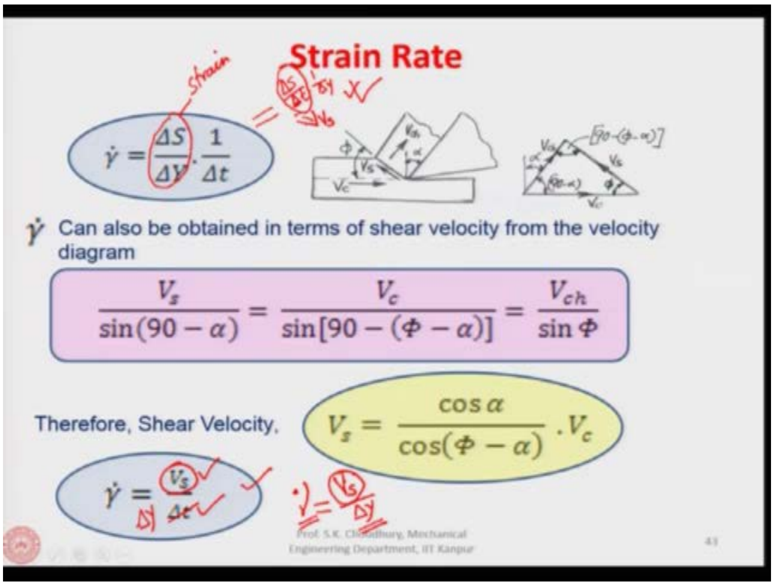
Machining Science - Part I
Prof. Sounak Kumar Choudhury
Department of Mechanical Engineering
Indian Institute of Technology Kanpur

Lecture - 09

Hello and welcome to the 9th lecture of the Machining Science course. Let me remind you that we have discussed the Merchant's circle diagram and how to find out the stress, strain, strain rate and how to find out the shear plane angle - how to measure the shear plane angle and how to analytically find out the shear plane angle.

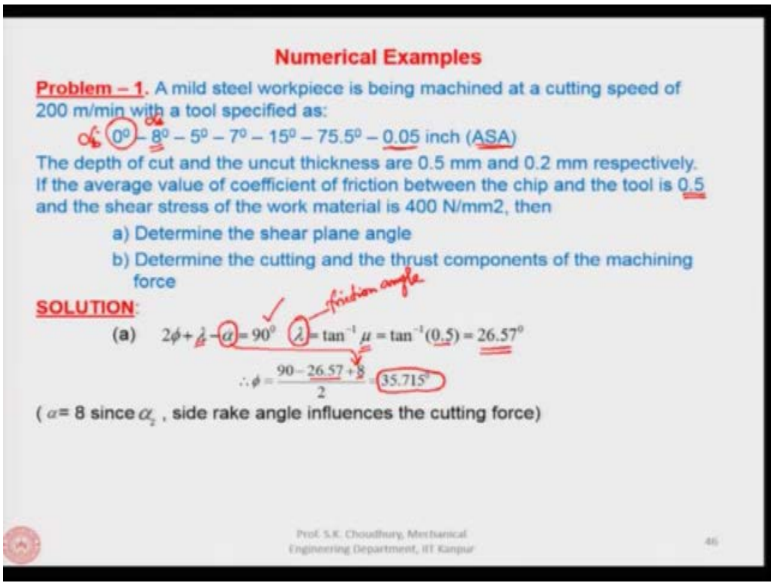
(Refer Slide Time: 00:43)



I would like to make a small correction here that in the shear strain rate we said that this is $\frac{\Delta S}{\Delta Y}$ into $\frac{1}{\Delta t}$. So, we can write this not like this, but $\frac{\Delta S}{\Delta t}$. I am correcting that, please make that correction. $\frac{\Delta S}{\Delta t}$ is actually the V_s, it will be divided by ΔY. So, please make this correction that this can be written as $\frac{\Delta S}{\Delta t}$ which is V_s that is how we can find out this is the V_s and this has to be instead of Δt this has to be therefore, ΔY.

So, what we get finally is that the shear strain rate is equal to $\frac{V_s}{\Delta Y}$. So, if we know the V_s for any given ΔY you can find out what is the strain rate.

(Refer Slide Time: 01:45)



So, now based on whatever we have discussed so far I would like to discuss some of the numerical examples, so that you could find out that how the Merchant's theory or whatever we have learned so far can be applied in practice.

