression like this.

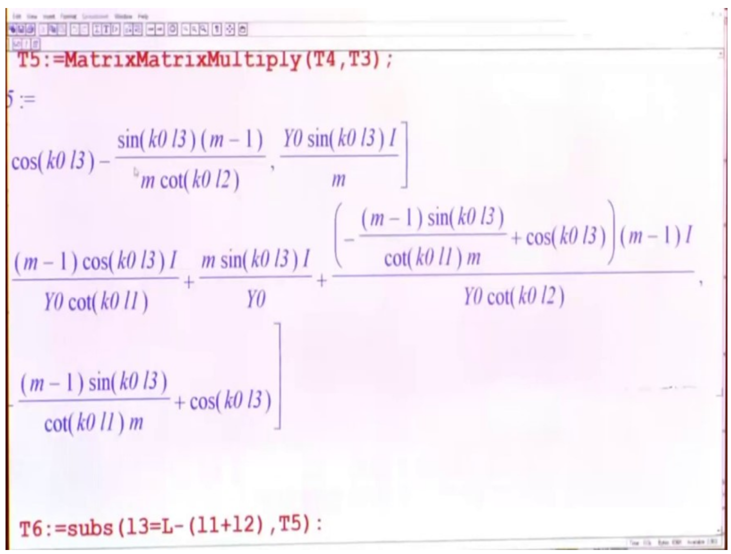
The one here let me call this as equation (4), to find out the transmission loss of the overall system ok. That we can do. But remember one thing, all this while we have not really considered the effects of mean flow which have which is important in lot of practical applications. So, we have to account for the convective effects of mean flow in the expression for evaluating the transmission loss and that would also show up in the T matrix parameters.

But for now, what we will do in the remaining part of this lecture, lecture 4 that is that revert back to the test case of our element that we promised to study here, the transmission loss.

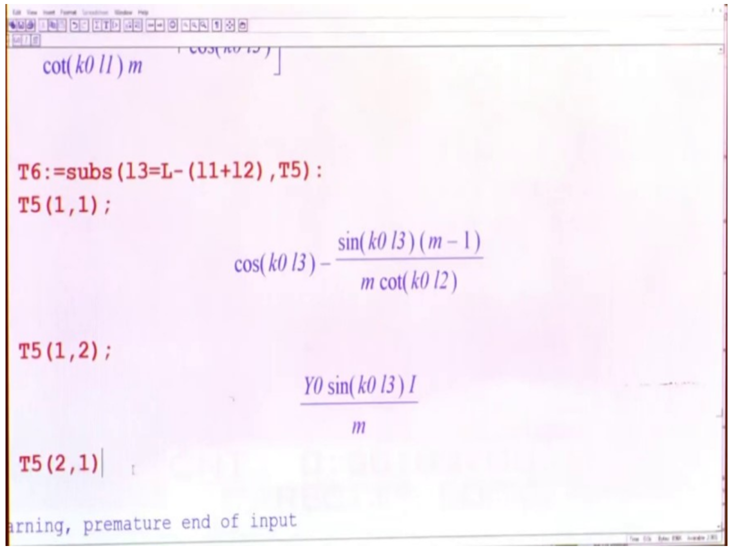
So, let me quickly go to maple and probably show you some expression and we can first we will discuss some qualitative features; why are we getting or why should we anticipate a peak at certain frequencies and probably relate to the troughs or the resonance frequencies of the chamber in all. So, let us go to maple then.



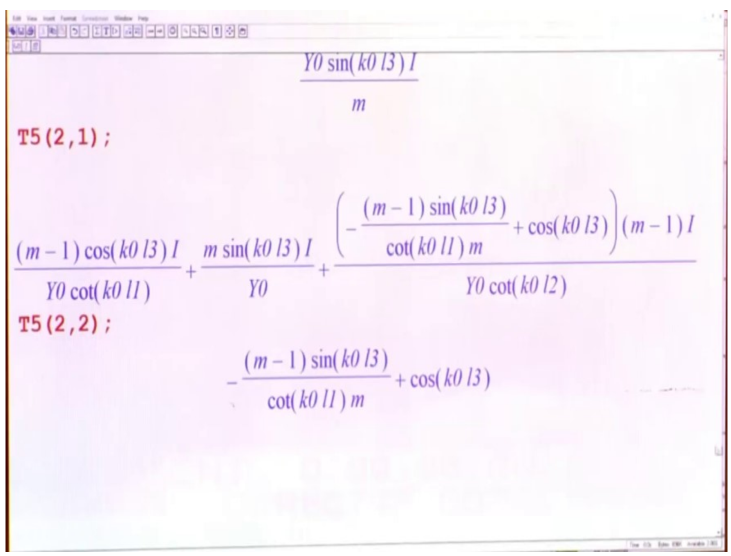
We are in maple, what we were discussing in the last in the last class, last lecture.



Look at the individual elements. So, you know the one that is highlighted here, I am so sorry. Let us say T5.



So, if I type T5 (1, 1). Well, we get this. Now, we need to every time, we open a maple script, maple file; we need to restart or re-run it again so that all these expressions the ones that are validated before this particular expressions are also evaluated and known to the function or to the program. So, now, we get this T11. Let us nicely write down all the T matrix parameters sequentially; T12 is like this.

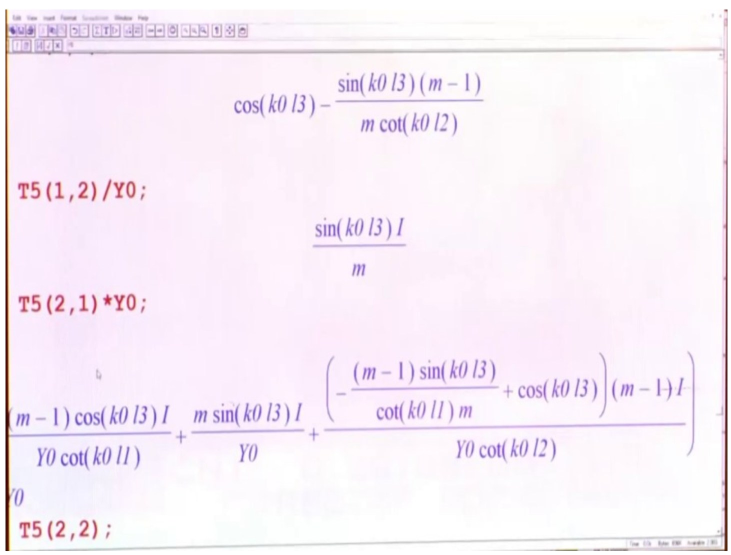


And T5 (2, 1) is like this; this is a bit nasty expression and, but nevertheless important. So, each of these four pole parameters are there in front of you. Now, based on the transmission loss expressions that we just saw just now, it was basically a summation of T11s and T12, T21, T22 obviously scale some of the expressions are scaled by a certain factor. Now, it does not matter those were basically your Y_0 terms. I Y_0 of the inlet and outlet pipe.

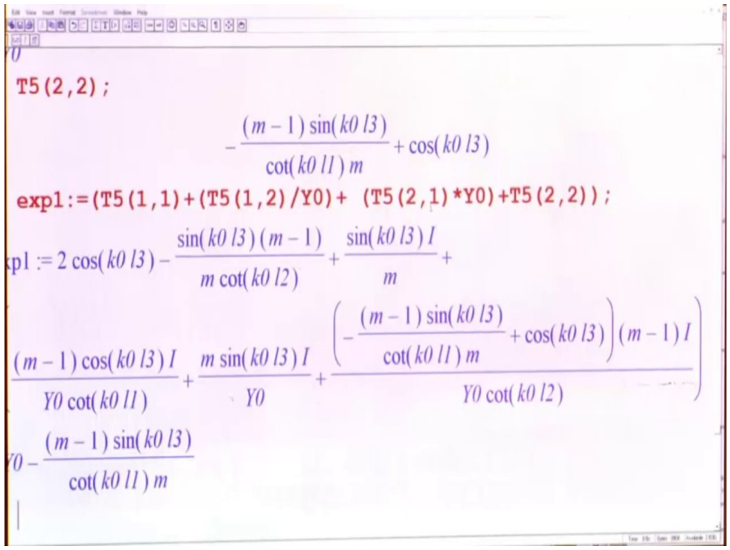
Now, let me go back to the slide here. So, here you are you have your T_{11} term, let me underline with the orange color T1, T_{22} terms and T_{12} is divided by Y_0 which is your characteristic impedance of the port and while T(2,1) is multiplied by Y_0, what does it mean? Let me get back to the maple script. Now, recall T5(1, 2), this is your element T(1,2) and it was divided by Y_0 and in the numerator, we have Y_0.

When you basically multiply this out completely, Y_0 will tend to cancel out, leaving us with only the sin k_0 l 3 term divided by m into J or I ok. T5(2, 1) or T(2,1) is basically has a contribution of Y_0 in the denominator and in the transmission loss expression, it is multiplied by Y_0.

