Foundations of Cognitive Robotics

Prof. Bishakh Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 13

Good morning students, welcome to the course of Foundations of Cognitive Robotics. In this final week of the third lecture, I am going to introduce you to a little bit of a mathematical concepts of how the neurons develop networks among themselves. You see in the last few lectures while discussing about intelligence and intelligent system development, there was one concept that we had continuously discussed that started from the Minkowski's concepts of agents and agency development.

And these agents they themselves are nothing, but in the terms of the brain, they are nothing, but the group of neurons and similarly of course, from the other side for cognitive robotics perspectives one can develop a group of artificial neurons in a networked system. Now what is important here is that each neuron having its own way of responding to the environment which is in terms of as we have earlier studied in terms of firing sequence of the neuron, it can be modeled as an oscillator we have seen that.

Now, if there are group of neurons which are connected; that means, there are actually a group of connected oscillators together what will be the characteristics or what will be the behavior of such group of neurons when they start to fire and oscillate together.

Is there any mathematical model possible so, that we can you know see that whether there are concepts like synchronous or totally incoherent working things like that whether it is possible from the behavior of the neurons, that is what we are going to look into today through certain classical models.

So, let us look into the behavior of group of neurons the mathematical model of such kind of a system.

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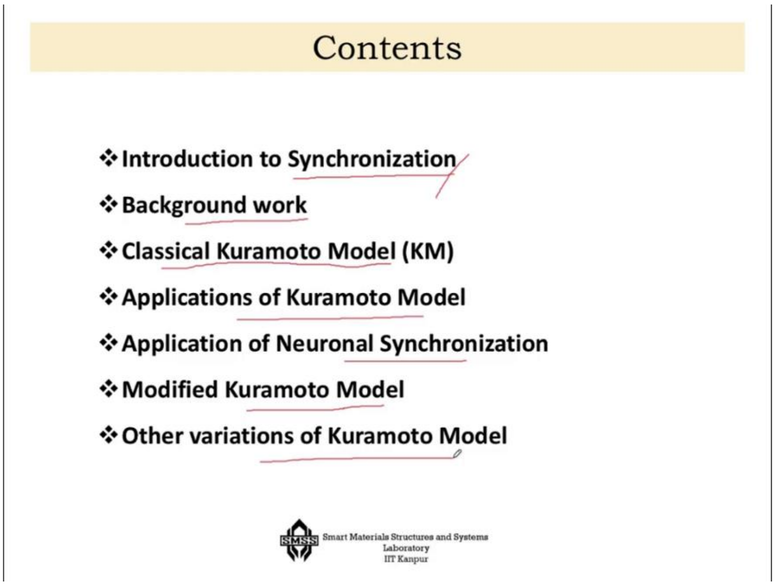

The outline of this lecture is that I will first talk about the concept of synchronization which is very important; that means, as I told you that a group of neurons working like oscillators do they synchronize or not what do we mean by synchronization? I will talk about some of the background models that has been developed in order to explain this concept of synchronization and then I will talk about classical Kuramoto model which is the standard model in terms of such neural behaviors.

We will also touch about the application of the Kuramoto model and the neuronal synchronization we will talk about some modifications that are possible in the Kuramoto model and some of the other variations of the Kuramoto model. So, that is the outline of the talk.

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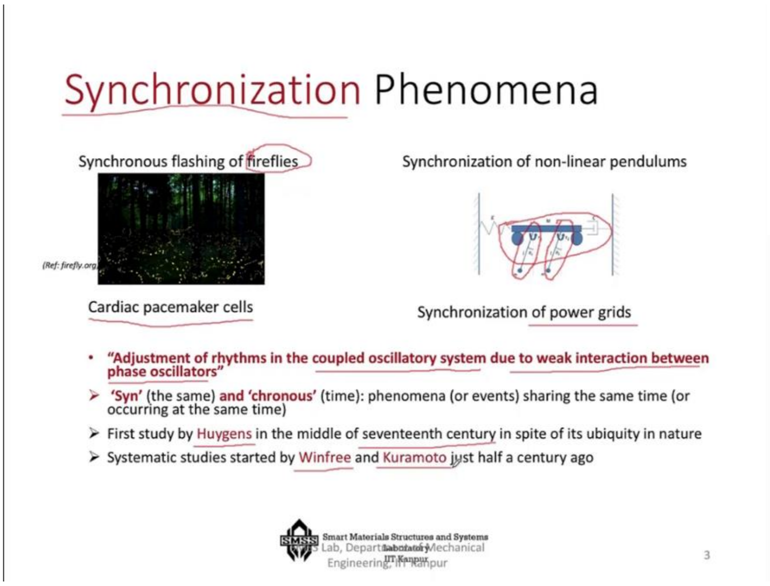

Now let us first discuss about that what we mean by what we call the synchronization phenomena. So, we have talked about synchronization.

Now, this word *syn* means same and the *chronous* part of it; that means, the time. So, synchronization refers to that about the oscillators a set of oscillators which has a similar timed response sharing the same time; that means, their response is very much the same and this would actually in our case it would refer to what we will be calling that adjustment of the rhythms in any coupled oscillatory system and that is possible due to weak interaction between the phase oscillators.

So, if each one of the oscillators like this is one oscillator and this is another oscillator, now they if they are weakly linked through this system they are weakly linked it is possible that under certain conditions they will develop synchronization in their response. Now this is not only possible in terms of a mechanical model, but also in terms of many natural events like synchronous flashing of fireflies.

It's a very interesting one I will show you and then cardiac pacemaker cells or say synchronization of the power grids, there are many such applications where you will see that the similar phenomena happens in the system. This is the phenomena which was first studied by Huygens in the mid of 17th century even though you know this kind of phenomena like synchronous flashing of fireflies etcetera they were quite ubiquitous in nature.

However, a systematic mathematical study only started in about last 50 years or so, starting from Winfree and then Kuramoto model.

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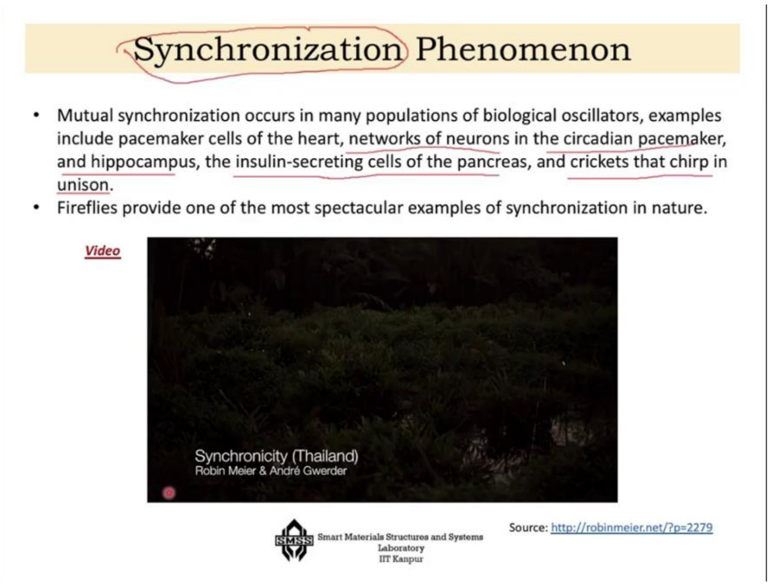

Now let us look into the synchronization in nature first. So, here what I am going to show you is the mutual synchronization of certain biological oscillators. For example, in this case if this is about the fireflies, but similar things you can also you will be able to appreciate in the networks of neurons when we will discuss particularly in the context of circadian pacemaker and hippocampus.

And also you will see similar concepts in insulin secreting cells of the pancreas and crickets that chirp in unison you will see particularly this monsoon season will be coming up and you will see suddenly the crickets are all chirping at times in unison. So, these are all examples of synchronization of different types of synchronization in nature or synchronization in manmade systems.