For example, a mild steel workpiece is being machined at a cutting speed of 200 m/min with a tool which is given in this way. So, if you understand that this is in the ASA system - if you remember we have discussed, it is in the coordinate system. So, first angle in the nomenclature is the back rake, next is the side rake and so on. So, all these angles are given; 6 angles and this is the tool nose radius in inch.

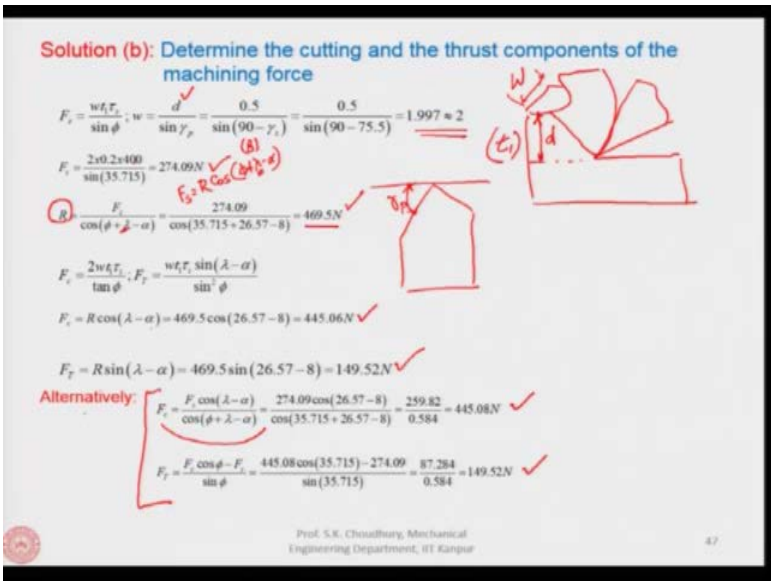
The depth of cut and uncut thickness are 0.5 mm and 0.2 mm respectively. If the average value of coefficient of friction between the chip and the tool is given to be 0.5 and the shear stress of the work material is $400 N/mm^2$, then determine the shear plane angle and determine the cutting and the thrust components of the machining force.

We know the Merchant's First Relationship. We also know that the friction angle λ is $\tan^{-1}\mu$, μ is the coefficient of friction which is given as 0.5. So, you put the value of 0.5 and you find out what is the value of the λ which is the friction angle. By the way, there it was β, but this is the friction angle λ used here.

So, from here we can find out that the ϕ is equal to $\frac{90-\lambda+\alpha}{2}$. λ has been found out as 26.57^o and the rake angle is given as the 8^o.

Now, why not to take that 0^o as the rake angle. This is because the first angle is the back rake angle, next one is the side rake angle and as you understand that the side rake angle we said influences the cutting force and the power, not the back rake angle. So, the α_s is taken as the α value and this is put in here as 8^o. λ we found out as 26.57 that we have put in here. From the Merchant's first relationship we can find out the ϕ.

(Refer Slide Time: 04:19)



We have to find out the ϕ, because then the $F_n = F_c \sin\phi + F_t \cos\phi$ will be according to the formula that we have seen earlier, $w = \frac{d}{\sin\gamma_p}$ we have seen that also in the cutting diameter, this is the plane like this shear plane and here we have said that this is the d, this is the depth of cut and this is equivalent to t_1 in the orthogonal case, all right. This is the tool and here we said that the width would be the width of the material, all right.

