Muffler Acoustics - Application to Automotive Exhaust Noise Control

Prof. Akhilesh Mimani
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 21
Transmission Loss (TL) Graph for a Simple Expansion muffler (MATLAB)

Welcome to week 5 lecture 1 of our course on Muffler Acoustics. So, we will begin lecture 5.

So, if you recall what we did in the immediately preceding lecture was we stopped at the Transmission Loss expression for a **Simple Expansion Chamber** which is shown here. So, we were just beginning to analyze the salient or the important features of transmission loss spectra.

$$\frac{1}{Y_0}\left(Ae^{-jk_0x} - Be^{jk_0x}\right) = \tilde{v}$$

$$A = B_1\, B = Be^{2jk_0L}$$

$$= 1 + sin^2 k_0\, L\, \left(M - \frac{1}{M}\right)^2$$

So, our primary job in this lecture will be to discuss in greater detail about the transmission loss performance the curves and possibly about the sequential connections of the muffler elements.

So, let us begin today's lecture. So, we have got a simple expansion chamber by now you would have understood what do you mean by a simple expansion chamber which is something like this the flow comes in here it goes out and you have your this thing here.

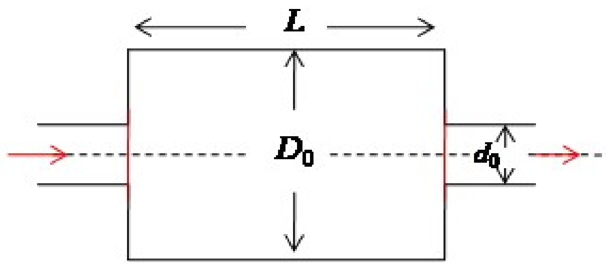
Now, as we saw from the last lecture. The transmission loss analysis was given by finally, it was given by

$$TL = 10log_{10}\left\{1 + \frac{1}{4}\left(M - \frac{1}{M}\right)^2 sin^2\, k_0L\right\}$$

Where,

$$M = \left(\frac{D_0}{d_0}\right)^2$$

whole square ok and k naught L is the Helmholtz number of the non dimensional frequency, non dimensionalized with respect to the chamber length L.



So, this was your D naught and this was this. So, as we can clearly see from the expression here when is a transmission loss maximum and when it is minimum. So, clearly it is a function of frequency which is represented by the sin sinusoidal square or the sinusoidal function. So, when TL when TL is maximum then of course, this the circle term must be unity that is 1 1. So, now, under such a situation we can just put it 1 and simplify it further.

$$TL = 10log_{10}\left\{1 + \frac{1}{4}\left(M - \frac{1}{M}\right)^2\right\}$$

So, we get this thing let me just get rid of the same thing. So, when does this happen? When

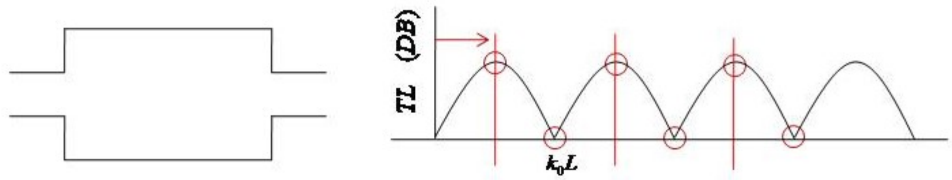
$$\sin k_0L = 1 = \sin\,(2\pi + 1)\frac{\pi}{2}$$

where n n starts from 0 0 1 2 3 4. So, let us simplify the expression,

$$k_0L = (2\pi + 1)\tfrac{\pi}{2}$$

for crest or domes.

So, if you recall a transmission loss spectra look like this. So, it was like this on the x axis is your Helmholtz number or the non dimensional frequency and this is your TL in DB.



So, by this expression we mean that the maxima is occurring at the first frequency of occurring is

$$k_0L = \frac{\pi}{2} \quad => \quad \frac{2\pi f}{C_0}\, L = \frac{\pi}{2}$$

So, when you simplify this thing further you get,

$$=> \quad f_{dome} = \frac{C_0}{4L}$$

So, this is the frequency at which domes occur. So, this is the frequency at which your maxima occurs, what is the next frequency? When you set n is equal to 1, 2 ,3, 4 like this. So, for the next frequency you get it will be just 3 times that is.

Let us let me call this as f n. So, it will just be obtained by

$$f_n = \frac{(2\pi + 1)C_0}{4L}$$

So, where n is equal to 0 that is your first thing you get,

$$f = \frac{3C_0}{4L}, \frac{5C_0}{4L}$$

So, that is where the frequency or the occurrence of the first dome that is this one occurs.

So, this will correspond to this particular frequency where we circle where n is equal to 2 it will be $5C_0$ / 4 L. So, that will be this one. So, like this pattern keeps on occurring, but what is more important to us and to all acoustic designers for a muffler designer is basically where the trough occurs.

So, where does the trough occur trough means by what do you mean by a trough? By trough we mean whether transmission loss is 0 or it is probably a minimum. So, that almost all the acoustic power goes away. So, now, let us look at this figure. Now, if you look at the transmission loss graph wherever this let me use a different color may be purple. So, wherever this trough is occurring that is characterized by 0 transmission loss. So, then at such a frequency,

$$TL = 0$$

So, when does that happen?

Clearly if you look at the transmission loss expression in the last lecture when sine k naught L. When this is equal to 0 you have this entire term goes away when sin k naught L is 0, then you are just left with log of 1 and obviously 10. So, this will become 0 this will happen when,

$$10 \log 1 = 0 \mid sinkx = \sin nx$$

So, let us simplify this thing.

$$k_0 L = n\pi$$

$$\frac{2\pi f}{C_0} L = n\pi$$

$$f_{\text{trough}} = \frac{nC_0}{2L} \qquad n = 1,2,3$$

$$\frac{C_0}{2L}, \quad \frac{C_0}{L}, \quad \frac{3C_0}{2L}, \ldots$$

So, why is this trough important? Let us analyze this because you see from the principles of ah anti logarithmic addition no matter if you are getting a very high transmission loss very high attenuation at a certain frequency, but not such a good performance at other frequency range. So, and it so happens that most of the noise is concentrated in near in this frequency band and in here and so on.

So, although you are getting good performance at other frequencies, whether dome is happened. But not so good performance at where the trough occur obviously, and then unfortunately the engine noise is concentrated in these frequency bands somewhere here, here then the muffler is pretty much useless because 0 transmission loss means all the acoustic power is going away it is transmitted as it is the muffler is transparent or is useless. So, the by the principles of anti logarithmic addition.

If you add the noise reduction occurred somewhere here somewhere here, here everywhere, you will see that the net transmission loss that one would get over different frequency band is basically given by the frequency where least attenuation is produced.

So, basically there is some kind of a bottle necking or a some kind of a limitation that is imposed with a trough and this is especially important because you know automobiles are basically a variable speed drive.

Because the speed changes and the number rpm of the engine changes and basically automobile engine exhaust noise is dominated by the first firing frequency and its multiples. So, if the speed changes the firing frequency the rpm of the engine changes and the firing frequency; obviously, changes.

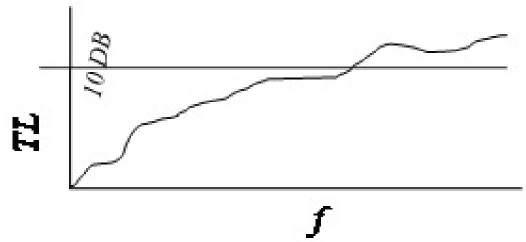
So, the idea is that the frequency spectrum keeps on changing based on different rpm and for such a variable speed drive the noise spectrum continuously changes.

So, what does one do to solve this problem? What does one do to basically design such a muffler that is that is effective for automobile or exhaust engine noise which is running at different rpms? So, one principle is to basically design I mean the best the idealistic situation would be to design a muffler which should produce a broadband transmission loss. So, this is the term that we are using perhaps for the first time let

BROAD BAND TRANSMISSION LOSS or also insertion loss broadband or also known as **BROADBAND PERFORMANCE.** We keep hearing this term broadband performance or broadband attenuation in several con text.

In the context of mufflers it means that the muffler is effective for a large range of frequency such that well if you are considering a complicated muffler and you are this is the TL and here is frequency and you are getting something like this kind of a curve.

