**Machining Science - Part I**
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**Lecture – 18**

Hello and welcome to the 18th session of the discussion on Machining Science course. I will remind you that we were discussing the forces in oblique cutting. We also said that forces in oblique cutting will be different than in the orthogonal cutting; because in case of oblique cutting the tool is inclined with respect to the cutting velocity vector and the forces are spreaded on all three axes and we have to resolve the resultant force into different planes; particularly in the plane normal to the cutting edge.

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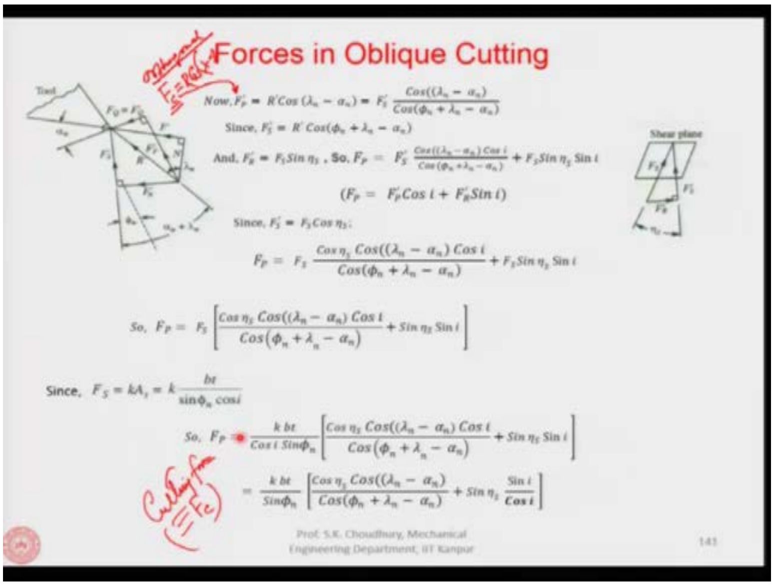


Let us look at this slide. Here we have determined analytically the cutting force, $F_P$. This is the cutting force along the cutting velocity vector and this $F_P$ is equivalent to the cutting force in the orthogonal cutting. We found out the following expression and this expression can be further simplified by taking $\cos\eta_S = \dfrac{1}{\sqrt{1+\tan^2\eta_S}}$. This is the formula that is known to us.

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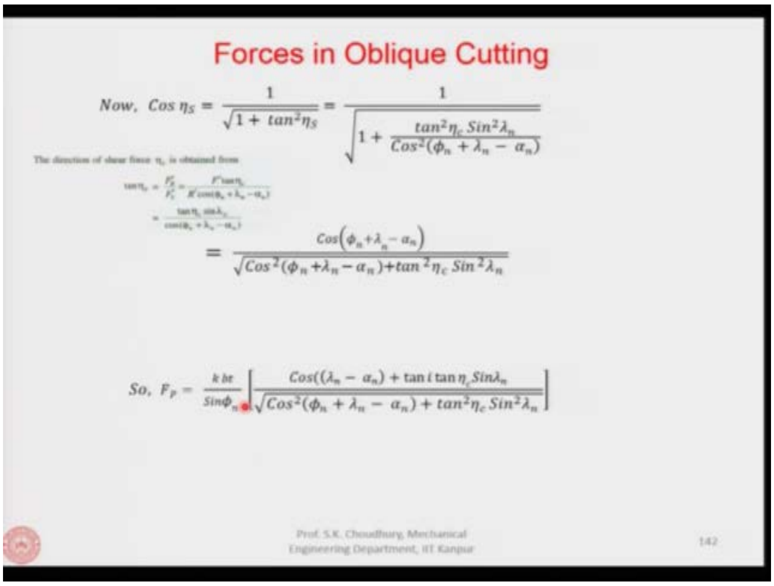


Now, if you remember, we found out the value of the $\tan\eta_S$ earlier like $\tan\eta_S = \frac{\tan^2\eta_c \sin^2\lambda_n}{\cos^2(\phi_n+\lambda_n-\alpha_n)}$. So, this expression we put in place of $\tan\eta_S$ and we find that this can be simplified in the following way.

