Muffler Acoustics - Application to Automotive Exhaust Noise Control
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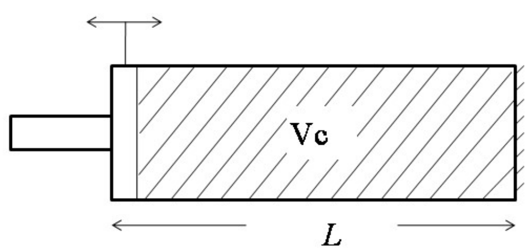
Lecture – 14
Lumped Analysis of a Uniform Pipe Closed/Open at an End, Concept of End-Correction.

In the last week, we were discussing about the Lump System Analysis.We discussed different elements such as inheritance, compliance and how do we arrive at the simplified expressions where lumped analysis really mean; that all the continuum of 1D or 3D continuum.They are approximated as one the entire continent is approximated as one particle which is behaving in a certain manner.

So, we consider the case of a constriction; a sandwich between two large volumes and then the case of a short cavity which is closed at one end. So, we derived certain expressions like I was mentioning.And in this week, we are goingto proceed further in that direction and for that end, we will revisit the case of acavitywhich is closed at one endand we have a sort of piston excitationat other end and the length is L.

So, we saw thatthe impedance of such a system wasgiven by



$$Z_{ac} = \frac{\rho_0 C_0^2}{j\omega V_c} = \frac{\gamma p_0}{j\omega V_c}$$

the volume ofthe system and p naught is your atmospheric pressure.

Now, how did we arrive at this expression?

$$\tan k_0 L = k_0 L + \frac{(k_0 L)^3}{3} + \cdots$$

So, how about if we retainthis termand neglect the higher order terms. So, higher order terms are neglected and we retain the underlying terms in the expression and see the magic what happens.

Well, in such a case what happens is the reason why I am doing this and it will be soon evident why this is important. So, Z input impedance then, given by

$$Z_{in} = \frac{Z_0}{jk_0L(1+\frac{k_0^2L^2}{3})} k_0L \ll 1$$

So, basically once we simplify this thing further, this expression we will see that this is nothing,

$$= \frac{Z_0}{jk_0L}\left(1+\frac{(k_0L)^2}{3}\right)^{-1}$$

$$\simeq \frac{Z_0}{jk_0L}\left(1-\frac{(k_0L)^2}{3}\right)$$

So, this is what we are going to get.And

$$Z_0 = \rho_0 C_0$$

So, what does it mean?What does this expression saylet us number this as1.what does this mean?

$$Z_{in} = \frac{Z_0}{jk_0L} + j\frac{k_0L}{3}Z_0$$

So, it means that the netor the actual impedance that,

$$Z_{ac} = \frac{\rho_0 C_0}{j\omega V_c} + \frac{j\omega\rho_0 V_c}{3S^2}$$

$$\frac{1}{j\omega C} + j\omega(L)$$

Then, this termthe second term is then a correction to the first term. So, this term is well acquainted with we saw that in the last lecture, and this clearly is a compliance.

$$j\omega L$$

$$L = \frac{\rho V_c}{3S^2}$$

So, this is you remember j omega L. So, L has this formrho naught times Vc.I would say this is Vc by 3S square S is a cross sectionalarea. So, it will satisfy the dimensionality things.And what basically it means is that the correction term which is the underlying term here, this is a positive reactance that is associated one third mass of the fluid in the cavity.

So, this is not quite a very unexpected result this is not very surprising, because the moment it is like a spring and if you have a bob or a mass on a spring and the mass of the bob is say M and the spring also hascertain mass M s.

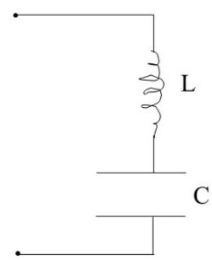
So, we typically from our elementary courses in vibration, we see that the net effective masses given by rootthen,I am sorry the net frequency then, is given by root over kby M plus M s by 3 or something like that.

$$\omega = \sqrt{\frac{k}{M + \frac{M_0}{3}}}$$

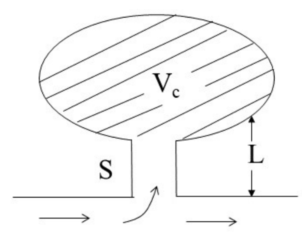
So, this is just for discussion that this factor of 1 by 3 that is coming here, that is exactly like a additional mass that is seen by the cavity.The impedance seen by the piston here, is if you consider the additional term is like the additional mass of the air that is also accounted for and that shows as the inheritance.That is a first order correction to the low frequency approximation.

