Machining Science - Part I
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Lecture - 08

Hello and welcome to the 8th lecture of the course on Machining Science. I will remind you that in our last session we started discussing the Merchant's circle diagram. And, we said that Merchant's circle is the circle which is drawn with a diameter of R or R' which is the resultant force acting on the chip, from the tool through the rake face or to the chip from the shear plane.

So, in this case, the components of these resultant forces, that is the cutting force, thrust force, shear force, normal to shear force, friction force and the normal force - the tips of these components will be lying at the periphery of this circle.

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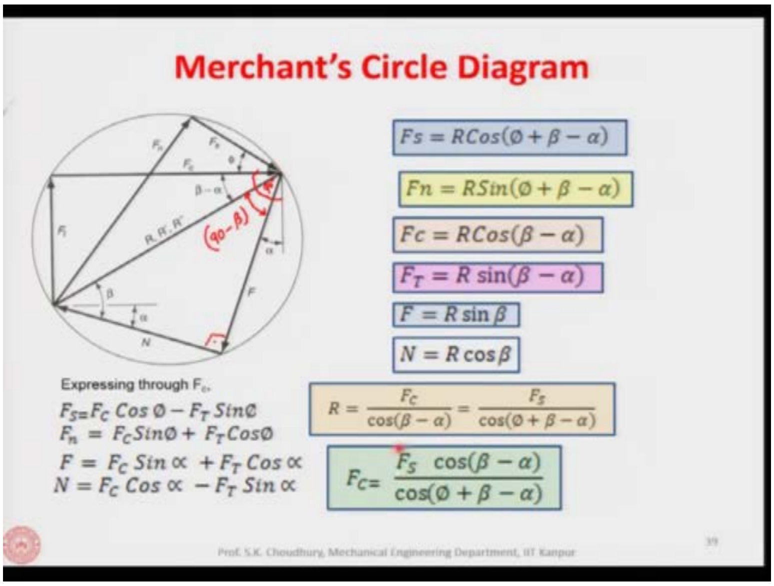

This circle was shown, which is known as the Merchant's circle. From here, knowing that ϕ is the shear plane angle, this is the rake angle and this is the friction angle, we can correlate the F_s, F_n, F_c, F_t, F and N with the resultant force and the angles - these three angles. From here, that we have derived last time, we can find out that

$$R = \frac{F_c}{\cos(\beta - \alpha)} = \frac{F_s}{\cos(\phi + \beta - \alpha)}$$

This is what is written here.

So, from here F_c which is the cutting force, can be found out as

$$F_c = \frac{F_s \cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)}$$

So, this is the component F_c that we can find out by this way. Here, it means that if we know F_s and if we know the friction angle, sheer plane angle and the rake angle, we can find out analytically the value of the F_c analytically without machining.

Now, here F_s is very difficult to find out, because we can measure the forces during the machining process and through the measurement we can measure only the F_c and the F_t.

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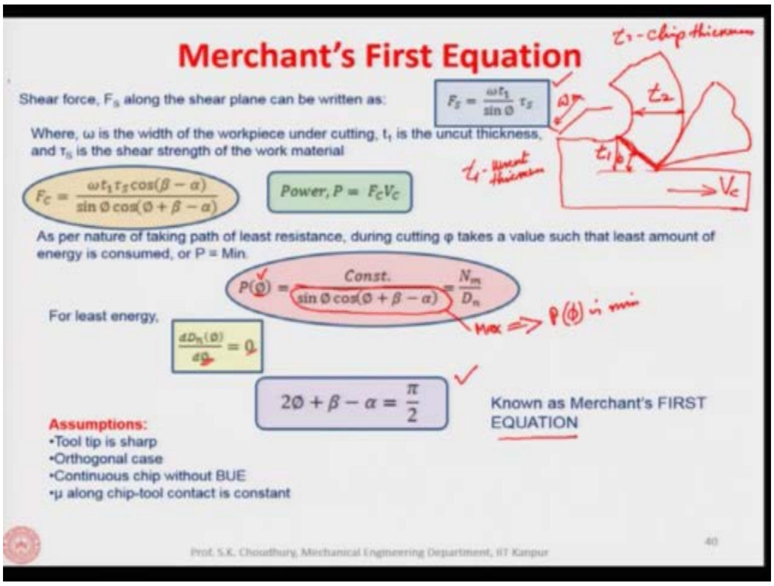