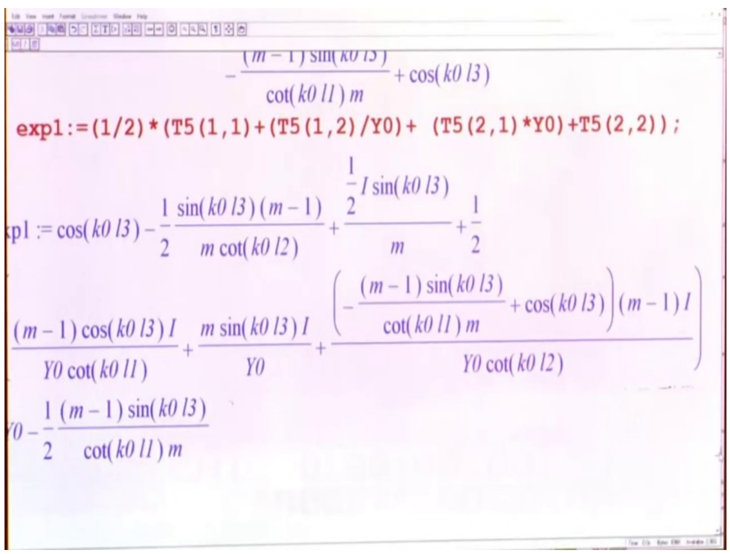
So, this term will eventually cancel out. All this will sort of cancel out and T22 probably will not have such a problem. So, basically the idea is just a summation of these expressions, when in the transmission loss when you sort of do that.



So, if we were to simply divide this by Y_0 and we need not ask it to simplify; but and similarly, we can sort of multiply this by Y_0 and this is like this.



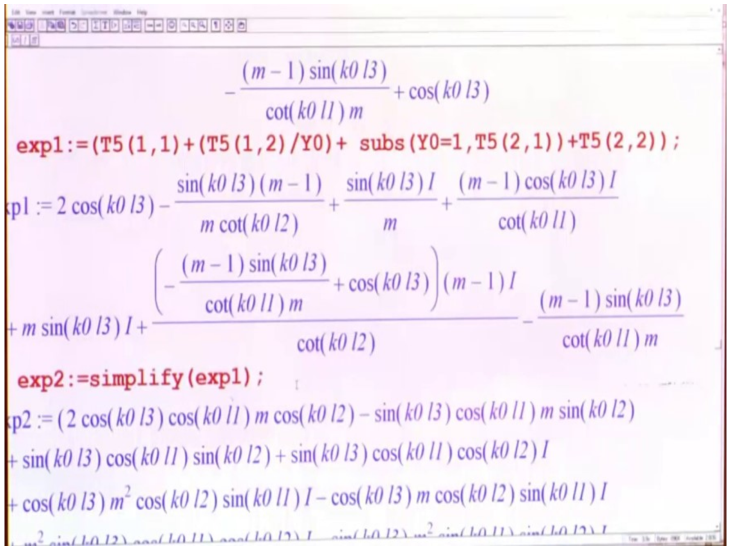
And let us call an expression exp1 and which is just nothing but the summation of T5(1, 1) and your each of these small expressions and this one. So, we get this thing.



And let us multiply this by 0.5 or half, probably a better thing would be multiply this by half and go about doing things. So, this is your expression that you get. This is eventually; we need to take the log of this tedious expression log of this reference to the base 10 and multiplied by 20. So, all those things are good.

But just looking at this expression, I guess we should be able to tell something about where the peaks would occur and whether they can annihilate the troughs. So, T5 to 1 multiplied by Y_0.

So, let me try to do something about this one; T5, this gets multiplied by Y_0. So, once you do that, we are getting Y_0 here. That should not really be the case. Possibly, we can get rid of half here and try to go about. So, Y_0 still occurs.



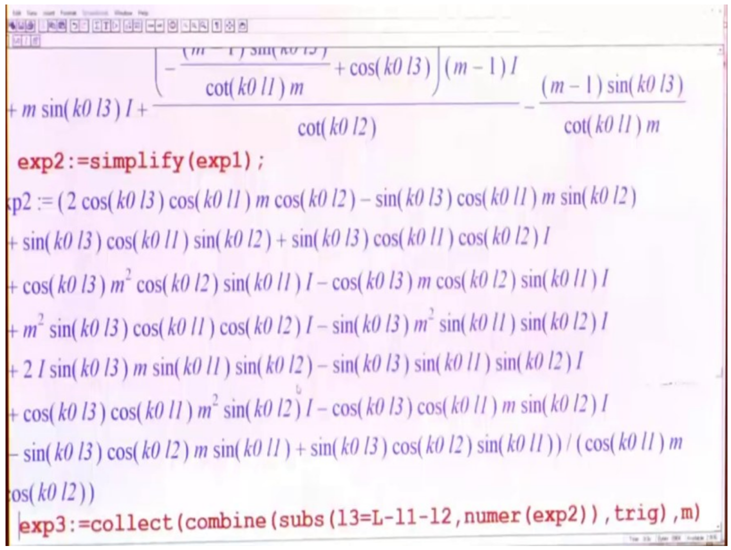
So, why is that? So, so basically what we intend to do is that simplify these expressions T5(1, 1); T5(1, 2); T5(2, 1); T5(2, 2) as much as possible and you know as I have been mentioning, we need to is basically the addition of these terms and they are being suitably scaled. So, one clever trick that we probably can figure out, look the thing is that this is this is multiplied by Y_0.

So, Y_0 is coming out here, but when you simplify things becomes even more you know it becomes a little more tedious.

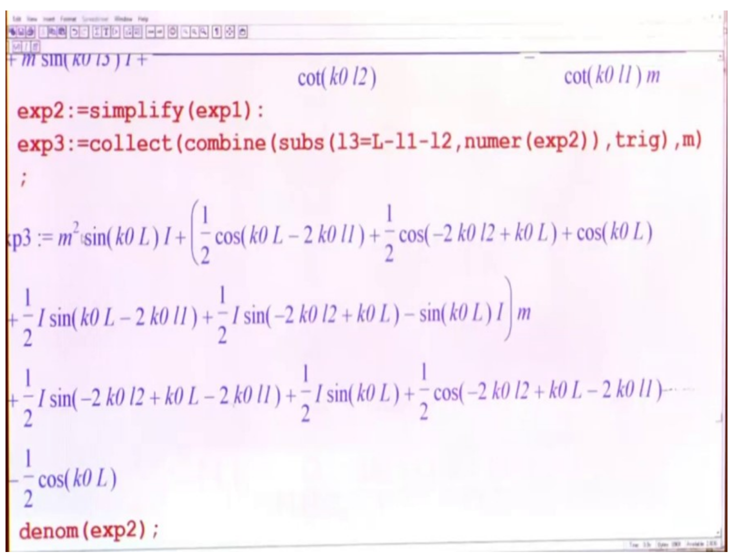
So, basically one clever trick of getting rid of Y_0 in the denominator, look this is bound to be cancelled the term in here and the term here, this will get cancelled. So, basically if we substitute Y_0 is equal to 1 and directly, add up all these terms. So, we will be getting expression 1.

So, this is the entire expression if we recall or the argument of logarithm in the transmission loss expression, obviously, there is a factor of half also which I have purposely omitted to keep things simple because all the dynamics, all the system behavior can be reflected by considering or simplifying these terms only.

So, what we do is that once we substitute Y_0 is equal to 1 here and simplify, we see Y_0 term and this thing has completely gone away and m is nothing but the ratio of the cross-section area of the chamber, without any extensions to that of the port. So, in other words, is the square of the ratio of the diameters. Now, we probably can simplify expression 1.



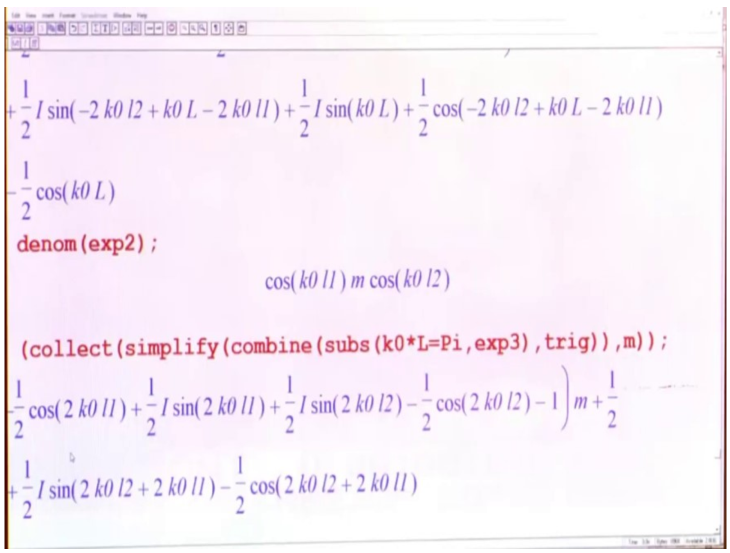
But then, you know a better way, we would be getting this thing. So, there is no point or is probably a little too much.



So, what we will do? Probably, put expression 3 here and substitute l3 = L - l1 - l2 that is to say we will substitute the intermediate length *l*3 as a difference of the neck extensions *l*1 and *l*2.

So, we will put that here and substitute that in the numerator; number here, what I am highlighting now, this term this is the numerator of the expression 2 which is step above. So, we substitute l3 = L - l1 - l2 and in this numerator expression that is this thing and combine trigonometric term because this is going to be a little tedious algebraically.

And then, collect terms in the order of m that is to say now if you basically have a look at this expression, we will get the following, where m square is the term associated leading order term associated with sin k_0 L that is L is the capital L is the chamber length and you get other terms. Denominator is much simplified, but very important which is cos (k_0 l1) m cos (k_0 l2).



Before we move ahead, we need to carefully analyze the numerator, simplified numerator part and the denominator part, where we have actually we now have only a few parameters or dimensions at our hand at our disposal namely l1, l2, capital L that is the chamber length, frequency of course k naught and the expansion chamber ratio m.

So, if we carefully now all this while I was saying that you know by putting the impedance right at the interface of the extended inlet and chamber junction and in extended outlet and the chamber junction, by setting the impedance is 0, we can get the attenuation; we can get the resonance of the annular cavities and that is where the peaks will occur based on the plane wave limit.

