Machining Science - Part I
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Lecture – 19

Hello and welcome to the 19th session of the discussion on Machining Science course. In our previous session we were discussing the economics of machining and we said that the aim of any production is to produce the final part with the accuracy, size, finish along with the minimum cost. So, we have to select the optimum process parameters to get the minimum cost.

Now there are three criteria - minimum production cost criteria, maximum production rate criteria and maximum profit rate criteria. By this we mean that for any set of machining parameters, we will always get a minimum cost or maximum production rate or maximum profit rate. So, the idea is for each of these criteria we have to find out these parameters which will be optimum at which these are satisfied.

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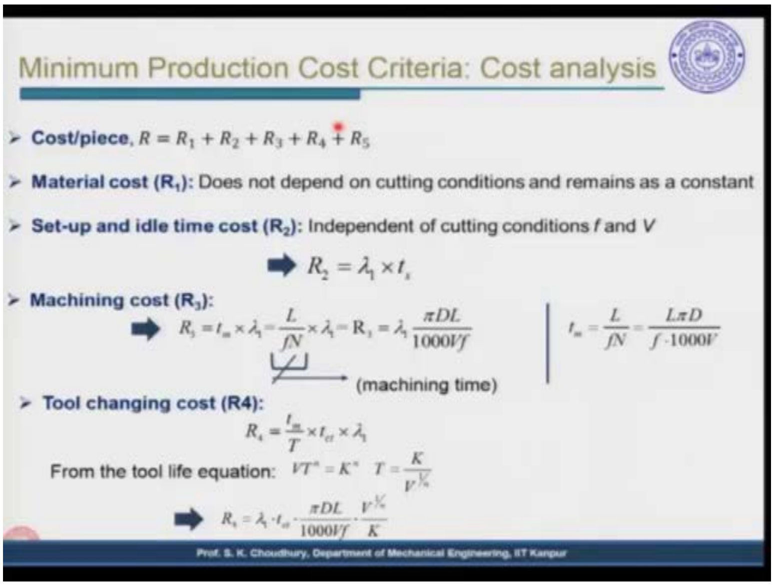

Let us say first the minimum production cost criteria which we started looking into the last class. Here we have to find out all the costs involved in the process in the machining of a particular part and then we have to minimize that cost. For any component, cost per piece will be consisting of 5 costs - material cost, then the settling and idle time cost per piece, machining cost per piece, the tool changing cost per piece and the tool cost per piece.

Now the material cost does not depend on the cutting conditions because we are particularly looking into the cutting condition and in this particular case we said there for simplicity we will look into the cutting velocity. So, we will find out the optimum value of the cutting velocity at which particularly this criteria that is the minimum production cost criteria can be satisfied.

Now, the setup and idle time cost also remains independent of cutting conditions like either cutting velocity or feed or depth of cut and this can be defined as the time taken for settling and the ideal time into the λ_1 which will be the overhead plus the labor cost. Now, the machining cost will be the machining time, t_m multiplied by the cost that we are incurring for that time, that is the λ_1 which is overhead plus the labor cost per piece.

The t_m , we know that this is $\frac{L}{fN}$ and N can be substituted because πDN is equal to velocity. So, this can be substituted by the velocity and it will be $\lambda_1 \frac{\pi DL}{1000.Vf}$. 1,000 we have taken to convert meter into millimeter, so that the unit finally could be in rupees. Now here it is shown that the $t_m = \frac{L}{fN}$ which is $\frac{\pi DL}{1000.Vf}$.

The tool changing cost R_4 is equal to the number of times we are changing the tool into how much time it is taken each time that has to be multiplied by the λ_1 which is over head plus labor cost. So, number of times we are changing the tool can be given by the t_m that is the machining time and if we divide that by the tool life; tool life is T.

Now, from the tool life equation we know that $VT^n = C$, C constant and C can be taken as K^n. Now from here we can find out that the tool life is equal to $\frac{K}{V^{\frac{1}{n}}}$, V is the cutting velocity. Now R_4 can be equal to if we put the value of T here which is $\frac{K}{V^{\frac{1}{n}}}$, it will be

$\lambda_1 \dfrac{\pi DL}{1000.Vf} \dfrac{K}{V^{\frac{1}{n}}}$ expression; so, this expression we have taken only here we will be considering the cutting velocity V.