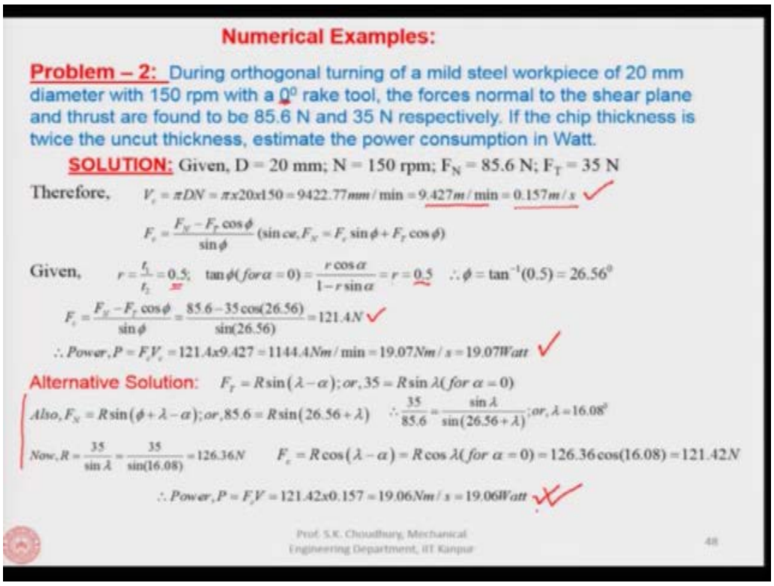
So, w will be the depth of cut divided by $\sin\gamma_p$, γ_p is the principal cutting edge angle and if this is the tool, then this is the principal cutting edge and this is the γ_p. So, from here we can find out what is the value of the w, this is the width and knowing the width

ϕ we found out since all other values are given, t_1 is given, τ_s is given, and we know that then F_s will be this. If we know the F_s, then the resultant force from the Merchant's we can find out, which is equal to F_s divided by $\cos(\phi+\lambda-\alpha)$, you remember that we said $F_s = R\cos(\phi+\lambda-\alpha)$. This is the friction angle, there we said as β by the way, the same thing β or λ; so, this is the friction angle.

Now, from here we can find out what is the resultant force, and if we know the resultant force, we know the formula of the F_c and the F_t from the Merchant's, then the F_c can be found out as $R\cos(\lambda-\alpha)$, R we found ou, λ we found out as 26.57^o and α is given as 8^o. So, this is the F_c.

F_t will be R of this and R we found out, λ we found out, α is given so it can be found out. There is an alternate way. You can see that this way also we can find out and the result will be the same, that is you take the formula which we have derived during the Merchant's circle diagram discussion. And from here, all these values will be known because F_s we found out put the value of the F_s and so on and we will find out that these values are the same. So, this is an alternate way of solving this problem.

(Refer Slide Time: 07:15)

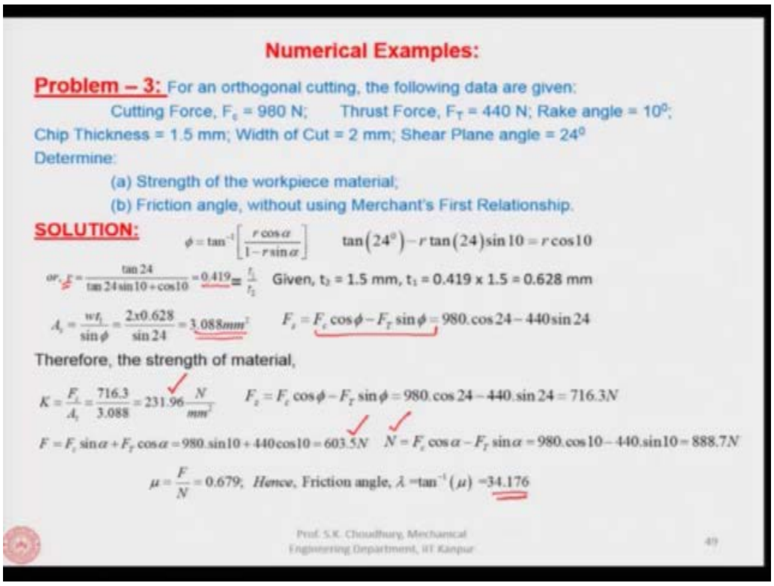


Next example, problem 2: during orthogonal turning of a mild steel work piece of 20 millimeter diameter with the 150 rpm and with a 0^o rake angle, the forces normal to the shear plane and thrust are found to be 85.6 and 35, normal force is 85.6 Newton and thrust is 35 thrust force. If the chip thickness is twice the uncut thickness, estimate the power consumption in Watt.

Now, first let us find out what is the cutting velocity, this is π, D, N - all these are given. So, it will be this value, either it is in meter per minute or you can convert it into meter per second. Then F_c which is from Merchant's, this formula F because the $F_n = F_c \sin\phi + F_t \cos\phi$ we have derived. From here, the F_c is equal to this, this is given that chip thickness ratio is 0.5 because the chip thickness is twice the uncut thickness. So, it is this therefore, for 0^o rake angle it is given here, we can find out the $\tan\phi = r$ which is 0.5 which is we found out from here.