So, why is that? Because you know this transmission loss argument consists of this entire big expression divided by the denominator.

So, when the denominator is going to 0 that is cos (k_0 l1) = 0, we are getting the peaks due to the first annular cavity that is the extension peaks due to the extensions at the inlet. Similarly, cos (k_0 l2) =0, the highlighted term here would basically give you the attenuation peaks due to the extended outlet part. So, that is the formulas are now it should be very clear. It is your (k_0 l2) = 2 n + 1 whole into $\pi/2$.

So, you can evaluate all those resonance frequencies for the extended inlet as well as for the extended outlet. Now, important question here is that what do we do to tune the peaks, tune the peaks with the troughs of the first and second resonance frequencies of the chamber. So, for that we have to look into the numerator and there are some special cases of course. So, now, the first actual resonance or the trough due to the first actual resonance will occur when k_0 into L = π.

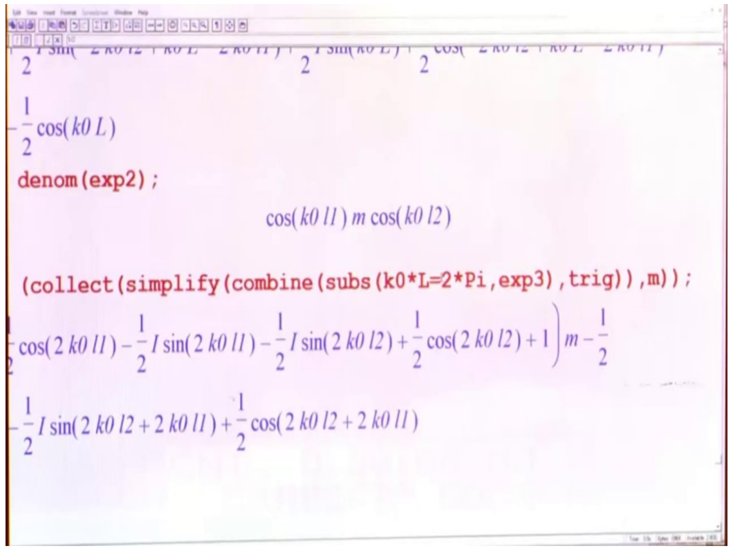
So, we substitute k_0 L = π in expression 3 which is the numerator given above here and combine again follow the standard procedure that is to combine terms using simple trigonometric identities and then, simplify and collect terms in the orders of m. So, after all this you know algebra which is fortunately done by maple, we do not have to worry about it. We get a much more simplified and a manageable expression.

So, ah special case of course, I would like to point your attention towards the special case, when you know when there is no extensions that is $l1$ is equal to $l2$ is equal to 0 that is it is a simple expansion chamber. So, what do we get? We get this is 1, this is 1 and this is just m, denominator is just m. Now, what happens to this thing at the first resonance frequency? We get this term is – 1/2 because this term goes to 0 and this term actually cancels out and so, does this term.

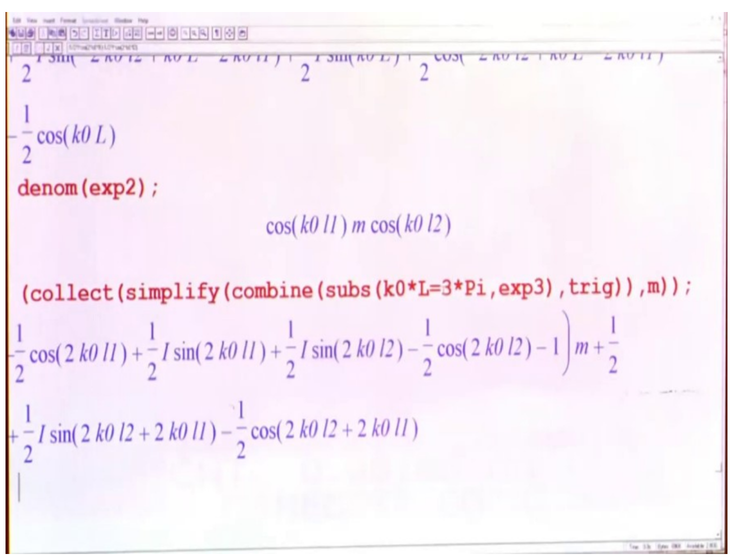
So, these two terms basically cancel out, the ones that is the highlighted here. But these two terms are minus half and minus half because cos of 0 is 1. So, minus half minus half minus 1. So, you get minus 2 times m and these two terms, this term will go to 0 again because sin of 0 0 and cos, this entire term will be minus half; but there is also half sitting here.

So, that will cancel. So, eventually, what we are left with is basically you know minus 2 m in the denominator, I am sorry in the numerator and m in the denominator. So, basically m m gets cancelled and you are just left with minus 2, but you are taking a mod of that. So, it is just 2 and in the denominator is also 2.

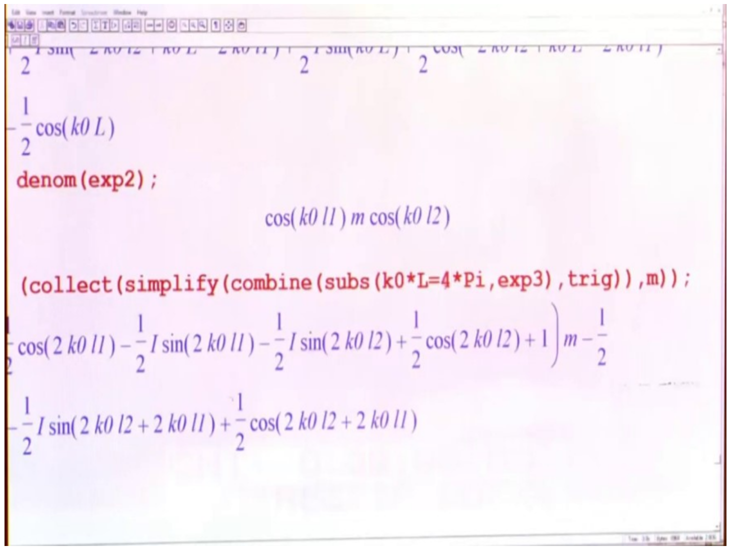
So, eventually, you are getting log of 12 to the base 10. So, that is how your at first actual resonance frequencies in the limiting case when $l1$ and $l2$ is tending to 0, we are able to again retrieve back the case of simple expansion chamber and that is important to validate our expression.



Now, let us see what we are getting here. If we substitute 2 π that is your second actual resonance peaks. Basically, you know the same argument would apply. Here, we are able to cancel these two terms, but these this term and this term will be half; half, half is 1; 1 plus 1 is 2. So, 2 times m, other terms will cancel out. So, 2 m and in the numerator m in the denominator. So, again the same logic log of 1 that is 0.



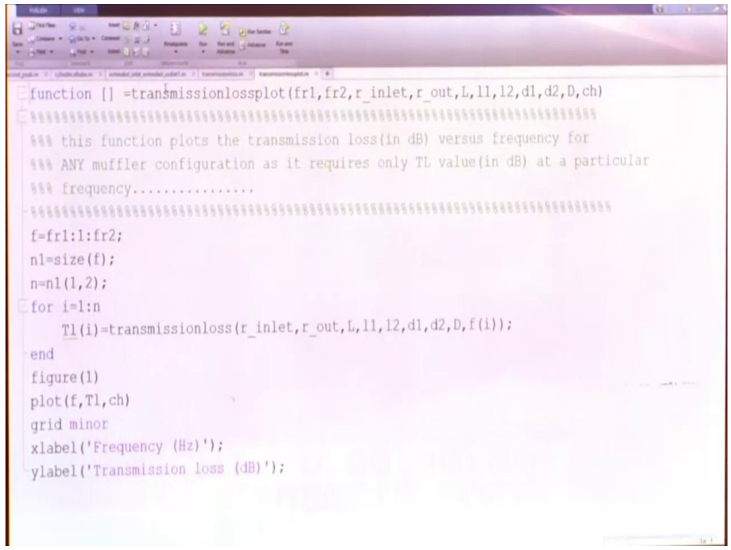
Like this, we can do it for any integer number 3; if we do we get the same thing.

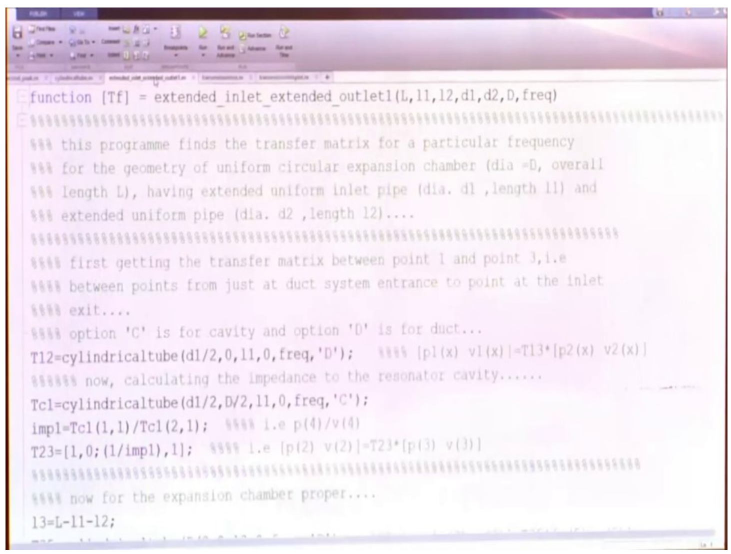


If we do it for 4, we again get half. Like this, we can keep on evaluating for the special cases. Now, it is a bit difficult or a little bit tedious to work out from just this expression that let us consider that *l*1 and *l*2 are non-zero in general, that is to say the chamber has a certain extension it need not be equal to L / 2. If it is L / 2 and L / 4, then you know it will be very interesting to see that when it is L / 2 here and this is L / 4 here and k_0 has a certain frequency.