Now, if you look at this particular system for example. So, this is a particular case in which you know this happens in actually you know in Thailand and there are these you know fireflies in a jungle and if you look at the video what you would see is that there are laser pointers which are actually kind of oscillator. So, they are oscillating you know or LEDs they are oscillating and the fireflies respond to that oscillation and they start to synchronize.

So, let us look into this video and then we will be able to see that you know what is the significance of this kind of a system is.

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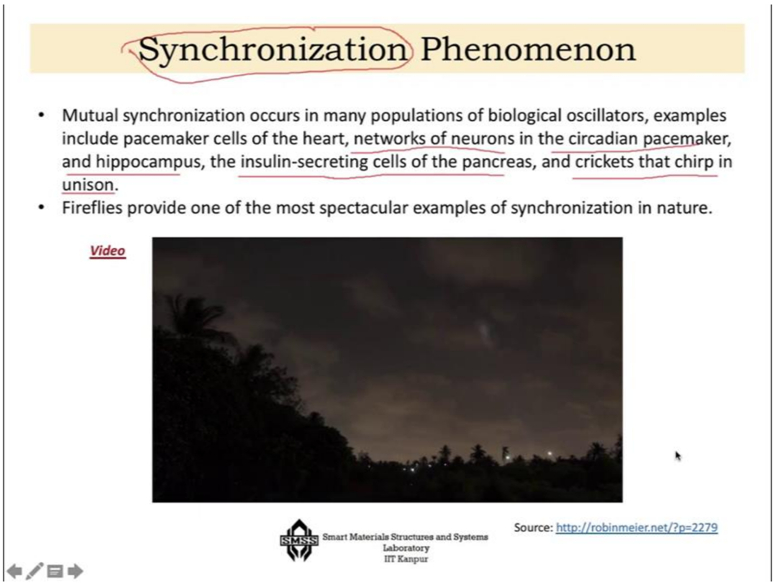

So, let us look into the video. As you can see that this is a remote village area and you can see that there are lots of these fire flies along with there will be some lights which will be oscillating in nature.

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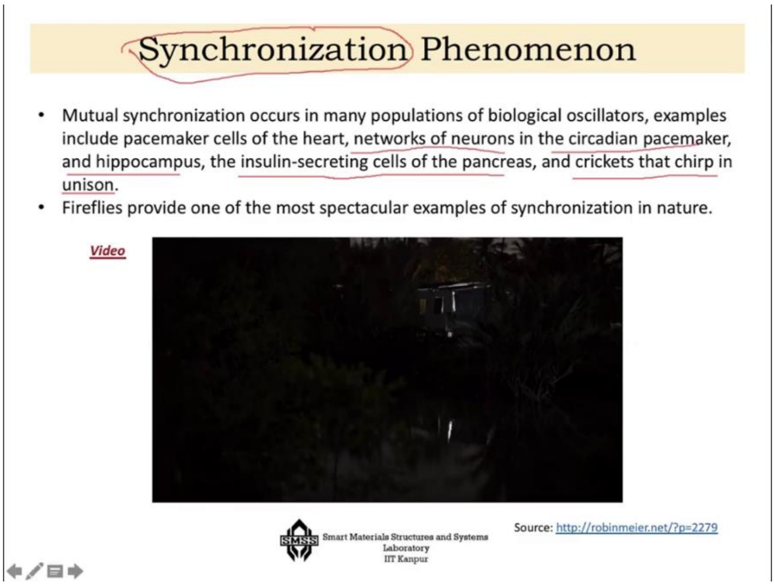

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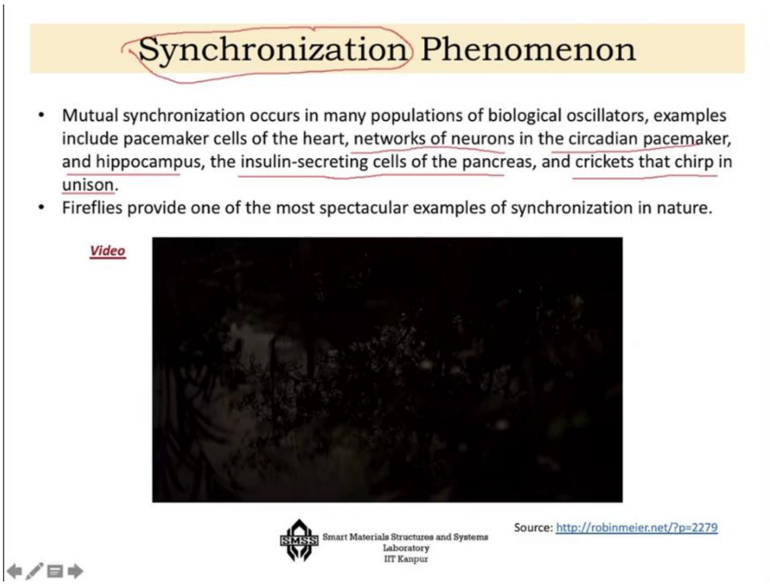

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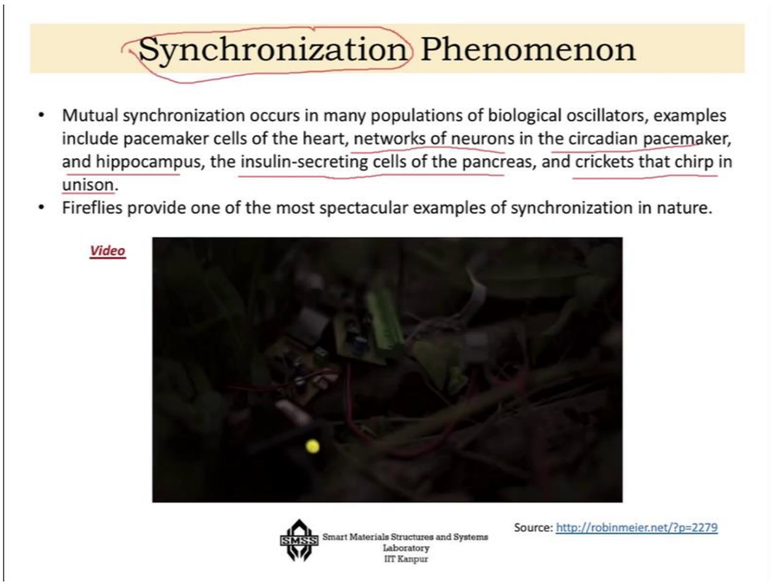

And you can already see that there is these lights which are oscillating in nature all around you can see the oscillatory lights and if you wait a bit. So, this is the oscillatory LEDs.