But physically, what it also means is that the short end cavity if it is not all that short you have two things that are acting in parallel. So, this has a form of j omega L this is j omega C. So, this inheritance term is present when you consider a correction term. So, this inheritance acts like a correction to the compliance which is like a more fundamental behavior of such a short end gravity and in a way, both your compliance and inheritance are sort of active or operative and they act in series.

So, thecircuit electrical circuit electro acoustic circuit of a short end cavity when youconsider such a correction is then given by inheritance and your compliance which both act in series. So, if you consider the points here something like this it is likeah inheritance L and your compliance C; they both act inseries and this of course, is a correction to this term.



And if we recall our discussion for the Helmholtz resonator which was something like a big some irregular cavity and this kind of a thing.The same thing is going on you have your basic behavior is that of inheritance jωL and for this thing is 1 by jωC. So, both add up.



So, net impedance like what we discussed in the last class.Let me go to the slide the last class, we saw this was the system j ωL/S and +γ p_0/jωVC. So, when we simplify the matter simplify this thing and find out the frequency at which the impedance goes to 0; this is also imaginarythis part is imaginary this part is imaginary.

So, when we simplify the algebra and find out the frequency where the imaginary part of the impedance is going to 0. So, we will get the resonance frequency for the lumped system analysis, but before we go aboutdoing that business of finding out the resonance frequencies of a Helmholtz resonator, it is probably prudent to first talk about the end corrections for the first time. So, let us introduce the concept of end corrections.

$$\int_0^t U^2 Z_0 dt$$

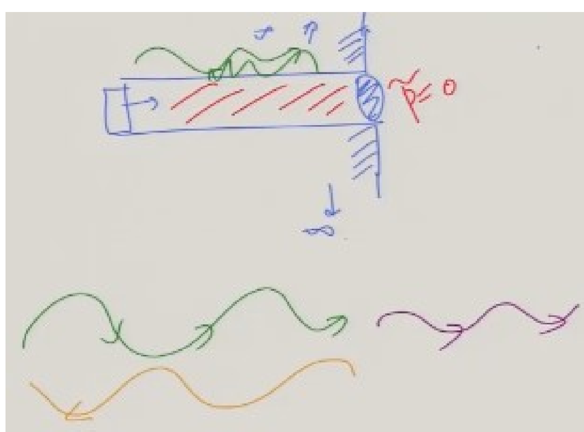
$$Z_0 = R_0 + jX_0 = \frac{\rho_0 C_0}{S}[R_1(2k_0 a) + jX1(2k_0 a)]$$

So, what do you mean by end correction?Let us consider in the first lecture of this week, we discuss very briefly all the very briefly there is a research matter in itself.If you have a open ended tube with a flangewith an infinite flange going to infinity here and going toinfinity hereand you havewaves that sees the impedance at this cross section area what I just drew.

So, what is going to be the effect. what is going to be the impedance that is seen. So, basically the idea is to get some sort of arealistic expression for the impedance.One can actually argue that there is fluidin the duct and we can simply put the conditionp tilde 0.

So, it is an open-ended condition while this is not all that inaccurate; the fact is that when a wave is incident sayfrom this part;and open-end condition would mean the entire waves is reflected in the opposite direction.

So, if you have wave going like this in its entire and the wave is completely reflected back; however, this is usually not the case, because suppose, you have a wave coming in from here, and you have a certain wave that is reflected back some amount of acoustic power alsopropagates downstream into the open end and that is what causes all the drama all the mathematical things start to fall then, because this is really the same amount of partial transmission is going on.



And so, it is necessary to account for the impedance seen here at impedance imposed by the atmosphere all thisatmospherewhatever impedance it imposeson thewave that is incident at this point we are going to account for that.

And what it physically would mean we will see by some simple algebra that it means there is an extra column of air just hypothetical column of air sitting here and it is like an addition of an extra mass. So, it is like an inheritance term that is added.

So, before we do that let me justwrite downthe expression for the impedance.

$$\mathrm{Z_0} = \mathrm{R_0} + \mathrm{X_0}$$

impedance for a open tube with infinite flange on both sides. So, in such a half space it is also called half space mathematically.

So, R_0 and X_0 take certain forms. So, the entire expression would basically

$$= \frac{\rho_0 C_0}{S}\{R_1\,(2k_0a) + jX_1\,V(2k_0a)\}$$

$$S = \pi a^2$$

$$R_1(2k_0a) = 1 - \frac{2J_1(2k_0a)}{2k_0a}$$

$$= \frac{(2k_0a)^2}{2 \times 4} - \frac{(2k_0a)^2}{2 \times 4^2 \times 6^2} + \cdots$$

So, we can very safely approximate this by retaining only the underlying term.When we do that what are we going to get.We are going

$$\simeq \frac{(k_0a)^2}{2}$$

this is approximation.And similarly, what about the term that is present here X naught.