What it means is that you know beyond a certain frequency beyond a low frequency region the performance is really catching up and for most of the frequencies you get you are guaranteed to get a constant say maybe this is like 10 dB this is say maybe this is 10 dB line.



So, beyond a certain frequency you are guaranteed to get at least 10 dB of noise reduction and there is no chance or no chance of muffler failing at any frequencies beyond this thing. Because the transmission loss or the attenuation performance is never 0, beyond a certain frequency.

So, by ensuring a broadband performance of a muffler we really are ensuring that there is no trough like what we saw in the graph presented here. Of course, simple expansion chamber it is too simple to be used, but probably the most well studied element and the probably the first element that we have studied in this course and all the courses in noise control engineering a simple expansion chamber is always studied.

Because it is analytically the simplest to understand, but it is not used in practice obviously, there are much more additions to it like extensions and dissipative mufflers, slow reversal and vertical perforates, bulk heads and what not. So, the point is that the point of introducing all the complicating features and more elements is to basically improve the performance with respect to a simple expansion chamber that is your troughs.

So, by the idea is to uplift the troughs or to increase the transmission loss performance for as wide frequency band as possible.

So, before we move much before we finish the discussion of TL analysis of a simple expansion chamber I guess, we probably would like to do some approximations to the TL expression that we obtained so far in terms of the area expansion ratio as well as conduct some basic parametric studies which I will present you using MATLAB in a while to further understand how attenuation performance happen.

And actually incidentally there is one more thing that I forgot to mention sorry. So, essentially a muffler a simple expansion chamber muffler what we are seeing here it can be considered like a duct.

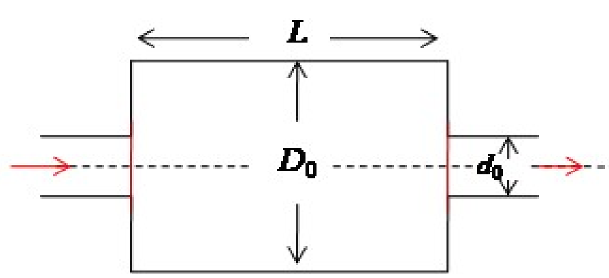
Say if the length is closed at both ends and you have your the acoustic pressure field here, well I am just making use of the some space here to write down the acoustic pressure field here its is something like this if you recall our discussions this is the expression for pressure. Velocity is nothing but minus and you will have your characteristic impedance at the bottom denominator.

So, why am I telling you this because to get the resonance frequencies or the actual resonance frequencies of a simple expansion chamber one needs to put a rigid wall condition at the left end and also the right end. So, basically once you do that you will get A is equal to B and another condition that you are probably going to get is

$$\frac{1}{Y_0}\left(Ae^{-jk_0x} - Be^{jk_0x}\right) = \tilde{v} \quad | \left(1 - e^{ijk_0L}\right)B = 0$$

So, once you substitute this in here. So, instead of B I will write b A I will write B here making use of this expression I will get rid of this.

And what I probably, but otherwise you will get a trivial solution. So, this entire thing has to be 0.



$$\frac{1}{Y_0}\left(Ae^{-jk_0x} - Be^{jk_0x}\right) = \tilde{v} \quad | \left(e^{jk_0L} - e^{k_0L}\right) = 0$$

So, once you simplify this thing further by multiplying by

$$C - js - C - jS = 0$$

So, then basically your cos will get cancelled eventually we left with nothing, but

$$\sin k_0 L = 0$$

Eventually you are going to get that. And this is your condition what we got from the transmission loss expression just a while back when we were dealing with the troughs, isn't it.

So, this is this discussion is important because the troughs always occur at the resonance frequency first, second, third fourth and so on resonance frequencies of a simple expansion chamber. So, this is one point I have thought of mentioning.

And finally, before we end the discussion on TL analysis of such a of this basic configuration just want to mention that

$$TL_{Max} = 10log_{10}\left\{1 + \frac{1}{4}\left(M - \frac{1}{M}\right)^2\right\}$$

So, since M typically we saw M like if we consider diameter to be 350 mm or even if it is 300 mm and the port diameter is roughly say 50 mm. So, you are going to get

$$M = \left(\frac{300}{50}\right)^2 = 6^2 = 36$$

So, 1 by M is 1 by 36 which is very small compared to M.

$$\frac{1}{M} = \frac{1}{36}$$

So, the idea is that we might want to drop the other term pertaining to 1 by M from the above expression to get a bit simplified expression which

$$TL_{Max} = 10log_{10}\left\{1 + \frac{M^2}{4}\right\}$$

$$\simeq \ 10\log\left(\frac{M}{2}\right)^2$$

So, this is an approximate expression for TL dome or TL max that we get from the analysis. So, this is what we are going to get and what about the expression for TL minimum that is of course, 0 like we discussed now.

So, using this estimate one can very cleverly get a feel of the transmission loss actually in many cases if you want to do a further simplification you could also perhaps try out putting dropping 1 you could also drop out 1 if,

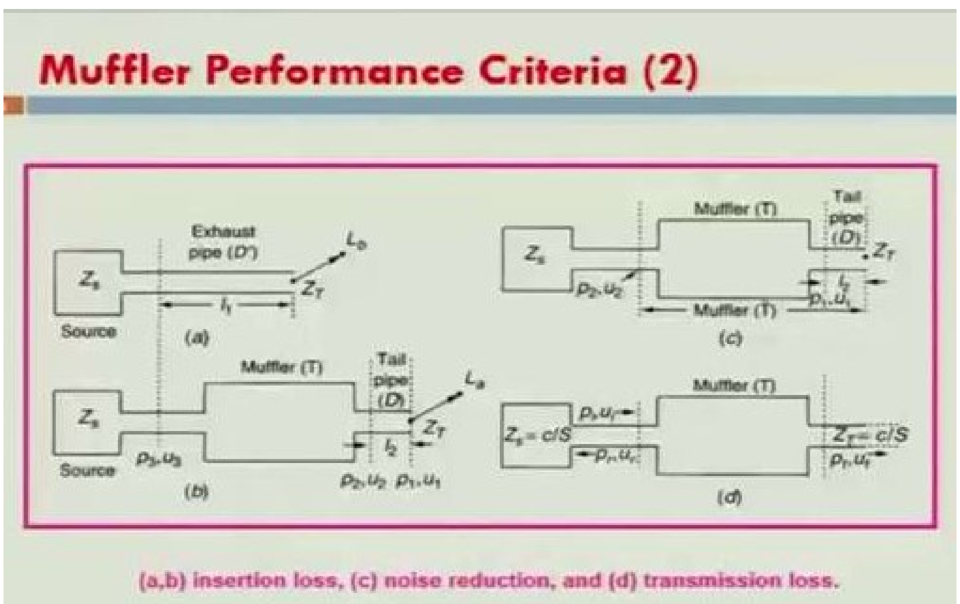
$$TL_{dome} \simeq 10log_{10}\left\{1+\frac{M^2}{4}\right\}$$

So, this would become 20 and this would become 20 and this would be 10.

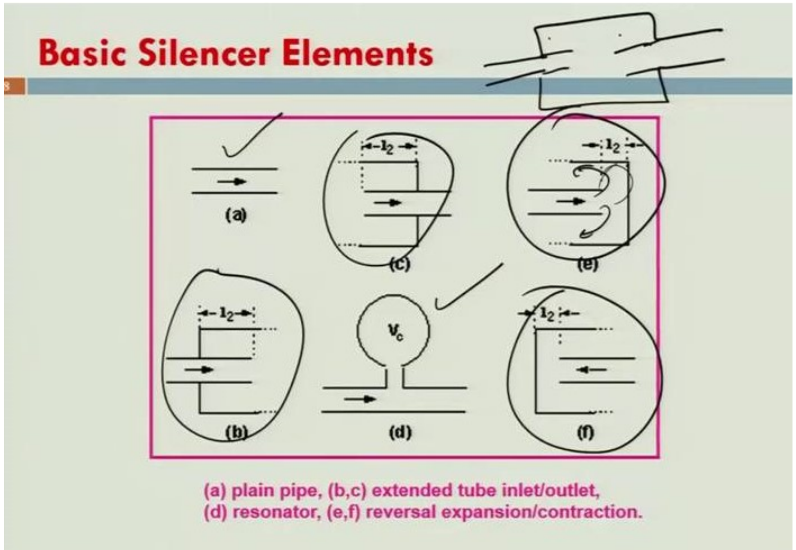
$$TL_{dome} \simeq 20\log\left(\frac{M}{2}\right)$$

This is I will would get rid of 10s understood. So, this is a even further simplified expression to get a very quick estimate of the maximum transmission loss that a simple expansion chamber can produce given the area ratio, given what will be the ratio of the cross section area of the chamber to the that of the pipe or the port.