So to find out the F_s, we can say that shear force F_s along the shear plane can be written as the F_s is equal to stress into area. Force is equal to stress into area and the area of the shear plane will be the product of width of cut and uncut thickness divided by $\sin\phi$.

So, this you can find out from the diagram. So, this is the shear plane ϕ, extending from the tool point to the point on workpiece where it started being plastically deformed. t_1 is the uncut thickness, because this is not yet cut. And this thickness is the chip thickness t_2.

Now, the area of the shear plane can be found out if we know the width of cut multiplied by t_1 which is projected here divided by $\sin\phi$. From this diagram you can find out how the F_s can be found out. $\frac{wt_1}{\sin\phi}$ is the area of the shear plane and τ_s is the stress.

Now, the τ_s is basically nothing but the strength of the workpiece material. t_1 is the uncut thickness, w is the width of the workpiece and τ_s is the shear strength of the work material. So, in F_c we can put these values - $\frac{wt_t\tau_s}{\sin\phi}$ which is F_s, multiplied by $\frac{\cos(\beta-\alpha)}{\cos(\phi+\beta-\alpha)}$.

So, having the value of F_c, which is the cutting force component, we can find out the power which is a product of F_c and the Vc; F_c is this cutting force, Vc is the cutting velocity. F_c is along the direction of the cutting velocity vector, Vc .

Now, like nature takes path of least resistance, during cutting, ϕ takes a value such that least amount of energy is consumed or P is minimum. This means that during the cutting process, by nature, the ϕ or the shear plane angle adjusts itself in such a way that for the entire process the minimum power is consumed. This is the natural process, like in nature everything goes to the least resistance. So, in here also it happens. So, that the ϕ adjusts itself so that the minimum power is consumed or the P remains minimum.

So, if we say that P is a function of the shear plane angle, ϕ then the numerator will be constant since there is no ϕ in it. Because, we are taking with respect to ϕ, and a function of the ϕ divided by the denominator which is a function of ϕ. So, this in the denominator, we have the $\sin\phi$ and the $\cos(\phi+\beta-\alpha)$.

Now, for least or minimum energy, the denominator has to be maximum. For this, the first derivative of the denominator has to be equal to 0.

So, if this denominator is maximum, the power P is minimum. So, for this maximum, we have taken the first derivative of the denominator and we have made it with respect to ϕ, this is equal to 0. So, if we do that, the first derivative of $\sin\phi\cos(\phi+\beta-\alpha)$ is $2\phi+\beta-\alpha=\pi/2$. This you can find out. You take the derivative of this with respect to ϕ and this will be the $2\phi+\beta-\alpha=\pi/2$. This is known as the Merchant's first equation, but you have to always keep in mind that this equation is valid only when those eight assumptions that I have shown in the last slide, are satisfied.

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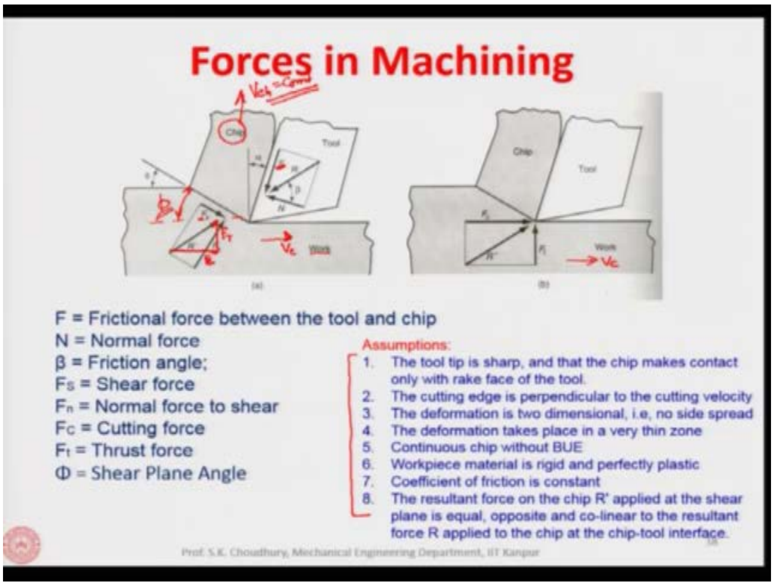