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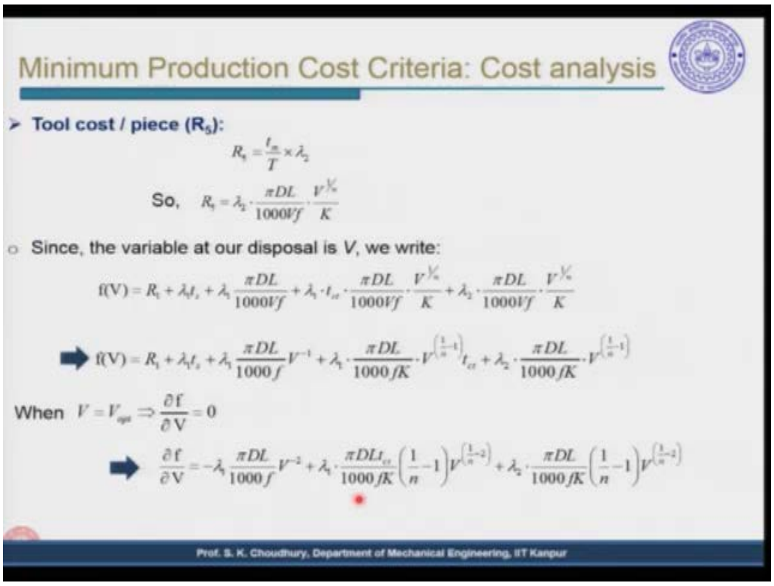

Now, for the tool cost per piece again we have to calculate how many times we are changing the tool and that has to be multiplied by the tool cost per piece which is λ_2. Here you have to be careful that λ_1 will not be involved, because here it is the tool cost which you are incurring. The number of times we are changing the tool, and that has to be multiplied by the tool cost per piece.

$\dfrac{t_m}{T}$ is the tool changing frequency; T we have found out earlier, t_m we found out earlier that is the $\dfrac{\pi DL}{1000.V_f}$. Value of T we found out as $\dfrac{K}{V^{\frac{1}{n}}}$. So, here for 1/T it will be $\dfrac{V^{\frac{1}{n}}}{K}$ and that has to be multiplied by the tool cost per piece. Now, the variable at our disposal as we said that we are considering the cutting velocity; that means, for the cutting velocity we will try to find out the optimum value for which the cost will be minimum. So, the function that will be having that is the R, cost per piece that will be a function of the velocity.

That is equal to R_1 which is independent of the cutting velocity, R_2 which is independent of the cutting velocity, which is given by delta λ_1 into t_s, R_3 this is the machining time that we have already calculated, R_4 this is the tool changing time cost per piece and the tool cost per piece. So, all these we found out. We sum them together and we say that this is a function of the cutting velocity which is basically the cost per piece.

So, we will segregate the V in terms of V^{-1}, $V^{\left(\frac{1}{n}-1\right)}$ and so on. Then we will get V optimum that is the optimum cutting velocity. We will take the first derivative of this equation or, cost per piece equal to 0.

So, if we take the first derivative of this equation or first derivative of the function f with respect to cutting velocity, we will get this. V^{-1} we have, so it will be minus. This term will not stay because it is with respect to cutting velocity and R_1 and R_2 they do not depend on the cutting velocity, so they are going out.

Now this term will be $-\lambda_1 \dfrac{\pi DL}{1000 f}$ which is constant, because it is not dependent on the cutting velocity. So, cutting velocity is here; so, it will be V^{-1}. Similarly, the derivation of this will be $\dfrac{1}{n}-1$ coming out here, in this case $V^{\left(\frac{1}{n}-2\right)}$ and here also it will be the similar one.

So, here it is the λ_2 which is the tool cost per piece and here in these two factors it will be λ_1 which is the overhead plus labor cost.