So, this let me spend a little time here. So, what is R_0 term is like a real part of the impedance and that is basically, a lossy that represents loss and this is the reactance term is also called the resistive term or the real part of the impedance and it denotes loss in the fluid or not in the fluid due to the radiation and X_0 is the reactance term that. So, if you integrate the power, there will be certain power that will be lost.

So, if you do something like U^2times Zdtand integrate it over a certain timeand you put this thing Z_0 here.The entire quantity will be non-zero, because of non-zero behavior of this; that means, certain amount of energy or acoustic power is lost per cycle because R_0 is non-zero.

So, the reactance does not contribute to thepower loss.It is just like energy being acoustic power being shifted in the acoustic potential energy or acoustic kinetic energy of the particles, but it is R_0 that contributes to the loss.

And X_0 the reactance the expression for that

$$X_0 = \frac{4}{\pi}\frac{(2k_0 a)}{3} - \frac{(2k_0 a)^2}{3 \times 5} + \cdots$$

$$\simeq \frac{8k_0 a}{3\pi} \simeq (0.85k_0 a)$$

So, the net impedance that is seenat this point by a wave is can be written safely in the low frequency that is k_0 a <<1.

So, if you have a pipe whose radius is a and for low frequencies.

$$Z_{0c} = \rho_0 C_0 \frac{k^2}{2\pi} + \frac{j\omega\rho_0 S}{S^2}\left(\frac{8a}{3\pi}\right)$$

So, we are trying to simplify things and plusj omega timesrho naught S by S square into 8 a by 3 pi. So, this of course, follows from certain simplification you have your 8a and3 piIhave club together; k naught I have taken and have put omega by Cnaught andsome simplifications were done to finally, arrive at the actual impedance seen by the wave here. So, the idea isthat this is the expression. So, the right-hand side then, is dominated by the positive reactance that is yourthis part underline part.

$$\rho_0 S \left(\frac{8a}{3\pi}\right) - 0.85a$$

$$\frac{kg}{m^3} m^2 \,.\, m$$

$$t + (0.85a) \times 2$$

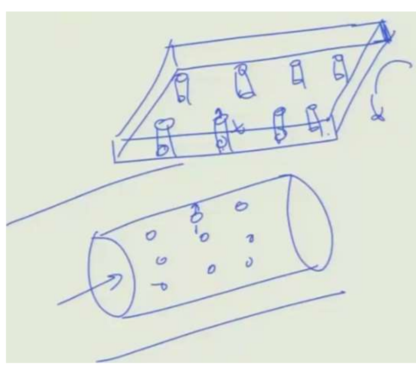
$$t = t + 1.7a$$

So, the idea is that at this pointwe have an additional mass or a small.The moment this wave arrives atthis point at the point at the interface of the atmosphere and the end of the tube, it does not see a zero load; zero load means we have this condition p tilde 0, which is not the case when you do this thing.

We have a finite load, and the load consists of mainly a short continuation of the tube.It is like the tube is not really ending at this point.It is slightly extruded. So, if you have a certain tube here and you have an infinite flange here,here the tube does not just end here. This small amount of small amount of tube that is protruding out hypothetical tube. So, there is a length.What is the length of such a small tube of the same radius?Radius is the same.S is the cross-section area.

So, the length is 8 by 3 pi that is basically, 0.85 times a. So, if a is very small like typically, say 40 mm for automotive mufflers. So, we can easily work outthe small protruding length hypothetical length the wave would see. So, this is like your end correction and we will see why this is important as far as resonance frequencies are concerned.

Similarly, now, if you have. Suppose typically you have things like big sheets or let me draw things for you or you have things likeyou have things like this. So, you have holes that are distributed across the face. So, you can call this as plates with perforated hole. So, typically you can bend the sheet.



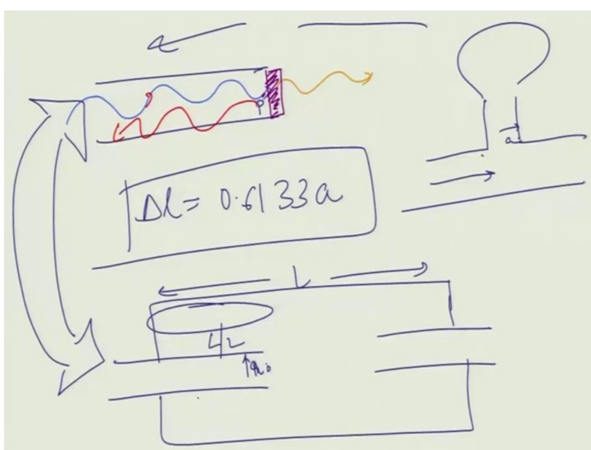
I will show some photographs in due course.Maybe, you will have some dedicated lecture which shows you nice photograph you can relate that.This is just a rough sketch ofthis thing.If you bend the sheet across and you have small holes also called perforates.