If any of these assumptions is not satisfied, in that case the Merchant's first relationship is not valid. This you have to keep in mind all the time.

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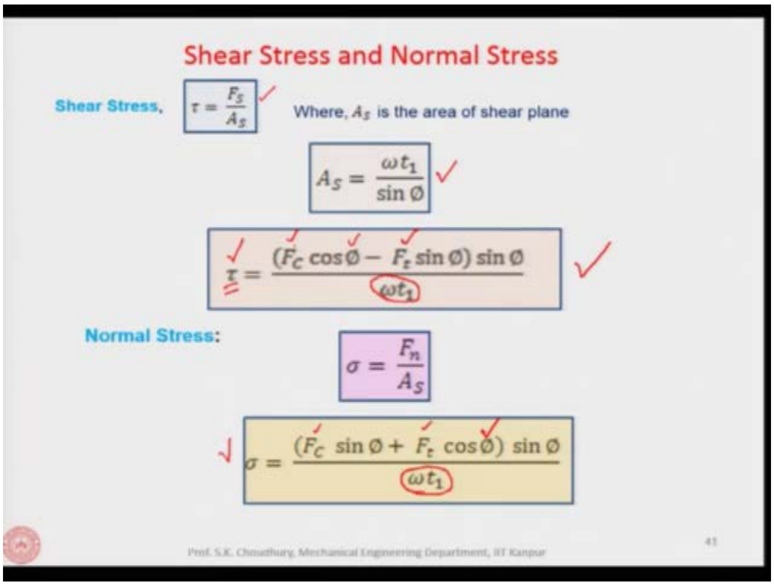

Now, that we found out the F_c, we can find out the shear stress as $\frac{F_s}{A_s}$, and the A_s, which is the area of the shear plane. We have already seen that this is $\frac{wt_t\tau_s}{\sin\phi}$, this is the area. Area is equal to width into the length, length is $\frac{t_1}{\sin\phi}$, I have already shown it to you; so width into length is that area and therefore, the τ shear stress, τ can be found out. F_s we found out already, $F_s = F_c\cos\phi - F_t\sin\phi$ it will go here added by A_s, and in the denominator, it will be wt_1.

Now, this is the shear stress that we can find out analytically. As I said earlier that when you are measuring the cutting forces in the machining zone, you can measure the F_c and the F_t through an instrument called a dynamometer, we will be discussing that little later.

So, if we know the F_c and F_t and if we know the ϕ, the shear stress, τ can be determined analytically knowing the physical parameters such as width of cut and uncut thickness. Similarly, we can find out the normal stress which is $\frac{F_n}{F_s}$ like we have done it here. This was $\frac{F_s}{A_s}$ for shear stress and this will be the normal force F_n because it is the normal stress; so, the normal stress can be found out by putting the value of F_n. F_n, if you see from the Merchant's, we found out as $F_c \sin\phi + F_t \cos\phi$; this will be multiplied by A_s, where $\sin\phi$ is going up and the wt_1 is here. So, the normal stress analytically can also be found out if we know the F_c, F_t, ϕ and the physical parameter such as width and the t_1. This is the advantage that when you do not have the cutting process, analytically you can find out the F_c and the F_t.

So, the advantage is that when you are designing the machine tool, you do not have the cutting process. But, you have to make the machine tool where, this machine will be capable of removing this much material or providing this much power.

