Muffler Acoustics - Application to Automotive Exhaust Noise Control
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Lecture – 12
Acoustic Impedance and Reflection Coefficient

Welcome to the second lecture of week 3. In this class in this lecture, we are going to talk about for the first time about the Constituent Elements of a Muffler, what are the basic elements that constitute a muffler. And, but before we do that, we need to again revisit some expressions, for the acoustic pressure planar waves for propagation in one dimensional duct as well as the expression for acoustic particle velocity along the same direction.

And develop a relation between acoustic pressure and acoustic particle velocity and the progressive wave variables. And talk about impedance relate impedance at different sections of a tube and derive some meanings. And, then talk about different elements like a lumped mass approximations and different parameters that constitute, different measures basically metrics that using which we evaluate the performance of mufflers.

So, let us begin with the 1D duct case, one dimensional duct. So, let us say, we have a wave propagation in a 1D duct, and you have certain waves, which go like this. So, the acoustic pressure then

$$\tilde{p}(x) = Ae^{-jk_0x} + Be^{jk_0x}$$

So, like I have been mentioning A and B are your amplitude of waves that propagate in the positive x direction this propagates in the negative x direction ok, alright. So, we have seen that using Euler equations, you have certain things. So, this will basically lead us and once we assume time harmonicity and all that,

$$\rho_0 \frac{\partial \tilde{U}}{\partial t} = -\frac{\partial \tilde{p}}{\partial x} => \tilde{U}(x) = \frac{1}{\rho_0 C_0}\left\{Ae^{-jk_0x} - AB\right\}$$

So, let me write down this thing very clearly, this is something like this. So, we need to make some meaning out of it this is what it is. Now, this thing can be written in the form of mass velocity. If, you recall the last weeks lecture.

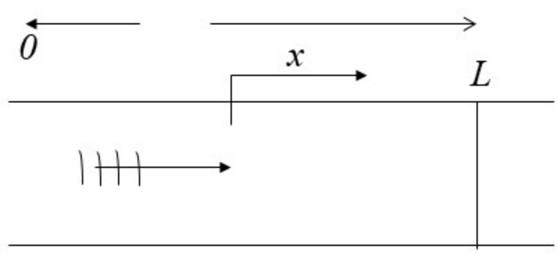
$$\tilde{p}(x) = Ae^{-jk_0x} + Be^{jk_0x}(1)$$

$$\tilde{U}(x) = \frac{1}{\rho_0}\left(Ae^{-jk_0x} + Be^{jk_0x}\right)$$

$$\tilde{V}(x) = \frac{1}{Y_0}\left(Ae^{-jk_0x} + Be^{jk_0x}\right)(2)$$

$$Y_0 = \frac{C_0}{S}, k_0 = \frac{\omega}{C_0}$$

So, we will use this relation this one and this one to derive certain relationship. Equations (1)and (2). Basically, the impedance at any point let us call this distance as x and this is 0, this is L perhaps.



$$Z(x) = \frac{\tilde{p}}{\tilde{v}}$$

$$= Y_0\frac{Ae^{-jk_0x} + Be^{jk_0x}}{Ae^{-jk_0x} - Be^{jk_0x}}$$

$$Z(0) = Y_0\left(\frac{A+B}{A-B}\right)$$

we derive some meaningful things. And it is easy to see, that at x is equal to L, this end might be open or it might be closed. It might be close with a rigid termination we will see what happens by giving it different values of Z.

$$Z(L) = Y_0 \frac{Ae^{-jk_0L} + Be^{jk_0L}}{Ae^{-jk_0L} - Be^{jk_0L}}, e^{j\theta} - C + jS \quad \cos\theta + j\sin\theta$$

Euler formula.So, once we do that and you know do the Math's do the algebra, what are we going to get? We can simplify this further to get.

$$= Y_0 \frac{(A+B)\cos k_0L - j(A-B)\sin k_0L}{(A-B)\cos k_0L - j(A-B)\sin k_0L}$$