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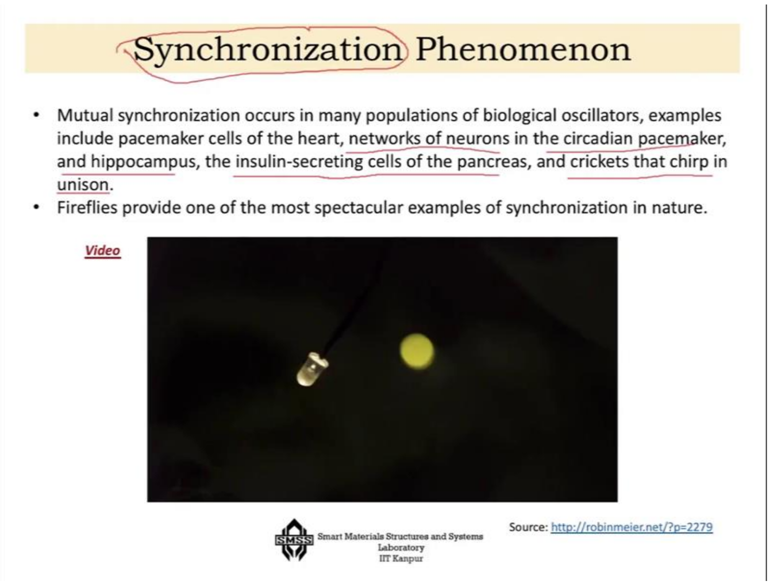

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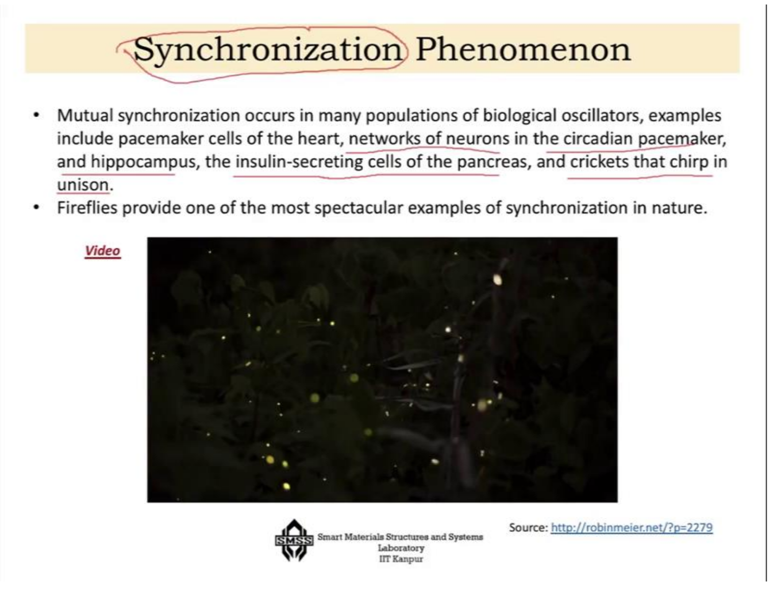

You would soon see that the fireflies they would start to respond to this oscillatory light now you can see them.

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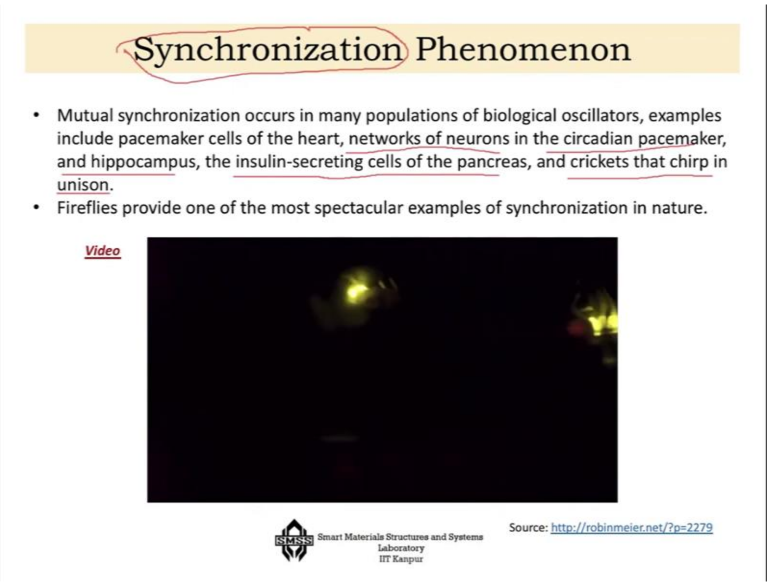

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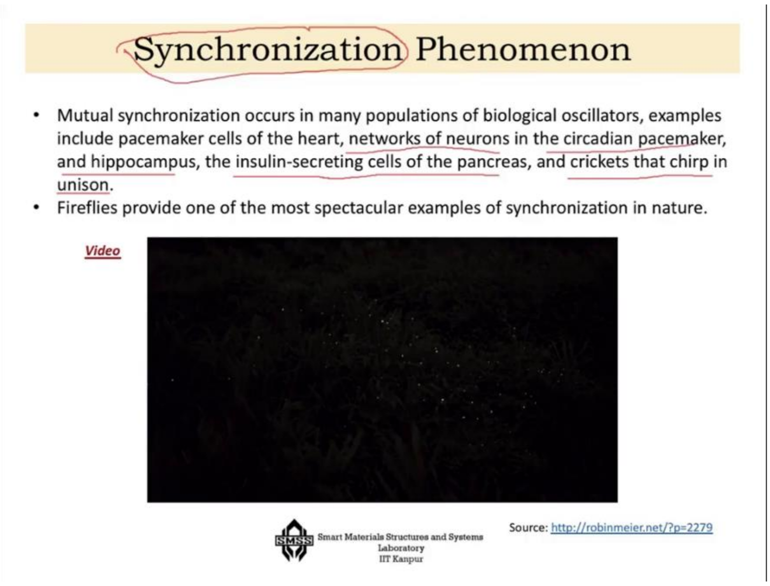

All around they are oscillating and very interestingly if you wait for some time you would see that their oscillation frequency is very much the same as the LED induced oscillation frequency.

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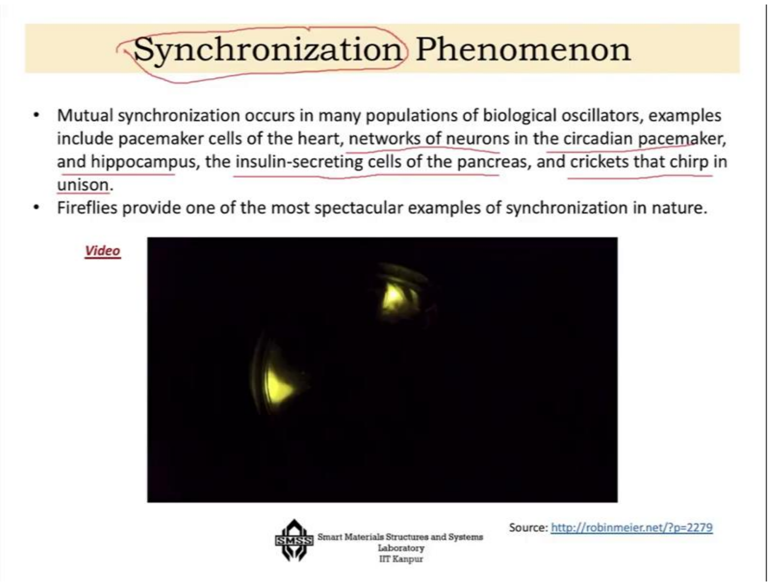

So, this is an example of oscillation synchronization in nature you can very clearly see it.

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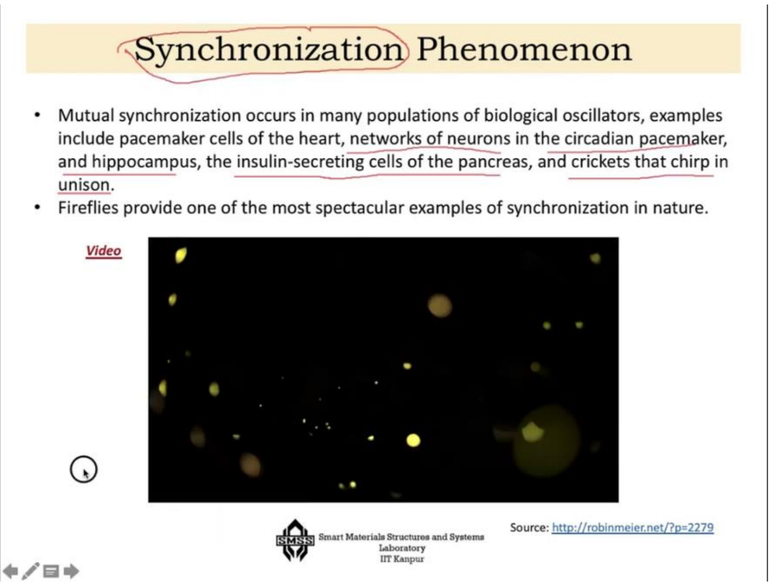

Now you have seen that how in nature you know the fireflies they start to synchronize with respect to some kind of an induced excitation you know frequency of light. A very simple example you would see I will show you from one of the UCLAs you know laboratory demonstration on metronomes. Metronomes are the systems which are used in order to measure the frequency or to drive an oscillator at a particular frequency.