So, you have to select the prime mover for that machine tool. Now the power of the prime mover has to be more, little more than the power which is consumed during the machining process. And the power consumed during the machining process can be found out by the product of F_c and the V_c. F_c you can find out analytically from the Merchants as we said and the V_c is you are giving that V_c, because V_c is the πDN; D is the diameter of the workpiece.

So, if you know that you have to machine a workpiece of a certain diameter and with a certain revolution RPM, then you will be knowing what is the power it will consume if we can find out analytically the value of the F_c which is the cutting force, this is clear.

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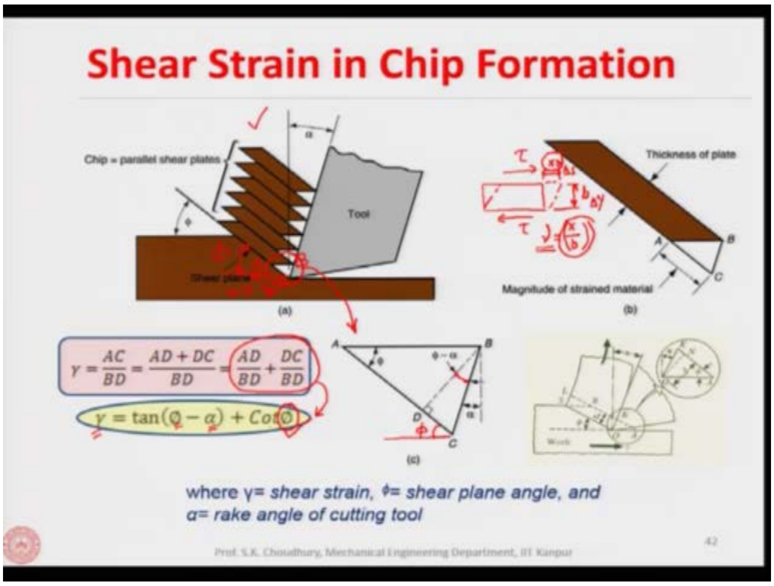

Next is how we can find out the shear strain in the chip formation. As you understand that the shear strain concept I have shown to you earlier. Suppose you have a piece of material like that and this has been deformed; for example, like this by a shear force or the shear stress which is applied.

Now, this movement because of the shear stress applied is x. I am repeating that I have already shown it to you earlier and this width is, suppose, b. So, the strain which can be denoted as ν is $\frac{X}{b}$. So, this is the strain occurring when we have the τ applied and because of the τ we have the movement of the material. Permanent movement which is the plastic deformation as x and the distance between the two layers will be b.

Similarly, in case of machining you can actually assume this chip which is flowing along the rake face of the tool as a stack of cards, for example. Let us say each card is a layer of the chip which is flowing along the rake face of the tool.

Now, when the force is applied from the tool, this stack will be moved and it will move as shown in this diagram. Now here what you can see is that, this is the movement which is equivalent to the x that we have shown. And the distance between the two layers will be perpendicular to the lines of the layer.

Now, we draw this triangle ABC. This point is let us say B, this point is A and this point is C. So, AC here is the movement, this is the movement which is let say equivalent to x. AC is the movement of the material because of the shear stress which is applied from the tool.

And the distance between the two layers, that is equivalent to B will be if we draw a normal from the point B to AC which is, let's say intersecting at D. So, BD is normal to AC, and this is the 90^o angle. Once again, this is the triangle which you have taken from here with AC being the movement equivalent to the x because of the shear force or shear stress that is applied through the tool.

Therefore, we can say that the strain which is AC upon BD that is equivalent to $\frac{x}{b}$. So, $\frac{AC}{BD}$ is the strain and $AC = AD + DC$, divided by BD. So, this can be assumed as $\frac{AD}{BD} + \frac{DC}{BD}$. And from here, you can see that this is the angle between BC and the line normal to Vc , which is rake angle, which is here. And this is the angle ϕ between the AC and the direction of V_c or the angle BAC meaning that this is the angle which is ϕ, shear plane angle. Angle between the AC and the direction of V_c and the angle BAC are the same angle because this line from point C along the direction of V_c and the line AB are parallel and the line AC is intersecting them.

