Muffler Acoustics - Application to Automotive Exhaust Noise Control
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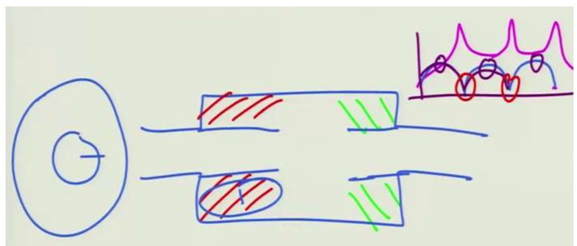
Lecture - 22
Extended-Inlet and Extended-Outlet Muffler Analysis

Welcome to Week 5, lecture 2 of our NPTEL course on Muffler Acoustics.

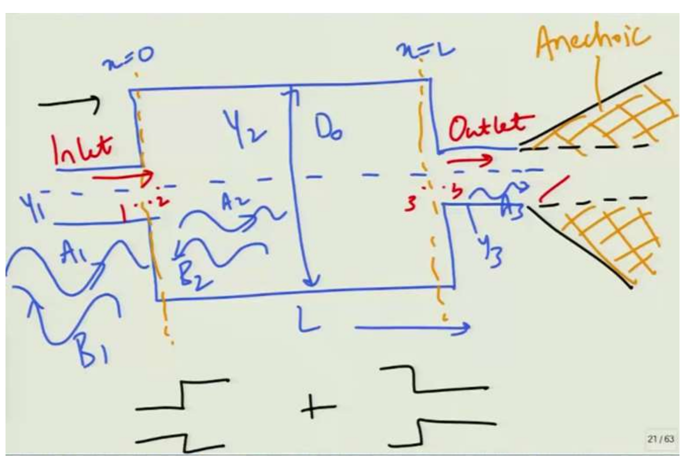
Before we begin this week, let us very briefly review what we intended to do in this lecture-2 that is the things that we discussed back in lecture-1 of week-5.



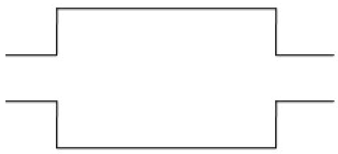
$$-jY_{ann1}\ cot\frac{\pi}{2} = 0$$

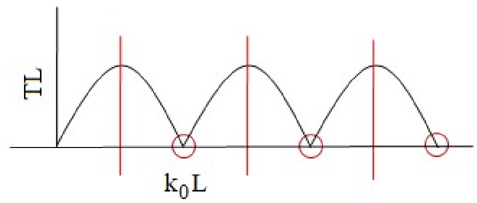
$$\sin(k_0L) = 0$$

So, basically we just stopped at the analysis of the transmission loss of extended inlet and outlet element. If you recall, we what we basically did was that we actually considered in detail the transmission loss properties that is the characteristic features of the transmission loss of a simple expansion chamber muffler.



So, this was the simple expansion chamber muffler. And we saw that I mean after a little bit lengthy derivation, we saw that the transmission loss of such a system then is given by the following expression which is of course we studied that in detail.

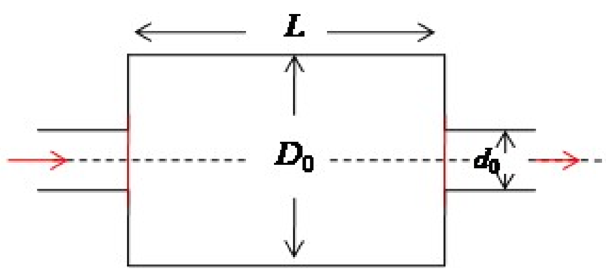




$$= 1 + sin^2 k_0\, L \left(M - \frac{1}{M}\right)^2 \qquad M = \left(\frac{D_0}{d_0}\right)^2$$

$$TL = 10log_{10} \left\{1 + \frac{1}{4}\left(M - \frac{1}{M}\right)^2 sin^2 k_0 L\right\}$$

where m is your square of your diameter ratio; m is nothing but $(D_0/ d_0)^2$. And you get periodic domes and trough. These are called troughs, these are called domes.



$$\frac{1}{Y_0}\left(Ae^{-jk_0x} - Be^{jk_0x}\right) = \tilde{v} \ \mid \left(e^{jk_0L} - e^{k_0L}\right) = 0$$

$$C - js - C - jS = 0$$

$$\sin k_0 L = 0$$

And we studied in quite a bit of a detail that why are we getting all these and some simple approximations and all that.