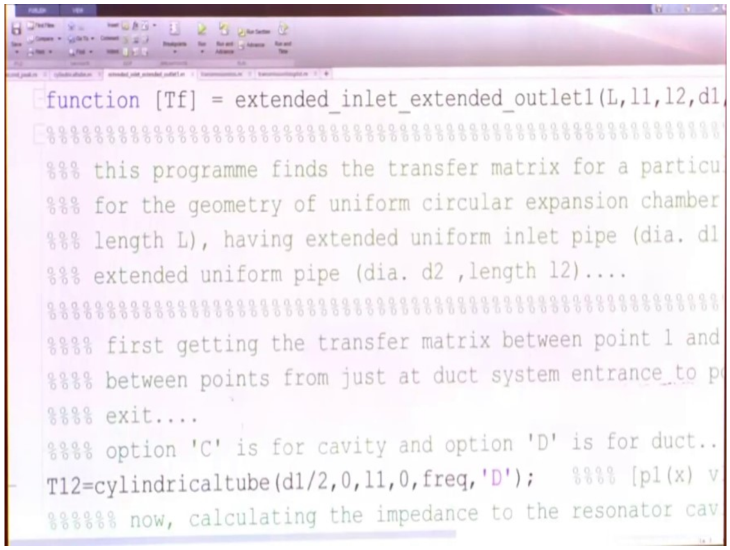
So, it will so happen that the numerator and the denominator will behave in such a manner that the transmission loss, the argument of the logarithm will never be equal to 1. If that is to say it will be sum-up, it will be a large quantity and that is why you get a peak there, a very sharp peak, transmission loss theoretically tending to infinity. So, that will happen when *l* 1 = L / 2, l 2 = L / 4 and for the frequencies π k_0 L = π or 2 π or 3 π.

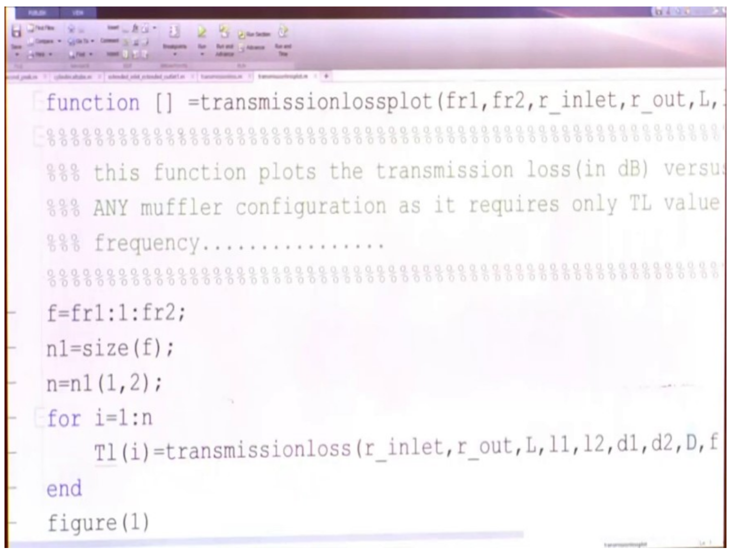
However, at k naught L = π, but *l* 1 is not equal to L / 2, you would see that theoretically you would be getting a peak. So, although it is a bit tedious to explain it using here, we have done decent in terms of explaining the basic behavior of the extensions, what extensions can do in terms of uplifting the trough.





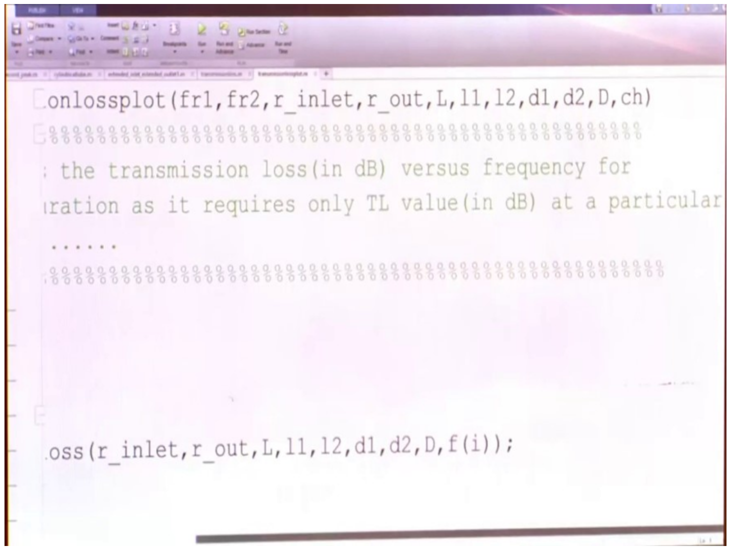
So, now, I guess is the time to go to a MATLAB code that was written here. Let me work out the MATLAB code.





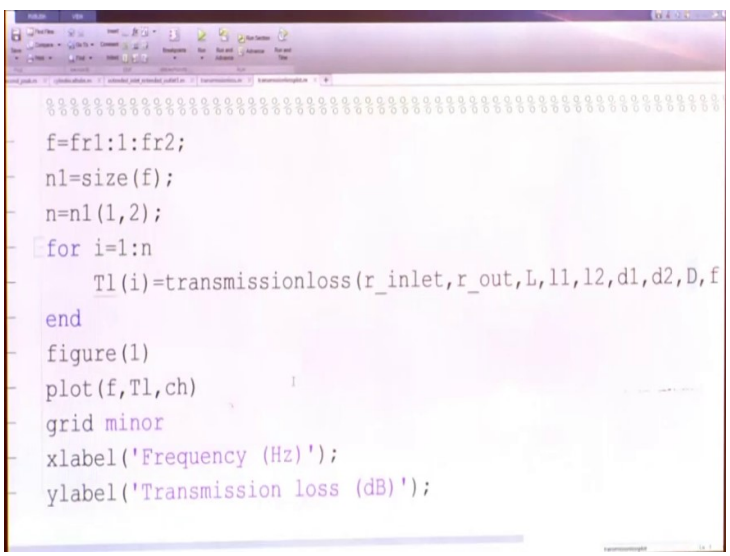
So, what we do basically let me explain to you very quickly, but in a sufficient details. We write MATLAB. MATLAB like Fortran or C is a programming language, especially useful for all scientific calculations. It is possible that a lot of you would be acquainted with MATLAB.

It is very easy to use language and lot of inbuilt commands are there, which makes a matrix multiplication or certain linear algebra operations very simple. So, the name of MATLAB in fact, its matrix laboratory, MATLAB.

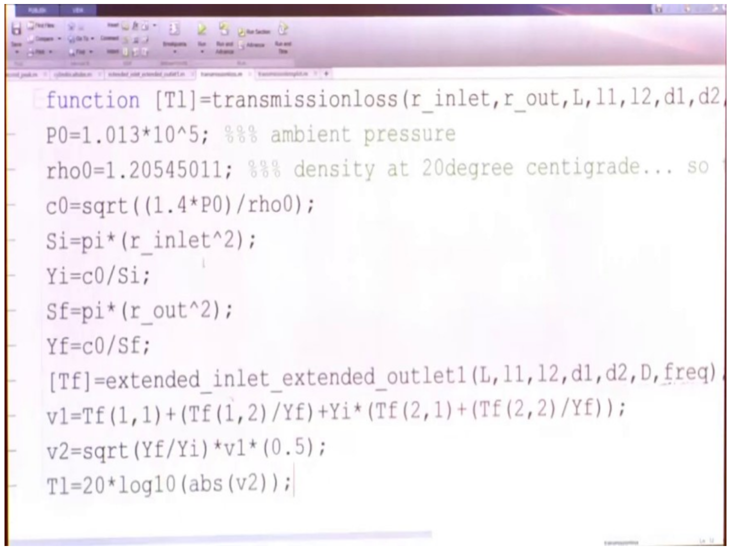


So, what we do? We write functions to evaluate certain quantities and each function will invoke the another function. So, let us do that. So, here we are writing a function to plot the transmission loss. Transmission loss plot that is the name of the function file.

It is calling two parameters; the upper and the lower limit of frequency. Inlet and outlet pipe diameters that is your port diameters or radius I am sorry, length of the chamber, length of the extension at the inlet that at the outlet and I guess d_1 and d_2 are pretty much the same as inlet and outlet respectively. d is the chamber diameter.

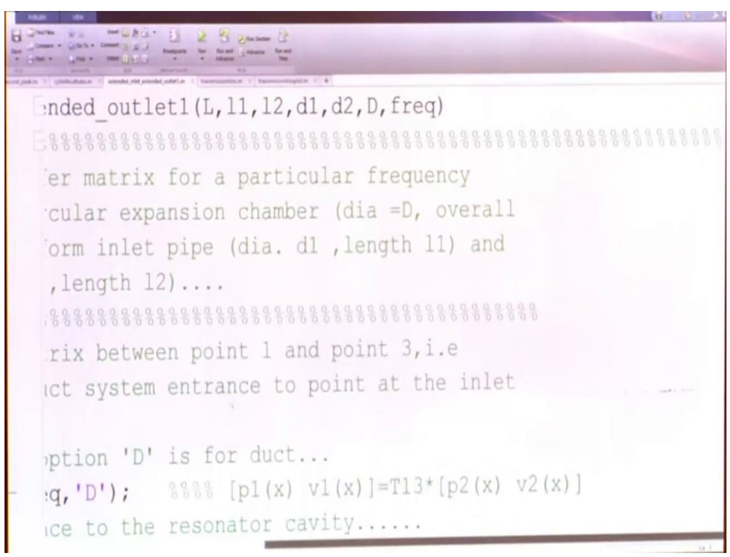


So, basically by suitably setting all this, we can get the transmission loss plot.