Finally, $F_P = \frac{kbt}{\sin\phi_n} \cdot \frac{\cos(\lambda_n-\alpha_n)+\tan i \tan\eta_c \sin\lambda_n}{\sqrt{\cos^2(\phi_n+\lambda_n-\alpha_n)+\tan^2\eta_c \sin^2\lambda_n}}$ is the final expression of the $F_P$ we can write. This you can now compare with the $F_C$ that we have determined in case of the orthogonal cutting during the Merchant's relationship and you will find some similarity except some terms.

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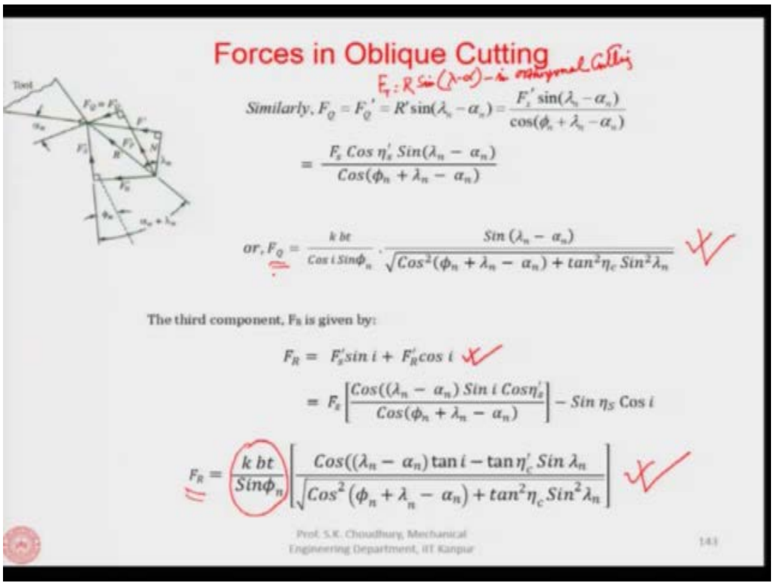


For example, $\eta$ is introduced, because this is due to the tool inclination in the oblique cutting. Similarly, we can find out the $F_Q$ which is the $F_Q^{'} = R^{'}\sin(\lambda_n - \alpha_n)$. So, if you remember that in the orthogonal case $F_T$ was equal to $R\sin(\lambda - \alpha)$.

Similarly, we are writing here that this $F_Q^{'} = R^{'}\sin(\lambda_n - \alpha_n)$. For orthogonal cutting it was $F_T = R\sin(\lambda - \alpha)$ . Here in this oblique cutting it will be $R^{'}$ and $R^{'}\sin(\lambda_n - \alpha_n)$ can be written as $R^{'} = \dfrac{F_S^{'}\sin(\lambda_n - \alpha_n)}{\cos(\phi_n + \lambda_n - \alpha_n)}$ and $F_S^{'} = F_S \cos\eta_S^{'}$.

Now, $F_Q = \dfrac{kbt}{\cos i \sin\phi_n} . \dfrac{\sin(\lambda_n - \alpha_n)}{\sqrt{\cos^2(\phi_n + \lambda_n - \alpha_n) + \tan^2\eta_c \sin^2\lambda_n}}$ and the third component, $F_R = F_S^{'}\sin i + F_R^{'}\cos i$, because they are mutually perpendicular.

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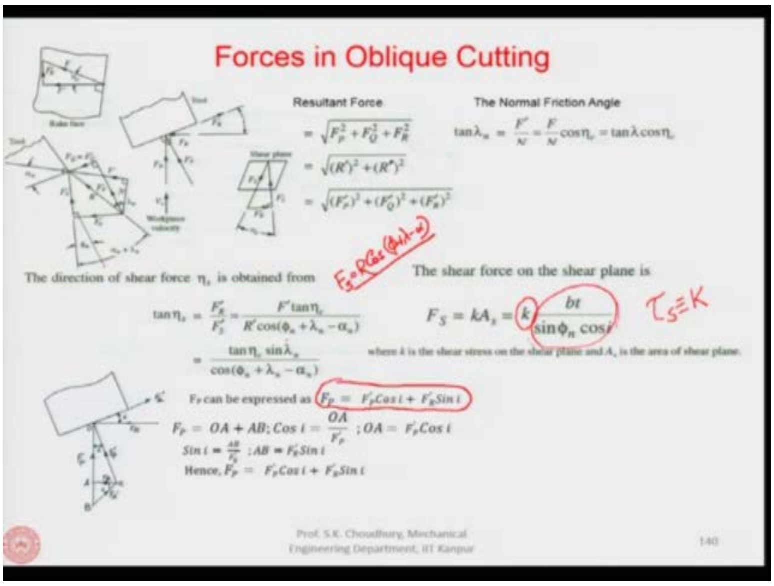


