Machining Science - Part I
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Lecture – 12

Hello and welcome to the 12th lecture of the Machining Science course. Now, let me remind you that we were discussing the practical machining operations. The idea of discussing the practical machining operations is to show you that in different operations like turning, shaping, planing, drilling and milling, the tools used are different and the process is different.

Although the turning shaping and planning, as we said that this is very similar because the tool used is similar; although the force distribution and the direction of the forces is different. Now in case of drilling for example, we said that here it is different than what we have discussed in case of turning or shaping, because the thrust and the torque both are given to the tool which is the drill.

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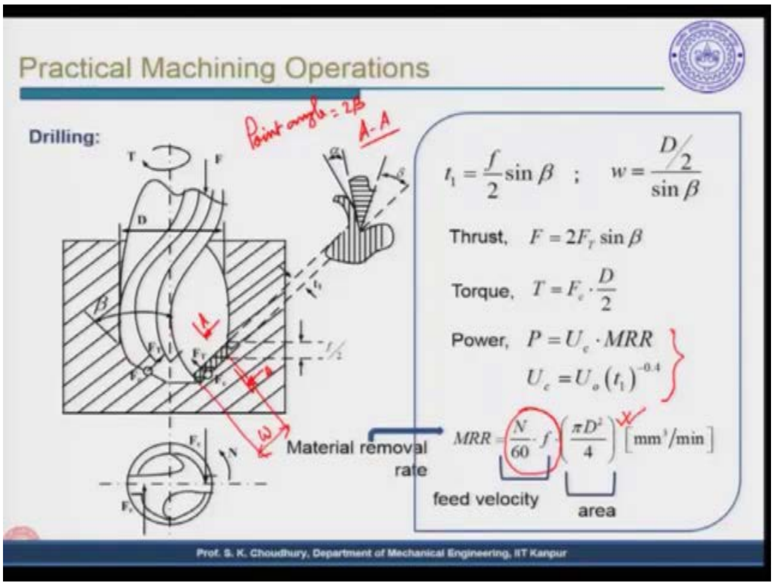

If you see this diagram here - the torque is given here and this is the force which is the thrust, both are given to the tool which is the drill here. Now, if you take a cross section A-A, we said that it will be visible like this. So, here we have the tool with the rake face and this defines the inclination of the rake face of the tool.

So, in this case the tool is the drill. The drill has two teeth and we said that the feed is distributed to both of them. Therefore, the movement of the tool towards the work piece that is penetration of the tool by one rotation of the drill, will be defined as the feed. And this distance between this point and this point is given by $\frac{f}{2}$ because, this is the total feed f which is distributed to both of these teeth. Now, the angle from this tooth to this is the point angle, as we said.

So, let us take β as the half of the point angle and in this case we said that the t_1 which is like in case of turning this is the distance and this $t_1 = \frac{f}{2}\sin\beta$, since $\frac{f}{2}$ is here. Now, w which is the width of cut is given here, that is the width, i.e. how much width we are removing. And this is given by $\frac{D/2}{\sin\beta}$, β is the point angle by 2.

The thrust force will be directed from these teeth towords the center of the drill and this will be perpendicular to the teeth. The F_C will not be visible in this view, F_C will be visible in this view, the top view and this will be resisting the torque which is applied to the tool, that is to the drill. The thrust is given by $2 \times F_T$ 2 of this F_T into the $\sin\beta$ and the torque is given by $F_c.\frac{D}{2}$ we are assuming that this is half of the diameter.

Let us say it is acting at the half of the diameter and the total distance between these two will be half of the diameter; diameter of the drill or diameter of the hole as I said. So, I also like to remind you that the diameter of the hole becomes little bigger than the diameter of the drill because of the vibration during the drilling process. Power is given by $U_c \times MRR$ and $U_c = U_o(t_1)^{-0.4}$.

