Machining Science - Part I
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Lecture – 16

Hello and welcome to the 16th session of our discussion on Machining Science. In our previous discussion, we talked about the Grinding Process, particularly the mechanics of grinding. And, we estimated analytically the total grinding force and, from the total tangential force on the grinding wheel we can found out the average force per grit.

Now, we said that the average force per grit is important because we want the worn out grains to be dislodged from the surface of the grinding wheel. When a hard wheel is used on the hard work piece material, then glazing occurs and the sharp grains on the grinding wheel are worn out very quickly.

The wheel is hard means the bonding is strong. So, the worn out grains will not be easily dislodged from the surface of the grinding wheel. In that case the worn out grains will cover the entire surface of the grinding wheel and the wheel will not remove material efficiently, it will rub more and it will consume more power.

Surface finish will be bad in the case of glazed wheel. That phenomenon is called the glazing of the wheel. We want the average force per grit to be increased so that the glazing does not happen. In case we have a hard wheel used on the hard work piece material we need to remove that worn out grains, for which we need to increase the $F_c^{'}$.

We have to know analytically how to estimate $F_c^{'}$ and then we can regulate or manipulate the parameters such that the $F_c^{'}$ can be increased. We have seen that $F_c^{'}$ has been found out through the power consumption in grinding and to estimate the power consumption we need to know the volume of a chip, then how many chips are removed per second and the material removal rate.

While estimating the volume of the chip, the number of chips and the MRR, we found out the maximum uncut thickness. Maximum uncut thickness is used to find out the specific energy and from the specific energy, we found out the power consumption.

While estimating the volume of a chip we have used the length of a chip. You understand that the length of a chip cannot be measured experimentally because chips are very small and chips fly away. Therefore, there is an analytical estimation of the chip length which is important because we have to find out very accurately the length of a chip, only then we can find out very accurately the volume of a chip and therefore, accurately you can find out the $t_{1\max}$, the power consumption and finally, the average force per grit.

Now, we will discuss how to estimate analytically the chip length in the grinding process. You understand that the chip length in the grinding process will be different in case of, for example, surface grinding or external cylindrical grinding or internal cylindrical grinding. Let us discuss how chip lengths differ from each other.

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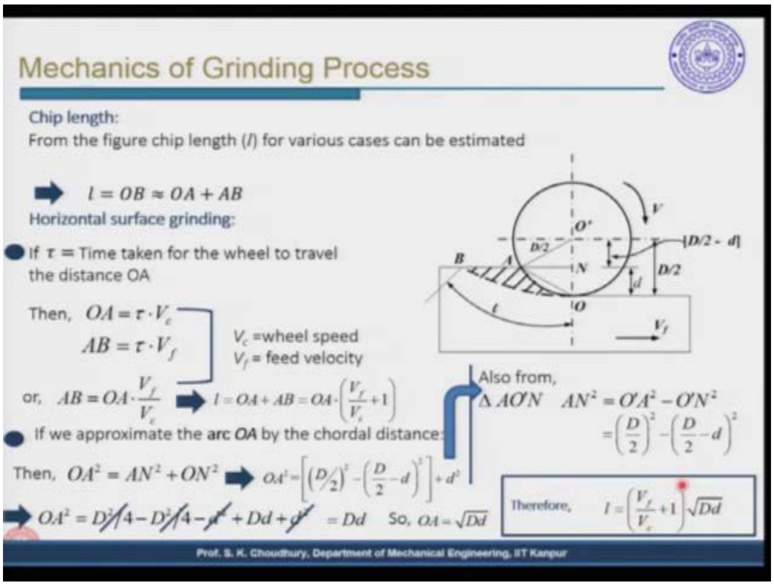

First we will discuss the horizontal surface grinding. In the above slide schematically it is shown that the grinding wheel is rotating at a grinding wheel speed of V and the work piece is given the feed velocity of V_f.

The distance from the grinding wheel center up to the point where it is in touch with the work piece is $\frac{D}{2}$. ON is the depth of cut, d. So, $ON' = \frac{D}{2} - d$.