In case of $F_R = F_S^{'} \sin i + F_R^{'} \cos i$, because they are perpendicular. Similarly, here you put the values of $F_S^{'}$ and the $F_S^{'}$ that we found out earlier and the final expression can be written. So, these are similar, if you can see that here the $F_Q$, this is the $F_R$ they are mutually perpendicular.

Shear force on the shear plane can be expressed as a product of stress and the area and the expression can be written as $F_s = k \dfrac{bt}{\sin\phi_n \cos i}$. And, once again I will repeat that here you will find that there is a similarity between the expressions of the $F_Q$ and $F_R$ in oblique cutting with the $F_C$, $F_T$ that we have done in case of the orthogonal cutting.

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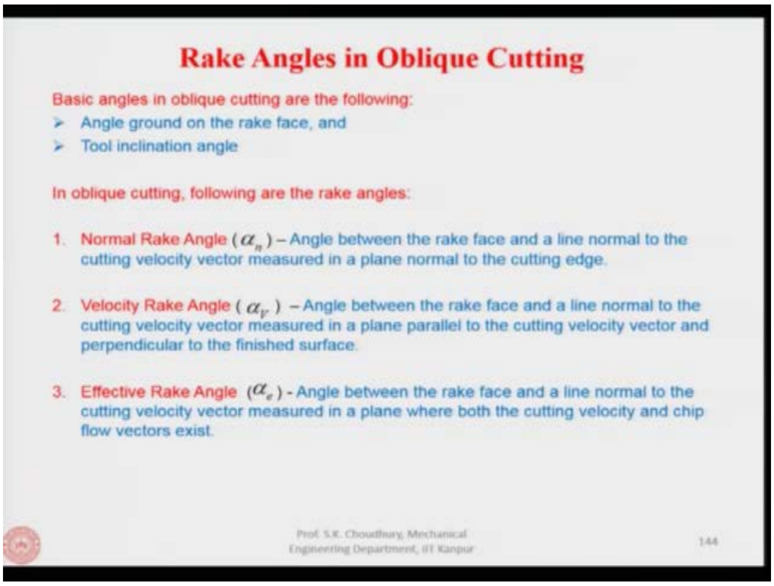


Now, let us see the rake angles. If the forces we have seen now how the rake angles can be determined in case of the oblique cutting. The basic angles in oblique cutting are the angle ground on the rake face and the tool inclination angle. Now, since the tool is inclined with respect to the cutting velocity vector, the chip does not flow exactly perpendicular or normal to the cutting edge. It will flow to that side of the normal to the cutting edge where it will get less resistance to the flow.

In oblique cutting this means that the chip will flow will flow to that side of the normal to the cutting edge where the rake angle or the inclination of the rake angle will be more than it is made on the actual rake face or the rake face of the tool. So, in oblique cutting the rake angles are the following.

One is the normal rake angle; this is the angle between the rake face and the line normal to the cutting velocity vector measured in a plain normal to the cutting edge. Another angle is the velocity rake angle. The definition is the same that it is the angle between the rake face and a line normal to the cutting velocity vector, but it is measured in a plane parallel to the cutting velocity vector and perpendicular to the finished surface. And, the third angle is the effective rake angle which is by definition same, but measured in a plane where both the cutting velocity and the chip velocity vectors exist.

Now, the angle is the same, but the value will be different because they are measured in different planes. Normal rake angle is measured in a plane perpendicular to the cutting edge. Velocity rake angle is measured in a plane parallel to the cutting velocity vector and perpendicular to the finish surface. The effective rake angle is measured when both the cutting velocity and the chip velocity vectors exist.