What next we could study regarding the simple expansion chamber is the cascading of mufflers, this many more many interesting elements.



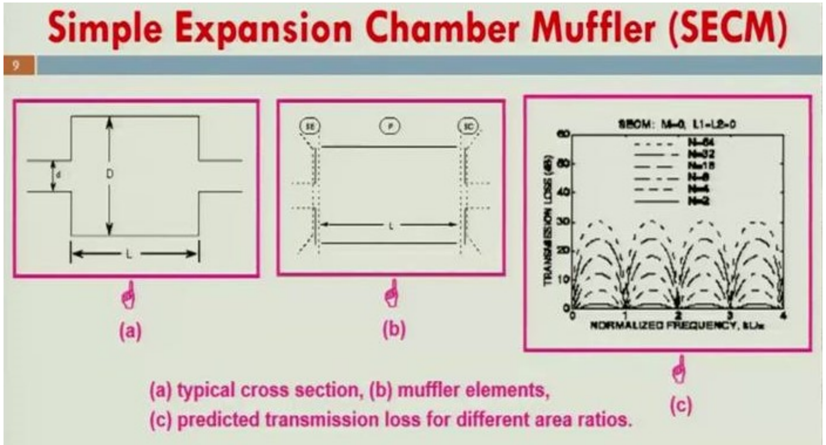
Actually let me present to you some important slides where we probably can consider different elements. And, but we will probably analyze it a bit later.



So, this is what it is. So, this presents to you this is something that we have analyzed a simple pipe or a pipe of uniform cross section and this is something that we know Helmholtz resonator, what we probably have not really seen so far are these elements. So, what are these as the legend or as the figure caption suggest this is an extension extended inlet element and this is a extended outlet element.

So, basically, so a simple expansion chamber is something like this, isn't it? So, instead of the ports being flush mounted if it is like this and if it is like this, then what happens? So, we have quarter wave resonators formed at the inlet and at outlet and you see that they will be instrumental they will be, they play the main role in uplifting the trough we will probably have a look at it in the next lecture perhaps.

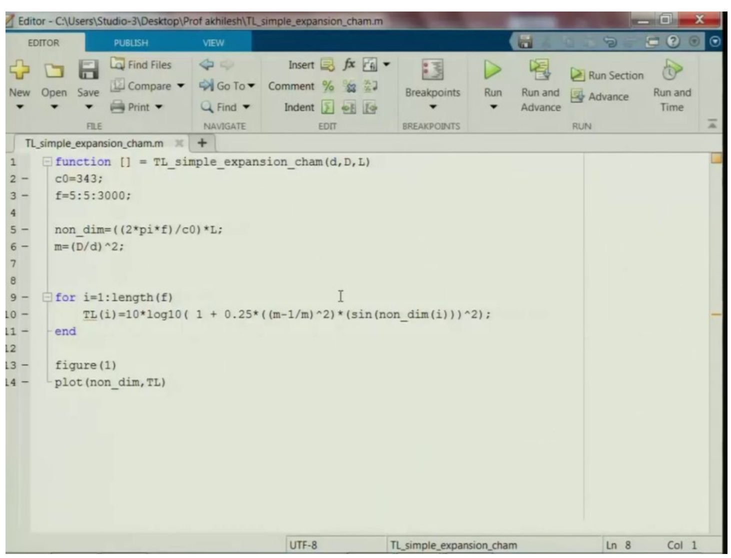
But I guess it is important to just introduce these elements and another one is of course, a flow reversal expansion and flow reversal contraction where the flow comes in here and it is forced to move like this it produces a lot of back pressure though, but this cavity acts like a resonator. So, does this one depending upon the diameter and this length it might be a short cavity or long cavity. So, the wave propagation might be we might cleverly choose the direction of wave propagation.



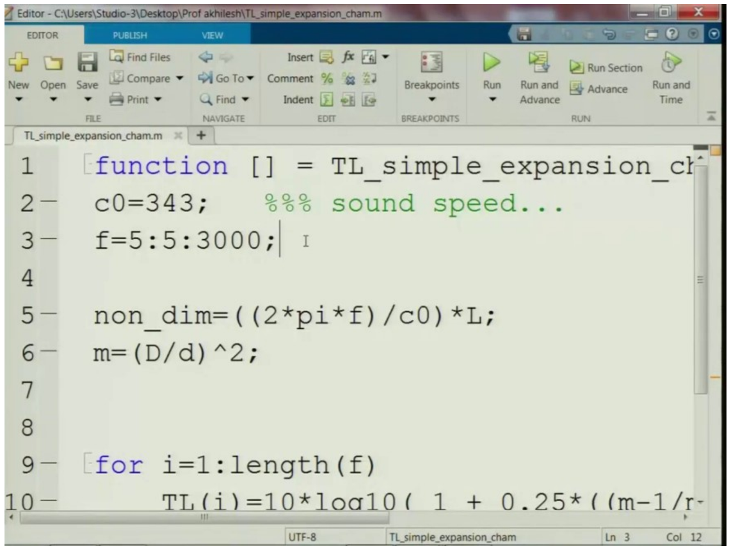
$$TL - 10 \log \left[1 + 0.25 \left(N - \frac{1}{N} \right)^2 \sin^2 kL \right], dB$$

Where N = area ratio, $(D\backslash d)^2$

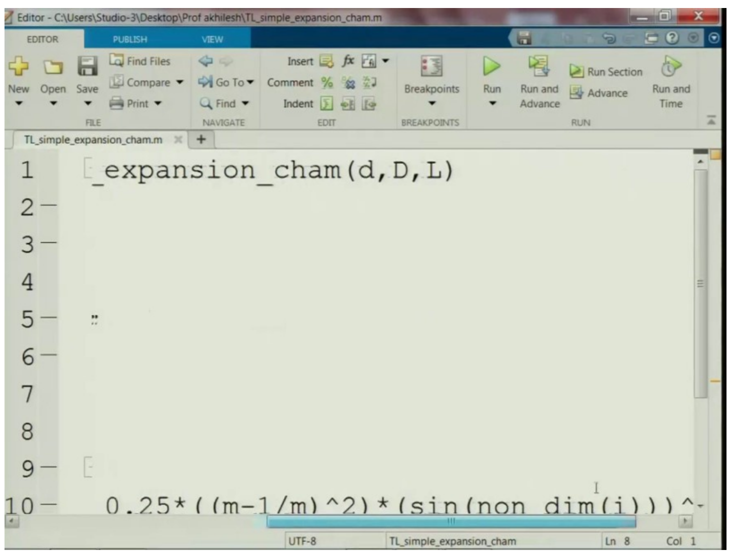
Now, before we analyze such elements I guess there are elements we actually we might just want to consider doing a parametric analysis of the transmission loss of such chambers. I will very shortly present you that and also talk about something like a cascading of transfer matrices. So, let me briefly talk about that; let us say we have written a code in which a simple algorithm.



I would not say even a code functions are used in MATLAB. So, say TL underscore simple expansion chamber.