Now, OB is the chip length which we have to determine analytically. Let us take an example of the grinding process and say that in one rotation during the feed, as a result of the grinding wheel velocity V and the V_f , the point has moved from A to B. The position of the grinding wheel will be changed because of the V_f given and the V existing.

Consider the chord OA by approximating the arc by a chordal distance of OA. Then from the triangle ANO, we can find out the geometrical relationship. Now, let us say that τ is the time taken for the wheel to travel the distance OA.

The distance traveled by point O on the grinding wheel in τ time with a velocity of V_c will be $OA = \tau . V_c$ where V_c and V are same.

Now, the distance AB is the distance on the workpiece and this distance has been moved because of the feed velocity, V_f given to the workpiece. Time taken for the wheel to go from O to A will be the same as when it is going from A to B, but A to B distance is traveled because of the V_f . So, AB we will be writing as the time τ into V_f .

As $OA = \tau . V_c$. $\tau = \dfrac{OA}{V_c}$ can be put into the equation of AB. The chip length OB, which can be said to be as $l = OA + AB = OA\left(\dfrac{V_f}{V_c} + 1\right)$.

Now, OA has to be found out. We take the triangle OAN by approximating the arc OA by a chordal distance OA. Now, $OA^2 = AN^2 + ON^2$. And $AN^2 = O'A^2 - O'N^2$ where $O'A = \dfrac{D}{2}$ and $O'N = \left(\dfrac{D}{2} - 1\right)$ and $ON = d$.

Therefore, $OA^2 = \left[\left(\dfrac{D}{2}\right)^2 - \left(\dfrac{D}{2} - d\right)^2\right] + d^2$. After solving we get, $OA = \sqrt{Dd}$, where, D is the diameter of the grinding wheel and d be the depth of cut. Put value of OA into $l = OA\left(\dfrac{V_f}{V_c} + 1\right)$ to get $l = \sqrt{Dd}\left(\dfrac{V_f}{V_c} + 1\right)$.

So, analytically we can find out the chip length in case of the surface grinding process and as I said earlier that the chip length in the surface grinding will differ from the chip length in the external cylindrical grinding or internal cylindrical grinding. So, let us see what happens in case of external cylindrical grinding.

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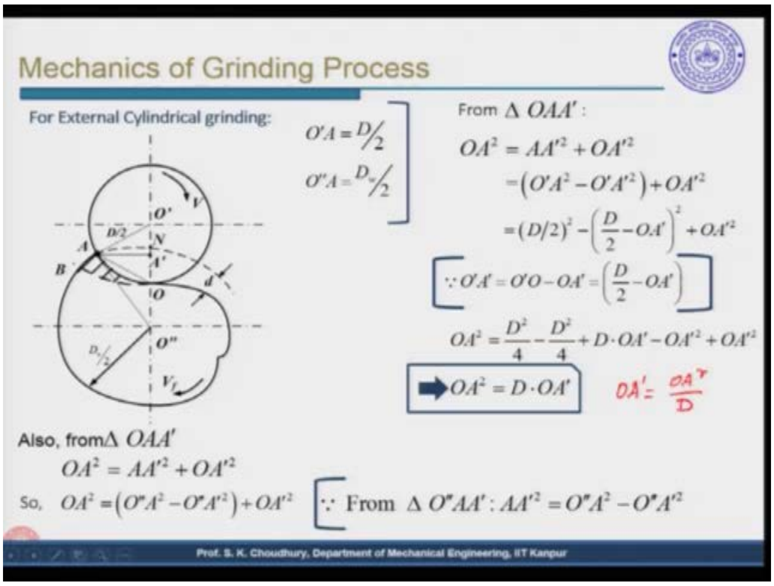

Let us take an example of the grinding wheel in this way. Body with center O' is the grinding wheel rotating at a speed of V or V_c and the body with center O'' is the workpiece and external surface of the workpiece is being ground. Grinding wheel is in contact with the work piece. The feed velocity V_f is given to the work piece as we have seen earlier in case of the horizontal surface grinding process. Because of the result of the V and the V_f , the point O will go from O to A.