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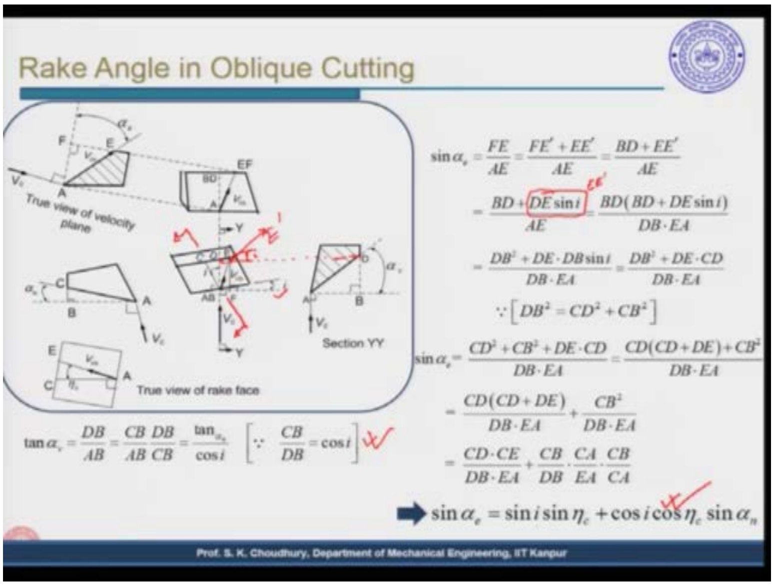


Now, let us look at the diagram. We will take the tool and this is the projection of the tool here in this plane. Now, this is the cutting edge which is inclined to the normal to the cutting velocity vector by an angle of $i$. Here, we will take the side view; so, this view will be normal to the cutting edge; if you take the normal to the cutting edge, you will get this view where this is the rake face of the tool.

If you look at the tool from this side, perpendicular to the rake face you will see a true view of the rake face where this is the cutting edge and the chip velocity will be along the rake face. Chip velocity will make an angle of $\eta_c$ with the normal to the cutting edge. In the other view, which is along the $V_c$, if you take a section along the $V_c$, let us say Y Y; this will be the Y Y section and here this will be the cutting velocity vector.

Here is the chip velocity vector, chip will be flowing along the rake face and this E will be here, F will be here at that point. And, if we take a section which is along the AE line or AF line then we will see a view which actually will be true view of the velocity plane. Because in here this is called a velocity plane along the chip velocity vector, but here the cutting velocity is perpendicular; here it is not visible because cutting velocity is here in this way. So, in this view it will be the direction as shown in the slide.

So, this is called the true view of the velocity plane, and this is the true view of the rake face, this is the section Y Y. This is a projection of this view given here. Now, here from the geometry the $\sin\alpha_e$ will be $\frac{FE}{AE}$. $\alpha_e$ is the effective rake angle. So, effective rake angle will be measured in a plane where both the chip velocity and the velocity vectors exist. If you see this plane in here we have the cutting velocity vector here and the chip velocity vector here.

In no other plane you will find both the vectors exist. This is the angle which is the effective rake angle and sin of effective rake angle will be $\frac{FE}{AE}$. From the geometry here you find out that with that FE we are taking from here to here and F from here to this view. And, FE can be found out as FE' and FE' actually; I will show it to you; it is not shown exactly here. So, if you take this view and from here if you take the line which is actually going to D, this is the intersection which is the EE'. This point of intersection is the EE'.

So, EE' along this line, along the EF line. Now, this can be represented as FE' , FE' is BD here plus EE'. This EE' can be represented as DE and the sin of this *i* because this angle between this line and this dotted line is the *i*.

Tool inclination angle is *i*. From this small triangle you will find out that EE prime is equal to DE and the $\sin i$ this is EE. And, this then can be taken as BD plus DE sin *i*. And finally, what we get is the $\sin\alpha_e = \sin i \sin\eta_c + \cos i \cos\eta_c \sin\alpha_n$, $\alpha_n$ is the normal rake angle.