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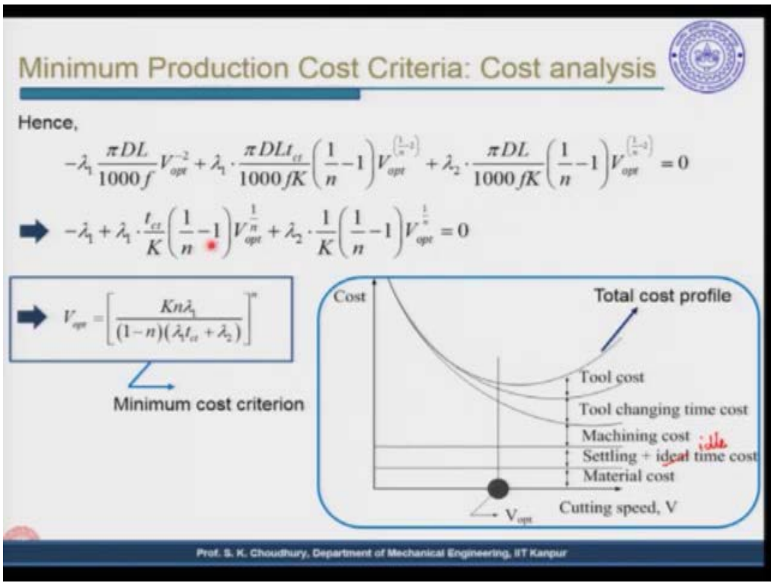

If we rearrange them and make them equal to 0, in this case all the V, velocity that we will be getting here are the optimum velocities, because we have taken the first derivative of this equation and we have we are equalizing to 0. So, we will write V optimum in all those cases, we will reorganize them and since it is equal to 0 so, these terms will be cancelled and we are getting $-\lambda_1 \frac{\pi DL}{1000 f} V_{opt}^{-2}$.

So, this will be now remaining, will be $\frac{1}{n}-1$ and this will be V optimum of course and similarly this factor, here it will be also $V_{opt}^{\frac{1}{n}}$, because we have taken V_{opt}^{-2} and $\frac{\pi DL}{1000 f}$ common. So, this is equal to 0 and if we solve that equation we will find that the $V_{opt} = \left[\frac{Kn\lambda_1}{(1-n)(\lambda_1 t_{ct}+\lambda_2)}\right]^n$.

Here of course, the K is constant, n is constant and those constants are coming from the Taylor’s tool life equation. These constants will depend on the combination of the tool and the work piece material of course. So, this is the minimum cost criteria and as per minimum cost criteria, this is the optimum value of the cutting velocity. If we can machine with this cutting velocity then we will be getting the optimum cost and how to get this V optimum that we already found out here.

So, now if we plot the curve - in y-axis if you put the total cost per piece and here along the x-axis if you put the cutting speed, that is the V velocity, in that case the material cost will be constant; it does not depend on the cutting velocity.

Settling plus idle time cost which is R_2 that also remains constant along the cutting speed meaning as the cutting speed is increasing there is no effect on the material cost or the settling plus idle time cost. Once again, please correct it that this is not ideal, but this is idle time when it is stopped, when it is not working - idle.

So, these two costs will be remaining constant, then the machining cost; if you see the equation of the machining cost it will be a non-linear equation because we have seen that the machining cost will be $R_3 = \lambda_1 \frac{\pi DL}{1000 fV}$ into V^{1} . So, this is the non-linear equation; it is varying in this way. As the cutting speed is increasing the machining cost will be reduced. And next is the R_4 which is the tool changing time cost per piece and then this will also be a non-linear equation and the tool cost which is also a non-linear equation. All these three costs - tool costs, tool changing cost and the machining cost and if we draw the total cost that is the sum of all of them, it will be a curve like this.

This is the total cost profile and in here there will be a minimum and this minima indicates that this is the point where we will have the V optimum this is the value of the V optimum and at this V optimum value the cost that we will have is minimum. In the sense that if we reduce this value of V, cost will be higher or at a higher velocity than the optimum velocity the cost will be again higher.

Only at this value of the velocity which is equal to this we will have the minimum cost. So, this is what we mean by the minimum cost criteria and this is the optimum velocity by which we can get the minimum cost. Mind one thing that here we are taking care of only the velocity as machining component.

Similarly, we can also find out the optimum value of the feed and the optimum value of the depth of cut at which the cost will be minimum. In that case you understand that the first derivative has to be taken with respect to f , feed or with respect to depth of cut and equal to 0; in that case the value of the f that will be getting or d that is the depth of cut that will be getting will be the optimum values. So, here we have neglected those cutting parameters, we have taken only the optimum velocity and similarly you can also get the optimum value of the feed and the depth of cut.