$$TL_{Max} = 10log_{10} \left\{1 + \frac{1}{4}\left(M - \frac{1}{M}\right)^2\right\}$$

$$TL_{Max} = 10log_{10}\left\{1 + \frac{M^2}{4}\right\}$$

$$TL_{dome} \simeq 20\log\left(\frac{M}{2}\right)$$

So, it turns out that we do get these domes and troughs that which was mentioned somewhere here. We get these domes somewhere here at certain frequencies. If you recall those, what were those frequencies?

$$k_0L = \frac{\pi}{2} \quad \frac{3\pi}{2} \quad \frac{5\pi}{2}$$

So, we could basically readily work out what where those frequencies. I have not would derive that again it. I will just probably mention it.

$$k_0L = \frac{\pi}{2} \;=>\; \frac{2\pi f}{C_0}\,L = \frac{\pi}{2}$$

$$=> \quad f_{dome} = \frac{C_0}{4L}$$

$$f_n = \frac{(2\pi + 1)C_0}{4L}$$

$$TL = 0$$

$$10\log 1 = 0 \mid sinkx = \sin nx$$

So, those were the frequency is given C_0 / 4 L, and of course, the first frequency is C_0 / 4 L that is your C_0 / 4 L, we get all these things where we get the maximum transmission loss. And on the other hand, when this thing is there, we get zero transmission loss.

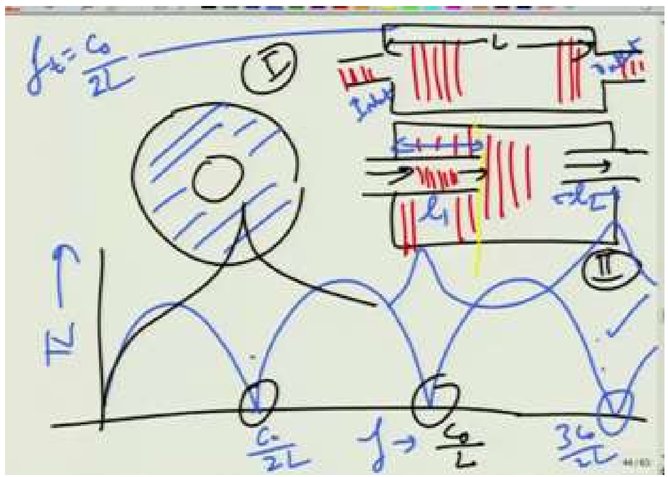
$$k_0L = n\pi$$

$$\frac{2\pi f}{C_0}L = n\pi$$

$$f_{\text{trough}} = \frac{nC_0}{2L} \qquad n = 1,2,3$$

$$\frac{C_0}{2L}, \quad \frac{C_0}{L}, \quad \frac{3C_0}{2L}, \ldots \quad \frac{L}{4}$$

Now, just when we stopped at the last lecture, we figured out that there could be a smart way to eliminate these problematic areas, you know the ones that I am sort of circling, then let me draw it cleanly let me draw it rather cleanly for you.



So, this was our thing is not it? And these were the problematic areas is not it? So, our focus was then how to resolve, how to completely kind of kill those troughs, is it possible somehow to do that? Well, it turns out that there is indeed a smart way to kill these troughs that is why if you consider a structure or a configuration like this, instead of a simple expansion chamber with flush mounted tubes if we consider this kind of a thing.

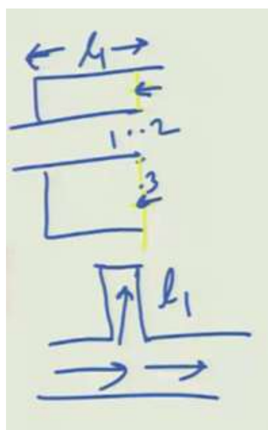
So, what does it do? How is this different from perhaps let me just get rid of this part how is this different from this? Let us try to understand the theory underlying this completely in this lecture. So, the length L remains the same. And the only difference that we have is that the pipes are inserted into the chamber in the configuration well. Let me say this is configuration 2 and this is configuration 1.

So, such a chamber in which the inlet and outlet ports are kind of protruded or extended in inside the chamber with the view to generate peaks at certain frequencies such that it is coincident with the natural frequencies of the chamber overall chamber. And when it so happens, then the troughs are uplifted the troughs are cancelled by the attenuation peaks and then you get a much better transmission loss performance. Overall the transmission loss performance would enhance.