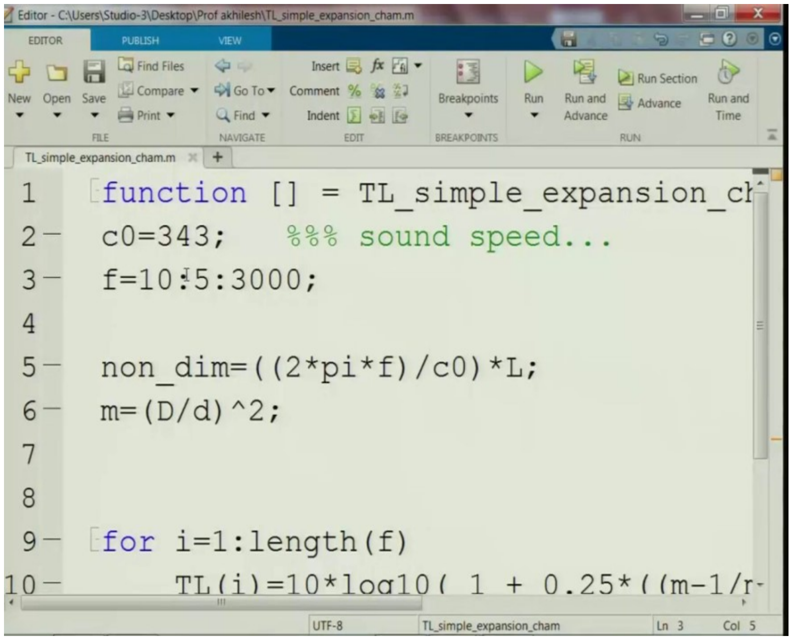


We have just increased the font size and what we are doing is writing a function.

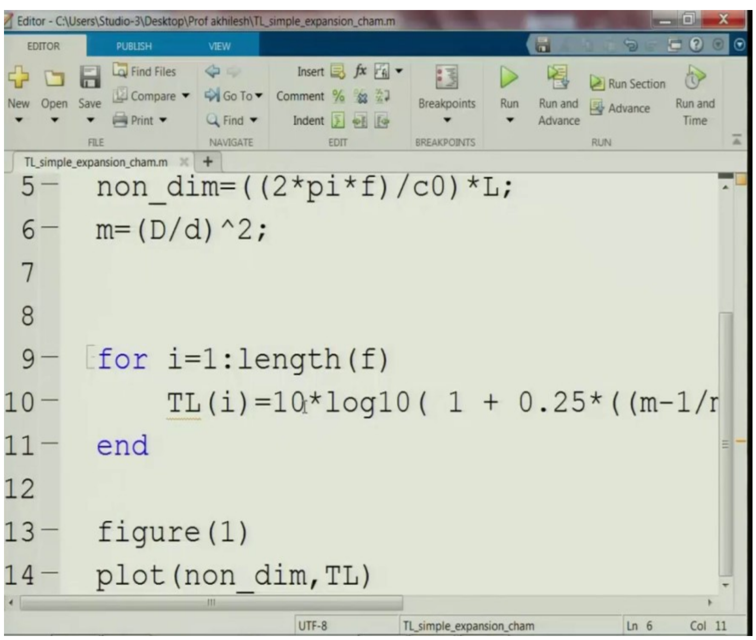


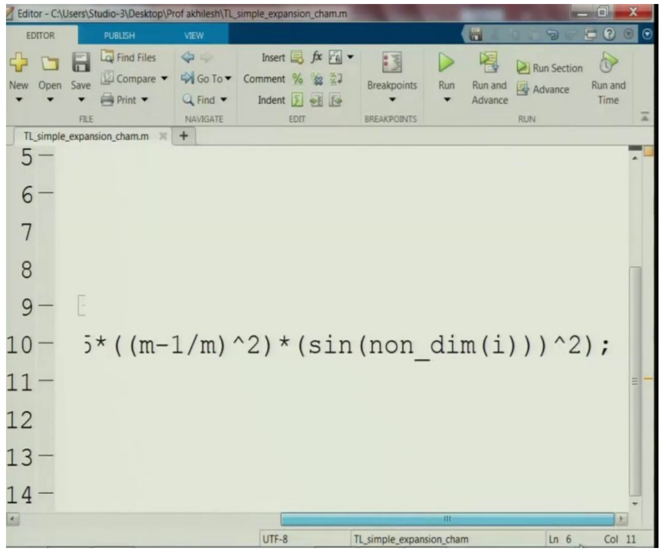
So, simple expansion chamber is just a name and it the function really wants 3 inputs you could write your own function and have different inputs including sound speed frequency range and so on. I have given it the option of choosing the port diameter the diameter of the chamber and the length of the chamber.

Note that the length of the inlet and outlet ports are right now sort of immaterial anechoic termination is imposed and there is nothing upstream what, that is what we assume sound speed. So, we could always comment things like this. So, alongside we will also try to learn some MATLAB hacks and this is your frequency. So, 5 is the lower frequency and 3000 is the upper frequency.

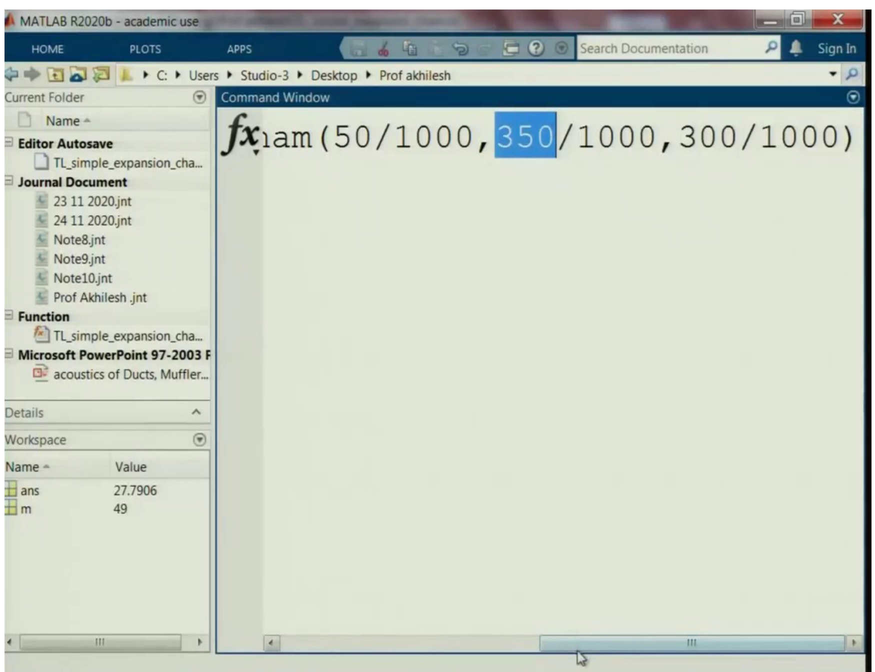


You could also do it something like 10 not a issue and 5 is the increment. So, then you do with 10, 15, 20, 30 so on. And you right now we have non dimensionalize the scale k remember this is the wave number and this is the square of the diameter ratio or basically area expansion ratio.