So, this is ϕ which is the shear plane angle. This angle DBC will be $\phi - \alpha$; if you see this which is equal to $\tan(\phi - \alpha)$ which is $\frac{DC}{BD}$. $\frac{AD}{BD}$ will be $Cot\phi$, because $\frac{BD}{AD} = \tan\phi$.

Now we can find out the strain, if we know the ϕ and the α. This means that analytically you can find out how much strain will be required for machining a particular material, if we know the ϕ. Rake angle, α is known because we are using a tool where we will be knowing what is the rake angle of the tool. So, we have to know the ϕ only to find out what is the strain involved in here.

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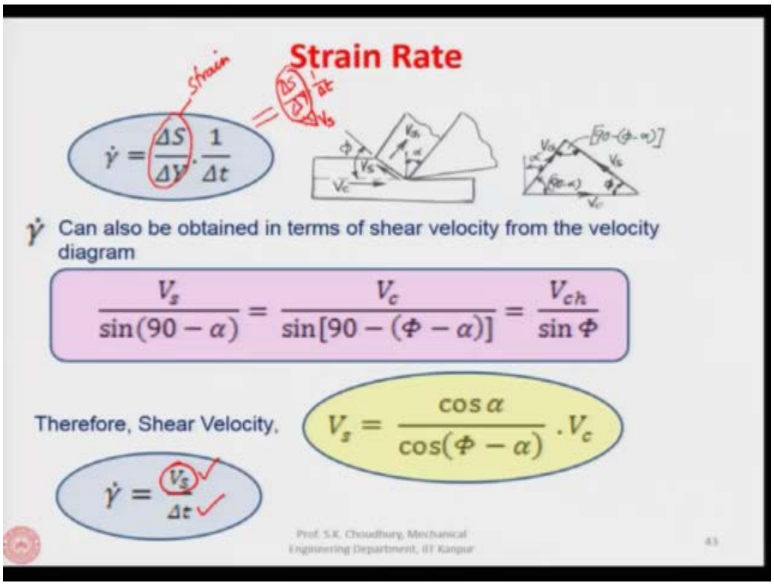

Now, let us see how strain rate can be found out. In strain rate, there is a time component. So, strain rate is strain $\frac{\Delta S}{\Delta Y}$ multiplied by $\frac{1}{\Delta t}$. So, $\frac{\Delta S}{\Delta Y}$ is this strain and this this is happening at the Δt time. Meaning that this movement of ΔS or x (these are equivalent) is happening at a time Δt. So, the strain rate will be strain into $\frac{1}{\Delta t}$.

Now this can be found out if we have the velocity diagram. Chip is flowing along the rake face of the tool and the chip velocity is here. So, the shear velocity is along the shear plane and cutting velocity is in the horizontal direction, as we have shown earlier.

So, if we join them, let us say this is the direction of the chip velocity; this is the direction of the cutting velocity, and the direction of the V_s is the along the shear plane. So, we have the V_{ch}, V_c and the V_s and we know the angles because between the V_s and V_c here, this angle we have seen that this angle is ϕ. Between the V_{ch} and the line perpendicular to V_c is the α. So, this is α; so therefore, this is $90-\alpha$ and this is the $90-\phi+\alpha$.

So, if we have this triangle you can use the sine rule and we can find out that V_s divided by sin of this angle equal to V_c divided by sin of this angle equal to V_{ch} divided by sin of this. That is what we have written here, $\frac{V_s}{\sin(90-\alpha)}$ which is the $\frac{V_s}{\cos\alpha}$, equals to $\frac{V_c}{\sin(90-\phi+\alpha)}$ equals to $\frac{V_{ch}}{\sin(\phi)}$; Therefore, the shear velocity V_s, we can find out through the V_c as the $\frac{V_c \sin(90-\alpha)}{\sin(90-\phi+\alpha)}$. Once we found out the shear velocity, you can write it in the form of $\frac{\Delta S}{\Delta t}$ into $\frac{1}{\Delta Y}$. So, $\frac{\Delta S}{\Delta t}$ is nothing, but V_s which is the shear velocity and this is divided by ΔY.