So, once we simplify this thing further what are we going to get? We will

$$Z(L) = Y_0 \frac{\left(\frac{A+B}{A-B}\right) - j\ tan\ k_0L}{1 - \frac{A+B}{A-B}\ j\ tan\ k_0L}$$

So, we can nondimensionalize it and

$$\frac{Z(L)}{Y_0} = \frac{\frac{Z(0)}{Y_0} - j\tan k_0L}{1 - j\ \frac{Z(0)}{Y_0}\tan k_0L}$$

So, basically if we were to simplify this thing further, we will get.

$$\frac{Z(0)}{Y_0} = \frac{\frac{Z(l)}{Y_0} + j\tan k_0L}{1 + j\ \frac{Z(L)}{Y_0}\tan k_0L}$$

So, where are we heading towards or

$$\frac{Z(0)}{Y_0} = \frac{\frac{Z(l)}{Y_0}\cos k_0L + j\sin k_0L}{j\ \frac{Z(l)}{Y_0}\sin k_0L + \cos k_0L} \qquad (3)$$

So, this expression is of significance to us and we will probably call this as number 3, because now if we again recall our system that we have got, this is the duct at x is equal to 0 and x is equal to L. Suppose, if we fix up this end. So, if we consider a duct like this.

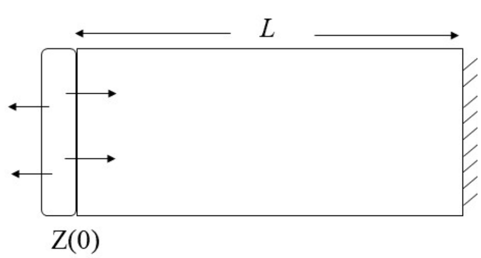
If we have a duct like this, and if we and if we have a duct like this, and if we block the end here, so, and do a piston excitation here or loudspeaker, so, what does the rigid surface do? What does the rigid surface do? It basically means that the velocity normal to the surface is 0, that is u n or u here is 0. So, when u is 0 what does it mean acoustically? It means, p by u is Z.

So, if u standing to 0; that means, Z standing to infinity. What we see here? If, we have a piston doing to and for motion and if you have a rigid surface on the other side, we will see the infinite impedance at this surface. In terms of the impedance perceived at x is equal to 0 or Z is equal to 0 end, that we can nicely evaluate by taking this particular term tends to infinity. So, does this one and so, these terms the underlying terms they are negligible in comparison to these ones.

So, what will we get,

$$Z(0) = -jY_0 \cot k_0 L$$

So, here we are taking j in the numerator and Y naught goes also in the numerator cos by sin so, this is what we are getting. So, for a closed cavity of length L and we are doing piston excitation back and forth.



So, at this stage at this point we are going to get this Z(0) is pretty much given by this expression. So, we can draw a lot of conclusions out of it. So, at this point the expression is this thing. So, if Z(0), that means, at those frequencies we

$$Z(0) = 0$$

$$\cot k_0 L = 0$$

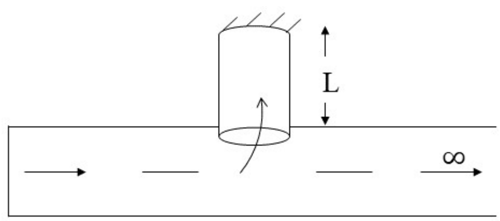
$$\frac{\cot k_0 L}{\sin k_0 L} = 0 \quad \cos k_0 L = O = \cos(2\,\mathrm{n} + 1)\frac{\pi}{2}$$

$$\mathrm{k_0 L} = (2\,\mathrm{n} + 1)\frac{\pi}{2}$$

$$\frac{\mathrm{2nf}}{C_0}\mathrm{L} = (2\,\mathrm{n} + 1)\frac{\pi}{2}$$

And, once we simplify this thing further such a frequency,

$$f_R = \frac{(2n+1)C_0}{4L} \;;\; n = 0 \quad f_R = \frac{C_0}{4L}, \frac{3C_0}{4L}, \ldots.$$



So, this is what we are going to get. So, what it means is that we have a certain side branch like this. And, if we have a tube system which say let us say goes to infinity and there is a certain wave coming from this direction and we have a blocked end here.