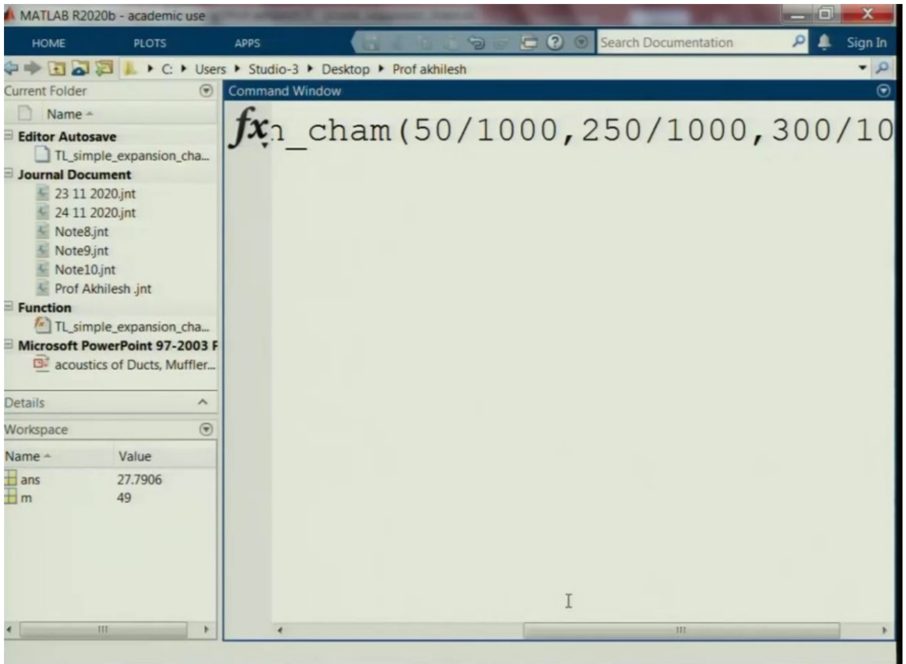




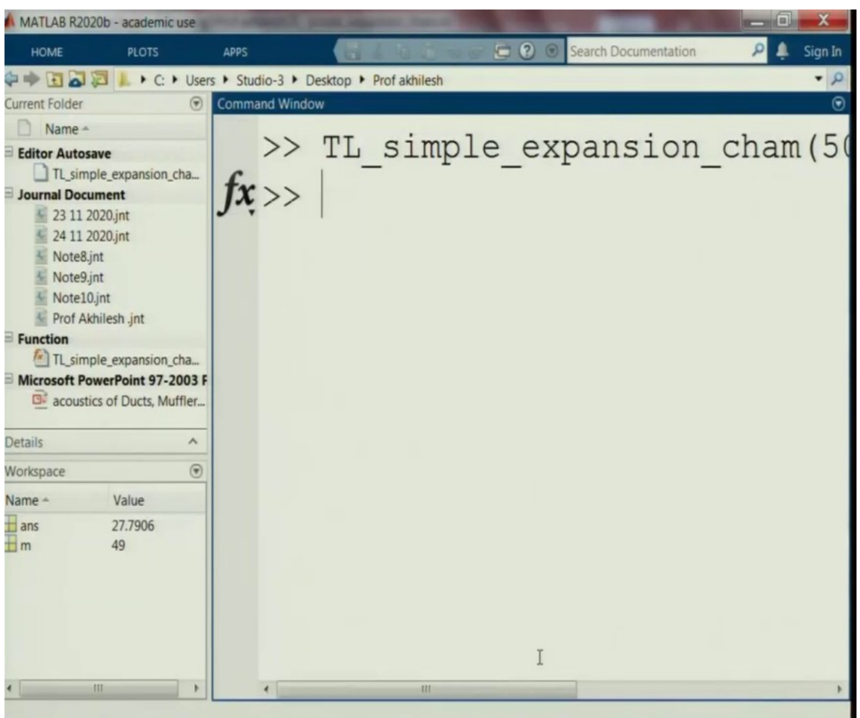
So, right now what we are doing is that using MATLAB to plot the analytically obtained transmission loss expression which is this one.

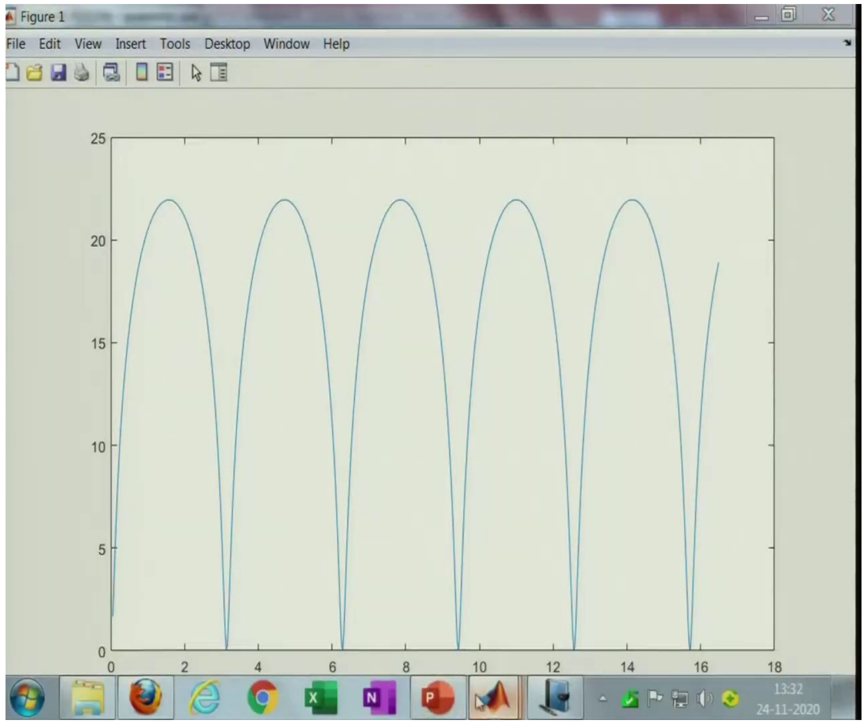


So, the idea is just to use that. So, once we do that we probably will get.

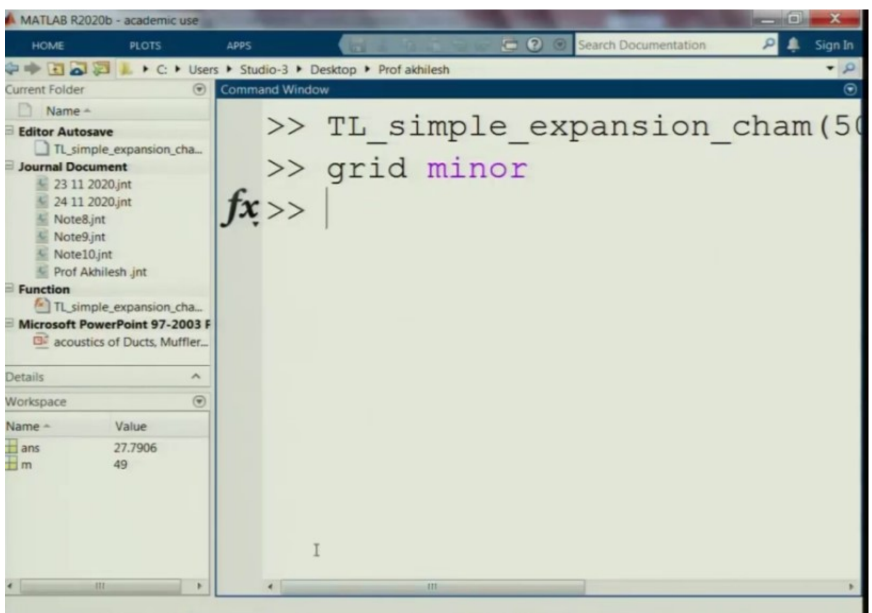


Let us say the diameter is chosen as 50 and the expansion ratio let us start with a smaller expansion ratio. That is given by let us say it is 250 and the length continues to be 300.