So, this expression is found out from the diagram here and the geometry. Now, the velocity rake angle is measured in a plane parallel to the cutting velocity vector and normal to the work surface.

So, in this plane we can find out that this is the rake angle because this is the rake face and, the $\tan\alpha_v$ from this triangle we can find out that this is $\frac{DB}{AB}$, multiply and divide $\frac{DB}{AB}$ by *CB*. And, $\frac{CB}{AB} = \tan\alpha_n$ and $\frac{CB}{DB} = \cos i$. , the $\tan\alpha_v$ can be expressed as $\frac{\tan\alpha_n}{\cos i}$, *i* is the inclination angle.

So, overall you can see that you have to represent the tool in the oblique cutting in this way, it is different because the tool inclination angle is there. And, we can find out the relationship between the angles like in here through the geometry. Once you understand the diagram and how these angles are derived particularly, these three angles once again - this is the normal rake angle. Normal rake angle is measured in a plane normal to the cutting edge, this is the normal to the cutting edge. And, along this plane this will be the view or this will be the section.

Now, since here it is a view shown so, it is not hatched and in this view or in this section if it is taken as section, you know the angle between the rake face and a line perpendicular to the cutting velocity vector will be given as alpha n which is the normal rake angle in the normal rake system.

The effective rake angle, once again, is the rake angle between the rake face and the line perpendicular to the cutting velocity vector measured in a plane where both cutting velocity and the chip velocity vectors exist. So, after that through the geometry you can find out the relationship between the angles alpha e and the alpha v. This is the purpose of discussing the rake angles in the oblique cutting overall.

I hope this is clear although this may be this looks like a little complicated, but you have to understand the diagram. In this diagram overall, once again I will show you that geometry: FE is the projection of E on the line perpendicular to the cutting velocity vector.

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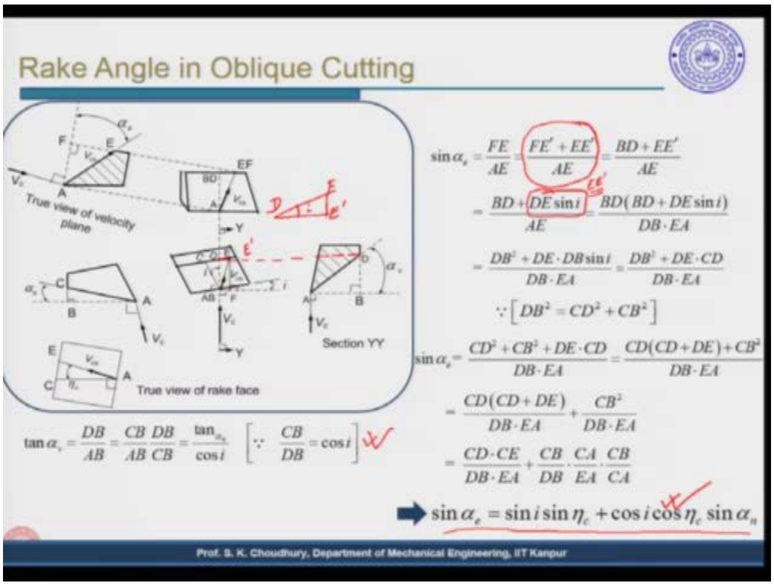


So, if you extend this line, it meets the D point. Now, this line is intersecting with this vertical line EF at a point which is EE prime. So, this is E this is E prime and this is D and this angle is i. So, from here we are finding out that this is DE sin i which is equal to EE prime; this is what we got in here.

Similarly, you can actually look at this expression; look at this derivation from the figure that has been given here. You can find out easily how this equation or the relationship between the angles can be found out.