For certain frequency is given by f R and this is the length the length is L. So, if we have length L and waves are coming here; and, so, for certain frequencies given by $C_0/4$ L 3 / 4 into $C_0/$L and so on, all the ways will be diverted here. And nothing will propagate downstream, that is the path of least resistance what most of us do?

Basically, all the acoustic power goes in the along the side branches and very less power is transmitted downstream this thing. So, these are also called quarter wave resonators. Because for certain frequencies, which is given by $C_0/4$ L, and it is multiples, and it is odd multiples, most of the all the acoustic power goes in the direction of the duct and very less power is transmitted. So, using this principle you can control you can achieve certain tonal noise control.

So, in a way we can also think of this particular expression.

$$L = \frac{C_0}{4f_R}$$

$$L = \frac{\lambda_R}{4}$$

$$\lambda_R = \frac{C_0}{f_R}$$

So, we are just trying to put this particular thing in here and so, length should be quarter of a wavelength.

So, wavelength can; obviously, vary it will vary with frequency. So, for specific wavelength or for specific tones certain tonal frequencies complete noise control can be obtained theoretically. However, life is generally not simple, and you have typically no material is rigid, what I mean to say.

So, whatever you are assuming this as rigid it is usually not rigid, the end plates usually no matter how thick they are after all they have some finite impedance. That is acoustic velocity, or the structural velocity will never be 0, you will always have certain structural certain velocity of vibration of this.

So, as a result we will probably experience an impedance, which is not infinity here at this point or finite and then we have to consider the effect of end plate vibration. So, that will basically complicate problems. And this is important because not only in the side branch resonators the noise is radiated by in this direction, in all directions, we also have typically things like the end plates of mufflers.

So, no matter how thick or properly you weld them, it is going to vibrate a bit. And that is going to have a major effect on the structural acoustics part of the breakout noise what it is called. We will talk about that probably sometime later, but another expression we can probably derive quickly for the impedance.

So, if you were to define the reflection coefficient,

$$R = |R|e^{j\theta}$$

This is your reflection coefficient, reflection coefficient.

$$R(0) = \frac{Be^{jk_0x}}{Ae^{-jk_0x}} = \frac{B}{A}$$

So, with this thing if we simplify if you put in our old expression for Z is generally like this. We are going to get,

$$R(L) = \frac{B}{A}e^{2jk_0x}$$

And, if you simplify this thing further reflection coefficient,

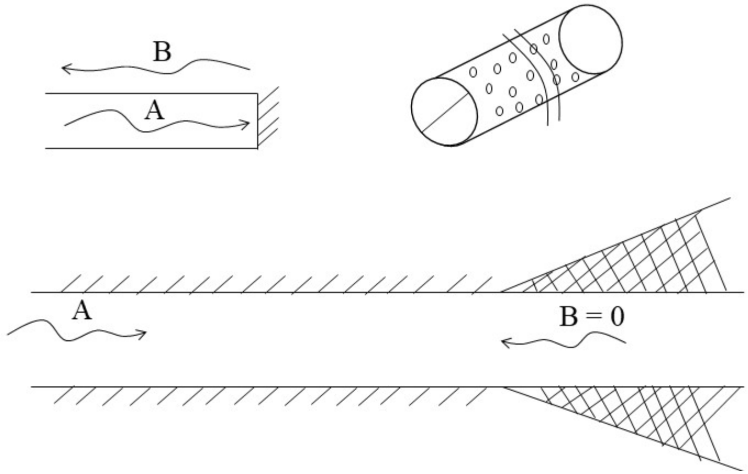
$$Z = Y_0\left(\frac{1+R}{1-R}\right) \;=> R = \frac{Z-Y_0}{Z+Y_0}$$

So; obviously, when we have

$$R_{\text{rigid termination}}, \rightarrow 1,$$

we have this tends to 1, because then your Z thing that is tending to infinity. So, this only this term will contribute will be dominant.