Now, in this experiment there will be many such metronomes and these metronomes are weakly coupled over a plank which is supported in a very you know movable manner ok. And you would see that as all the metronomes will initially be at their own frequencies own characteristic frequencies and you will see that what happens after a few minutes are passed. So, let us look into this example also.

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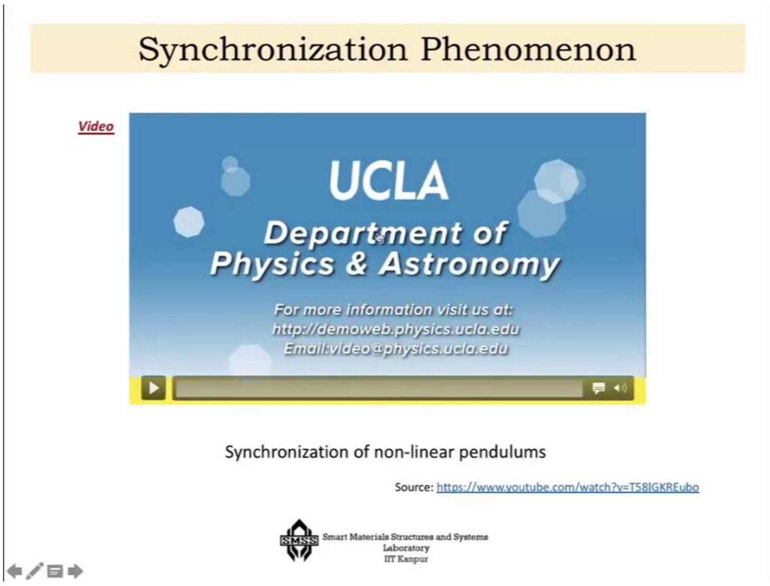

So, in this example this is from UCLAs department of physics and astronomy let us look into this example.

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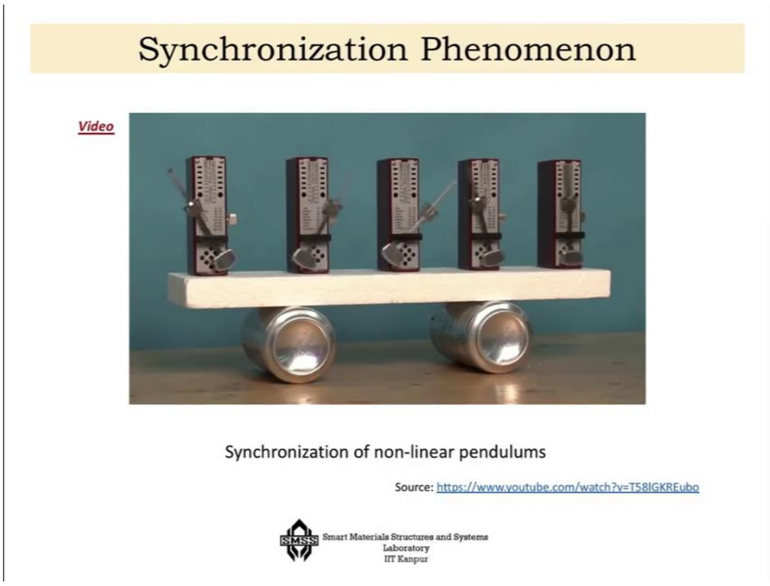

As you can see that there are metronomes here, each metronome is actually excited at their own frequencies and initially they are all out of phase you can see one is going right another is going left etcetera and you can also see that they are weakly coupled because this whole base is moving.

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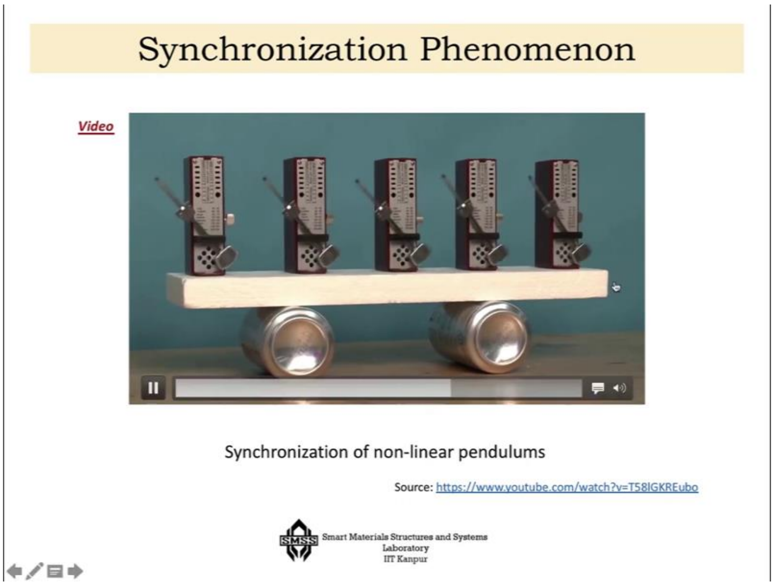

Now, as you can see gradually you see all of them are going towards the same direction at the same time. So, which means that they have actually synchronized. So, imagine that each one of these metronomes we are talking about each one of them is a neuron and imagine each one of these neurons are connected by some kind of a weak link then this weak link can actually induce a situation based on certain conditions where all the neurons will be synchronized to each other. So, that is the problem that we are going to look at.

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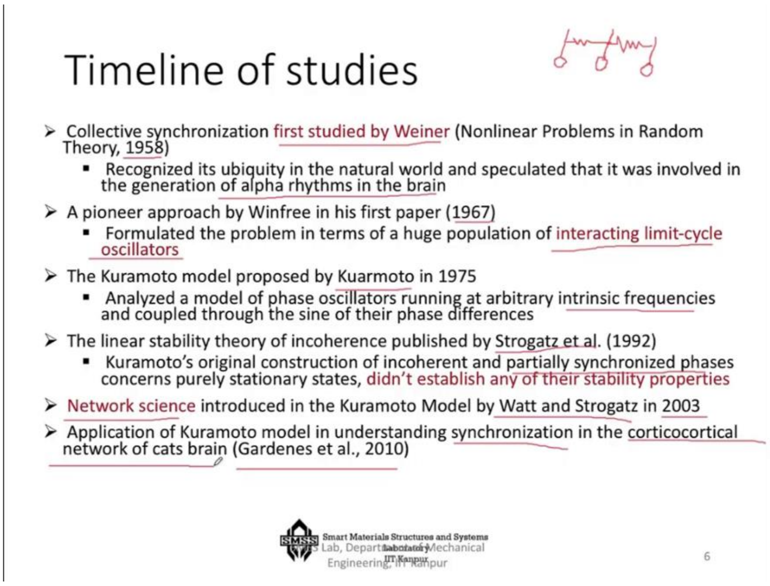

Now, there is a timeline as I told you the evolution of the study 1958 this was first studied by Weiner mathematically we will soon see Weiner's way of you know putting this he recognized the ubiquity in the natural world and speculated that it was involved also in the generation of alpha rhythms in the brain. So, that was the for the first time that somebody speculated that the source of alpha rhythm in the brain maybe that is because of neuronal synchronization.

Now, later on Winfree in 1967, he has formulated the problem in terms of large population of interacting limit-cycle oscillators. So, he has considered that problem of each one of these oscillators and they are weakly coupled. So, how they will exchange the energy with each other that was the problem that he has actually considered weakly coupled oscillator problem.