Therefore, $\phi = \tan^{-1} 0.5$ is 26.56^o and then we can find out the F_c because the F_n is given, F_t is given, ϕ we found out and we can find out therefore, the value of the F_c; V_c we found and F_c is here. So, product of them will be the power. So, this is in Newton meter per second which is Watt. There is an alternate solution. You can also see that this way using the Merchant's diagram or the Merchant's formula we can also find out and the result will be the same.

(Refer Slide Time: 09:21)



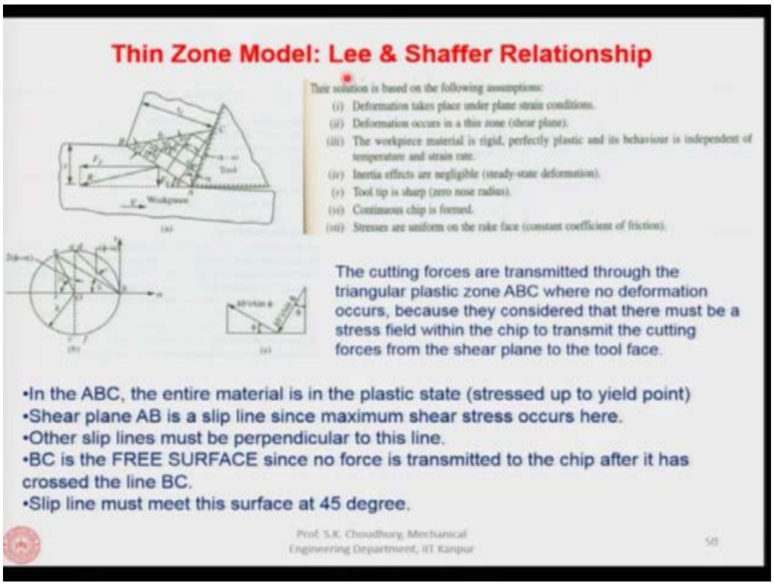
Third problem is for an orthogonal cutting. The following data are given: cutting force is given, thrust force is given, rake angle is given, chip thickness and width of cut shear plane angle - these are given. Determine strength of the workpiece material, friction angle without using Merchant's First Relationship.

Now, that the ϕ we know, we can find out that the value of the r is 0.419. Now, it is given that the t_2 is 1.5 millimeter, t_1 is this therefore, the product of chip thickness ratio into t_2.

So, the area is this which we can also find out as 3.088 mm^2 and then the F_s can be written in this way, this is the formula taken from the Merchant's and we have the values of F_c, F_t and ϕ known. Therefore, the strength of material will be F_s force upon area. We know the F_s, so we can find out the strength of material, F_s we can find out from here this is given how to find out the F_s.

$F = F_c \sin\alpha + F_t \cos\alpha$ this is directly taken from Merchant's and all these values are now known we can find out what is the value of the F which is the friction force and similarly the normal force. So, friction force divided by normal force will be the coefficient of friction and here the μ can be found out by $\frac{F}{N}$, hence the friction angle which is the $\lambda = \tan^{-1}\mu$ which is this. So, whatever has been asked this we found out simply knowing the relationship that we have already derived earlier.

(Refer Slide Time: 11:39)



Next to this we will be discussing another model which is based on another principal rather than the Merchant's principal. Once again let me remind you that the Merchant's principal is based on the minimum power consumption and here this is a thin zone model also, because they are assuming that the deformation is occurring in a thin zone and as I have said earlier that normally the plastic deformation occurs in a zone and not in a plane.

So, they are saying that, this zone is very thin and this can be assumed to be a plane like in the Merchant's. So, this solution is based on the following assumptions that the deformation takes place under the plane strain condition and the deformation occurs in a thin zone. Another assumption is that this is the shear plane and not the shear zone, the workpiece material is rigid, perfectly plastic and its behaviour is independent of temperature and strain rate.

Next they are assuming that the inertia effects are negligible, which is a steady state deformation, tool tip is sharp; that means, there is no nose radius and the continuous chip is formed, and the stresses are uniform on the rake face as in case of the Merchant's. So, as you can see that this is also a 2-D; I mean, this is not a 3-dimensional, it is a 2-dimensional because they are saying that the tool is sharp, μ is constant and the stresses are uniform and so on.