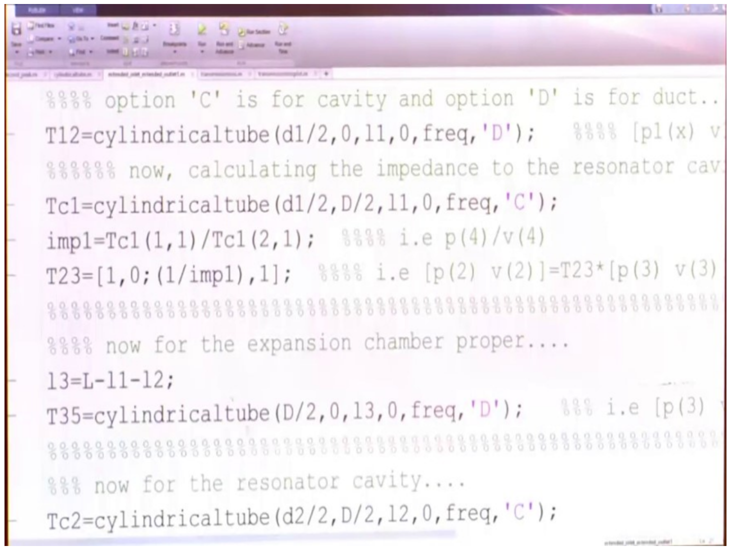


So, transmission loss, then its basically we assume a standard atmospheric conditions and density and all that. So, we evaluate the sound speed and inlet outlet pipe, characteristic impedance and then, there is a function file to evaluate the transmission loss sorry transfer matrix for the extension in inlet and outlet the that is basically based on cascading.

And this is the expression that we have been talking about all this while some of the T matrix parameters scaled by certain characteristic impedances. So, this is something that we have been discussing. So, I will quit this part and directly, move on to the function that computes the transfer matrix.

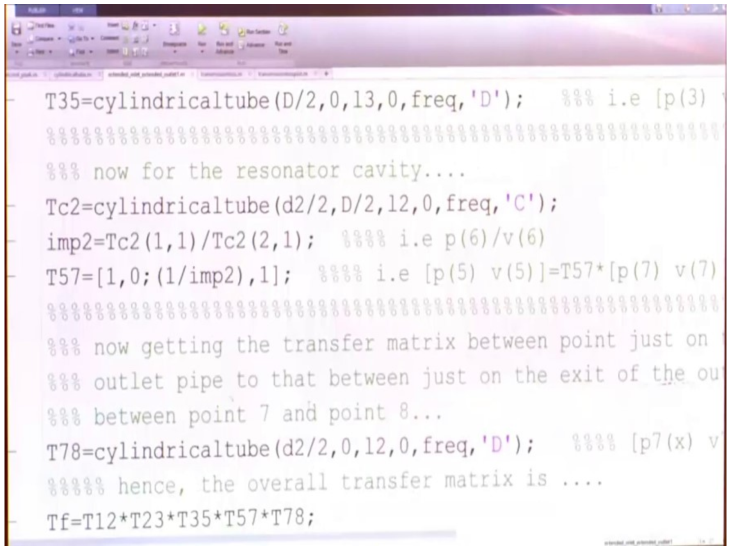


So, as usual, we have a length of the chamber extensions at the inlet-outlet diameters of the inlet-outlet pipe which is equal to the that of the r in r underscore inlet and r underscore outlet respectively.

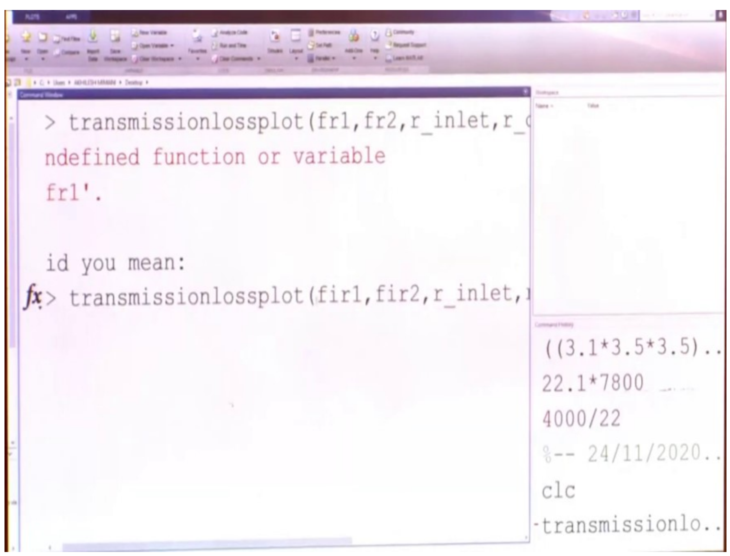


Frequency is a given frequency at which the transfer matrix parameters have to be evaluated. Remember transfer matrix parameters are functions of frequency. So, here is a simple x cylindrical tube. So, we know the transfer matrix for the portion of the inlet pipe, where this is evaluated and similarly, Tc1 is a cylindrical tube, where basically the chamber is evaluated.

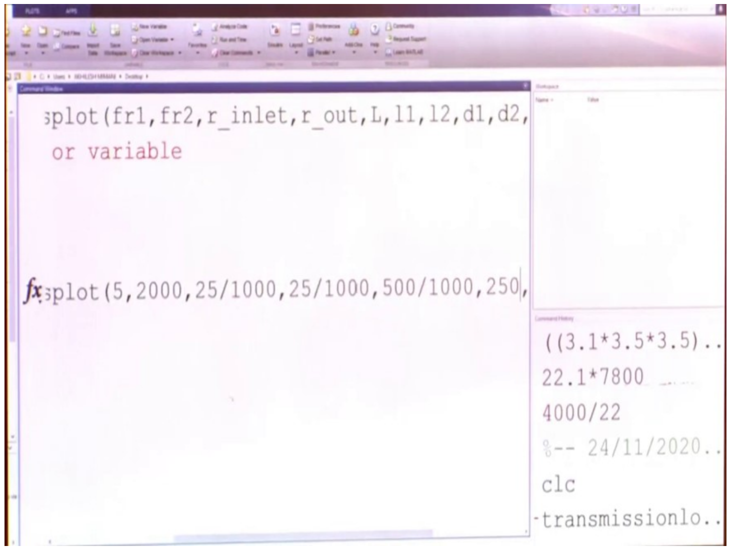
So, probably this for the cavity. I am so sorry. So, by doing all these by suitably multiplying the transfer matrices T 12 Tc1, so we evaluate the impedance here and T 2 3 is the transfer matrix at the sudden area expansion due to the extended inlet and l 3 is the length of the propagation length in which the protrusion is not there and we evaluate the transfer matrix for that portion.



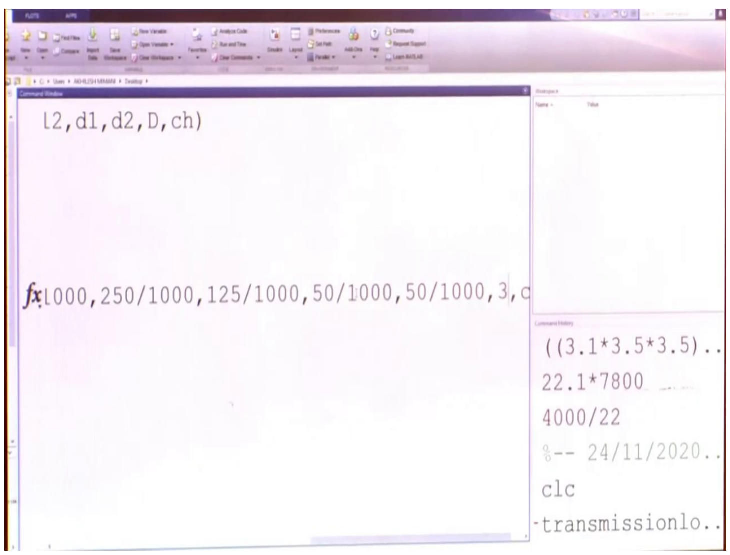
And this is again that of the cylindrical pipe and we are doing nothing but sequentially multiplying points 1 to 2, 2 to 3, 3 to 5 and all those sorts of arrangements. Here of course, we do something like evaluate the impedance at the extended outlet. So, then, this transfer matrix is basically passed on to this function and I guess we are in a good position to do some report some parametric studies.



So, what we will do basically is that we will basically write down certain things. Let me increase the screen space for you. So, let us consider some realistic dimensions.