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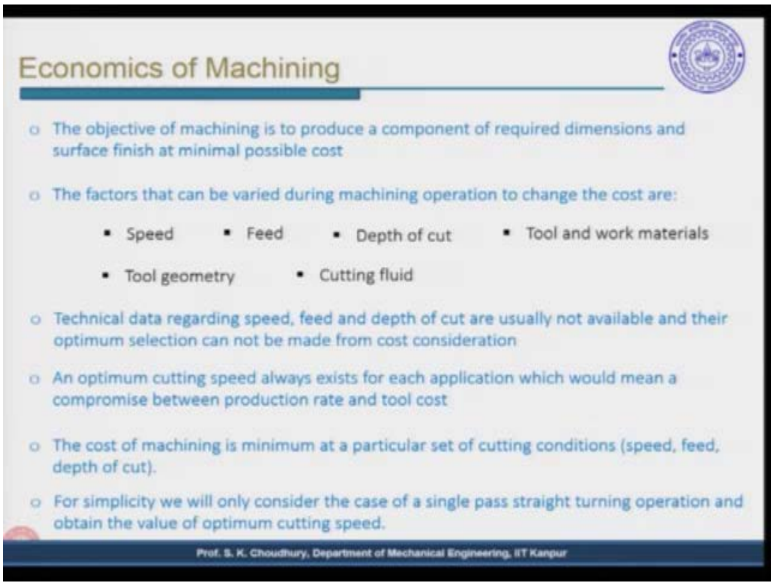

Next topic will be economics of machining and economics of machining as you understand is one of the most important factors in machining. Because, in the definition of machining we said earlier that our aim is to obtain a component, final component with the proper shape, size, finish and accuracy. What we have not mentioned is that our aim also should be to make a component with the minimum cost. So, how to make a work piece with optimum costs that is what we will be studying in the economics of machining.

So, the objective of machining is to produce a component of required dimension at minimum possible cost. The factors that can be varied during the machining operation to change the cost are speed, feed, depth of cut, tool and work material, tool geometry, cutting fluid. This we have discussed.

Now, the technical data regarding speed, feed and depth of carter usually are not available and their optimum selection cannot be made from the cost consideration only. Now, an optimum cutting speed always exists for each application which would mean a compromise between production rate and the tool cost; what does it mean let me explain it to you.

When you are producing at a higher speed you will always get more material removal rate or less time required to produce that part. So, your production rate will be higher. If the production rate is higher in that case you know that the profit will be more. But what happens when you are using a high speed is that the tool wear is also very high. So, you have to use a certain value or certain cutting speed in which the production tool wear will not be very excessive and the production rate also will not be very less.

So, that is what we are calling as compromise between production rate and the tool cost. The cost of machining is minimum at a particular set of cutting conditions. So, it is not only the speed, but it will be speed, feed and depth of cut and when their values are at an optimum level; then the cost will be minimum and that is the idea of showing in the economics of machining. For simplicity we will only consider the case of a single pass straight turning operation and obtain the value of optimum cutting speed.

However, you understand that the optimum cutting feed and the optimum cutting depth can also be found out. But, here for the simplicity let us explain how to get the optimum value of the cutting speed.

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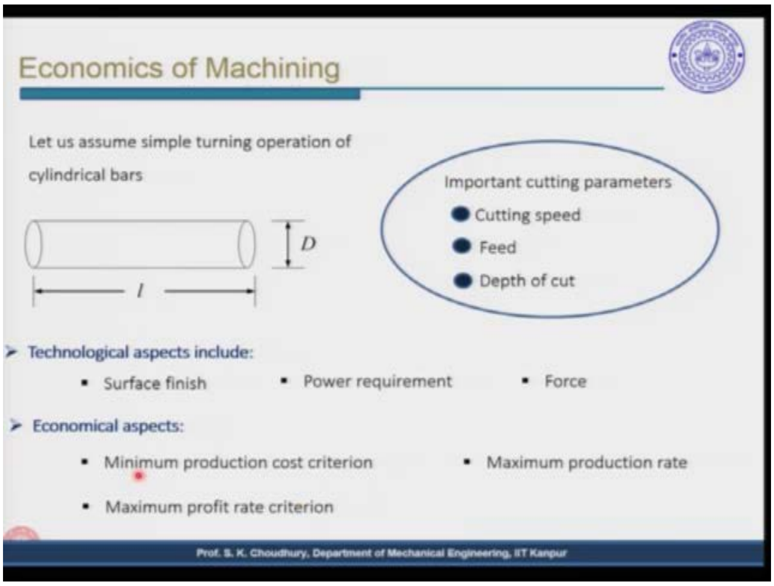


Let us assume that we have a cylinder and in that cylinder the length is *l* and the external diameter or outer diameter is capital D. Now, the important cutting parameters are the cutting speed, feed, depth of cut, The technological aspects include the surface finish, power consumption, power requirement force, but the economical aspects will be the following: what is the minimum production cost criteria, maximum profit rate criteria and the maximum production rate criteria. So, what I mean is that if we know the important cutting parameters from the machining that we have already discussed, we have discussed the technical, technological aspects, surface finish, power requirement etcetera.