So, let us understand the math's underlying it. Suppose, we do not know what would be the length. We assume arbitrarily the length is l_1 of this extension and l_2 of the outlet the port that is also outlet port that is also extended inside the chamber. So, this is the inlet, this is the outlet. And when you have these extensions, you got certain things like this, then it is possible that we can create resonances by suitable insertion by tuning the lens.

Now, all this while we are assuming an only planar wave to propagate that is here the planar waves propagate. And here also in the chamber there is plane wave. So, when the acoustic wave is incident just at the interface of the extent inlet in the chamber, a part of that is going or is diverted towards the annular cavity and part of that is going here.

So, but before that let us figure out the resonance that the waves sees right at this section at this section mark in yellow color what is the impedance. So, remember this is a quarter wave resonator. So, essentially what we have; essentially what we have is a cavity, is an annular cavity. So, we are trying to figure out the resonance frequencies of this cavity.



So, with this in mind let,

$$Y_0 = \frac{C_0}{S_L}, Y_p = \frac{C_0}{S_p}$$

So, we will keep these relations aside that is to say if you have extensions like this. So, what is the characteristic impedance of this annular section ok? What is the characteristic impedance?

$$Y_{ann} = \frac{C_0}{S_e - S_p}$$

We know this now, let us figure out what is the impedance that is seen by the wave when it is just incident here that is looking into the cavity of length l_1. It is

$$\frac{\tilde{p}_S}{\tilde{V}_3} = Z_{ann1} = -jY_{ann} \cot k_0 l_1$$

that is the impedance that is seen by the wave when it is incident right at the interface. So, now, let us this.

So, let us introduce some nomenclature or knowing thing now naming the point. So, this is 1, let us say the point just outside the interface is 2, and this is 3. So, this is then p_3 tilde by volume velocity or mass velocity V_3.

What do we do with this particular expression? How do you use that?

So, remember if you recall our last lectures, if we had a certain tube such that it was closed at one end, and it was there was a wave coming here. So, if you could tune the length l_1 such that most of the acoustic power is diverted in the side branch and very little or negligible power is transmitted downstream.

Then you can significantly abate or control the noise or the sound that is propagated downstream at least in that frequencies or for tonal frequencies. And it turns out in this case it is going to be is going to significantly not only kill that particular trough or that problematic frequency, it is overall going to improve the transmission loss performance in the vicinity of such a problematic frequency.

So, when is the impedance Z_1 of the annular cavity going to 0? When is that going to happen? I should have probably named it as Z_{ann1}, annular cavity 1, because there is analog cavity 2 also at the extended outlet.

So, when this term is going to

$$\cot k_0 l_1 = 0$$

$$n = 0, 1, 2, 3$$

$$=> \quad \cos k_0 l_1 = \cos\ (2\pi + 1)\ \frac{\pi}{2}$$

$$k_0 l_1 = (2\pi + 1)\ \frac{\pi}{2}$$

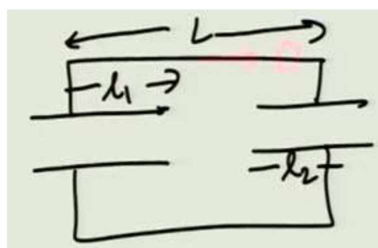
So, what does this mean? It basically leads us to the familiar thing. Actually this in a more general sense can be written like this where,

$$\frac{2\pi f_p}{C_0} l_1 = \ (2\pi + 1)\ \frac{\pi}{2}$$

So, let us start cancelling terms. π, is cancelled and And you can multiply both sides by C_0 to get rid of the denominator and f let us say this is peak f_p. And let us divide this by l_1 further to get this.

$$f_p \quad \frac{(2\pi + 1)C_0}{4l_1}$$

So, that is what we are going to get.



Now, let us look at the lowest possible frequency. This expression I am sure you would have come across. So, n is equal to 0, this would give us

$$f_p = \frac{C_0}{4l_1}$$

is not it? And let us recall from our last lecture what is the trough frequency. What is going to happen to the frequency of the trough when you have a chamber of length L. So, let us go back to our last slide. So, here we were.

So, when l_1 and L, so this the, so naturally the trough will happen let us recall the last discussion. So, this was what we had C_0 / L I am not sure why this was there C naught by L is the first frequency, first actual resonance frequency. So, going back to our slide here,

$$f_t = \frac{C_0}{2L}$$

this is the frequency at which the first trough occurs.