So, you have a pipe filled with all the holes and if you have a pipe like this with small holes everywhere. So, this is like you know you are trying to bend the sheet into a pipe and there is a fluid flow here or air is flowing through the airway then, outside there is an annular cavity. So, on this side you can consider this as a hole with a infinite flange on both side and similarly for the hole inside.

So, the basically when you evaluate the thickness of this thickness t. So, it is not just tit is something like 0.85 times amultiplied by 2, because you see the end correction that is adding here as well as here; meaning that it is the end correction is adding at this side and also this side. So, the net effective thickness becomes 1.7 timesthe radius a.It is important in a Helmholtz resonator context.We will soon see why this is important.

So, this is the net effective length you can call this t e f ft effective length. And for small tubes it may so happen that this term the correction term dominates the physical thickness of the tube or a sheet of metal.

So, this term is important end correction term and that is, because the fluid is seeing a virtual or a hypothetical kind of a thing in here. So, this thing adds on both sides.I hope I am clear, but we will probably have aquestion answer session in which you can answer this question in a more upfront manner. There are; obviously, situations in which we do not have a case like this rather we might have a case where you haveyour tube which is just plainly exposed to the atmosphere.



So, when a wave is coming here, some part is being reflected back, but other part is also going. This is much more complicated, because the point I am going to make is thatthis thing does not existthis infinite flange businessis no longer there.This has this really simplified the matter, but the fact that it is notthere poses considerable difficulty in the mathematical analysis.

So, just to keep it simple,I would just say this has been again subject matter of the past researchquite a few papers are there for acoustic radiation impedance seen at the end of a un-flanged open pipe. It isnow, we have this space also and this space also.It is no longer a half space.Half space means you have only infinite space in one direction, other is constrained.You have almost a full sphericalspace minus this duct of course.

So, the impedance like I said was much more complicated to derive there has been papers, but the important result that we will be using in this course you might well use is that; you know typically when you have things like this the end correctionlength of the hypothetical fluid cylinder that is added right at the point p say at the end of the tube is the length of such a thing is given by $\Delta l = 0.6133a$.

So, you can have situation in which one end is open and the other end is flanged. So, then, the end correction will be this one plus 0.85 times awhatever it is. So, we can consider the corresponding cases as well. So, it depends on the application of the end connection then is we must carefully choose that.

I will present to you one example before I close this particular discussion.Just like we have a Helmholtz's resonator things like we sawthings like thisbeinganalyzed. So, here also it can be considered flange and this part also is flanged. So, the length is your 1.7 times the radius of this part, but you also have a very important muffler element called extended inlet and outlet element.

So, you typically havethis kind of a structure. So, our analysis would say that this is say if the length is L; the length of the tube is L and this is L by 2. So, the impedance or the resonance frequency of this quarter wave resonator would actually effectively cancel out the trough or the first actual resonance of this chamber.

We will do all the match later, but the point I am trying to make here, is that you see carefully analyze this small pipe; this small pipe and you have a big chamber.Something like you have a pipe which is being extended into a big cavity muffler cavity. So, this can be considered as a case this case.

These two cases in a wayare equivalent, because you have a open ended pipe discharging into a almost un-flanged domain or a space. So, the end connection. So, what we see essentially say the radius isr naught.

So, the net effective length let us say,the length of this thing can be l1 or the extensionnet extension. So, the acoustically.

$$l_{eff} = l_1 + 0.6133\left(1 - \frac{R_0}{R_0}\right)R_0$$

It would not be that simple, because you also have some bounded.It is after all the bounded media.

So, when r naught is equal to R then,of course, there is no meaning to this term, but when basically r_0 is very small; actually, there should be additional things maybe perhaps it could be r naught somewhere sitting out here.

So, then when r naught is very small then, this term can be neglected.Basically, the idea is that the inside term kind of acts like a correction term to the classical end correction approach.We will probably worry about these things a little later as we go into the transfer matrix approach and matching conditions and all that kind of a thing.

For now, we will probably end this lecture.And in the next lecture, we are going to talk about the resonance frequencies of Helmholtz resonatorto start off withand then, probably your concentric hole cavity resonators and a few important constituent elements.

And then, we will worry about probably about the electro acoustic circuits and Thevenin's theorem,Norton's theorem and those kinds of things that will probably be taken up in the next lecture.

So, thanks a lot andthanksfor attending. I will see you in the next lecture.