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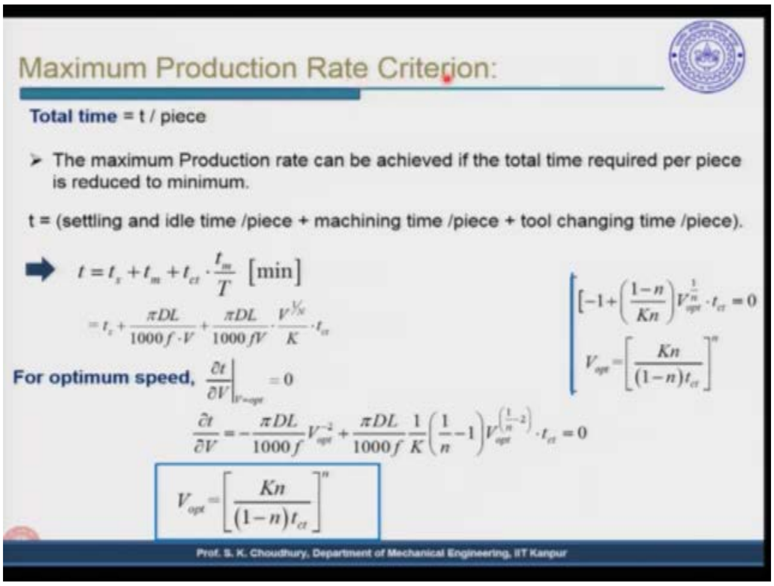

Let us see the next criterion which is the maximum production rate criterion; here also we will take care take only the velocity component and we will see what will be the value of the optimum velocity at which the production rate can be maximum. Now to have the production rate maximum, you know that the time taken to make that part or produce that part should be minimum.

So, the idea is that we will sum up all the times which are taken for fabricating that part and we will minimize that time so that the production rate could be maximum. The maximum production rate can be achieved if the total time required per piece is reduced to minimum. Now, total time taken to fabricate the part is equal to the settling and idle time per piece plus machining time per piece plus tool changing time per piece. These are the 3 components of times which are involved in the total time.

So, the settling time is t_s, machining time we said is the t_m and the tool changing time is t_{ct} that has to be multiplied by the number of times we are changing the tool. Because, t_m we have taken as machining time per piece, but t_{ct} we set the tool changing time, and tool changing time per piece will be tool changing time into the number of times we are changing the tool and which as we have seen that will be t_m machining time divided by tool life.

So, we have said earlier that t_m can be found out by this way that this is equal to $\frac{\pi DL}{1000Vf}$ and $\frac{t_{ct}t_m}{T}$ can be written as $\frac{\pi DL}{1000\,fV}.\frac{V^{\frac{1}{n}}}{K}.t_{ct}$. This also we have seen earlier. Now, for optimum speed, this equation that is the total time t that is taken, if we take the first derivative of that and we equalize to 0 that will give us the optimum velocity with respect to cutting velocity.

If we now solve this equation and make it equal to 0 like in the case of the minimum cost criteria, we will get the optimum velocity which will be $\left[\frac{Kn}{(1-n)t_{ct}}\right]^n$. Now, this is also a value which is optimum value of the velocity, but at this value we will be getting the production rate maximum, but not the minimum cost. For minimum cost anyway we have to use the minimum cost criteria and with this value we will get the minimum cost.

This we have to keep in mind that for each criteria, the value of the optimum parameters are unique. In the sense that for with this value of V optimum we cannot satisfy the other criteria like maximum profit rate or minimum production cost.

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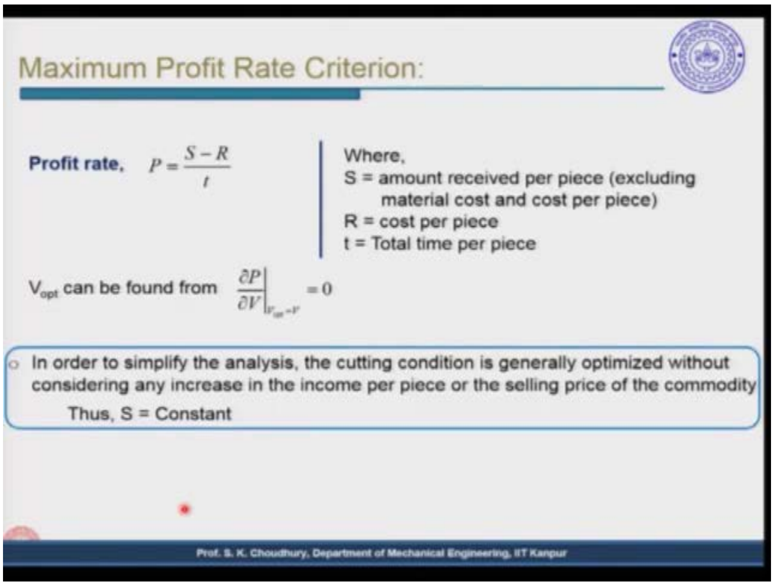