This remains same as in case of turning, shaping and planing. However, the material removal rate is different than in case of turning. Here the feed velocity is given as $\frac{N}{60}f$ and the area is $\frac{\pi D^2}{4}$, since it is a hole. So, $\frac{\pi D^2}{4}$ will be the area of the machining zone which is the hole and it is given in $mm^3/\min$.

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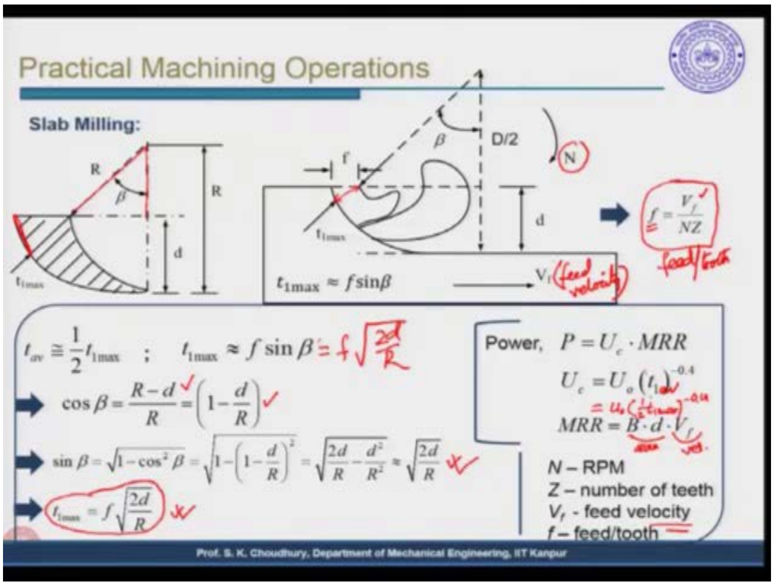

Now, in case of practical machining operations of slab milling, it is little different than the other processes because here in case of drill we had two teeth. In case of single point cutting tool we had one tooth and in case of milling it is more than 2, it is multiple. So, the basic difference is that here the feed which is given is feed per tooth.

So, per tooth the feed given is *f*, and this is equal to the $\frac{V_f}{NZ}$ and the V_f is the feed velocity as shown in the slide. *N* is the RPM of the milling cutter. *Z* is the number of teeth in the milling cutter. Now, suppose you take this line, look at the line diagram and this is one of the teeth of the milling cutter.

And in this case this much is the chip which is removed by one tooth, when the milling cutter is rotating at an RPM of *N* and the feed velocity is given as the V_f, this is the depth of cut, *d*. This much material has been removed. *D/2* is the radius of the milling cutter, diameter of the milling cutter is *D* and this defines the t_1 maximum. Here $t_{1\max}$ is important because if you look at the chip removed by one tooth of the milling cutter, the t_1 goes from minimum which is 0 to maximum.

So, we take the $t_{1\max}$ and this is the diagram which clarifies how the $t_{1\max}$ can be taken. This is the centre of the milling cutter and if you join this along this point and extend it, then this is the distance which is considered to be the $t_{1\max}$, these the maximum uncut thickness. Now, this is the radius of the milling cutter *D/2*, small *d* is the depth of cut. Now, the $t_{1\max}$ can be assumed to be $f \times \sin\beta$ and the β is the small angle that has been shown in the diagram.

This is because we can assume this chord to be a line and from this triangle you can find out that $t_{1\max}$ is equal to $f \times \sin\beta$. Now, here in milling process the average value of the uncut thickness is used which is t_{1av} which is normally taken as half of the $t_{1\max}$. Now, this $t_{1\max}$ is therefore, given as $f \times \sin\beta$ and the cos of this angle β is given as

$$\cos\beta = \frac{R-d}{R}$$

Now, this value can be written as $1-\frac{d}{R}$. So, the $\sin\beta$ can be written as $\sqrt{1-\cos^2\beta}$

$\cos\beta$ is $1-\frac{d}{R}$. So, we are getting $\sqrt{\frac{2d}{R}}$ which is $\sin\beta$. Therefore, we can say that

$$t_{1\max} = f.\sqrt{\frac{2d}{R}}.$$

Now, you can see here the difference between the milling operation and the turning operation or the shaping operation. Power is given in the same way as in other processes.