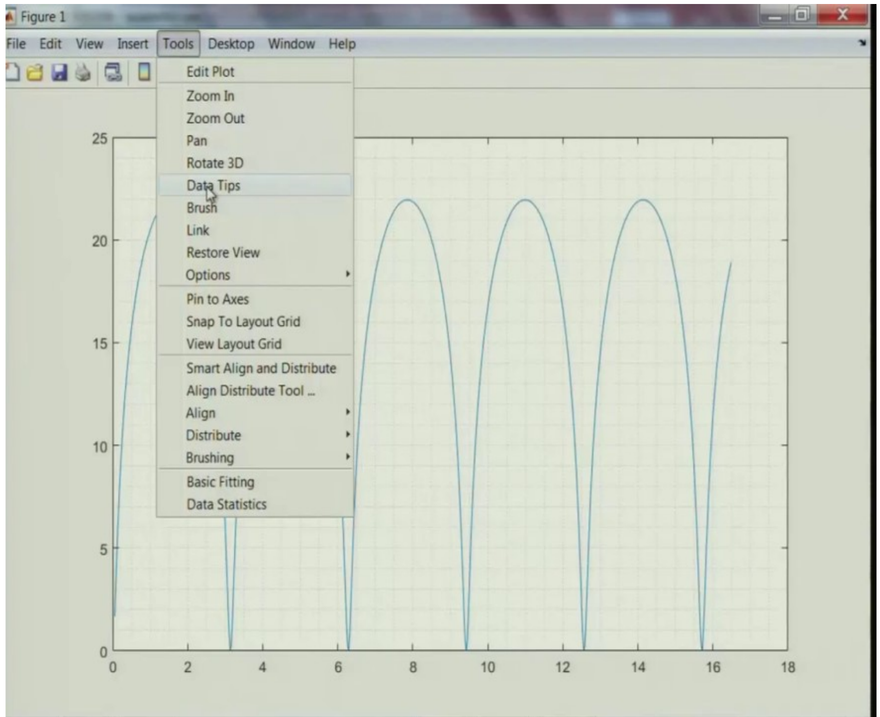




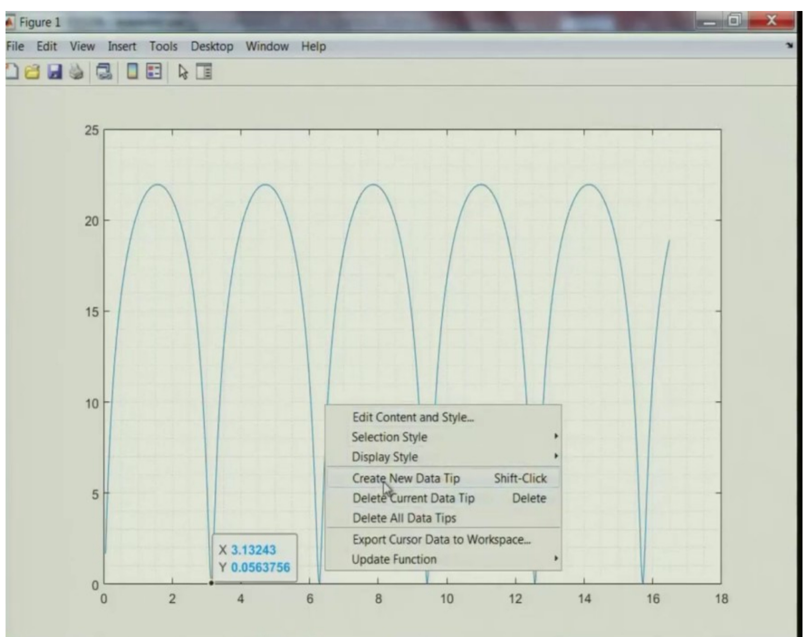
So, let us see what we get we get this.



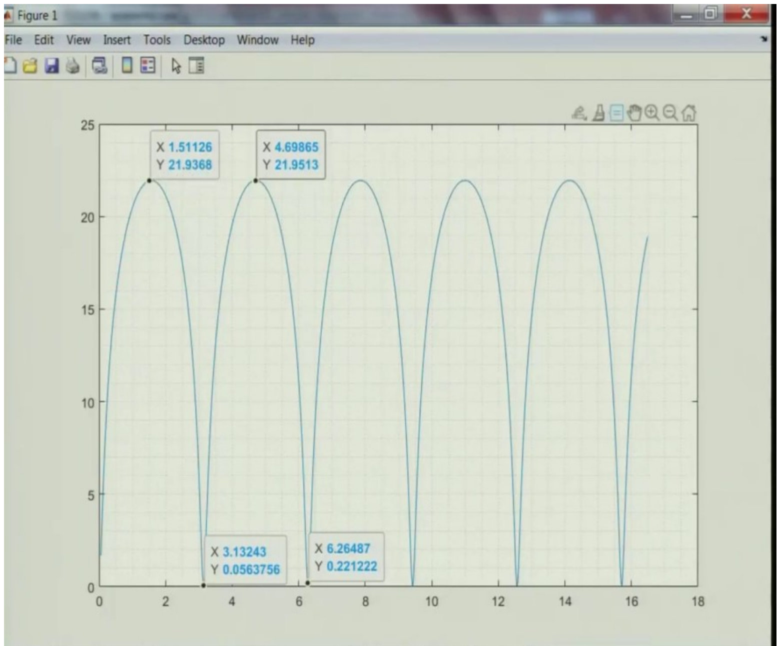
And let me draw the grids grid on for you a grid minor.



We will get this thing. So, we could use the data point data tips here and get an estimate of the value.

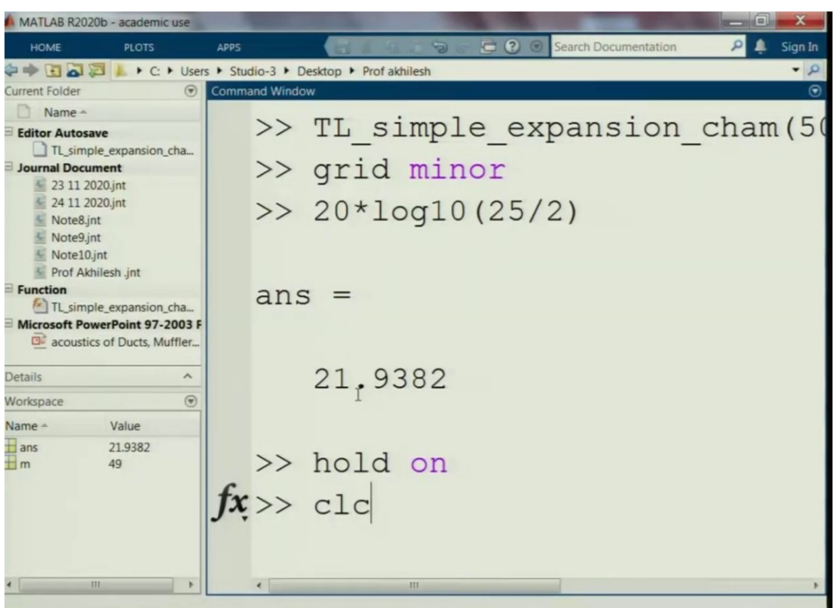


So, this is 3.14 approximately pi because of the placement of the mouse or frequency resolution we are getting not exactly pi, but very close to pi.

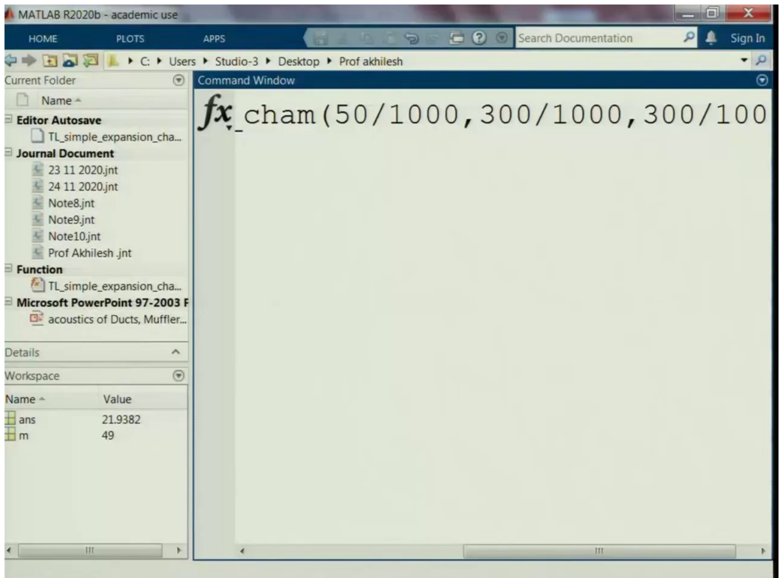


We can create another data point and continue to see that this is nearly 2 pi 6.2 something whatever it happens. And like this will continue to have 3 pi 4 pi and so on.

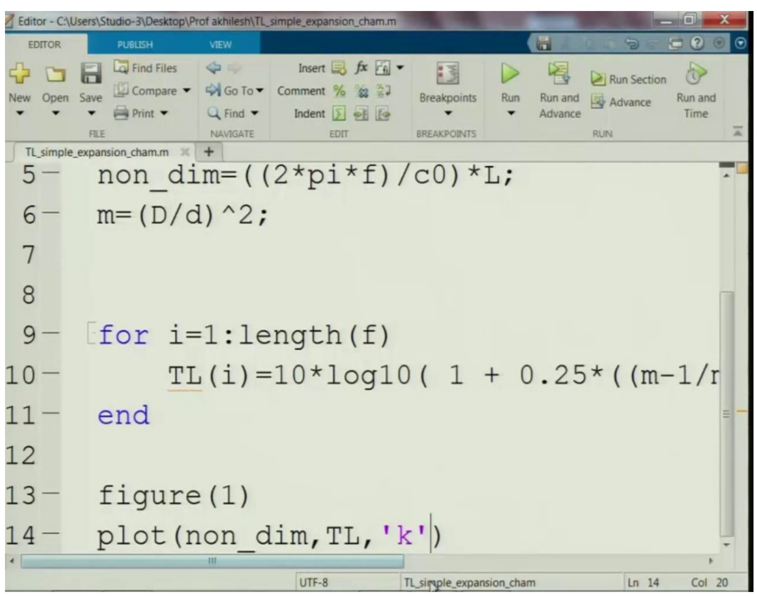
And the maximum frequency occurs domes occur when it is pi by 2 or that is close to 1.57 or 3 pi by 2. You know and like this we can continue to do that. And notice the important values 21.93 well 21.9 is the maximum TL and what did we get when I showed to you the this thing 20 times log M by 2?