And afterwards in 1975 Kuramoto came out with a better model of the oscillator and that has been globally accepted in the scientific community where he has shown actually that these oscillators when they were run at arbitrary intrinsic frequencies to begin with and they are essentially coupled through the sine of their phase differences.

And based on certain conditions and coupling at the degree of coupling etcetera it may be possible that there will be a synchronousness or partial synchronousness that will be developed in the system. However, Kuramoto did not address the stability problem and which has been later on addressed by Strogatz and he has also worked on the partial synchronization of the phases that was also another contribution by Strogatz.

And he has shown that the experimental results are more matching with respect to the certain connectivity which we are going to discuss that Strogatz has explained. If we consider that type of connectivity, then actually the time for synchronization matches with the experimental results. At a subsequent phase the concepts of network science were introduced by Watt and Strogatz in 2003.

And we have seen such works like in 2010 by Gardenes where these you know modified Kuramoto model is used for understanding synchronization in the corticocortical network of cat's brain. So, we can see that the mathematical concept is getting used in terms of modeling the network of neurons in brain.

Well we have seen the timeline of development of this type of you know modeling and it took as I told you that nearly from 50 to 70 years to reach into the stage in which we are today. So, let us start first with the Winfrees model and then to the Kuramoto model and see the mathematical modeling how it gradually evolved.

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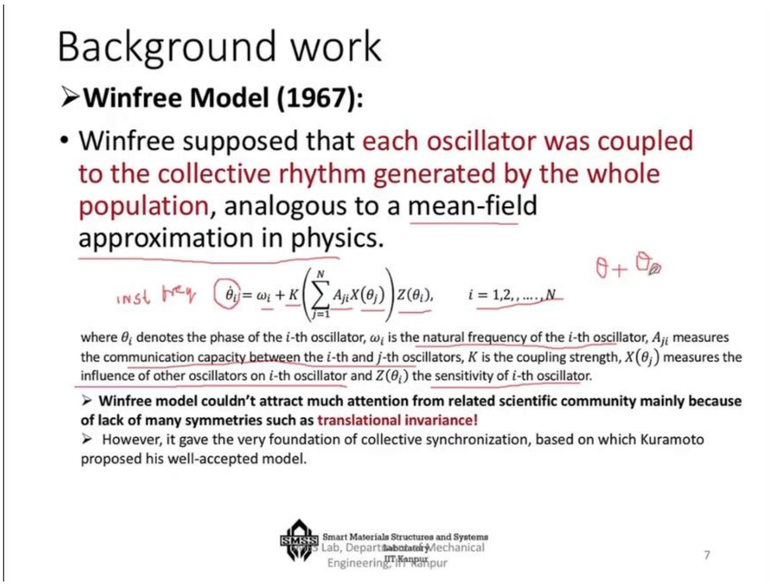


So, as I told you that Winfree was the first to develop the mathematical model of the oscillators which are coupled to the collective rhythm. So, that was Winfree model and this is analogous to a mean field approximation in physics.

So, what is mean field approximation? Well what it says is that if there are you know certain variables which are random in nature, then the simplified approach to actually model this is in terms of the mean of the variation and that is why in this case also the oscillators oscillating frequency even though it is random in nature, but by applying similar to the mean field theorem you can think of an average oscillating frequency in the system.

Now here he has developed certain interesting things for example, the oscillator is actually dynamics is described in terms of the phase of the oscillator. So, these are the phases

$$\dot{\theta}_i = \omega_i + K\left(\sum_{j=1}^{N} A_{ji}X(\theta_j)\right)Z(\theta_i)$$

And K is actually the coupling strength because if you remember that particular example where the oscillators the metronomes are connected through a wooden plank, now depending on the fixity of the wooden plank the coupling will be there.

So, that is what is the coupling strength and A_{ji} is a measure of the communication capacity between i^{th} and j^{th} oscillator and $X(\theta_j)$ is the measure of the influence of other oscillators on i^{th} oscillator that is $X(\theta_j)$and $Z(\theta_i)$ is actually the sensitivity of the i^{th} oscillator and in this model he has considered *N* number of oscillators which are networked together.

Now this model was not widely accepted by the scientific community simply because it lack certain symmetries like symmetries of translational invariance what happens if the phases are shifted if theta is shifted from theta to say theta plus theta 0 under such condition the model we need to remain invariant.

So, these are something that this kind of a model was not able to show. So, even though he has understood the crux of it that the oscillators are to be described in terms of its rate of change of phases.

But the relationship is not mathematically very rigorous in this case which has been later on improved by Kuramoto. Now the classical Kuramoto model which is a very successful model of the same system this was attempted by Kuramoto in 1975.

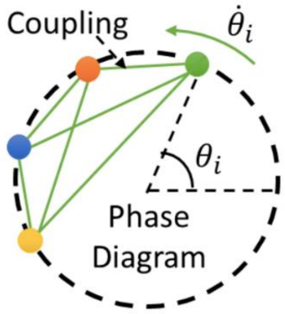
And this was very well acknowledged in the community as you can see that the relationship is very much the same here also we have these rate of change of the phase

Kuramoto represented a large number of oscillating subsystems by the set of N coupled differential equations:

$$\dot{\theta}_i = \omega_i + \sum_{j=1}^{N} \Gamma_{ij}\left(\theta_j - \theta_i\right), \quad i = 1, \ldots, N$$

$$\Gamma_{ij}\left(\theta_j - \theta_i\right) = \frac{K}{N}\sin\left(\theta_j - \theta_i\right)$$

[Γ_{ij}: phase interaction function]



- $\theta_i(t)$, phase angle of the ith oscillator
- $\dot{\theta}_i(t)$, angular velocity
- $\omega_{0,i}$, natural frequency
- K, is the coupling strength
- N, is the number of oscillators
- a_{ij} is the corresponding element in the adjacency matrix

So, what it is telling is that the rate of change of the phase not only depends on the natural frequency of every oscillator, but also the effect of other oscillator on this oscillator.

And this is defined in terms of parameters like coupling strength K, the number of oscillators that we are considering, the more the number of oscillators the lesser will be this effect and the $\sin\left(\theta_j - \theta_i\right)$ which is essentially the relative phase this is the measure of the relative phase with respect to the particular oscillator that we are talking about. So, this is a relative phase that we have to keep in our mind ok.

So, this is the you know very simple, but so, to say elegant description of the dynamics of a group of oscillators which are weakly coupled.

Now, if you look at these you would see that as K essentially increases. So, if you go back these coupling constant as it is increasing, you will be finding that its effect is more coming on the dynamics of the system.

So, you will see that the dispersion the basic dispersion of natural frequencies omega i could be overcome due to the increase of K and that will actually result in terms of coherence in the system. Initially if it is incoherent because of high K it will gradually become partial coherence and then it will gain full synchronization.

Now, the other important point is that in order to quantify this degree of synchrony we can consider a centroid vector. Once again apply the mean field theorem a centroid vector which has a magnitude of r and a phase which is ψ. Now naturally this depends on the average response as I told you by using the mean field theorem.

As K increases, the interaction functions overcome the dispersion of natural frequencies ω_i resulting in a transition from incoherence, to partial and then full synchronization.

To quantify the degree of synchrony, centroid vector of this phase distribution is considered as,

$$re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j} \dots\dots\dots\dots\dots (1)$$

Here, ψ: mean phase of set of θ_i,
r: order parameter, captures the degree of phase coherence in the system
Here r(t) with $0 \leq r(t) \leq 1$ measures the coherence of the oscillator population, and $\psi(t)$ is the average phase.