Now, if we were to somehow choose the length in such a manner that f_p is exactly equal to the f trough that is the; that is the frequency of occurrence of the peak due to the quarter wave resonator, that is when the frequency at which the impedance of the annular cavity is tending to 0 is exactly equal to the frequency of occurrence of the trough of or the first natural frequency of the chamber. Then you have the following condition.

So, let us start cancelling the terms and simplifying. So, l_1 it turns out its nothing but L / 2. Why? Because of course this cancels you get this and you know simple algebra would basically get,

$$f_p = f_t \quad => \frac{C_0}{4l_1} = \frac{C_0}{2L}$$

$$=> \qquad l_1 = \frac{L}{2}$$

So, basically if in other words, if you were to choose the extension in such a manner that it is half the length of the chamber. So, half of the length, so basically you need to insert the pipe for almost half the length, not, almost exactly half the length of the chamber based on the one dimension or planar wave propagation, then you can in principle eliminate the trough.

Now, we can actually do the similar exercise for the annular cavity at the outlet that is the cavity that is formed here. So, how do we, how do we go about doing it?

$$Z_{ann2} = -jY_{ann2}\, cot k_0 l_2 \quad \rightarrow 0$$

So, when similarly if you have one needs to find out really the frequency where which the annular cavity resonance occur, so this we need to find out f p. So, for that, we again set this term to 0 to finally obtain

$$Z_{ann2}(f_p) = 0$$

$$k_2 l_2 = (2n+1)\frac{\pi}{2}$$

So, when you get such a thing, so what is the first frequency? Of course, looking at this formula, it is easy to figure out that for

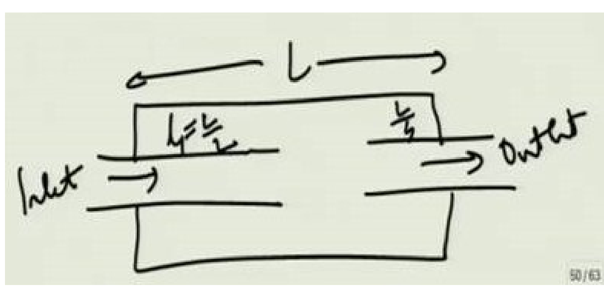
$$=> \qquad f_p = \frac{C_0}{4l_2}.$$

So, this was l_1. Let us say this is l_2; this is $l2$. So, when you have this kind of a thing, then when you have basically this expression, this frequency for a certain l_2 for the first actual resonance frequency first resonance frequency at which the peak will happen will occur for the annular cavity at the outlet is given by C_0 /4l_2.

So, let us try to eliminate the second trough that is the second natural frequency. So, if you recall, if you recall our last the slides these ones, this was the first actual resonance. And this occurred at C_0 /12. We are hopefully we should be able to we will do some MATLAB simulations and which we will show that if you have L / 2 this the trough is, this trough is completely eliminated by the peak here.

And for eliminating the trough due to the second actual resonance that is C_0 / L, one has to basically equate the second resonance frequency of the chamber with the actual resonance with the resonance frequency of the cavity formed at the outlet. To do that, this must then be equal to C_0/ L alright.

$$\frac{C_0}{4l_2} = \frac{C_0}{L}$$



So, let us simplify this these terms. So, you get

$$l_2 = \frac{L}{4}$$

these terms then are cancelled. And what essentially we get is l / 2 is equal to L / 4. So, these then are the relations, is not it? And so we have two relations

$$l_1 = \frac{L}{2}$$

So, you so basically the idea is, the idea is very simple then how to tune the performance and we will soon see some a transfer matrix derivations for this extended inlet element and we will see some MATLAB or maple based simplifications in a while.

So, basically what really one needs to do, if we go by the planar wave or axial plane wave propagation approach is that, the idea is to simply not just flush mount the inlet and outlet pipes we have to basically insert it by length L / 2 and L / 4.

So, if you are able to accomplish this, if you are able to get a configuration made such that the inlet is extended by half the length of the into the chamber equal to the extension length being half of the chamber, similarly and the outlet is extended into the chamber with extension beam quarter of the length, then the first and second actual resonance frequencies can be eliminated. We hope that and we will show that.

But this one more thing that I would like to point out that let us say we consider the extension length l_1 is equal to L / 2. So, let us see how we are able to; how we are able to in fact use the extension length L / 2 to also possibly eliminate higher order troughs.

So, now, if we recall our expression for the impedance for the annular cavity one we have

$$Z_{ann1} = -jY_{ann1} \cot k_0 = \frac{L}{2}$$

But we already chose l_1 to be L / 2. So, we get this. And let us also recall the troughs of the chamber that is the resonance frequency to the chamber.