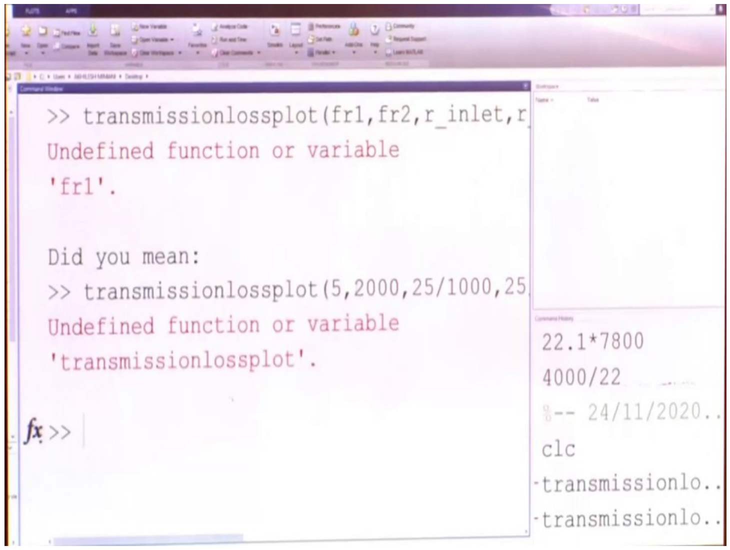


Frequency let us start with 5 Hertz and move on till you say 200 Hertz. It should be good enough the r inlet. So, let us say this is 25 mm / 100 because to convert mm to this thing. Length is 500. Let us say the length is 500, 500 mm and L is 250 mm.



*l*1 and this is 125, I guess and this is also let us say this is 50 mm because that was 25 and so was this and diameter of the chamber. Well, typically you know automobile mufflers have a diameter maximum diameter of say 300 mm or 350 mm max.

We will consider 300 mm. It can go from 150 to you know 300 or so and this is just the color in which the graph will be plotted.

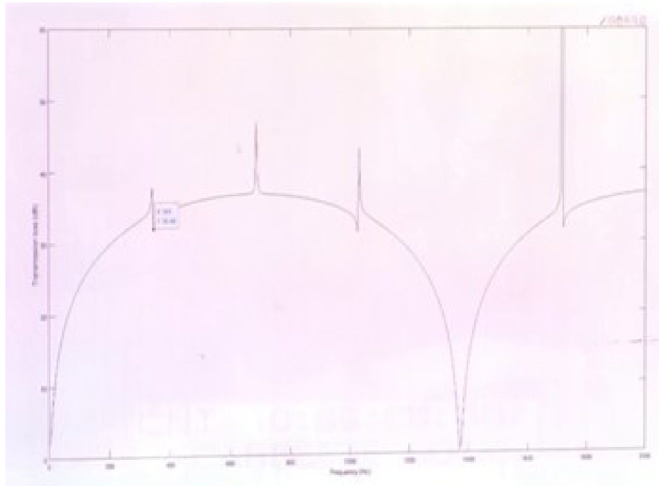


So, what we will do is that set the frequency parameters 5 Hertz is the big is the lower frequency limit, 2000 is the upper one. 25 mm, these are the two was at the inlet and outlet pipe extensions there.

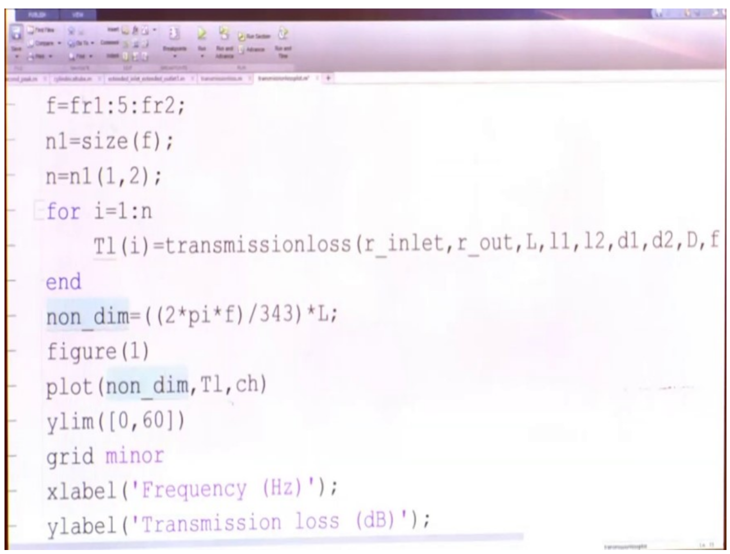
These are the same as the diameters of the inlet and outlet pipe. 250 mm is the length of the extensions at the inlet, extension of the inlet and 125 mm is the length of the extension at the at the outlet and 500 mm is the total length of the chamber. So, the intermediate length, where there is no protrusion is given by 125 mm sub.

Basically, capital L - $l1$ - $l2$ small and 50 mm each is the diameter of the extensions at the inlet and outlet and 300 mm is a diameter of the chamber which is typical of that of commercial automobile.

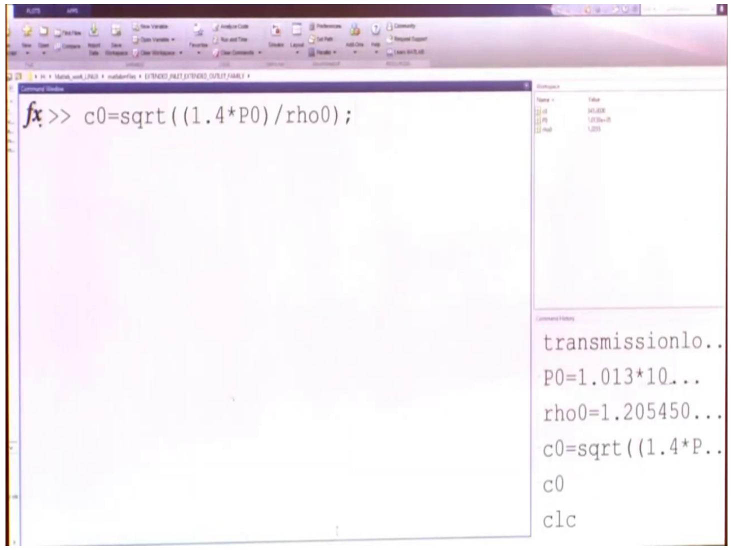
The maximum diameters that are used in commercial automobile mufflers is about 300 mm or so, in some cases, it might go up to 325, 350 mm and it ranges from well 150 to 150, 175 to all the way up to well 350 mm. So, we choose about 300 mm as the diameter of the circular muffler.



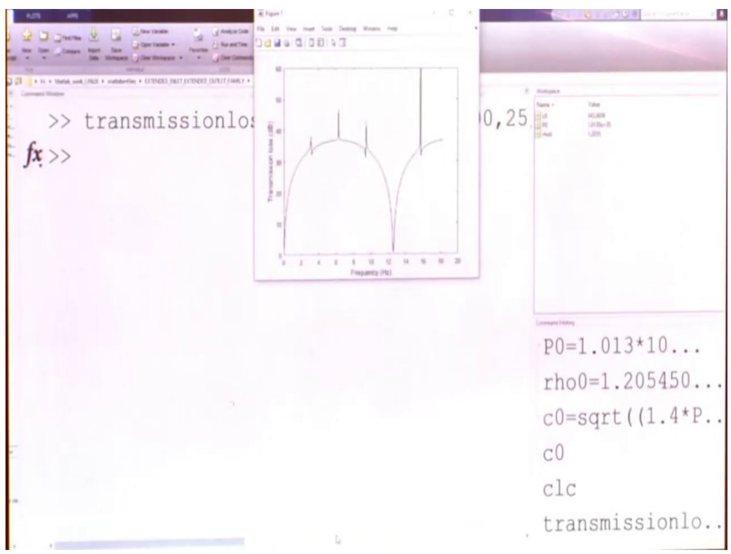
And let us see what we get. Well, we get the following thing. So, if we notice carefully, this was the frequency at which the trough would have occurred. So, you know we could actually do the following ah.



Just to make it make the explanation perhaps a bit better, what we could do is that we could quickly write some non-dimensionalized the frequency 2π f by sound speed which is 343. So, we will probably put it 343 times L. So, 2 π. So, non dimensional. So, non dime.

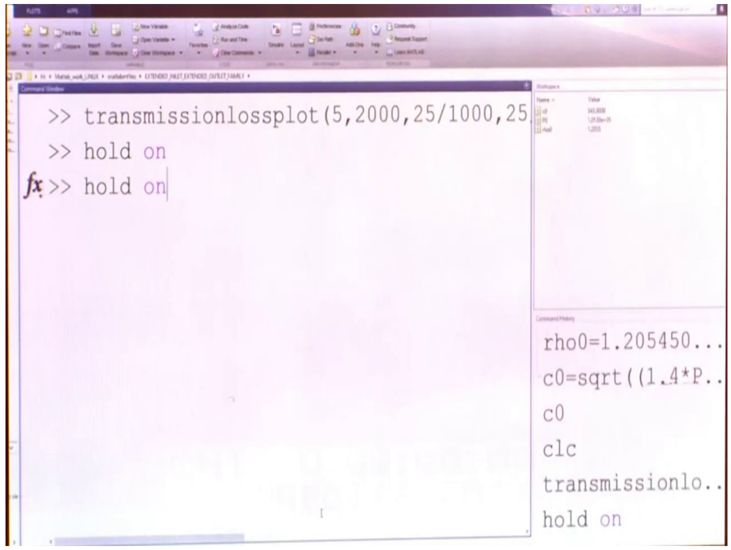


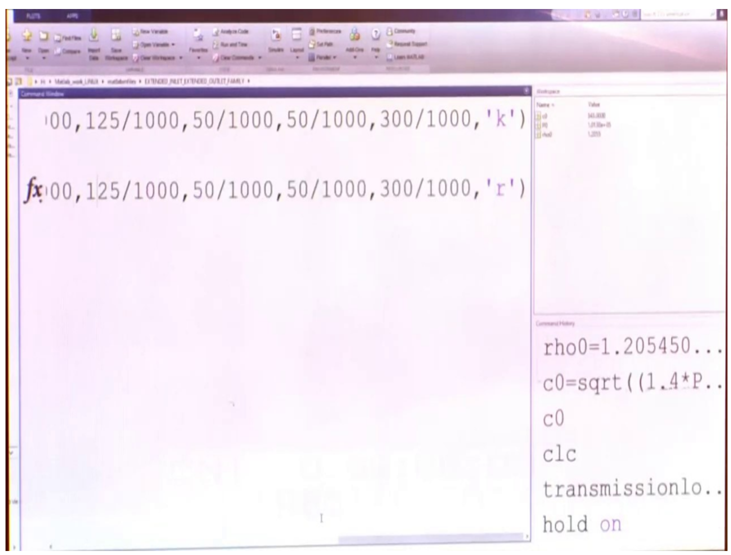
So, once we do that, let us see if our life is improved.