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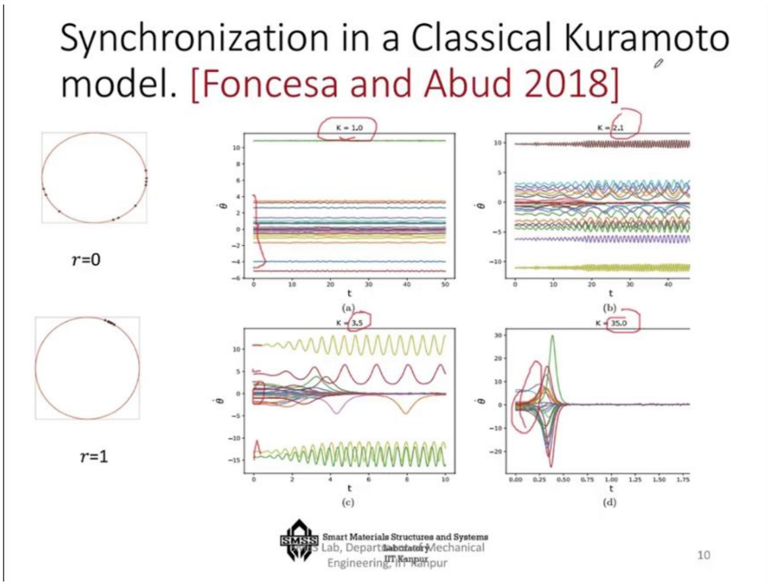

Now, a very nice way. So, first of all r equals to 0 would mean that all the oscillators in a phasor diagram they are arbitrarily spread and r equals to 1 means they are all very closely spaced as you can see here. Now Foncesa and Abud in 2018 has given a very nice description of the system.

He has considered 1000 neural oscillators and have chosen 25 of the neural oscillators arbitrarily from them and then he has plotted their phase rate of change of the phase corresponding to different coupling strength as you can see that there is coupling strength unity and then gradually increase it to 2.1 then to 3.5 and then to a very high value order of magnitude at 35.

Now what you can see is that initially the phase distribution is quite a big you know rate of phase distribution, but gradually you can see as you are increasing the coupling strength, there is a grouping that is happening. As you have increase the coupling strength further you can see that there are distinct groups most of the groups are somewhere here whereas, there are groups here there are there is a group here there is a group here.

And if you are increasing it to even more to K equals to 35 astoundingly that even if initially they are at a very wide you know natural frequency distribution, they will be all coming their instantaneous their instantaneous frequency of response that frequency will be very close to each other. So, that is what is very interesting. So, that you know has been studied by Foncesa and Abud that is one of the beauty of the Kuramoto model.

Now, if you further explore these mean field approximation of the Kuramoto model, then we can think of that this is the coupling K_{ij} is in some sense the average of the all coupling and which has to be positive in nature.

The original analysis of synchronization was accomplished by Kuramoto in the case of mean-field coupling, that is taking $K_{ij} = K/N > 0$

- Multiplying both sides of eqn. (1) by $e^{-i\theta_i}$, we get

$$re^{i(\psi-\theta_i)} = \frac{1}{N}\sum_{j=1}^{N} e^{i(\theta_j-\theta_i)}$$

- Equating imaginary parts,

$$r\sin(\psi-\theta_i) = \frac{1}{N}\sum_{j=1}^{N} \sin(\theta_j-\theta_i)$$

$$\dot{\theta}_i = \omega_i + Kr\sin(\psi-\theta_i), \ \ i = 1, \ldots., N$$

- Phase θ_i seems to evolve independently from each other here, but interaction is set through r and ψ.

- Effective coupling now proportional to the order parameter r, creating feedback relation between coupling and synchronization

So, you can see that even though in the last equation it was in terms of summation of some sinusoidal by using this average phase psi and with respect to that the deviation we can get an even simpler relationship by using the mid field approximation of the system.

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- Order parameter given by, $re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$ can be rewritten as

$$re^{i\psi} = \int_{-\pi}^{\pi} e^{i\theta}\left(\frac{1}{N}\sum_{j=1}^{N}\delta(\theta-\theta_j)\right)d\theta$$

- In the limit of infinitely many oscillators, they may be expected to be distributed with a probability density $\rho(\theta,\omega,t)$.

$$re^{i\psi} = \int_{-\pi}^{\pi}\int_{-\infty}^{\infty} e^{i\theta}\rho(\theta,\omega,t)g(\omega)d\theta d\omega$$

- If $K \to 0, \theta \approx \omega t \Rightarrow r \to 0\, as\ t \to \infty$ (Reimann-Lebesgue Lemma) (oscillators not synchronized)

- If $K \to \infty, \theta_i \approx \psi$, oscillators synchronized to their mean phase, $r \to 1$
- For intermediate couplings, $K_c < K < \infty$, part of oscillators are phase locked $\dot{\theta}_i = 0$, and part are rotating out of synchrony with the locked oscillators- **state of partial synchronization** $(\mathbf{0} < \boldsymbol{r} < \mathbf{1})$**.**

Stationary synchronization for mean field coupling

- A continuity equation can be written for the oscillator density as,

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial\theta}(\rho v) = 0$$

Where $v = \omega_i + Kr\sin(\psi - \theta_i)$ is called angular/drift velocity.
- The above equation solved together with an appropriate initial condition and the normalization condition gives:

$$\int_{-\pi}^{\pi}\rho(\theta,\omega,t)d\theta = 1$$

- Trivial stationary solution: $\rho = \frac{1}{2\pi}$, $r = 0$, corresponding to an angular distribution of the oscillators with equal probability in the interval $[-\pi,\pi] \Rightarrow$ **Incoherent solution.**

Stationary synchronization for mean field coupling

A simple solution corresponding to oscillator synchronization

- A typical oscillator moving with velocity v will become stably blocked at an angle such that
$$\mathrm{Kr}\sin(\theta_i - \psi) = \omega \text{ and } -{}^{\pi}/_{2} \leq (\theta - \psi) \leq {}^{\pi}/_{2}$$
- Oscillators with frequencies satisfying $|\omega| > \mathrm{Kr}$ can't be locked, their stationary density obeys $\boldsymbol{\rho v = \mathrm{constant}}$
- This is a **stationary state of partial synchronization**.

The corresponding stationary density is therefore,

$$\rho = \begin{cases} \delta\left(\theta - \psi - \sin^{-1}\left(\frac{\omega}{kr}\right)\right) H(\cos\theta), |\omega| < Kr \\ \dfrac{Constant}{|\omega - Kr\sin(\theta_i - \psi)|}, \; elsewhere \end{cases}$$

Equivalently, $\rho = \sqrt{K^2r^2 - \omega^2}\delta(\omega - Kr\sin(\theta - \psi))H(\cos\theta)$
for $-Kr < \omega < Kr$

Now, order parameter in the state of partial synchronization,

$$r = \int_{-\pi/2}^{\pi/2}\int_{-\infty}^{\infty} e^{i(\theta-\psi)}\delta\left(\theta - \psi - \sin^{-1}\left(\frac{\omega}{kr}\right)\right) g(\omega)d\theta d\omega$$
$$+ \int_{-\pi}^{\pi}\int_{|\omega|>Kr} e^{i(\theta-\psi)} \frac{Cg(\omega)}{|\omega - Kr\sin(\theta_i - \psi)|} d\theta d\omega$$

Second term vanishes because of symmetry of $g(\omega)$ and the first term becomes

$$r = Kr\int_{-\pi/2}^{\pi/2} \cos^2\theta \, g(Kr\sin\theta)d\theta$$

Two solutions, one $r = 0$ for incoherence and second branch of solutions, for partially synchronized phase:

$$1 = K\int_{-\pi/2}^{\pi/2} \cos^2\theta \, g(Kr\sin\theta)d\theta$$

This branch bifurcates continuously from $r = 0$ at a value $K = K_c$ obtained by setting $r = 0$,

$$K_c = \frac{2}{\pi g(0)}$$

For a general frequency distribution $g(\omega)$, an expansion of right-hand side in powers of Kr yields the scaling law

$$r \sim \sqrt{\frac{8(K-K_c)}{-{K_c}^3 g''(0)}}$$

The partially synchronized phase bifurcates super-critically for $K > K_c$ if $g''(0) < 0$ and sub-critically for $K < K_c$ if $g''(0) > 0$

We have seen the mathematical modeling of the basic Kuramoto model. Now let us look into some of the applications of this model specifically in the context of applying these for the neuronal networks.