So, OA plus AB we have to determine like in earlier case and the chip length will be OB which is equal to OA plus AB which remains the same as in case of the surface grinding process. If D is the wheel diameter then, $O'A = D/2$. D_w is the external diameter of the workpiece, $O''A = D_w/2$. External diameter of the workpiece is taken because we are considering the external grinding process.

From $\Delta OAA'$, $OA^2 = AA'^2 + OA'^2$ and from $\Delta AO'A'$, $AA'^2 = O'A^2 - O'A'^2$.

Now, $O'A = D/2$ and $O'A' = O'O - OA' = \left(\dfrac{D}{2} - OA'\right)$. Substituting these value, we have,

$OA^2 = \frac{D^2}{4} - \frac{D^2}{4} + D.OA' - OA'^2 + OA'^2$ and

$OA^2 = D.OA'$

Again, we will come back to the same $\Delta OAA'$ we will write that $OA^2 = AA'^2 + OA'^2$ as you have written earlier. *AA'* is common for $\Delta OAA'$ as well as for $\Delta AA'O''$. Therefore, $OA^2 = (O''A^2 - O''A'^2) + OA'^2$.

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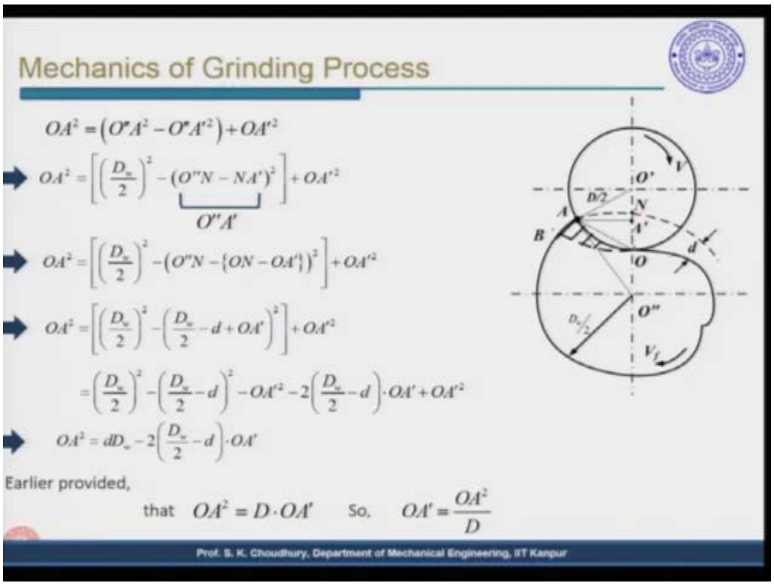

So, next we are manipulating the above equation. $O''A = D_w / 2$ where, D_w is the diameter of the workpiece and $O''A'^2 = O''N - NA'$.

The distance between the machined surface and the unmachined surface will be the depth of cut *d* like we have seen earlier in case of the surface grinding process. Therefore, $ON = d$ So, finally, what we are getting is that $OA^2 = dD - 2\left(\frac{D_W}{2} - d\right).OA'$.

Earlier we have proved $OA^2 = D.OA'$, or $OA' = \frac{OA^2}{D}$. Put it in the above equation and solve to get $OA^2 = \frac{D.d}{1 + \frac{D}{D_w}}$.

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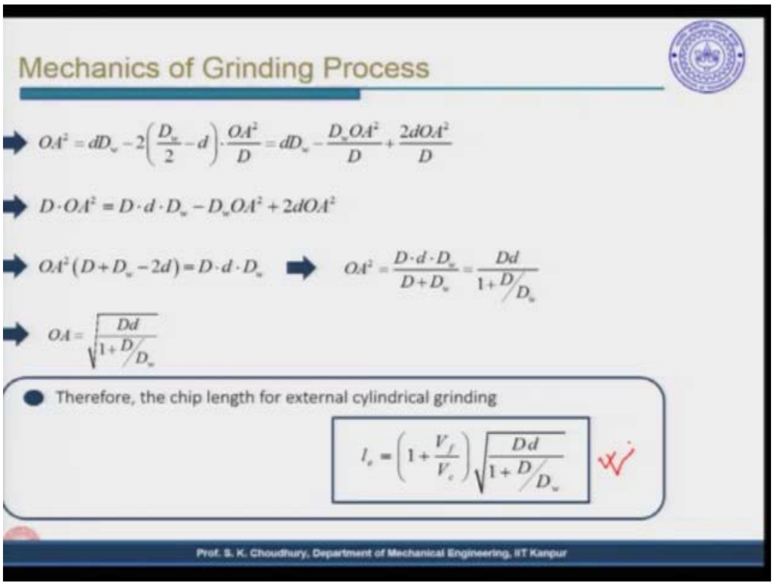