Now, the cutting forces are transmitted through the triangular plastic zone ABC. Meaning that if there is a resultant force acting on the chip from the workpiece through the shear plane, for this to be transmitted towards the rake face of the tool, it has to go through a triangular plastic zone ABC where no deformation occurs. Because, they considered that there must be a stress field within the chip to transmit the cutting forces from the shear plane to the tool face.

Once again, I will explain it to you that the resultant force has to be transmitted from the shear plane to the rake face. Now this transmission can take place through a plastic zone where no deformation occurs, because of this force, then only this can be transmitted. If this force creates deformation, it will be absorbed.

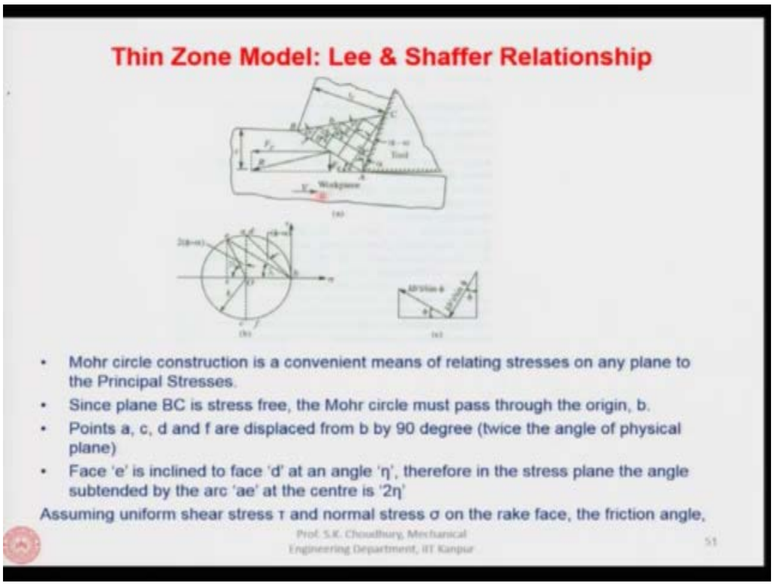
So, they are saying that this is the ABC which is a the plastic zone where the material is plastically deformed to the maximum. In the ABC, the entire material is in the plastic state, stressed up to the yield point, yield point B in that diagram that we have discussed in the very beginning. Shear plane AB, is a slip line since maximum shear stress occurs here. As you can see that AB is the shear plane and we said that the maximum shear deformation occurs along the shear plane.

Other slip lines must be perpendicular to this line, these lines are also the slip lines, because this is maximally stressed. As we said ABC is the plastic zone within the chip through which the forces are transmitted from the work to the tool. BC, is the free surface since no force is transmitted to the chip after it has crossed the line BC. You understand what I was saying is that this is maximally stressed and the force is transmitted up to BC and then there is no force transmitted.

So, BC is the free surface since no force is transmitted to the chip after it has crossed the line BC. Slip line must meet this surface at 45^o. This is the slip line field theory and in the theory you can find out that the free surface, here it is BC, makes an angle of 45^o with the slip line, that is the AB. AB is the slip line that makes an angle of 45^o with the free surface which is BC - this is according to the slip line field theory.

We are not discussing the slip line field theory in details. This is the application of the slip line field theory, which is called the Lee and Shaffer relationship. So, they have taken a principal which is different than the principal taken by the Merchant and Ernst. Here they are saying that there is a slip line field theory and according to that there will be a stress field through which the forces are transmitted from the shear plane to the rake face of the tool.

(Refer Slide Time: 16:55)



Now, if we have this, then the Mohr circle construction, Mohr circle diagram you all are familiar with, is a convenient means of relating stresses on any plane to the principal stress. So, I have also shown it to you that the Mohr circle diagram is placed on the, center is placed on the normal stress or the direct stress and perpendicular to that is the shear stress, all right.

Now, since plane BC is a stress free, Mohr circle must pass through the origin, b. So, as you can see that if it is a surface, plane BC the stress can be given as a point on the periphery of the circle. This is the 2 D and I showed it to you earlier also that in the 2 D case it can be shown on the periphery of this Mohr circle.