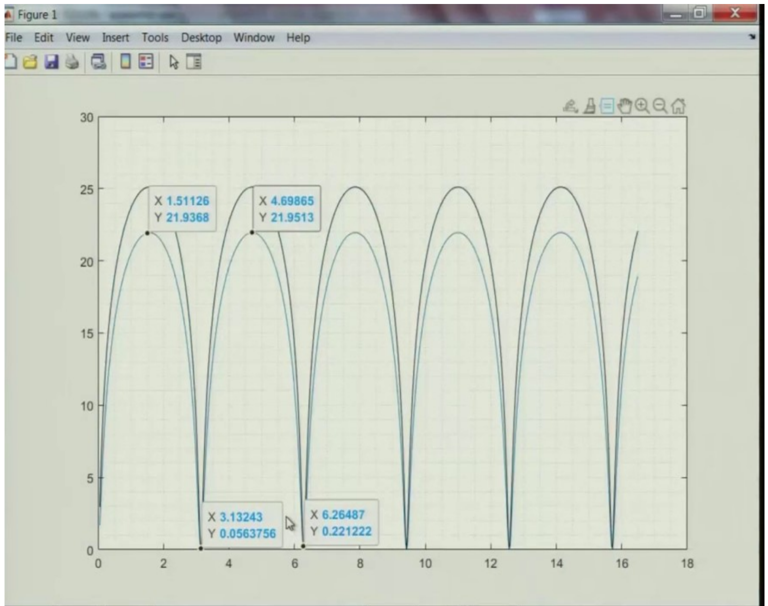


So, if you go to MATLAB and type 20 times log 10 and your M was what? M was 5 square 250 divided by 50. So, 5 5 square is 25 /2 if you do that it is 21.93 bang on and what we got is pretty much the same. So, this approximate thing gets you a very good result and what I will do just to drive home the message how important the expansion ratio is.

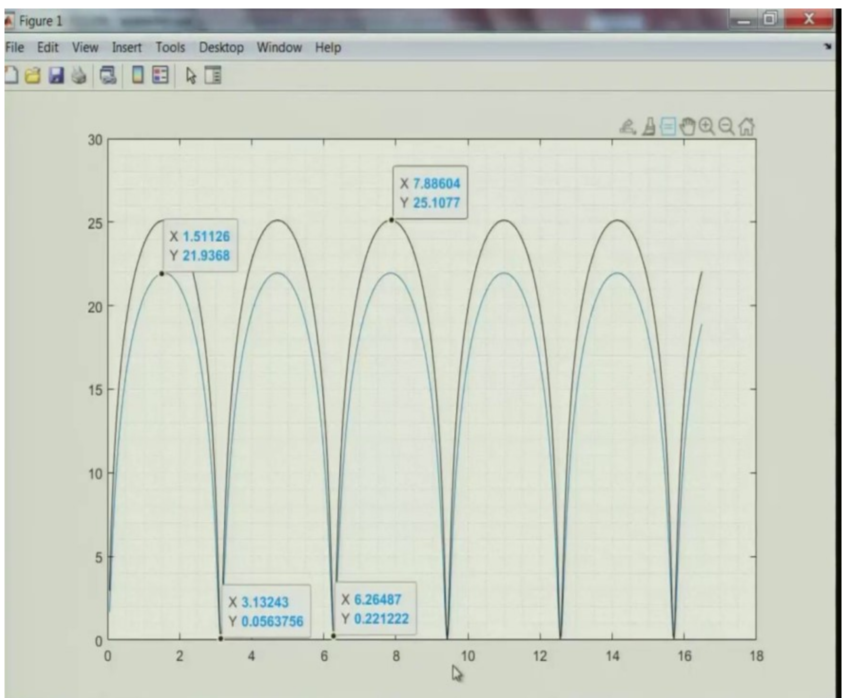


I have put this thing hold on that is basically to hold the figure and plot it using a different color which I am going to do now. So, let us say we have a slightly increased expansion ratio which is the like this

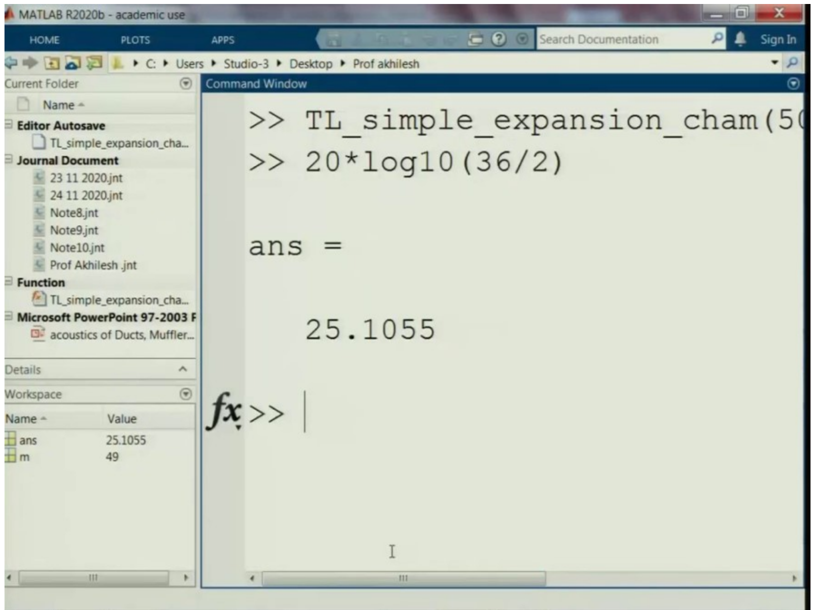




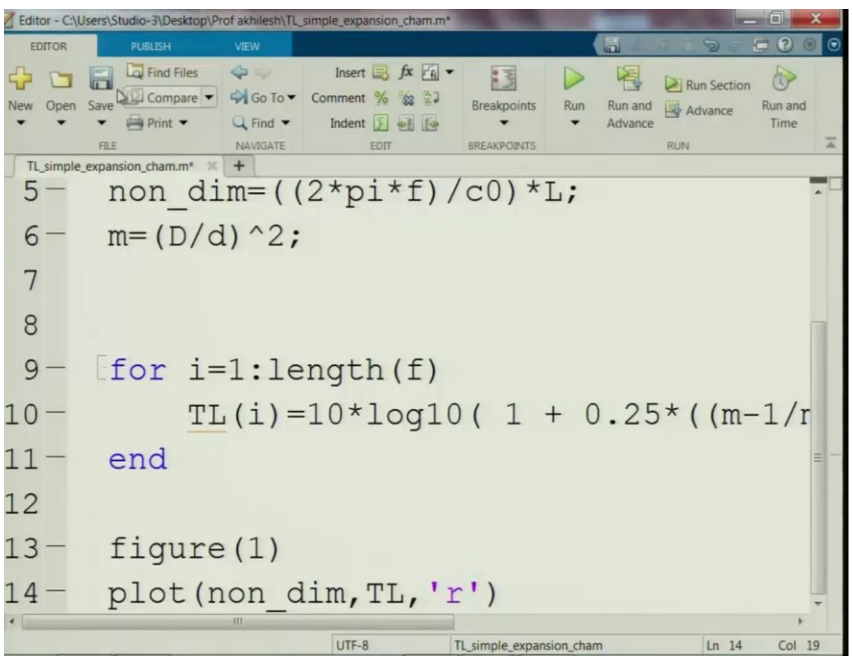
. So, you see the transmission loss has gone up. So, what exactly was it? It was well.