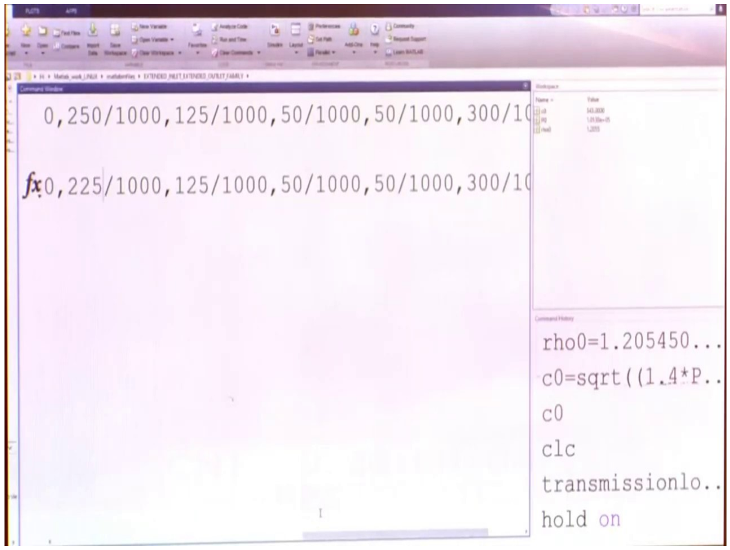


So, we get this ok. What do we see here? We see that if we if we go to the cursor thing and a mouse here, we get the peak almost at 3.14 k_0, L = π. So, that might just be because of frequency scaling. So, we get almost that we are able to almost kill the trough that would have occurred at k_0 L = π if then extensions was not equal to L / 2.

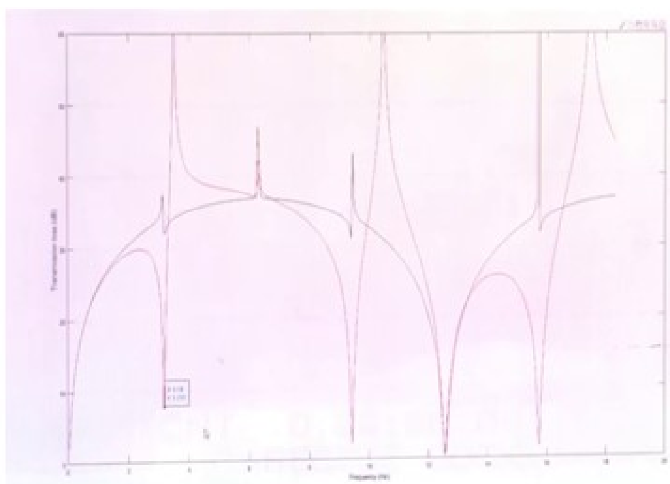
Similarly, if it was because of the fact that it is L / 4, the extension at the outlet we are able to kill the second thing which is occurs at 2 π that is 6.28 or something like that. So, we will what we will do is that we will keep this figure as it is both the extensions at the inlet and outlet have been tuned.

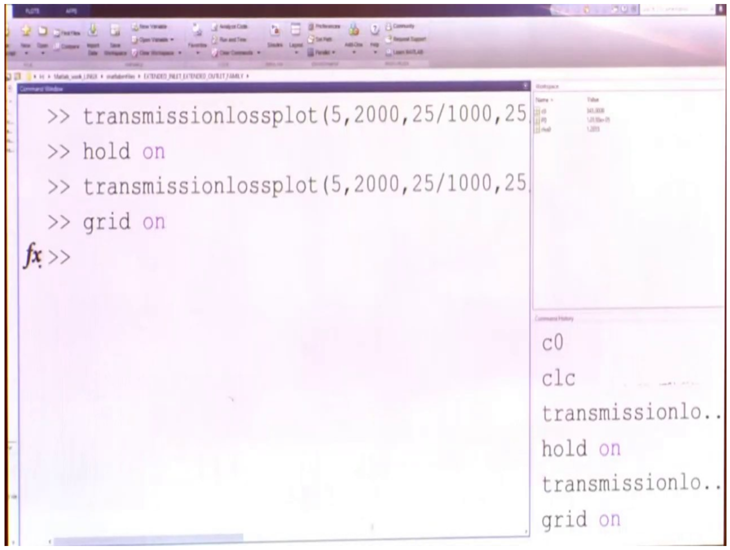




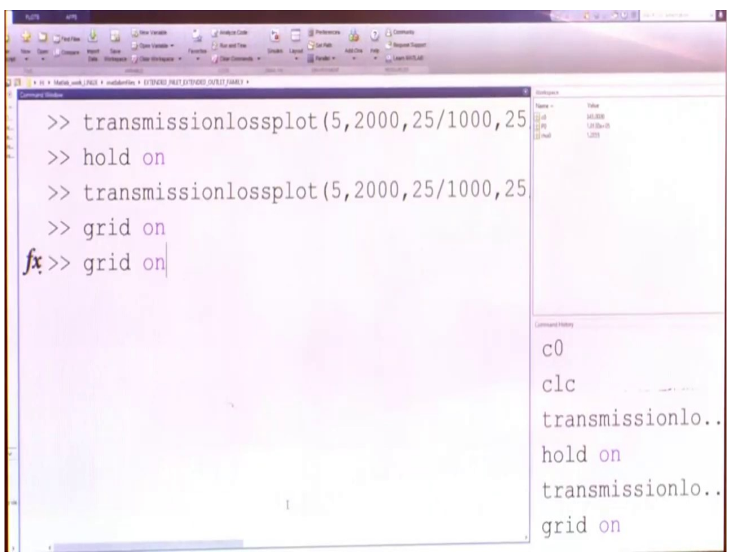


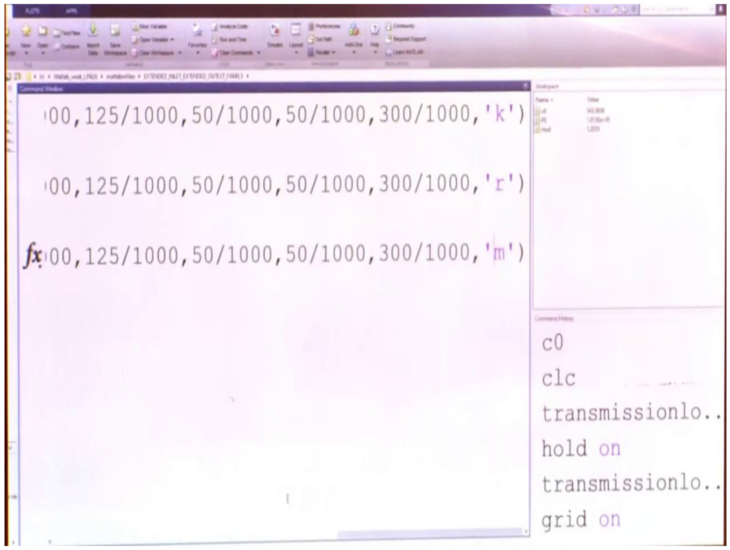
And we will basically, purposely miss tune or consider a non-tuned extension. Let us say we put 225.



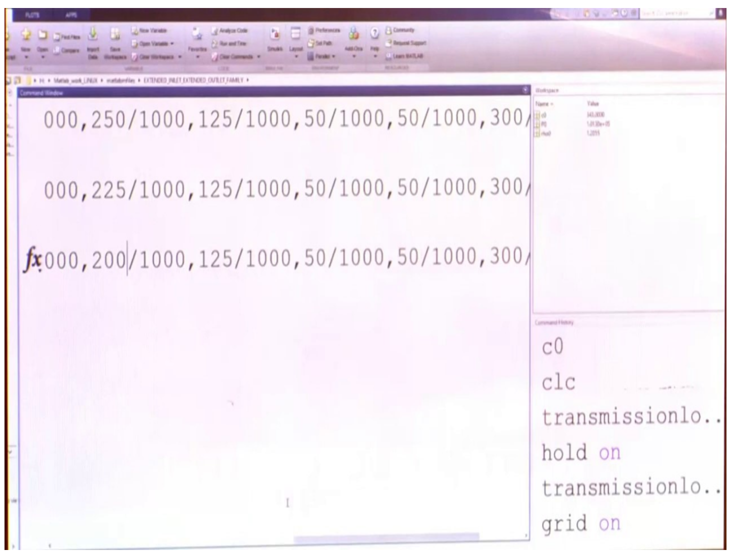


So, we see let us put grid on. So, we can look at the grids and so, you see, we are getting a trough here; we are getting a trough here. So, this trough occurs at well 3.14 close to that and that is because the length of the extension is less than L / 2. So, naturally based on the plane wave limit, the frequency thing is pushed up.