Now, the economical aspects that we have not discussed so far, will discuss in the economics of machining which is minimum production cost criteria; how to make, how to select the optimum cutting parameters to have the production cost minimum or the maximum production rate - how to select the optimum cutting processes so that we have the production rate as maximum or the profit rate as maximum, let us see this. Let us take first the minimum production costs criteria.

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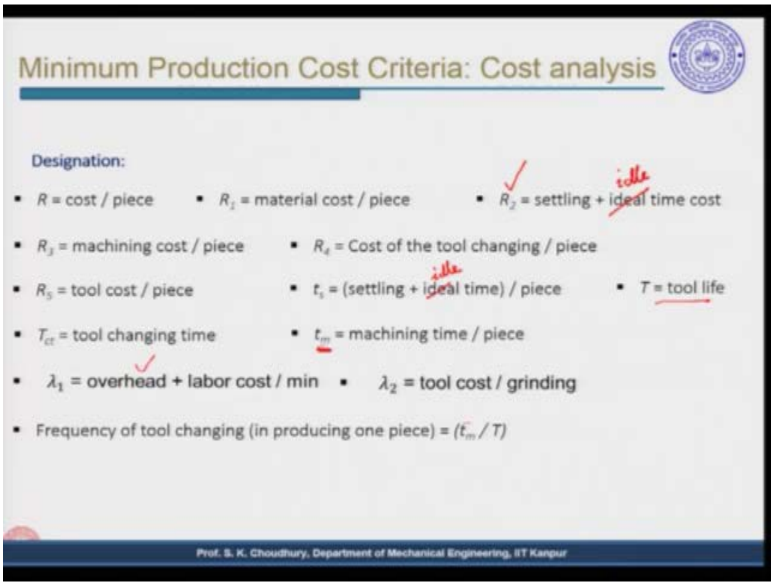


Let us designate that $R$ capital as the cost per piece. Now, in the cost per piece we will have the $R_1$ which is the material cost per piece, $R_2$ is the settling plus idle time cost per piece. Now, I will explain what those parameters are. Well, material cost you understand that you have a raw material; you are adding value to the raw material to get the final product. So, that raw material has some cost, this is the $R_1$ - material cost per piece, settling plus idle time cost is that before the machining starts, the operator has to set up the machine, set up the speed, feed, clamp the work piece.

And, there are some idle time. Idle time is when machine is not working, machine is idle. Now, like one pass has gone and in between if the parameters have to be changed then that machine does not work, but that time also has to be considered. So, that is called the idle time. So, this is $R_1$ which is settling plus idle time cost, $R_3$ is the machining cost per piece.

So, what I mean to say is that everything, all the time that is being spent either for machining or for setting up the machine or the idle time - for all these times we have to pay. So, those costs have to be considered to find out the final cost per piece. Now, $R_4$ is the cost of the tool changing per piece when the tool is being changed, either it can be changed for the regrinding or it can be changed to have another tool fixed in the tool post and so on. $R_5$ is the tool cost per piece. Suppose a tool costs let us say 100 rupees.

Now, with that tool you are machining let us say two pieces. So, per piece tool cost will be 50 rupees, this is an example, $t_s$ is the settling plus idle time per piece. Now, settling plus idle time we have seen here, this is the cost we are taking, but here this is the $t_s$ which is the settling plus idle time; how much time it is being taken per piece. Capital $T$ is the tool life, $T_{ct}$ we will take as a tool changing time, $t_m$ is the machining time per piece.