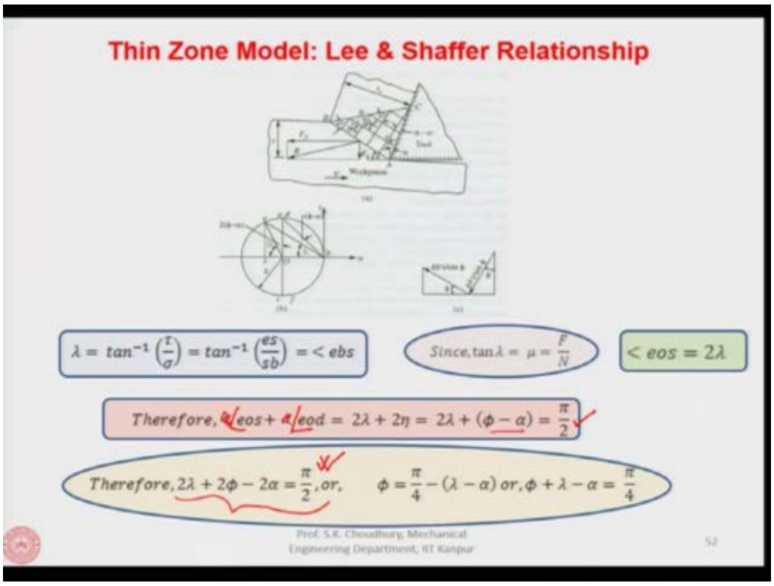
So, this BC, stress on this is b which is located here because it is a stress free, there is no stress - 0 normal stress 0 shear stress. Point a, c, d, f are displaced from b by 90^o which are point a, c, d and f - these are here. d is this plane, this plane is f.

So, these are the planes a, these are the lines either vertically here or normal to this, these are displaced from b by 90^o. This will be twice the angle of physical plane here it is 45^o. So, in this the a, d, c and f they will be at a 90^o angle from b, they will be displaced by 90^o angle.

Now, d is inclined to e by an angle of $\phi - \alpha$, you can find out that this angle is $\phi - \alpha$ because this is the rake angle, this is the shear plane angle. So, this angle will be $\phi - \alpha$ let us say this is $\nu = \phi - \alpha$.

So, if it is ν therefore, in the stress plane the angle subtended by the arc ae this one this will be at the center which is the 2ν. So, this angle is the 2ν angle, because the ν we are saying that this is the number $\phi - \alpha$, this is with respect to the point b. So, with respect to center this angle is the 2ν or the $2(\phi - \alpha)$.

(Refer Slide Time: 20:25)



Assuming uniform stress τ and normal stress σ on the rake face the friction angle we can find out here. And this will be the $\tan^{-1}\frac{\tau}{\sigma}$. From this you can find out that this will be equal to $\tan^{-1}\frac{es}{sb}$. Since $\tan\lambda = \mu$, you can find out the μ which is $\frac{F}{N}$.

Therefore the angle eos plus the angle eob is 2ν or the $2(\phi - \alpha)$.

So, $2\lambda + 2\nu$, $\nu = \phi - \alpha$. So, therefore, this is equal to $\frac{\pi}{2}$ and this makes $2\lambda + 2\phi - 2\alpha = \frac{\pi}{2}$. So, this is the equation which is derived by the Lee and Shaffer and you can see that this is very close to the Merchant's relationship. Although the

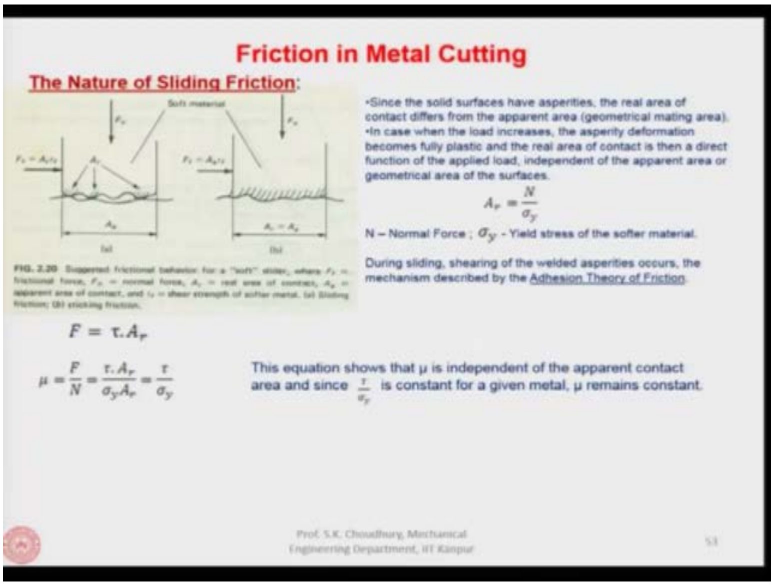
Merchant’s relationship is based on the minimum power consumption during the machining process, and here in this case the Lee and Shaffer say that there should be a stress field through which the forces are transmitted from the shear plane to the rake face of the tool.