Similarly, for the maximum profit rate criteria, we can write this equation as profit rate this will be $\frac{S-R}{t}$. Now S is the amount received per piece excluding material cost and the cost per piece. Meaning that we have fabricated a part or a unit and for how much money we are selling that which in one word is the selling price minus cost per piece how much we have incurred to produce that part and divided by the total time per piece which has been taken. Now the R we have found out, this is the cost per piece; we found out the total time per piece.

And let us say S is constant for the simplicity; meaning that selling price is not fluctuating and for the constant S knowing the R which we have derived earlier and knowing the value of the t which also you have derived earlier, we take the first derivative of this equation and equalize to 0, then you will get the value of the V optimum. Again with this value of the V optimum we cannot satisfy the other two criteria, but it will be unique for this criteria that is for maximum profit rate.

Now once we got these values of V optimum, it is not necessarily that we will be able to use these values. There are certain restrictions.

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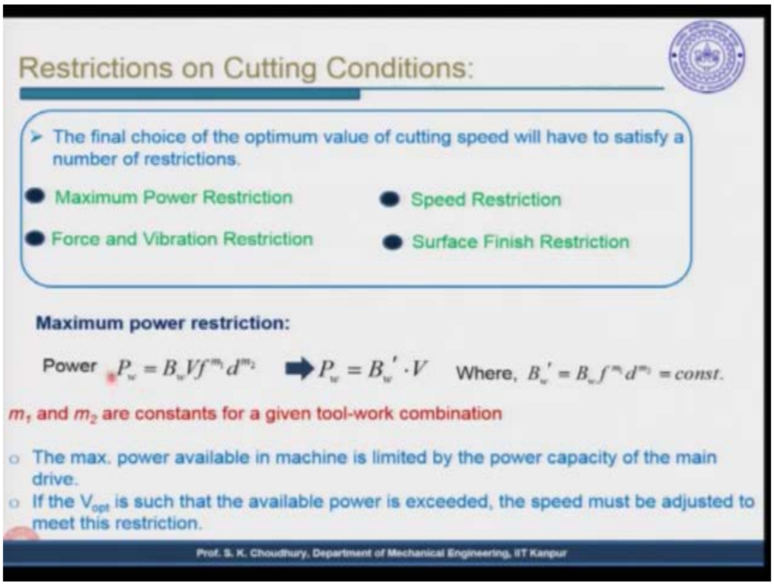

In the sense that there are certain maximum power restrictions, speed restriction force and vibration restriction, surface finish restriction. Just to give you an example that maximum power restriction - power can be calculated by this, that is constant Bw, V is velocity, f is feed and d is depth of cut, the Bw m_1 m_2 are constants. Now, let us say that the power required for the machining is Pw.

According to the optimum velocity if we are getting the power more then we have to reduce that value of the optimum velocity to get the value of the required power, P_W. Meaning that the power obtained by the calculated optimum velocity cannot be more than the power provided by the prime mover or the motor of the machine.

So, this is the machine power. Machine will be designed with this power and the power we can calculate required for the fabrication by the optimum velocity that we have calculated from the economics of machining. If that power calculated from the economics of machining is more, then we have to reduce the value or decrease the value of the V to get the decreased power to match with the power of the machine available - this is the maximum power restriction.

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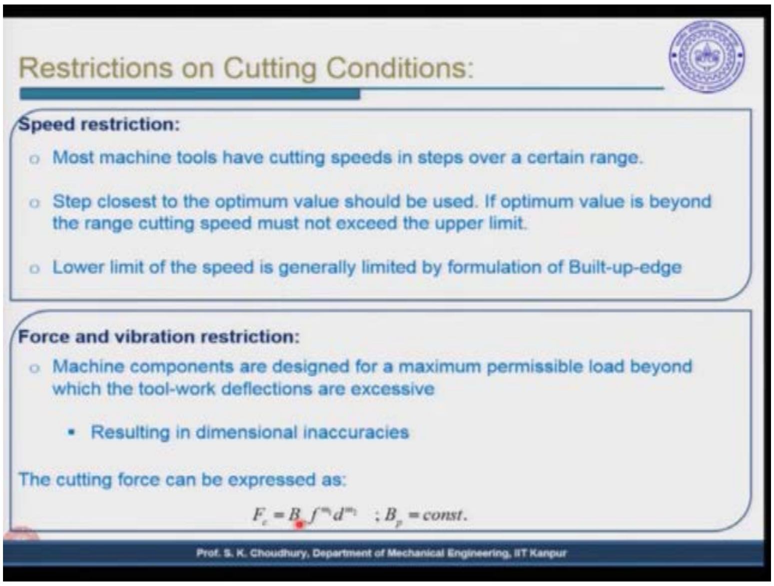