Finally, $OA = \sqrt{\dfrac{D.d}{1 + \dfrac{D}{D_w}}}$

So, if you notice that this value is different from the one that we have earlier found out for the surface grinding process. Now, the chip length for external cylindrical grinding will be $\left(1 + \dfrac{V_f}{V_c}\right).OA$. Using value of OA, we have $\left(1 + \dfrac{V_f}{V_c}\right).\sqrt{\dfrac{D.d}{1 + \dfrac{D}{D_w}}}$.

Now, let us see what happens in case of the internal cylindrical grinding. In case of internal cylindrical grinding because of the configuration of the grinding process and because of the various properties of the grinding process the chip length will be different than in case of the surface grinding process or the external cylindrical grinding process. Let us see what happens in case of the internal cylindrical grinding.

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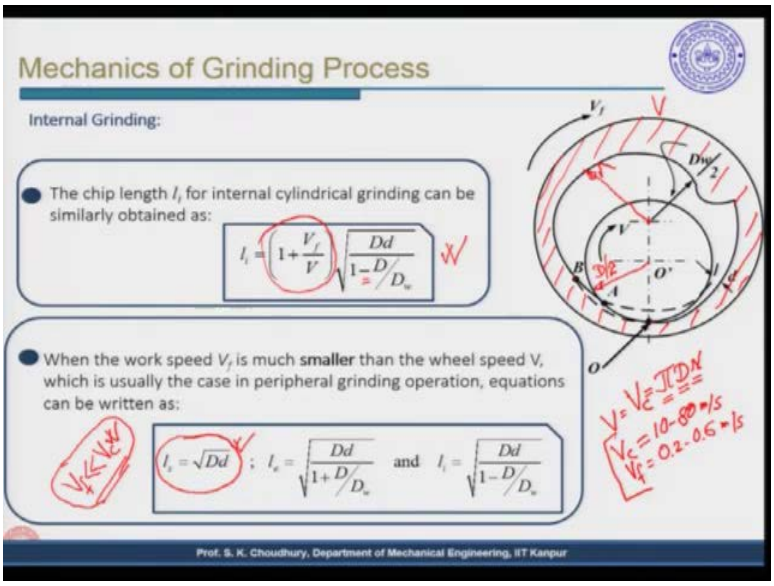

In the diagram, internal body is the grinding wheel and external is the workpiece which is sectioned. So, you understand that we are talking about the internal cylindrical grinding. In case of internal cylindrical grinding, the internal surface of the work piece will be ground. Internal diameter of the workpiece is D_w. So, the distance from center of the workpiece to its internal surface is $D_w/2$. Similarly, $D/2$ is the radius of the grinding wheel.

Now, like in the previous diagrams, *O* is the point where the grinding wheel has started grinding process and it is in touch with the workpiece and then if we extend the curve which is not machined it will go to B. So, this is equivalent to the B point in the case of surface grinding as well as in case of external cylindrical grinding so, in case of external cylindrical grinding, I will show that diagram once again to you.

In case of external cylindrical grinding we had the O, which is in contact with the work piece and O point is moving from O to A and then by extending the curve we are actually finding out that the point on the workpiece which is the point B.