So, I once again would like to repeat that there is a stress field. In this stress field these are all the slip lines where the maximum deformation occurs and each of these lines is given a name like for example, d, e, a, b, c, f and so on. So, here they are all maximally deformed and they are making an angle of 45^o with the free surface.

So, in the Mohr circle it will be twice that, which is 90^o. So, we said that a, d, c and f they are deflected from the b at a 90^o angle. Then what we said is that this e is inclined to this by an angle of ν and here it will be twice the ν. Then if you sum up angles eos and eob which is 90^o. From here you can find out this relationship.

So, the relationship is not very difficult to find out, only thing is that you have to construct the Mohr circle diagram, from the stress field and the machining diagram that the Lee and Shaffer has proposed. Now, please all the time you should also keep in mind that this is valid only when these assumptions are met, if any of these assumptions is not satisfied in that case the Lee and Shaffer relationship will not be valid.

(Refer Slide Time: 24:07)



After that we will be discussing the topic which is friction in metal cutting. Why we are discussing this, because in the normal case when we have the sliding friction there we have the F and the N, because of the normal force the friction force F is created. So, the $\frac{F}{N}$ we always assumed to be constant; that means, the F is proportional to N, this is what happens in the case of the sliding friction.

We will see the friction in metal cutting through a model how it differs in the case of the metal cutting and how it differs from the sliding friction particularly where the $\frac{F}{N}$ remains constant. In the nature of sliding friction we have to consider that the solid surfaces have asperities.

So, when two surfaces meet each other, they will actually not meet along the area which is the real area, but they will be meeting at some point on the asperities as you understand this. The real area of contact differs from the apparent area apparent area, which is the geometrical mating area. What we are saying is that the real area of contact is different than the area of contact on which they are meeting because they are meeting on the asperities and not the full area. So, the real area is different from the geometrical mating area.

In case when the load increases the asperity deformation becomes fully plastic and the real area of contact is then a direct function of the applied load independent of the apparent area or the geometrical area of the surfaces. So, what is said is that when the load is increasing then these points on which the two surfaces are meeting on the asperities, they become initially elastically deformed and then as the load is increasing further the elastic deformation becomes plastic deformation. In that case the real area of contact becomes a direct function of the applied load N and this will be independent of the apparent area or geometrical area of the surfaces.

So, we can write down that A_r which is the real area of contact is directly proportional to N it is equal to N by the yield stress of the softer material. Out of these two materials whichever is the softer material, the yield stress of that material has to be considered. So, the real area of contact will be equal to the N divided by yield stress of the softer material.

Now, during sliding, shearing of the welded asperities occurs, the mechanism of which is described by the addition theory of friction. Meaning that once the normal load is applied as we said that asperities become welded because the normal force is increasing and then when two bodies move with respect to each other, these welded asperities are sheared off and this is known by the addition theory of friction.

Now, the adhesion theory of friction will give you an equation of the F that is a friction force is equal to τA_r. A_r is the real area of contact and therefore, μ which is $\frac{F}{N}$ this can be given as $\frac{\tau A_r}{N}$; we found out as the A_r into the yield stress of the softer material. So, which is nothing, but A_r is getting cancelled so, the τ divided by yield stress of the softer material σ_y.

This equation shows that μ is independent of the apparent contact. That is the basic idea in the case of the normal sliding friction when the two materials slide on each other and then how it can be applied to the machining and how the friction behaves in the case of the machining that we will be discussing in the next class.

Thank you for your attention.