Similarly, we will have the speed restriction; because most of the machine tools will have the maximum speed and the minimum speed. You cannot go beyond the maximum speed, because this is not allowed by the machine. Minimum speed is limited by the built-up edge formation and if the V optimum that you calculated is more than the V maximum then you have to reduce it. If it is less than V minimum you have to increase that. So, you have to adjust these values to get the right kind of fabrication. Similarly, we have a force and vibration restriction; machine components are designed for a maximum permissible load beyond which the tool work deflection will be excessive.

So, you find out the cutting force by this formula. Cutting force which will be designed according to the machine and if this is coming out to be more or less by the optimum velocity that you have calculated then you have to adjust it. That is the basic idea behind the adjustment.

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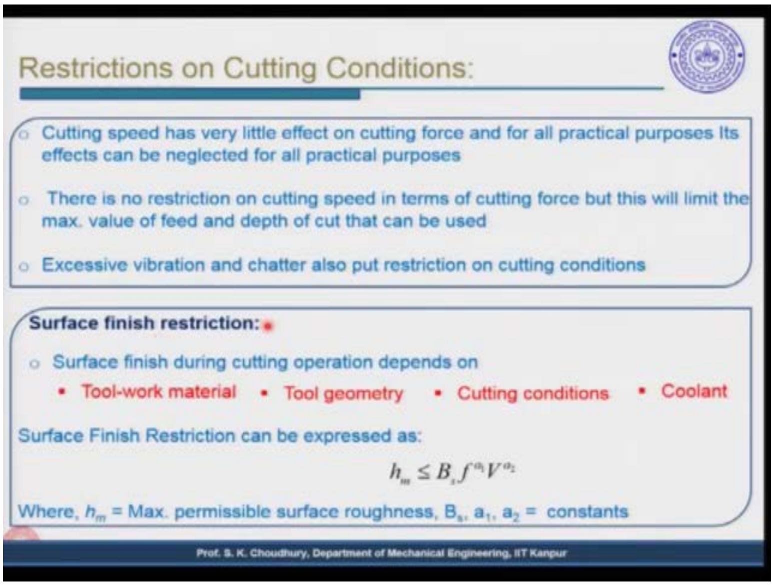

Finally, we have the surface finish criteria. Surface finish criterion says that the surface finish overall should be less than this value. And if by putting the V optimum calculated earlier from the economics you see that this equation is not satisfied, then you have to adjust the V optimum to satisfy this equation; that is, the surface roughness should be less than or equal to the constant Bs , feed to the power some power which is constant and V to the power a_2 which is some constant.

So, this is what we mean by the restrictions on the cutting components and whatever you have calculated from the economics of machining they have to be adjusted to certain extent, so that these criteria are satisfied.

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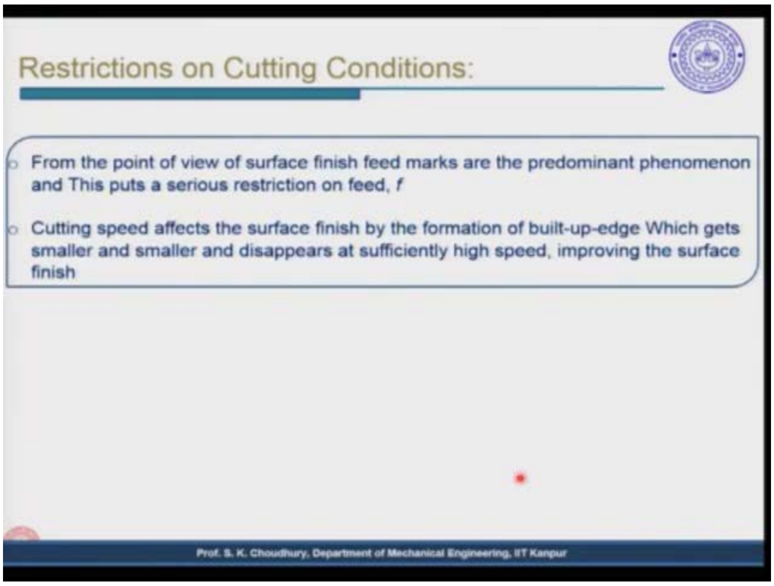

Here in these slides, whatever I said is written here. So, you can read them.