And, what it means that the entire wave is reflected back without suffering any phase change or something like that. So, stay for a perfectly rigid termination reflects the incident waves with the same amplitude and phase. Another termination that is very important in muffler acoustics is the anechoic termination.



So, we what we saw in this slide, let me draw it again for you rigid termination is of course, like this is blocked end completely no. Another termination so, whatever waves are going here, whatever waves are striking here same thing is coming back B ok Now, another termination that is used very often in muffler acoustics is basically your anechoic termination.

So, it is typically drawn like this you have wave like this. So, what it means? Nothing comes back. So, this motion is not allowed. Only A is allowed B is 0, that is no matter how loud the sound is or whatever the condition is whatever goes inside this termination is completely absorbed by these linings. Basically, they are nothing, but mineral wool glass or some absorbent material.

So, whatever all this while this is a rigid termination, rigid cylindrical duct made of steel, or brass, or whatever it is. And here you have a conical kind of a termination, conical annular area, filled with rock wool glass wool or some dissipative stuff. And, as the wave traverses through this path and obviously, I forgot to mention one thing here, they are not; obviously, solid tubes they are perforated.

So, you see perforated tube. So, typically by perforate we means, the small-2 holes will be there along the along the curve surface area. The waves going inside can interact with the stuff that is outside through these holes; that is what is going to happen here. The waves they enter these holes, and they get dissipated, most of the acoustic power is dissipated.

But again the termination being anechoic, or the principle or the ability of the termination to completely prevent or suppress the waves that travel in the backward direction, it depends on its construction the material the flare angle theta, or whatever the number of parameters, and; obviously, frequency. So, the point I am trying to make is that no termination is perfectly anechoic for all frequencies, it depends on the frequency. So, we need to design it properly based on a lot of experiments.

So, the idea is that we can consider a termination and a quick beyond a certain frequency, much in the same manner what acoustic chambers or anechoic chambers are designed. So, this termination is used in setting up a muffler testing rig, anechoic termination using some parameters which will cover later.

So, equivalent of a tube with anaerobic termination equals characteristic impedance of a tube. So, when we basically let us go back to this expression. So, when we put reflection coefficient to

$$R = 0 \qquad\qquad R = \frac{Be^{()}}{Ae^{()}}$$

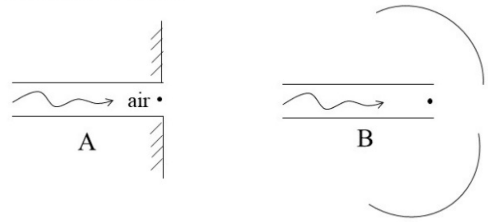
basically nothing but e to the power whatever it was and A to the power whatever it was. So, if you put the numerator 0, we are going to get R = 0.

Once, you put this R is 0 here, what are we going to get? We are going to get Z is equal to Y naught. That is the impedance at any point is equal to the characteristic impedance of the tube. Much like the free field you have rho naught C naught here you have Y naught by S cross section area. So, it depends on the sound speed as well as the cross-sectional area of the duct. So, because this is not surprising, because we have only here a progressive or the forward travelling wave.

So, another thing before we move on with the other elements is the radiation impedance, I am just going to talk about that very very briefly, radiation impedance is nothing but the impedance imposed by the atmosphere on the acoustic radiation of the end tube. So; obviously, a full-fledged 3D analysis is required.

So, if you have a certain tube lot of work has been done so on this. So, if you have a tube and asome wave is coming out here and this is your this is your termination. So, what happens here is that, this is a baffled termination.

BAFFLED PLATE



And, another termination that happens is your wave is coming out and is suddenly exposed to the big atmosphere. So, the radiation impedance is nothing but at this point, let me draw it with a black mark here.

$$Z = \frac{\tilde{p}(0)}{\tilde{U}(0)},$$

So, this impedance, radiation impedance for the baffled case as well as for the unbaffled case is a classical point of study, and a lot of work has been done in this regard. So, what I will probably do is that write, just present certain expressions for the radiation impedance without bothering to derive them.