So, now, let us say so these are the things. Now, hopefully we have already taken care of this and this. How do we ensure that just by choosing L / 2, this would possibly kill the higher order troughs as well? So, if we choose f_p or the frequency k_0 corresponding to this frequency that,

$$\frac{C_0}{2L}, \frac{C_0}{L}, \frac{3C_0}{2L}, \frac{2C_0}{L}, \frac{5C_0}{2L}$$

So, what do we get?

$$k_0 \frac{2\pi f}{C_0} = 2\pi \frac{\frac{36}{2L}}{C_0}$$

So, let us start cancelling the terms C_0 is cancelled 2π is cancelled and is

$$= \frac{\pi}{L}$$

So, this is what we are going to get for k_0.

Now, let us use another slide an annular cavity one at such a frequency is

$$Z_{ann1} = -jY_{ann1}\, cot\, \left(\frac{\pi}{L} \frac{L}{2}\right)$$

because k naught remember this was pi by L here and into L by 2 multiplied by L by 2, L by 2 and here it was pi by L. So, we cancel this and we get,

$$= -jY_{ann1}\ \ cot \frac{\pi}{2} = 0$$

So, what does it mean? It means that we can also hopefully take care of this frequency. Why? Because the trough occurs at $3C_0$ / 2L the third one that is in the sequence of troughs I guess which I presented somewhere here, this one this will again go. So, this one $3C_0$ / 2L frequency, this is what the frequency axis, this was the transmission loss. So, hopefully we should be able to take care of this as well.

But how far we can go with L / 2? Can, so L /2 is actually we anticipate that because the trough actual resonance trough here and the peak they are there the frequencies are mutually coincident. So, the peak would hopefully cancel out the trough, the first one as well as the third one that is here. So, we would get something like this and something like this, we will soon see. We will get the transfer matrices derived and show some results in MATLAB based on the planar wave theory.

And then the whole idea behind is to extend the broadband or extend the attenuation performance for as high frequency range as possible. Because remember let me reiterate here I have done, so in the previous classes that apart from the automobile or the engine exhaust system is really a variable is it is a device which produces noise sources at different frequencies based on the running speed.

So, ideally we would want to have a resonator that works well for a broadband or as wide range of frequencies as possible. So, we would want in general to have a broadband attenuation, and plus a good low back pressure drop, and pressure drop or things like that. But so just by L / 2 extension, we are able to take care of this one and this one. Can we go for higher order troughs that is $5C_0$ / 2 L, is it possible?

So, let us see if that is the case, because your other higher order frequencies would probably occur at this occurs at $3C_0$ this thing. So, this would probably occur at $5C_0$ / 2L and so on. So, if you have such a

$$\frac{\frac{2\pi 5C_0}{2L}}{C_0}$$

Then the term that you have on this side is L / 2. So, basically what happens is that L, is cancelled. And you get ,

$$\frac{\pi 5}{L}\frac{L}{2} = \frac{5\pi}{2}$$

So, this will also tend to 0, that means, theoretically if you take L / 2, it is going to eliminate even higher order trough that

$$cot\frac{5\pi}{2} = \frac{\cos()}{\sin()}$$

But then we probably should not celebrate a bit too early because there might be some problem when you choose L / 4. So, for this one, we know that when you choose L / 4, it is going to cancel out the second trough. But can it cancel out the fourth trough? Because the fourth trough would basically correspond to the frequencies given by, so here we were basically here we would get $2C_0$/ L, 5 / 2L into C_0.

So, by choosing the outlet length to be L / 4, we could get rid of we could match the resonance frequency with the second actual resonance. And now the job is ready to see whether we are able to match the two, 2nd or the 4th resonance frequencies because remember phi C_0 / 2L is the 5th actual resonance frequencies. And by choosing L / 2, we can possibly kill out or eliminate this trough.

But before we get too happy about this thing, it is very important to check whether by choosing L / 4, we can cancel out 2 C_0 / L that trough the trough corresponding to that because that occurs earlier than the 5th one, the 5th actual resonance frequency.

And we cannot change L / 4 because you have really you would want to eliminate the second trough that is your priority. And again because engine noise is concentrated at the first firing frequency of the cylinder as well as the its first few harmonics.

So, the lower frequencies are of always a priority. And unfortunately most mufflers at least the reactive mufflers, they do not; they do not seem to give a very good performance in very low frequencies unless you of course increase the expansion chamber area expansion ratio. Anyhow, let us get back to our discussion let us put the frequency as $2C_0/L$ in our slides that in our ensuing slide.