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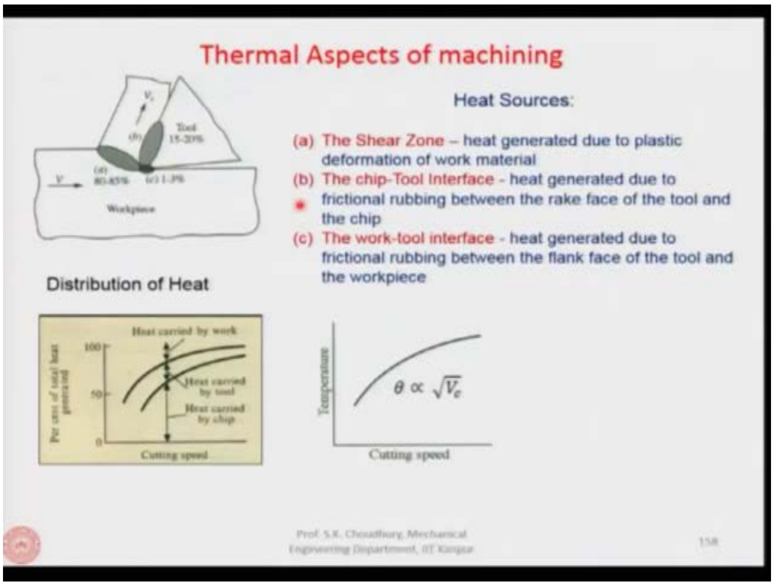

Now, let us talk in brief the thermal aspects of machining and here I will particularly discuss the sources of heat generation, how the heat is distributed and how to measure the temperature in the shear zone, because this is very difficult to measure. When you measure the temperature with the thermocouple for example, you will measure the temperature in the cutting zone, but you cannot individually measure the cutting temperature in the shear zone or the cutting temperature in the chip tool contact length for example.

So, you have to analytically find out through a model how to calculate the temperature. Well, first of all let us say from this diagram we know that most of the heat is generated because of the plastic deformation which happens in the shear zone. 80 to 85 percent heat is generated because of this.

Next source of heat generation is the friction due to chip moving along the rake face. So, in the chip-tool contact length about 15 to 20% heat is generated. And finally, we have the rubbing of the flank face with the already machined surface and the contribution of this friction is about 1 to 3% .

Now, the heat sources are the shear zone - heat generated due to plastic deformation of the work material, tool interface - heat generated due to frictional rubbing between the rake face of the tool and the chip, and finally, the work-tool interface that is heat generated due to frictional rubbing between the flank face of the tool and the work piece.

So, once again the major contribution is by the plastic deformation which occurs in the shear zone. Next is the chip tool contact area, here about 15 to 20 percent heat is generated and 1 to 3 percent heat is generated due to the rubbing of the rake of the flank face with the machine surface. Now, earlier also we have discussed that when the cutting speed increases then the temperature increases nonlinearly.

So, $\theta \propto \sqrt{V_c}$. When the cutting speed is increasing the heat increases. Along the y-axis we have the percent of total heat generated due to the machining. So, total heat meaning that this is due to plastic deformation, due to the friction between the chip and the rake face of the tool and due to friction between the flank face and the machined surface.

So, total heat 100% is here and out of the total heat, the heat carried by the chip is maximum. So, most of the heat is carried out by the chip. This is roughly about 80% of the heat, carried away by the chip. Now about 15% goes to the tool and the rest goes to the work piece.

So, as you understand that during the machining, the chip will be the hottest, next to that will be the tool and next to that will be the work piece. Chip is getting heated up because of the plastic deformation in the shear zone.

So, after it is heating up and maximum heat is generated in the shear zone, the hot chip will be further heated up by the friction between the chip and the rake face of the tool. This is the phenomena which actually you have to understand, that how the temperature occurs.