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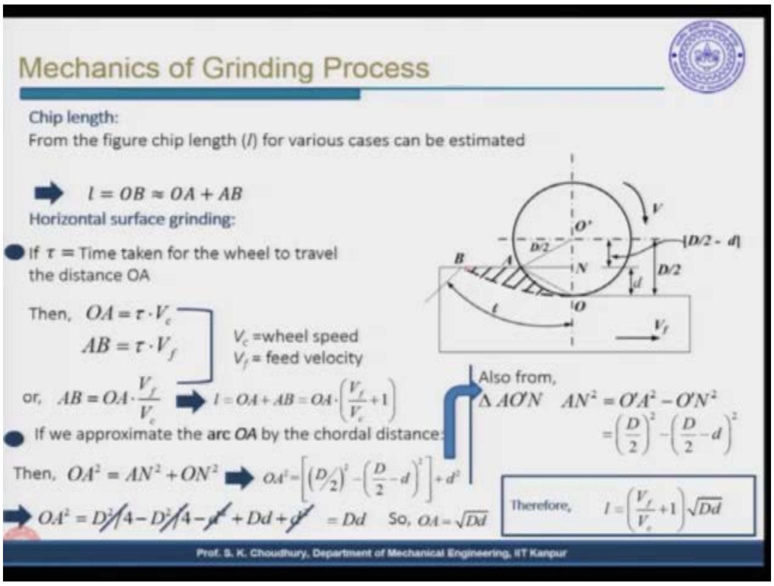

In case of surface grinding process, for example, it is easier to understand because it is visible that it is on the surface of the work piece. So, once again that O, A and B we are considering equivalent to the surface grinding as well as the external cylindrical grinding. Now, let us come back to the diagram of internal grinding.

O is the point where the grinding wheel is in touch with the workpiece. Now, if we extend this on the wheel OA, A point will be on the wheel and through the A point if we extend the curve, the extension of the curve of the machined surface will be touching the point B which will be equivalent to B in the case of external cylindrical grinding and the surface grinding.

So, we have that OAB. The chip length is OB which is OA plus AB which is $\left(1+\frac{V_f}{V}\right).OA$. V or V c is the grinding wheel speed that is found by the πDN. So, what I mean is that V or V_c is the same here which is πDN. N is the revolution per minute of the grinding wheel, D is the diameter of the grinding wheel and π constant.

Now, if we see the configuration then, when the work speed V_f is much smaller than the wheel speed V which is usually the case, the peripheral grinding operation equations will be written as $l_s = \sqrt{Dd}$; $l_e = \sqrt{\frac{D.d}{1+\frac{D}{D_w}}}$ and $l_i = \sqrt{\frac{D.d}{1-\frac{D}{D_w}}}$. Before that, I should tell you that if we look at the configuration of the internal cylindrical grinding we can actually say that it will be similar to the external cylindrical grinding except that in the denominator it will be $1\text{-}D/D_w$.

So, this sign is different in case of external cylindrical grinding it was $1+D/D_w$ and in case of internal cylindrical grinding it will be $1\text{-}D/D_w$. This is the only difference between the external cylindrical grinding and the internal cylindrical grinding.

However, you should appreciate that the value that we are getting for the chip length in case of external cylindrical grinding is different from the value that we are getting for the internal cylindrical grinding. So, you can judge which one is less which one is more. Now, what we are saying is that the V_f can be considered to be very small much smaller than the Vc that is the feed velocity given to the work piece is much smaller than the cutting velocity given to the grinding wheel.

In that case we can ignore this factor that is $\left(1+\frac{V_f}{V}\right)$ therefore, the chip length in the surface grinding can be considered as the root over $l_s = \sqrt{Dd}$. And in case of external grinding process it will be $l_e = \sqrt{\frac{D.d}{1+\frac{D}{D_w}}}$ and in case of internal cylindrical grinding a sign will be changed.

Now, if you remember when we were discussing the mechanics of grinding process initially we said that we have to take the volume of a chip. In case of volume of a chip we have taken that $\frac{1}{2}B_{\max}t_{1\max}l$ and that l we have taken as $\sqrt{Dd}$ because we have considered that the feed velocity given to the workpiece is much less than the cutting velocity given to the grinding wheel.

By the way if you remember, I said that V_c ranges between 10 to about 80 m/s whereas, the V_f ranges from 0.2 to 0.6 m/s. So, if you compare you will find that V_f is much smaller than the V_c in that case we can actually approximate these values.

But, when it is not the case then we have to very accurately find out the chip length in case of surface grinding, external grinding and internal grinding, so that we could find out the power consumption very accurately and the total tangential force on the wheel accurately and finally, the force per grit. That is important and that is why you are studying this chip length here.