Power is the product of the specific energy and the material removal rate. Specific energy is the same as in other cases that is the energy spent to remove 1 unit volume of material which can be expressed as $U_C = U_o(t_{1av})^{-0.4}$. t_1 here is the uncut thickness which is shown here. t_{1av} is taken as half of the $t_{1\max}$. Therefore, U_C can be expressed as

$$U_C = U_o(\frac{1}{2}t_{1\max})^{-0.4}.$$

So, this is the difference between the turning and the milling process because here the t_1 varies from 0 to maximum. So, the half of the $t_{1\max}$ taken as the t_{1av} as it is shown here. In this formula of $U_c = U_o(t_1)^{-0.4}$, this t_1 we take as t_{1av} which is half of the $t_{1\max}$. Now, the U_o, once again I will remind you that this is called the specific energy coefficient or the specific energy constant and it can be defined as the specific energy required to remove 1 millimeter of the t_1; t_1 is the uncut thickness.

So, here in this case will be average uncut thickness. Material removal rate is the same that is the area into the velocity, this is the $B \times d$ is the area and the velocity is the V_f which is the feed velocity as shown here. So, the concepts are remaining the same, that is product of the U_c and the MRR gives us the power, product of the area and the velocity gives us the MRR.

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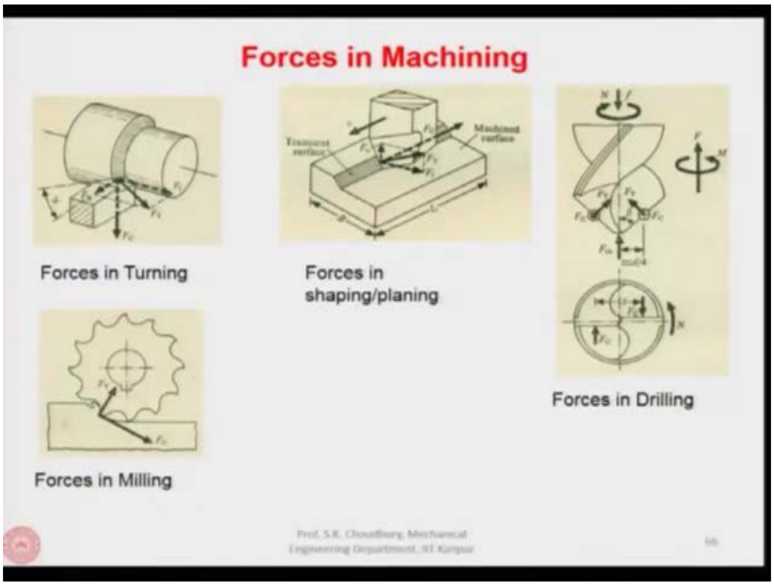

In the next topic we will discuss the forces in machining. The idea of discussing this is to show you that forces in different kind of processes like turning, shaping, drilling and milling are very different. And these forces are shown here once again although I have shown it to you when we were discussing the practical machining operations.

For example, forces in turning are the radial force, thrust force, feed force and the cutting force, although the thrust force is the summation of the feed force and the radial force. Now, in case of shaping it is once again repeated, that is the is here, F_N, F_t and F_r. In case of drilling it is the torque *N* and the thrust is the *F*. In case of milling, it is the radial or the total tangential force which is the F_c and the thrust force which is directed towards the centre of the milling cutter.

Now, here these are the forces for all the different operations and as you can see that they are different. Now, how to measure these forces? That becomes a problem because, if you can measure by an instrument the forces in turning, that instrument cannot be used for measuring the forces in shaping or for measuring the forces in drilling or milling because the forces are different, force locations and the directions are different.