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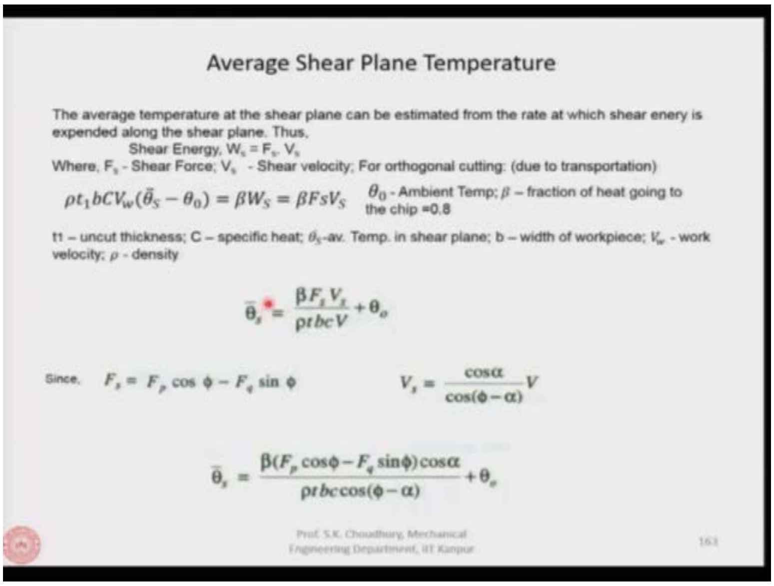

Next let us see how the average shear plane temperature can be found out. This is a thermal model by which we can estimate, analytically find out what is the temperature occurring at the shear zone.

Now, the average temperature at the shear plane can be estimated from the rate at which the shear energy is expended along the shear plane. So, we are applying the shear force, we are applying the shear stress and that shear energy is expended along the shear plane. So, along the shear plane where the maximum heat is generated, and we are here trying to find out the heat generated in the shear zone particularly. Here the shear energy is equal to the $F_s V_s$. We found out earlier that the specific energy is equal to the product of the velocity and the speed. Same thing here - this is the shear energy.

So, the shear energy will be the product of the shear force and the shear velocity. You now know that the shear force will be acting along the shear plane of the machining. So, shear plane is here and the F_s, if you remember, we have done that in the Merchant's circle diagram that the F_s is parallel to the shear plane and this is acting along the shear plane.

For orthogonal cutting due to transportation the heat flow equation can be written as $\rho t_1 bCV_w(\overline{\theta}_s - \theta_o) = \beta W_s = \beta F_s V_s$. Let us see what is it. Now, we are considering the orthogonal cutting and in orthogonal cutting we will find out how the temperature is produced.

Now, the temperature is produced due to transportation. And due to transportation this is the formula that is well known in the thermal engineering that the density into the t_1 is the uncut thickness. b is the width of work piece, C is the specific heat, V_w is the work velocity, $\overline{\theta}_s$ is the temperature which we are interested in that is the temperature which is generated in the shear plane. The θ_o is the ambient temperature.

In this equation what is expressed is that, the heat that is generated is equal to the β which is the fraction of heat which is going to the chip which is about 0.8, as I said that about 80 percent of the heat goes into the chip and the W_s as we said the W_s is the shear energy which is being expended.

So, once again for orthogonal cutting due to transportation, this is the equation that we know from the thermal engineering and this is the heat which is generated here by this equation, out of which fraction of heat going to the chip will be 0.8 and the rest into the shear energy which is expended. So, $W_s = F_s V_s$. So, we can substitute this.

Now from this equation we can find out that $\overline{\theta}_s$ which is the temperature which is occurring in the shear plane which will be equal to the 0.8, which is the beta we have taken there is a fraction of heat going into the chip. $\frac{F_s V_s}{\rho tb}$. This is the specific heat and the velocity of the work piece. So, this will be plus whatever will be the ambient temperature.

Now, F_s we know from the Merchant's that $F_s = F_p \cos\phi - F_q \sin\phi$. If you remember in the Merchant's Circle diagram we have said that F_s is can be expressed through the F_c and the F_t . So, this is equivalent to F_c and this is equivalent to F_t . So, V_s also we found out from the sine rule, if you remember, and then by putting the values of V_s and F_s we find out that this is the final equation of the heat generated or this is the average shear plane temperature. This is the only way to find out the average temperature in the shear plane analytically, because we cannot measure this temperature as I said earlier. Rest of the things we will discuss in the next session.

Thank you for your attention.