So, we will write it in this way that this is equal to $\frac{\Delta S}{\Delta t}$ into $\frac{1}{\Delta Y}$. So, $\frac{\Delta S}{\Delta t}$ this is actually V_s or the shear velocity, and this is into $\frac{1}{\Delta Y}$. Therefore, if we know the shear velocity V_s, we can find out the strain rate for a certain value of ΔY.

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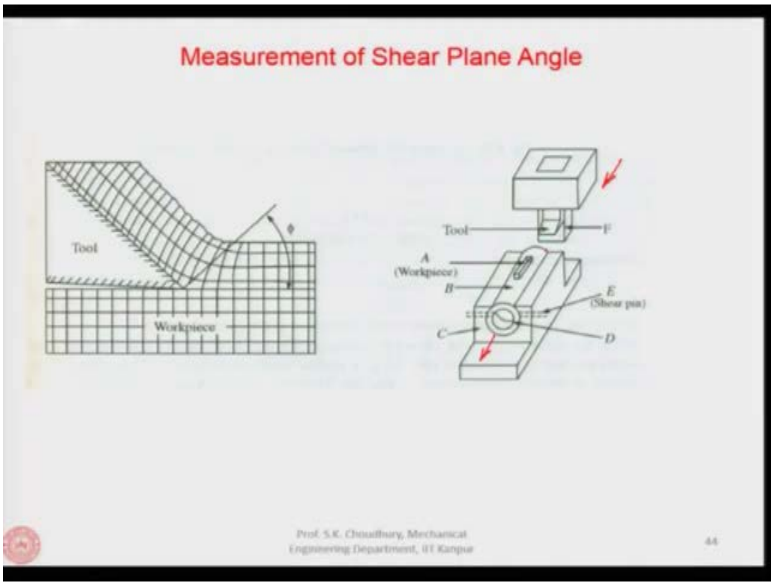

Now let see that how the shear plane angle can be measured. As you understand that all the time what we were telling is that, we can find out analytically the value of the F_c if we know the ϕ, because all other angles will be known. So, determining the shear plane angle ϕ is very important.

The shear plane angle can be measured. Shear plane angle is measured using a device which will actually make a partially formed chip. For example, here this is the device where you have a cylinder and this cylinder is fixed in here with a pin, shear pin. Meaning that if the cylinder is pushed to this side, the shear pin will be broken and this cylinder will come forward, it is stopped here by the shear pin.

On the cylinder there is the workpiece for which you are trying to find out the value of the ϕ which is the shear plane angle. And behind this there is a device where you have the tool and at the back of the tool there is a tongue or a surface. So, if it is moving forward in this direction, the direction of this workpiece; in that case the tool will touch the workpiece before this shoulder.

Therefore, as it is coming in contact with the tool, the shoulder is still at the back, it is still not touching this. So, this will actually remove a part of the material and then this shoulder will come and hit the cylinder. In that case the cylinder will come abruptly to this side, breaking the shear pin and we will get the workpiece with a partially formed chip like this.

If the workpiece is graduated as shown in the figure, we can find out how these graduations have been deformed and then we can clearly see a line through which the deformation has taken place. If we join these points, we can find out, we can measure under the microscope what is the value of the ϕ which is the shear plane angle. Of course, this is not a very accurate measurement there can be faults, there can be errors; and therefore, normally the shear plane angle is determined analytically and that is also a very easy process.