If we look at the application of these kind of Kuramoto model, as I told you earlier itself that there are many field where we can apply this concept.

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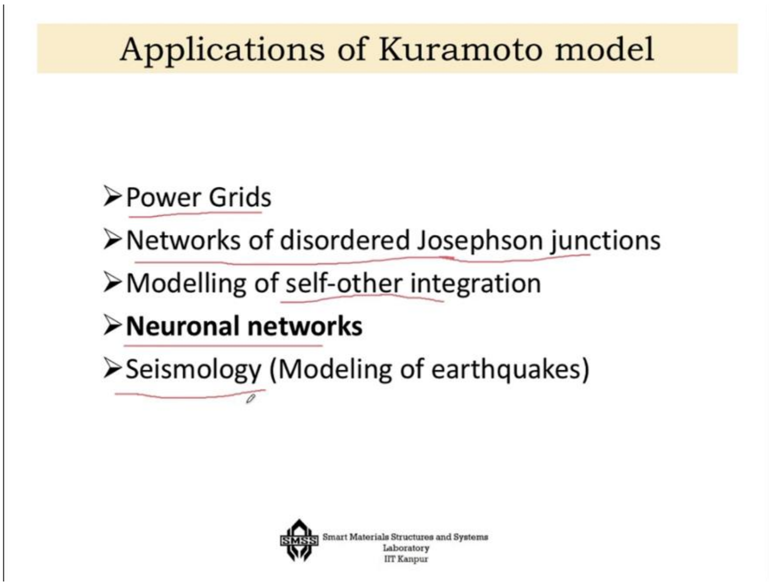

For example, for power grids for networks of disordered Josephson junctions for modelling of self other integration. Now self other integration means that when there are of number of agents these agents may be some living and some non living so, but a number of agents and they are interacting with each other.

So, it is a self and, but they are to be you know agents which are having the cognitive capability and there you can get a self other integration during the synchronous ness of these kind of oscillators and of course, the neuronal networks itself and also sometimes fields like seismology can get benefited by the same Kuramoto model.

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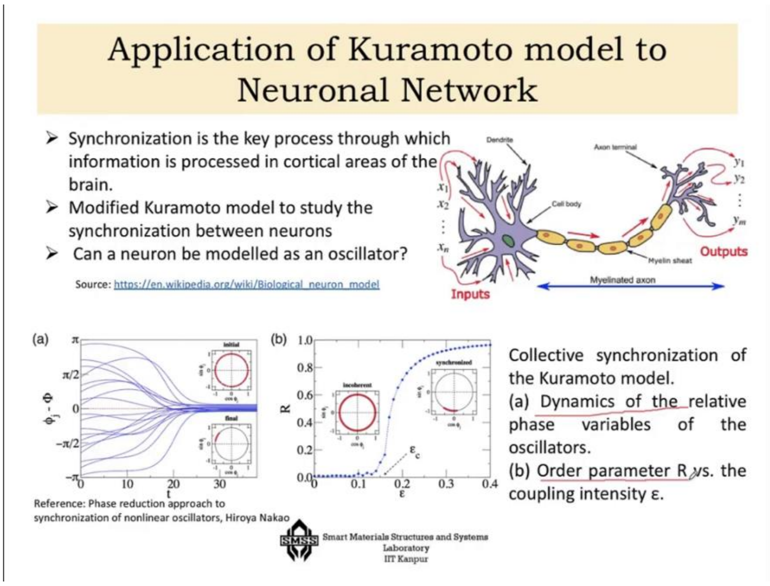

Now, if you look at the Kuramoto model for the neuronal network. As you can see that we have discussed about this particular figure many times that this is a single neuronal the dendron's this is the soma part of it and this is where is the axon part.

And you also know that there is this nodes of Ranvier there and this is this parts are myelinated that will increase the speed of the signal travel and then the signal comes to the axon terminals from here it starts to actually fire the chemicals which is absorbed by the next dendrite.

Now, the question is that clearly each neuron there is an oscillator. The question is that can they also have a kind of a synchronousness as has been predicted long before well indeed this is a possibility, I encourage you to look into this particular you know biological neural model.

Now collective synchronization of the Kuramoto model is certainly possible for this kind of neuronal systems and the dynamics of the relative phase variables of the oscillator plays an important role here and the order parameter R versus the coupling intensity this also plays an important role in this kind of a system.

Modified Kuramoto model to study the synchronization between neurons

$$\boldsymbol{\dot{\theta}_i = \omega_i + \sum_{j=1}^{N} \Gamma_{ij}\left(\theta_j - \theta_i\right)}, \quad \boldsymbol{i = 1, \ldots, N}, \quad \Gamma_{ij}\left(\theta_j - \theta_i\right) = \frac{K}{N}\sin\left(\theta_j - \theta_i\right)$$

- 'All-to-All' connectivity → Small world Connectivity
- Fixed natural frequency → Time-Varying natural frequency
- Fixed coupling strength → 80:20 coupling/ STDP Based coupling
- Time delay
- External stimulation

Time delay: finite conduction speed of electrical signals down the axon

$$\tau_{ij} = \frac{d_{ij}}{v} + \tau_{transduction}$$

External Stimulus: Models the foreign currents which are applied to the neuron

$$\dot{\theta}_i = \omega_i(t) + X_i(t)S(\theta_i) + \frac{1}{k}\sum_{j \in \vartheta(i)} K_{i,j}\sin\left(\theta_j(t - \tau_{i,j}) - \theta_i(t)\right), \quad i = 1, \ldots, N$$

Now, if you look at the modifications that are possible in a Kuramoto model, the first important modification that is needed is in terms of that the connectivity is in considered in Kuramoto model is all to all connectivity. Whereas, the connectivity that is generally feasible is actually a small world connectivity which means local neurons are connected to each other.

The fixed natural frequency of each neuron may not be also realizable. So, you need a time varying natural frequency and the fixed coupling strength may be changed by 80-20 coupling or STDP based coupling.

And then there is this time delay that happens in the system and external simulation effect all these things will actually modify the behavior of the neuron. Now if I take the time delay into consideration the finite conduction speed of electrical signal actually creates the time delay as you can see that there is a time delay that is involved in the system. Again if there is external stimulus in the system the that would model the foreign currents. So, this is where is the external stimulation in the system that can happen ok.

So, there are various such things that we can actually introduce into the Kuramoto model in order to make it more realistic in nature.

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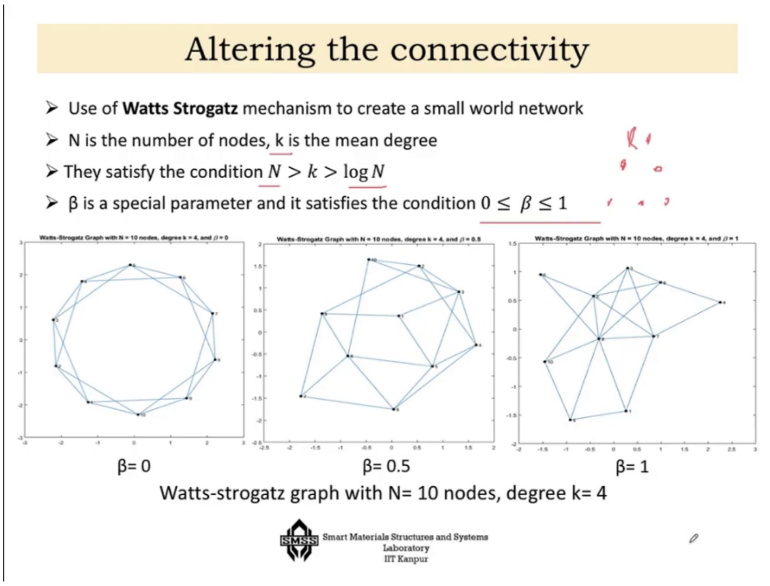

Now, if you look at one of the connectivity called the Watts Strogatz connectivity in terms of a small world network, this essentially tells is that the neurons which are close in the neighborhood they are actually connected more rather than the neurons which are very far apart.