So, I will get rid of this part. I will get rid of this part and

$$\frac{\frac{2\pi\ 2C_0}{L}}{C_0}\ \frac{L}{4}$$

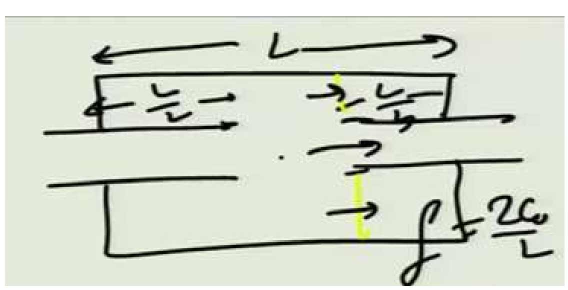
So, we cancel this part, we cancel this part and we are left with this multiplied by this. So, this is cancelled; this is cancelled.

So, this becomes 4 π/4. So, we get pi. So, then what is the argument of the angular cavity

$$2\pi\frac{2}{L}\times\frac{L}{4}=\frac{4l}{4}\ =\pi$$

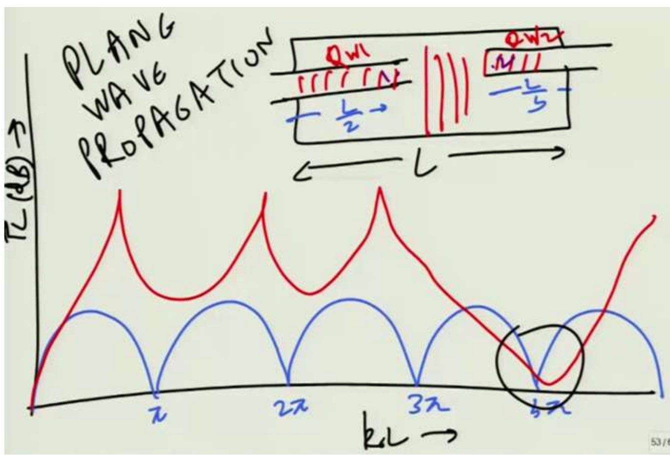
$$Z_{ann2}=-jY_{ann2}\,cot\pi\ \rightarrow\ \infty$$

So, it is actually tending to infinity theoretically.



So, that means, if you have a resonator with length, so if this length is your L / 2 and this is your L / 4. So, what would happen then? It would kill the first actual peak actual resonance, second actual resonance due to this, the third one as well, but fourth one it is not going to it cannot it cannot eliminate the fourth trough.

This for the simple reason because the impedance based on the plane wave consideration. The impedance seen by the plane wave at this section at frequency given by 2 C_0/ L at this frequency. The wave c is here infinite impedance or it tends to basically completely transmit the acoustic power by following the least path of least resistance or least impedance, and it goes directly in the downstream.



So, let us draw things a fresh. Let us draw it fresh. This was something like let me draw the simple expansion chamber thing with a blue curve 3, 4, 5 and so on. So, π, 2π, 3π, 4 π, these are all non dimensional frequencies. So, we can actually name this as k naught into L. And here you have transmission loss that is dB.

So, what will we get? We will get when you choose of course something like this, when you choose something like this, L / 2 and L / 4, let me make it more this thing. So, then we are possibly going to get things like it is going to kill the third trough as well, and here it is going to break down. It might increase at this thing, but then increase here is probably inconsequential because you already have a deep here at the fourth this thing.

So, basically the first by choosing this is the what is the moral or the take home message of this lecture that based on the plane wave propagation theory, plane wave propagation, if we choose a length of inlet pipe to be extended into the chamber by L / 2 and outlet pipe to be extended into the chamber by L / 4, we can get rid of the first three troughs that is important.

And of course, assuming here waves are planar. And these are all quarter wave resonators Q_{w1}, Q_{w2}. We probably by the simple trick, we can possibly extend the broadband transmission loss range a lot. We can definitely improve the transmission loss performance. It is not only we will see all these graphs for typical configurations using a MATLAB code and maybe some simplifications using maple algorithms, maple thing, maple worksheet.

And we can derive the transfer matrix of the entire thing right from the point here, let us say this is the point 1, and this is the point 2, we can derive the transfer matrix between these two points, and we can see all these things wonderfully happening. And then worry about basically simplifying things and plotting it and all those kind of things.

So, that will be the focus of our next lecture. And till that time stay tuned and I will see you in the next lecture.

Thanks.