Now, $\lambda_1$ we will take as overhead plus labor costs per minute. What is overhead is that apart from the labor cost that you are paying the salary of the workers, you have to also pay for the building where the factory is located. You have to pay for the electricity, you have to pay for the water, you have to pay for the hydraulics, pneumatics and so on. So, all these expenses are included in the over head. You have to also pay to those people who are involved in the design for example, who are involved in the advertisement.

And, all those costs are involved in the overhead plus labor cost per minute and $\lambda_2$ is the tool cost per grinding; the tool is ground each time it is worn out. So, let us say a tool can be ground 5 times maximum; If the tool that we have taken earlier as an example, if it costs 100 rupees, so, per grinding it will be 20 rupees lost because 5 times it can be ground, after that the tool cannot be reground. So, this is the tool cost per grinding.

Now, frequency of tool changing in producing one piece we can understand in the following way: we have the machining time $t_m$; $t_m$ is taken as a machining time that divided by tool life will give you the frequency of tool changing; how many times you are changing the tool.

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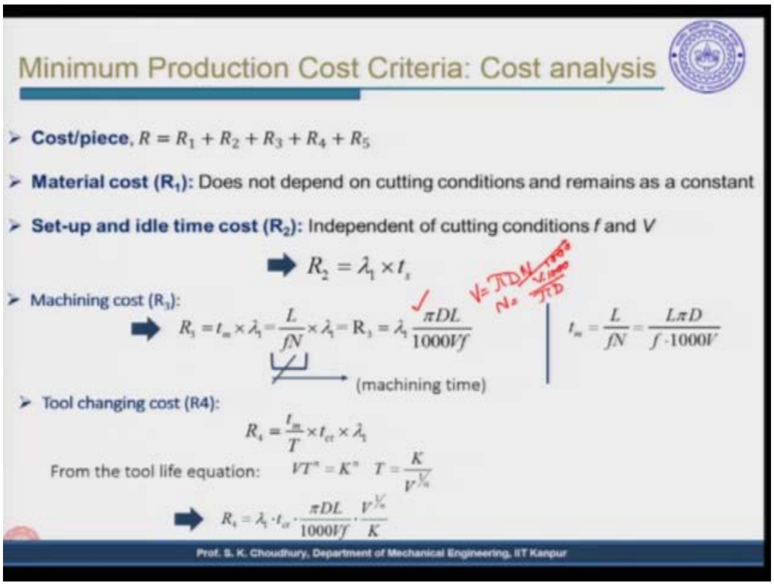


With these designations now, we have the cost per piece *R*, this will be equal to the raw material cost plus settling plus idle time cost per piece plus machining time cost per piece plus the we have $R_4$ , we said that this is the tool changing cost per piece and the tool cost per piece. So, all these costs are to be considered for calculating the tool cost per piece overall.

Now, the material cost that does not depend on the cutting conditions and remains as a constant. What is important here is that, what is the optimum cutting speed that we will be getting for making the cost minimum. In the material costs you know the velocity does not affect; so, that does not depend on the cutting condition and this remains constant.

Now, the setting up and idle time cost also is independent of the cutting conditions. Because, whatever cutting conditions you will have, a certain settling time and idle time has to be elapsed for which we have to pay. So, that is the cost of that time and this is not dependent on the cutting conditions, like cutting velocity, feed and depth of cut.

$R_2$ is given by overhead that you are paying into how much time you are spending for the settling and the idle, that is the settling and idle time per piece, $t_s$ and for that time you are paying the overhead and the labor costs. So, this is $\lambda_1 \times t_s$. Now, the machining cost is how much you are spending on machining; $R_3$ is given as machining time and we are paying for that $\lambda_1$ which is the overhead plus the labor costs. Now, the machining time we have seen earlier that this is equal to length of the work piece that we have divided by feed and the rpm N.

So, $t_m \times \lambda_1$ will be equal to the $R_3$. This can be written as $t_m$ is equal to $\frac{L}{fN}$. Now for example, the $V$ velocity is given as the $\pi DN$. And $N$, can be taken as $\frac{V}{\pi D}$ or $\frac{V.1000}{\pi D}$ to make it in mm/min unit. Now, the remaining in the cost analysis or the economics in machining we will discuss in our next session.

Thank you for your attention.