So, suppose N are these number of nodes which are there and k is the mean degree of connectivity of each neuron. So, the probability of each neuron connected with you know k such neuron on an average ok. So, that is what is the k is the coupling and the k satisfies the condition that its value is maximum k is N that is all to all coupling and between log N and N the value of k and beta is a special parameter which satisfies the condition that beta has to be in between 0 and the unity.

Now, under such a situation you see when beta equals to 0 you have actually all to all connection every single one is connected. If beta equals to 0.5 you start to develop a kind of a you know small world kind of a thing where one of the neuron is connected to your neighbors when beta equals to 1 you can see that almost there is no long distance connection. So, each one of them are connected to the neighboring neurons. So, there are various such possibilities that are there.

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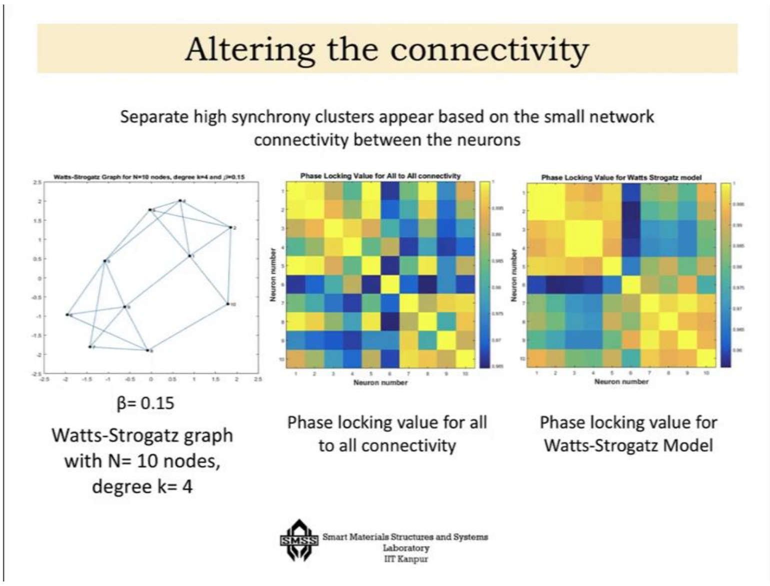

Now, beta equals to 0.15 once again you would see that there are high synchrony clusters which can appear based on this kind of a small network connectivity. So, a typical case for which beta equals to 0.15 in a Watts Strogatz graph with N equals to 10 and the degree as k as 4 you can see that if we would have considered all to all connectivity, then this would mean that there will be many yellow points yellow points actually are synchronous and they are distributed all over essentially.

On the other hand, you can see; you can see that there is a distinct banded nature and the phase locking has actually improved in terms of synchronousness of the neuronal oscillators.

Other variations of Kuramoto model

There are some other variations possible of Kuramoto model for example, we can introduce a bit of stochasticity in the model, where the it is not just in terms of the natural frequency variation, but in terms of this $\xi_i(t)$ which is an independent white noise process, an independent arbitrary input coming into the oscillator system.

Stochastic Kuramoto Model: The mean field model including white noise forces (accounts for the contribution of different stochastic forces)

$$\dot{\theta}_i = \omega_i + Kr\sin(\psi - \theta_i) + \xi_i(t), \;\; i = 1, \dots, N$$

- $\xi_i(t)$ independent white noise processes with expected values

$$\langle \xi_i(t) \rangle = 0, \ \langle \xi_i(t) \xi_j(t') \rangle = 2D\delta(t - t')\delta_{ij}$$

- The Fokker-Planck equation for the one-oscillator probability density $\rho(\theta, \omega, t)$ is

$$\frac{\partial \rho}{\partial t} = D \frac{\partial^2 \rho}{\partial \theta^2} - \frac{\partial}{\partial \theta}(v\rho)$$

So, you need to add that in the form of Fokker Planck equation in order to get a stochastic Kuramoto model. This is one possibility the other possibility is that in order to consider a second order Kuramoto model. So, here the model with frequency adaptation where both phases and frequency evolve in time and having synchronization that slowed down by inertia can solve such problems. So, that is a second order Kuramoto model.

- **Second order Kuramoto model**: The first order Kuramoto model approaches too fast the partial synchronized state compared to experimental observations, infinite coupling strength for $r = 1$

  - The model with frequency adaptation, where both phase and frequency evolve in time and having synchronization slowed down by inertia, can solve such problems.

  - $\ddot{\theta}_i = -\alpha \dot{\theta}_i + \omega_i + \lambda \sum_{j=1}^{N} A_{ij} \sin(\theta_j - \theta_i)$

So, in a summary in this lecture we have seen that how the dynamics of a group of neurons can be developed in terms of a set of first order differential equations and can be described in terms of a physical model called Kuramoto model.

We have also seen that the Kuramoto model has its weaknesses particularly in terms of all to all connectivity which can be changed by you know small world connectivity this each neuron is connected with only the neighboring neurons. So, that is a small world connectivity.

And we have seen that with respect to the small world connectivity you can develop a better partial synchronization in the system. We have also seen that you can extend in future this Kuramoto model in terms of many parameters, but some of them that we have discussed are like you know white noise excitation of the system or in terms of the coupling of the system or delay in the system. So, these are some of the things that in which you can actually extend the Kuramoto model.

I think we will put an end there, in the final lecture we will see an experiment where we will show you that how a robot and a kid can actually interact and some kind of a phase locking can happen in the system.

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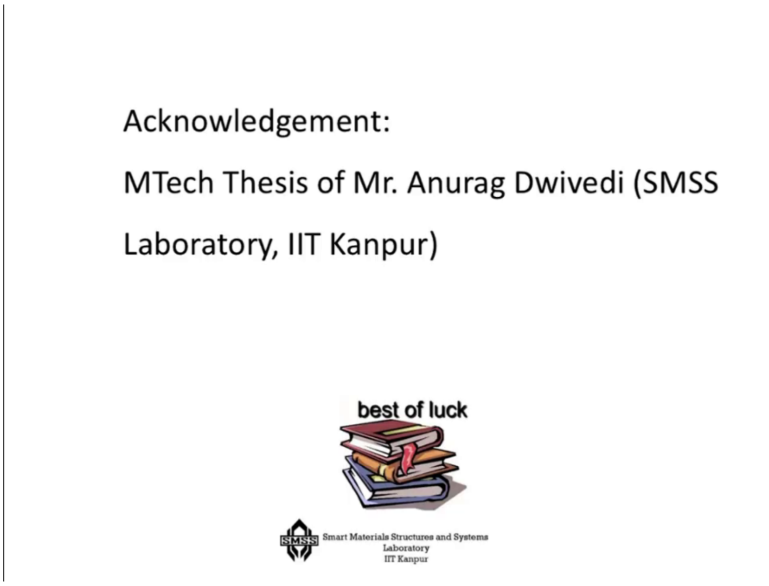

So, this is where we would like to put an end. I must acknowledge my student Mr. Anurag Dwivedi who has completed his MTech thesis in this direction recently for many of the slides.

Thank you.