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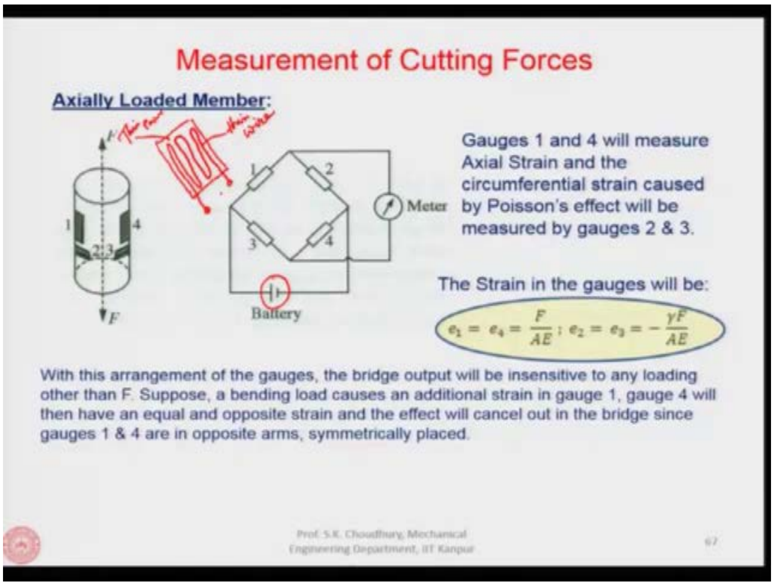

So, let us discuss the measurement of cutting forces. The easiest way of defining is the axially loaded member. Suppose this is an axially loaded member meaning that here this workpiece is exhorted a force which is *F* and this is axially located. So, how to measure this force? These are the gauges 1 and 4. Normally the gauges used are the strain gauges and these days in the dynamometer, as you know the dynamometer measures the cutting force, in dynamometers we also use the piezoelectric components or the elements. I will discuss little later what are those components and the strain gauges.

Now these strain gauges or piezoelectric elements 1 and 4, they will measure axial strain and the circumferential strain caused by the Poisson's effect will be measured by the gauges 2 and 3. So, if you can see that when the F is applied, the axial strain it will be affected, it will be sensed by the 1 and 4 gauges. And 2 and 3, they will not be able to sense the axial strain that is happening in this part in this component. The 2 and 3, the way they are located, perpendicular to 1 and 4, they will sense the circumferential strain which is caused by the Poisson's effect.

So, the strain in the gauges will be then e_1, e_4, this will be $\frac{F}{AE}$; F is the force, A is the area, and E is the Young’s modulus of the material of this component. And e_2 and the e_3 , since they will sense the circumferential strain this will be equal to $-\frac{F.\text{Poission's Ratio}}{AE}$. Now this is the simplest element, that is a cylindrical job and an axial load has been applied and these are the strains that we get.

So, as you know that these strain gauges are arranged in a particular way and this is called the Wheatstone bridge. The configuration of this bridge is shown here; it has an ammeter here and with the battery the power is supplied to the sensors. These are the sensor 1 and 4 which are located opposite to each other and this is because that when suppose apart from F if any axial load occurs perpendicular to F in that case an opposite load will be on the sensor 4.

They will mutually nullify. That is why they are placed opposite to each other. Now, with this arrangement of gauges the bridge output will be insensitive to any loading other than F. Meaning that 1 and 4 will be measuring only the axial strain or the axial force and it will not be able to sense 2 and 3 that is the circumferential strain. Suppose the bending load causes an additional strain in gauge 1, then the gauge 4 will have an equal and opposite strain and the effect will cancel out in the bridge since gauges 1 and 4 are in opposite arms, symmetrically placed.

So, they are actually placed opposite to each other and they are symmetrically placed. Now let us see and what is the strain gauge? Probably many of you must have seen that, a strain gauge is something where a very thin wire is placed on a very thin paper like this and these are the two nodes. So, this is a thin paper, very thin paper and this is a thin wire. This will be rigidly pasted on the material of the workpiece and when the workpiece will be deformed, in this case it is the axial load given.