That is beyond the scope of the present course, although classical texts are available in this one. I will define the meanings of the symbols.

$$Z_0 = R_0 + jX_0$$

Radiation impedance that is the impedance that is seen by a by the wave, which is just incident here that is p by u.

What is the impedance of the opposition by the atmosphere to the propagation of the wave? Is a complex number as usual? Because the reason is that no termination is anechoic, no termination is fully rigid. You know, if you consider a simple case, if you have a pipe and exposed to the atmosphere. Just at the exit of the pipe the atmosphere or the surrounding air will pose an impedance, some waves will be reflected back, some waves will be transmitted downstream.

So, it is a kind of a midway between a rigid termination as well as a anaerobic termination, of there are other terminations also of course, called open end termination where p is 0, where p is 0 then Z is 0. So, that is also an extreme case, but the more realistic thing is to develop suitable models.

Because, here also there is air here also there is air, the density might be different based on the temperature the exhaust gas. But, the point is that we have a pretty complex expression for R0 and x 0 in terms of some Bessel functions given in the classical text.

So, for the case of termination which has an infinite flange, basically a baffled termination like this one, I would say this as baffled. So, here you have only baffled plate. So, here you have only the hemispherical space available not the full spherical space.

So, for such a case

$$R_0 = Y_0 \left\{ 1 - 2\frac{J_1(2k_0r_0)}{2k_0r_0} \right\}$$

Bessel function is your Bessel function of order 1 and 1st kind. So, when you do this you will probably get.

$$\simeq Y_0 \left\{ \frac{(2k_0r_0)^2}{2 \times 4} - \frac{(2k_0r_0)^4}{2 \times 4^2 \times 6} + \cdots \right\}$$

Basically, you are doing nothing but writing down the first few terms of the Bessel series in terms of the Fabian series solution. So, like this you will have many more terms.

$$X_0 = \frac{4}{\pi} \frac{(2k_0r_0)^3}{3^2 \times 5} + \frac{(2k_0r_0)^5}{3^2 \times 5^2 \times 7} +$$

So, were,

$$Y_0 = \frac{C_0^2}{\pi r_0^2}$$

Course is your characteristic impedance of the duct and this is pi by r naught square and r naught is the radius of the duct, that is this thing here r naught. So, once we simplify it that is for low frequency some simplification is possible, because you see typically in automobile mufflers, the radius is not very large is typically about the diameter is about 50 mm at the max 50, 55 mm.

So, radius can be 25 to 30 mm something between that. So, and the fact that the engine noise is concentrated mostly at the firing frequencies of the first few harmonics of that,

$$k_0 r_0 < 0.5$$

Under such a condition the real part of the radiation impedance

$$k_0 Y_0 = \frac{k_0^2 r^2}{2}, \qquad X_0 = Y_0 (0.85\, k_0 r_0)$$

we are going to get. $Z_0 = k_0 + jX_0 = Y_0 \frac{1+k}{1-k}$

So, we can derive all sorts of this thing and this particular expression is useful, when if we were to evaluate the thing called insertion loss. So, we if we know the radiation impedance will be good. Similarly, another expression for the unbaffled case, that is basically this case, which is much more complex.

And, derived in a paper published a long time back this is sort of given by your for, of course, low frequencies, $k_0 r_0 < 0.5$unbaffled case, unbaffled case is given by,

$$Z_0 = \left(Y_0 \frac{k_0 r_0^2}{4} + j0.6 k_0 r_0 \right) k_0 r_0 < 0.5 +$$

So, you will have those sorts of arguments.

Now, we will stop in this lecture here and probably resume in the next lecture where we will talk about different elements that constitute the muffler, sudden area expansions. And, but actually before we do that, we will probably have a look at the different measures to evaluate the muffler performance. Like insertion loss, transmission loss, level difference and so on. This probably then will be a good idea to talk about the different constituting elements of a muffler.

So, I will probably stop here, and I see you in the next class.