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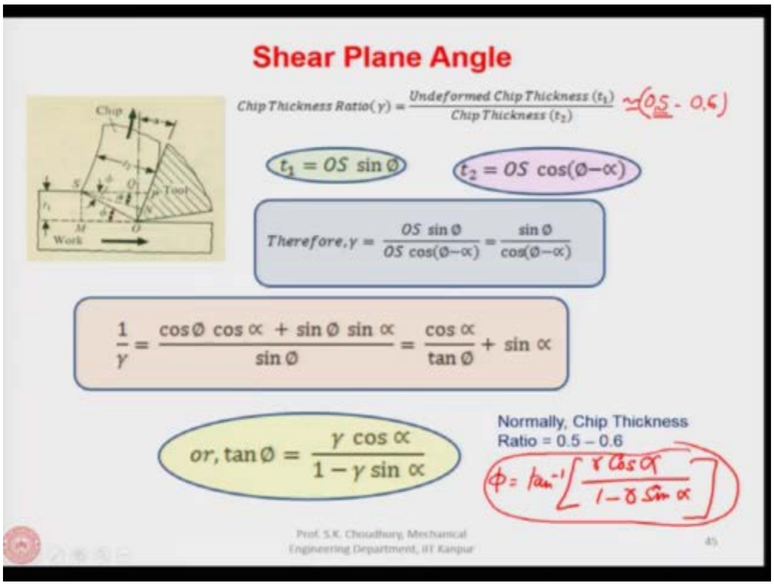

For example, here if you see the diagram, we have the tool here and this is the rake angle all right, this is the shear plane angle that we have seen.

Now, this angle if we see between the line SP and the SN, which is the perpendicular to the rake face of the tool, this angle is also α that is the rake angle; because this line SP is perpendicular to this line OQ and this line SN which is perpendicular to the rake face of the tool. So, between these two lines also the angle will be the same as in here which is the rake angle.

Now, if we have this diagram with the t_2 as the chip thickness and t_1 as the uncut thickness, then the chip thickness ratio concept is there, which is the ratio of the undeformed chip thickness by deformed by the chip thickness. Normally in practice it has been seen that this is roughly between 0.5 and 0.6; that means, after deformation the chips becomes wider, the thickness becomes more, thickness becomes more by almost 2 times, this is $\frac{1}{2}$.

Now, chip thickness ratio is the ratio of $\frac{t_1}{t_2}$. Now t_1 from this figure you can find out that this is OS into sin of this angle, ϕ. Similarly from this triangle you can find out that t_2 is OS into cos of this angle and this angle is the $\phi - \alpha$.

Now, from the triangle OSN you can find out that the t_2 which is this SN is equal to O S, this shear plane, multiplied by $\cos(\phi - \alpha)$. So, the chip thickness ratio can be found out as t_1 which is $\text{OS}\sin\phi$ divided by t_2 which is $\text{OS}\cos(\phi - \alpha)$; OS is cancelling, $\frac{\sin\phi}{\cos(\phi - \alpha)}$ is remaining. Then we can find out the value of 1 upon this chip thickness ratio, you can inverse it. And from here you can find out that $\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha}$.

So, from here you understand that analytically we can find out the value of the ϕ which is the $\tan^{-1}\left[\frac{r\cos\alpha}{1 - r\sin\alpha}\right]$. Chip thickness ratio we know and the α we know. So, this is the way we can analytically find out the value of the ϕ rather than measuring it, because measuring will be not very accurate.

So, as you can see here that from the Merchant's diagram we found out the F_c, we found out the values of the F_t. After that we can find out the shear stress, normal stress with these values, because here the stress it will depend on the F_c, F_t and this ϕ now we found out analytically. So, analytically now we can find out what is the value of the shear stress. Similarly we can find out the normal stress by analytically finding out the ϕ and we know the F_c and F_t and we can find out the value of the normal stress.

Similarly, we can find out the value of the shear strain, because now we know the ϕ; this is the shear plane angle and α of course, as I said that this the rake angle, which is known because the tool is known to us. Therefore, the strain rate as well can be found out from here, from this relationship. That means, these are the parameters which can be easily found out analytically after the F_c and the F_t can be determined from the Merchant's circle diagram. Rest of the things we will discuss in our next class.

Thank you for your attention.