So, there will be an axial strain. So, when this will be deformed then along with that these wires will be deformed. And when a particular voltage is passed through this, so this voltage, according to Ohm’s law, will be changed because this wire has been deformed and the resistance is changed. This is the principle which is used in the case of the strain gauges. Now, this deformation of the wire because of the strain will change the voltage which is passed here and that changed voltage will be proportional to the deformation that is taking place. By measuring the change in the voltage, we can actually measure the strain.

So, e_1 in e_4 will be equal to $\frac{F}{AE}$ and with Poisson's ratio it will be minus opposite Poisson's ratio into the strain that has been occurred in the e_1 and e_4 nodes. Now let us see the more complicated version of the strain gauges and the measurement.

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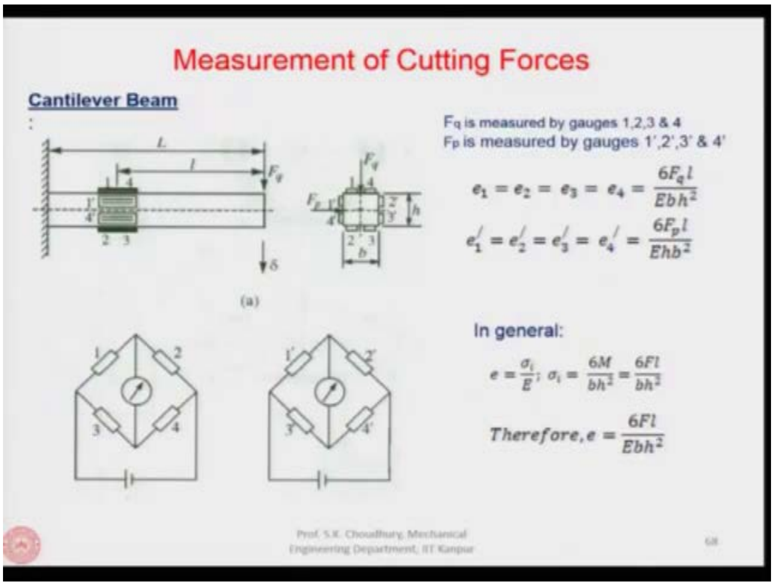

For example, for a cantilever beam, normally the load that has to be measured is sensed through the moment; because if the point of application of the load is known then the load into this distance will be moment. So, if you can measure the moment, from there knowing the distance from the application of the load we can find out what is the force. This is the principle by which the forces are measured in the cantilever beam.

Let us take an example; suppose this is the cantilever beam and in this cantilever beam there are two forces acting: one is the F_q, which is the thrust force and one is the F_p which is perpendicular to that. So, there are two, perpendicular to each other, forces F_q and F_p. If you see in this section, you can see the point of action of the F_q and the F_p. Now, here we have the strain gauges; there are four strain gauges, 1, 2, 3, 4 on this side; 1, 2, 3, and 4 in this section you can see, and on the 90^o there will be 4 more - 1', 2', 3' and 4'.

If this is the distance *b* and this is the distance *h*, then we can actually form two Wheatstone bridges, one comprising of 1, 2, 3 and 4 and the second Wheatstone bridge comprising of 1', 2', 3' and 4'. So, when the F_q will be acting on the cantilever beam, as you can see from here, the sensors 1, 2, 3 and 4 will be sensing this.

Whereas, 1', 2', 3' and 4' will be insensitive to this, insensitive to the thrust force F_q. F_q is measured by gauges 1, 2, 3, 4; and F_p, which is perpendicular to F_q is measured by gauges 1', 2', 3' and 4'. Now, e_1, e_2, e_3 and e_4 values will be given by $\frac{6F_qL}{ebh^2}$; since in general we know that the strain is equal to stress divided by Young's modulus.

Now, the normal stress can be found out by 6 M - the moment divided by bh^2. This is the formula taken from the solid mechanics. Now, the M is moment which is the force applied here into the length, *F* into *L*. So, $\frac{6FL}{bh^2}$, this will be the normal stress. So, the strain that we get will be $\frac{6FL}{ebh^2}$ in general.