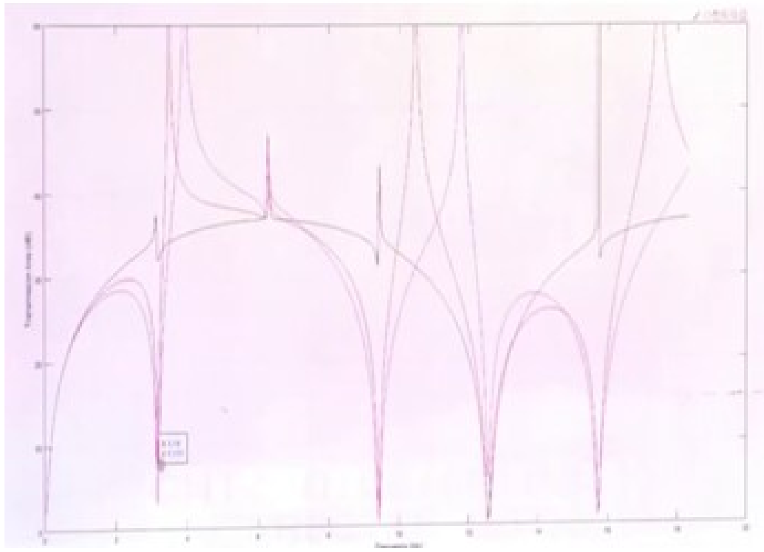




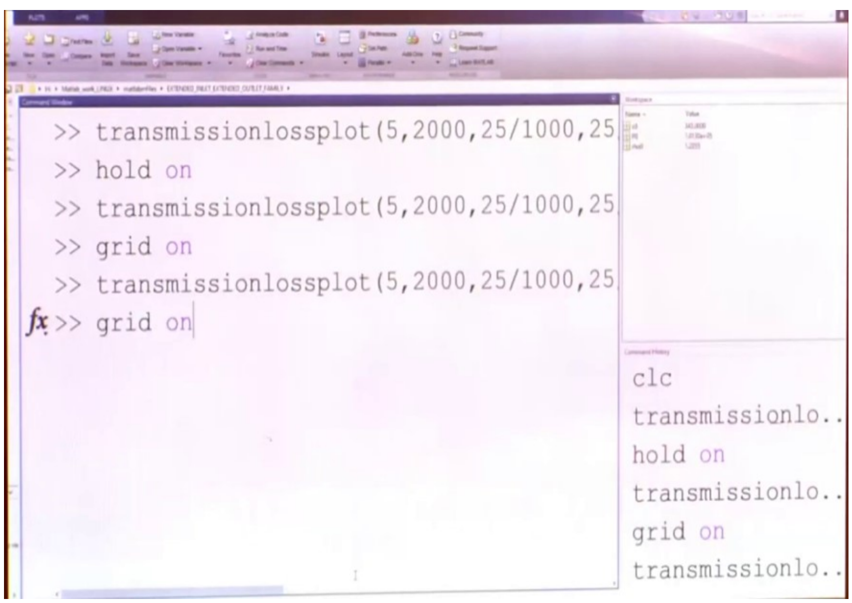
So, now let us consider a couple of more parametric studies at the inlet. Let us name this as use another color.

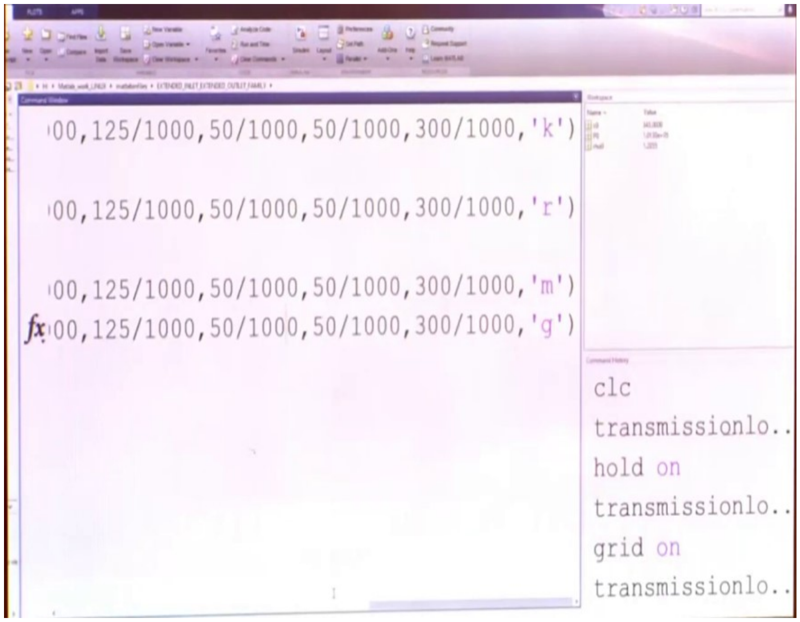


Let us instead of 225; let us say we put it at 200.

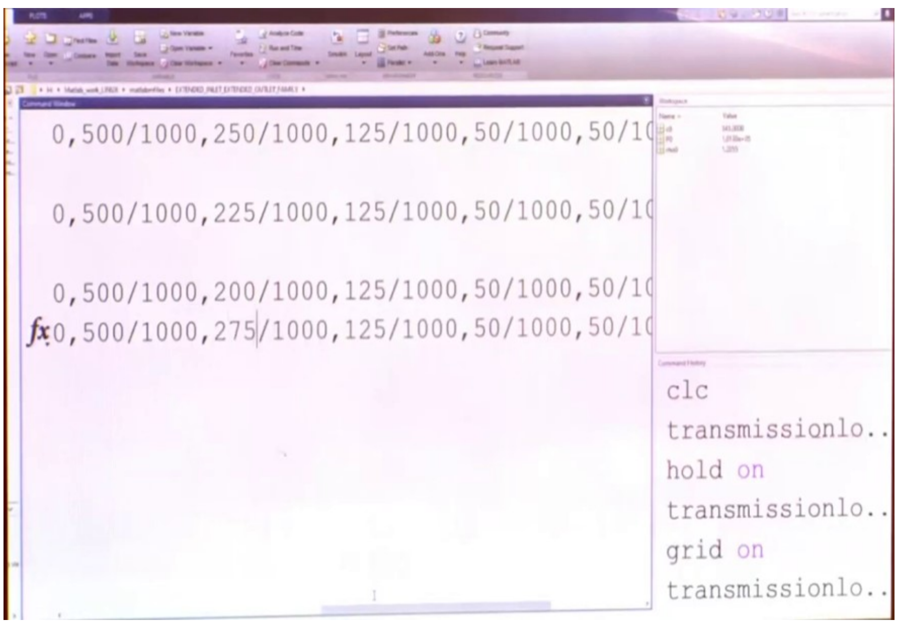


And let us see how bad the results are. We still get the trough at the first actual peak and this peak shifts towards the right because the chamber length now is the annular cavity length is less.

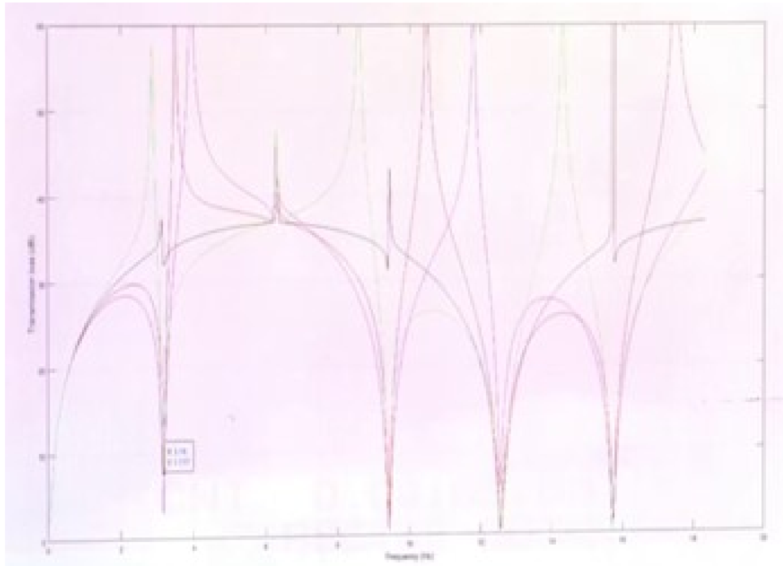




This is green and your.



And now on the other hand, if we consider an x increase length say 275, we will see the peaks occur slightly before the trough.



So, there is a shift in the peak and if we consider a further extension length, we will see the peak occurs at even lower frequency. The point being is that there is a miss tuning or the actual resonance trough or the first actual resonance trough is not coincident with the first peak of the quarter wave resonator formed at the extended inlet.

So, that is why it is very important to tune it to match the trough at the first actual resonance and then, that is how we will be able to completely kill a element the trough at the first actual resonance peak. So, by choosing the extension at inlet l 1 = L / 2, we are able to completely annihilate the trough that would otherwise occur if the extension was not tuned.

And the similar logic will apply for the extensions at the outlet that is at this point, if we choose a length different from L / 4, we will see a sharp trough occurring here. So, and also notice that when you are getting a peak here, this peak when I am pointing my mouse you are also getting the peak somewhere here, that is your third thing.

So, we are able to cancel two troughs, the draft due to the first action resonance and third action resonance. But the fourth one is not being able to improved, even when we have L / 4 extension. But the fifth one, we can improve.

So, this is what we basically get by this extensions. So, with this parametric study, I guess we will stop in this lecture, it was a bit extended lecture and we will meet for a little shorter lecture in the in the final lecture of this week, that is lecture 5 of week 5, where we will probably discuss about some other typical elements used in mufflers such as mufflers with side inlet and side outlet and also, before we begin that we will probably have a very briefly review this particular extend inlet and outlet element with regards to higher order plane wave theories, make some comments essentially.

So, with this, I would end here and thanks a lot for attending. Stay tuned.