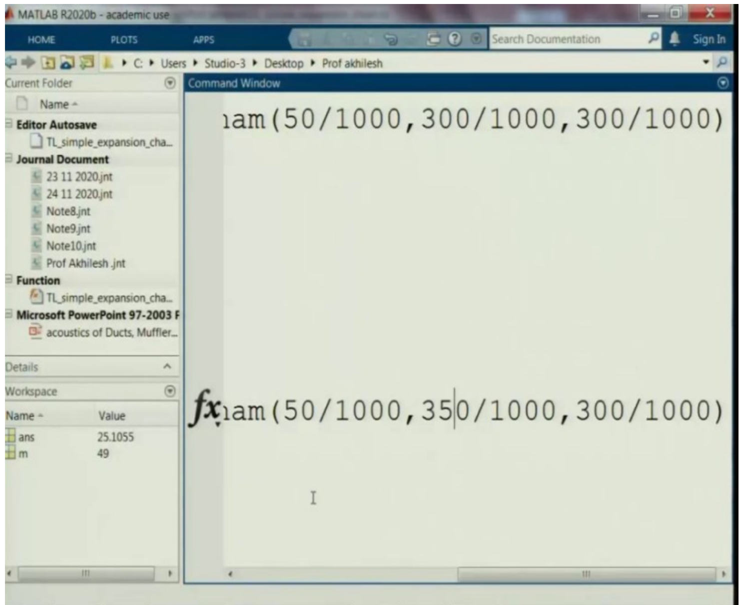
We can put your mouse here 25.107. So, 20 times log. So, 300 by 5 that is 6^2 is 36.

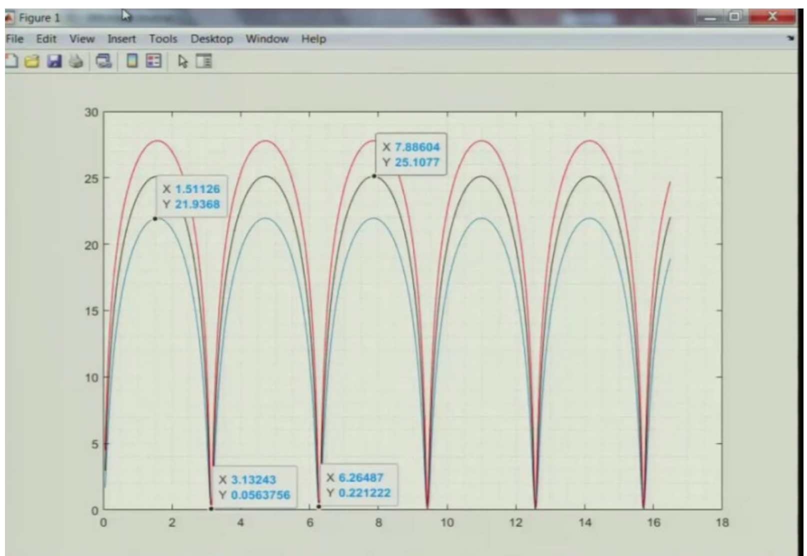


So, 25.10 and that is what we are; what we are getting. Now, if you continue to do like this.



Let us say let us consider another one final trial we probably can do.





Let us say if we put 350 we are going to get higher transmission loss well the and the troughs continue to occur at the same frequency. So, does the domes or things that are somewhere here where my mouse is.

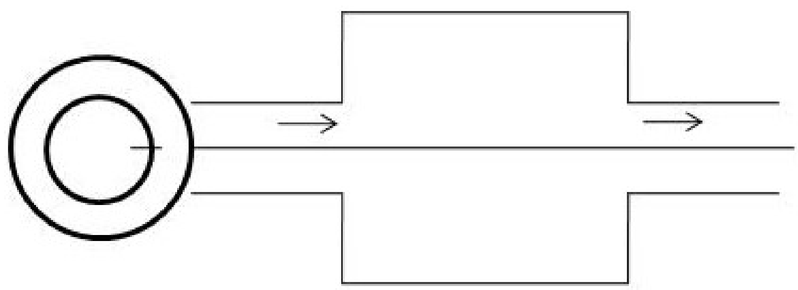
So, the idea is that by increasing the expansion chamber ratio we are going to improve the transmission loss efficiency we are going to get more attenuation, but one thing all this while we have not really bothered and that is what bothers me [Laughter]. We have not we have assumed only planar wave propagation so far. So, if you recall from our last lecture it was the limit for the plane wave propagation.

We see all these things later was

$$k_0 R_0 = 1.8412$$

for such a concentric simple expansion chamber with a concentric inlet and a concentric outlet that is you know if this is a center line and this is your thing. So, this is all coaxial or concentric for such a thing actually azimuthal modes or circumferential modes will not propagate.

$$k_0 R_0 = 3.83 \mid f_{cut}$$



Only the radial modes propagate and the first radial mode occurs at the frequency 3 point I guess if I am not wrong it is probably 3.83 or something like that for a circular chamber. So, we can work out that you know and we are seeing the non dimensional frequency axis. The point I am going to make is that we can obviously; calculate the frequency cut on frequency based on this expression at which the radial modes will start propagating.

So, the nice beautiful domes dome trough pattern this is the dome, this is a trough dome and trough and so on. So, this beautiful dome trough pattern will break down just before the radial mode starts propagating, the 0 1 radial mode starts propagating. So, what frequency does it happen?

It happens at a certain frequency which is given by the radius of the chamber. So, we can all obviously, show a line here which basically a vertical line to demarcate that beyond this frequency do not consider the planar wave propagation very seriously because it will eventually break down. So, right now our world is only about planar wave propagation. So, let it be like this till we are confident in analyzing a large well range of muffler elements like the ones we are discussing extensions and resonators and all that.

And with the understanding that the cut on frequency for the first higher order mode that is a limit up to which this predictions linear wave predictions can be considered seriously beyond which we should not. So, probably we have to wait till the time we consider 3D analytical formulation or analysis then we can consider all the 3D effects.

But for now we will be sticking to 1D and remember one thing that so far all the 3D techniques appear to be very attractive they are; obviously, much more accurate in 1D they are more realistic and so on. It is the 1 D analysis that really that really helps you to understand the physics of the system. For example, before we end today's lecture on a concluding note let me just share something with you.

So, we saw that you know this is the chamber and it has a beautiful dome trap pattern we would like to eliminate these troughs. So, by you know basically considering protrusions or inclusion extensions of the inlet and outlet port within the chamber we can actually eliminate this trough. How? Because the these annular cavities act like quarter wave resonators you know these cavities at the inlet and the outlet. So, why are we so interested in this? Because if we can choose the resonance frequency of this cavity.

Let us say one such that

$$-jY_0 C_0 t\ (k_0 l_1) = 0$$

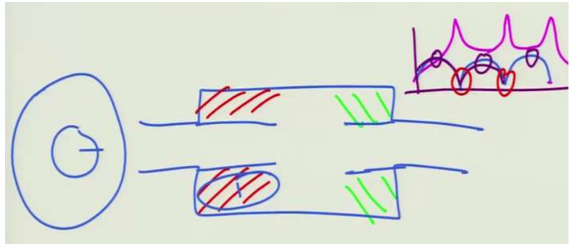
$$\sin\ (k_0 L) = 0$$

the frequency corresponding to 0 impedance remember this was the impedance. If this was tending to 0 at a certain frequency k naught and the same frequency corresponds to where the chamber was showing its first trough.

So, if these two frequencies can be matched, then the resonance peak due to this quarter wave resonator will annihilate or completely overcome the trough that is formed here. So, that the resulting pattern probably would be.

$$-jY_{ann1}\ cot\frac{\pi}{2} = 0$$

$$\sin(k_0 L) = 0$$



So, this is if this is the expansion chamber thing this thing would be something like this and we can similarly tune the length of this to get this thing and the third peak third trough will can also be uplifted. The idea is that resulting pattern would be a broadband attenuation using just a planar wave analysis and by wisely choosing the quarter wave resonator lens based on planar wave propagation.

But does that always help? Well for a planar wave we are assuming these are the theoretical limits and probably during the later parts of the course, when we do analytical formulations and all that. We will see that these extensions based on the planar wave limit $l/2$ $l/4$ as we will see in the next class are not very well they are useful to a certain extent, but they have to be supplemented by some end corrections or probably use of analytical models.

Nevertheless the 1D analysis has a has a special place is the beginning of the muffler analysis and helps in understanding the underlying physics.

Based on which we can you know do some further corrections like analytical 3D models or even numerical finite elements model based on actually a full 3D analysis or a mode matching techniques and all that. But for now this week and the next week we will consider only plane wave analysis at least for the next couple of weeks. So, before we move on to other topics.

So, for now I will stop here and I will see you in the next class.

Thanks.