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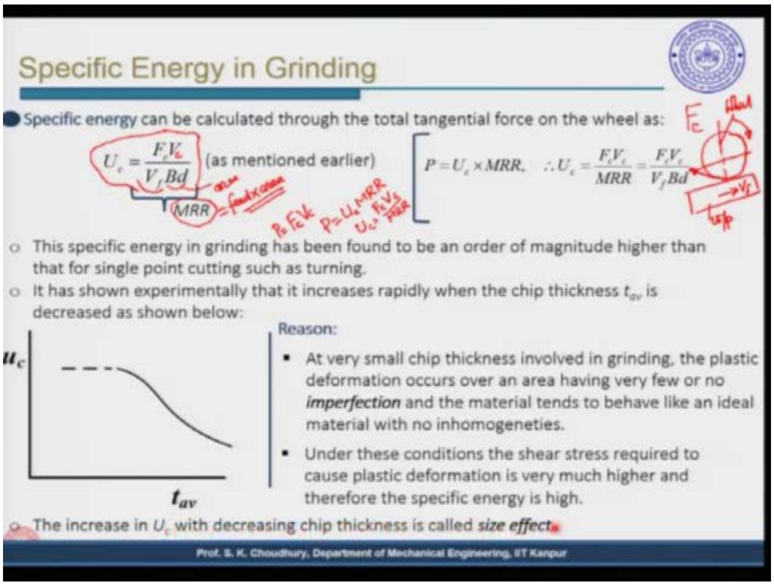

After that we will see the specific energy in grinding. The specific energy can be calculated through the total tangential force on the wheel. And, I said earlier the total tangential force which you have considered to be F_c which is acting on the wheel tangentially.

Let us consider the workpiece and the grinding wheel which is rotating clock wise. The feed velocity V_f is given to the work piece. In this case, the specific energy is equal to $\frac{F_c V_c}{MRR}$.

$F_c V_c$ is the power and power is also given as the product of Uc and the material removal rate. Therefore, from these two we can find out that $Uc = \frac{F_c V_c}{MRR}$.

So, we said that V and Vc is the same and the material removal rate is equal to speed into area. So, Bd is the area of cut and V_f is the feed velocity. Feed into area is equal to material removal rate. So, this is what we are doing here. So, we are finding out that the U_c is $\frac{F_c V_c}{V_f B d}$.

Specific energy in grinding has been found to be an order of magnitude higher than for the single point cutting such as turning. It has been shown experimentally that U_c , specific energy increases as the uncut thickness t_1 or the t_1 average is decreasing. From the curve we can see that as the t_{av} is decreasing the U_c is becoming very high and when it is increasing value of the U_c is low.

This is the reason that at very small chip thickness involved in grinding, the plastic deformation occurs over an area having very few or almost no imperfection and the material tends to behave like an ideal material with no inhomogeneities. I will remind you coming back to the defects and dislocation which we have studied in the very beginning. There we said that the actual shear stress or the theoretical shear stress which is required is $\frac{G}{2\pi}$ to create the slip.

And, that value of $\frac{G}{2\pi}$ is very high and it is almost impossible to provide, but in practice what we have seen that the stress required to make a slip or the plastic deformation is much less than that. And, this is because of the defects or the dislocations which happened in the lattice structure of the metal.

Same thing happens here that at a very small t_1 the imperfections do not lie there. The defects or imperfections lie little below that layer. In case of grinding t_1 which is uncut thickness is very small within that layer since there is no imperfection that layer can be considered as an ideal material.

And, the for an ideal material you know that the shear stress required is very high. Therefore, the power consumption is very high and the specific energy will be very high. That is the reason why this happens that in grinding the very high specific energy is required for removing the small t_1 which is the uncut thickness.

Now, this is the effect that when the t_1 decreases the U_c increases or the specific energy increases this effect is called the size effect. The increase in U_c with decreasing chip thickness is called the size effect. However, size effect is one of the reasons why this specific energy in grinding becomes very high. There are other two reasons that we will be discussing in our next session.

Thank you for your attention.