If we are measuring e_1, e_2, e_3 and e_4 , since they will sense the F_q so, this F will be F_q here. So, $\frac{6F_qL}{ebh^2}$. e'_1, e'_2, e'_3 and e'_4 which are sensed by these sensors measuring as we said the perpendicular to F_q which is F_p. So, here in this case in the general formula we are putting in the place of *F*, F_p because these are the sensors which are measuring that is 1', 2', 3' and 4' which are sensing the force F_p.

Only here it will be ebh^2 because it is in the other side. So, this is the principle which is used in case of finding out the strain or measuring the force through this strain of a cantilever beam. You understand that from the strain you have measured, if you know the *L*, *b* and *h* you can find out the F_q. So, this is how indirectly you can find out the forces by measuring the strain.

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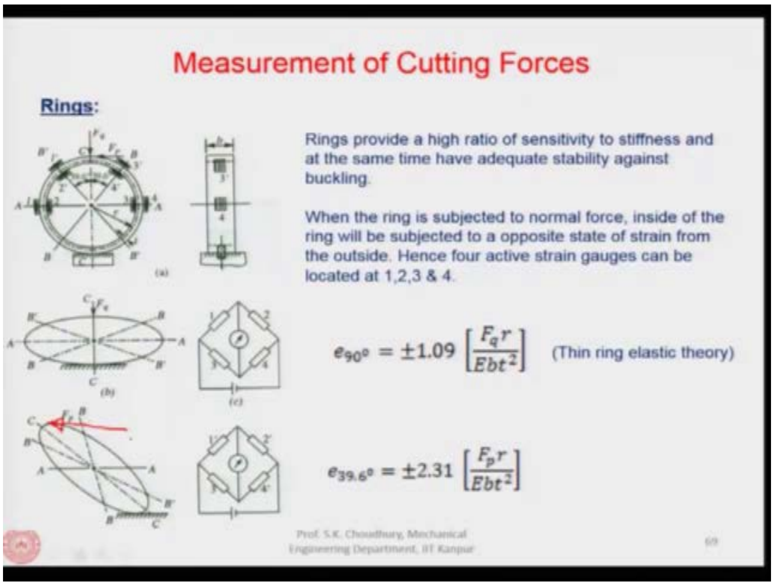

Now, let us see how it is done in case of a ring type dynamometer or ring type element. Rings provide a high ratio of sensitivity to stiffness and at the same time have adequate stability against buckling. These are the elements which are also used to measure forces. Now for example, if we have a ring like structure and the side view is here. So, it has been seen that these are the nodes when they are subjected to the forces like F_q and F_p as shown here.

It has been seen that the nodes where the maximum strain occurs, that would be at an angle of 90 ^{o} and at an angle of 39.6^o, this has been measured or this has been found out by the photoelastic experiments for the element. So, if we have the sensors along the 90 ^{o} that is 1, 2, 3 and 4 and at an angle of 39.6 which is 1', 2', 3' and 4' in that case this 1, 2, 3 and 4 it will sense the F_q.

And e'$_1$, e'$_2$, e'$_3$ and e'$_4$ will sense the F_p force. Now, for example, under the action of the F_q the deformation of the ring will be like this, ring will be deformed like this and under the action of the F_p the deformation of the ring will be like this. So, as you can see that these axes, that is cc, a a, bb and b prime b prime they are actually getting deformed. Now, when the ring is subjected to normal force F_q, inside of the ring will be subjected to an opposite state of strain from the outside. Hence four active strain gages can be located at 1, 2, 3, 4.

So, this is the justification why the 1, 2, 3, 4 are positioned or located in the way it is shown here, that is where once again I am repeating that when the ring is subjected to normal force, inside of the ring will be subjected to an opposite state of strain. You can see from here.

So, these are the four sensors or 4 strain gauges which are placed here. Similarly, 1' 2' 3' and 4' are placed at an angle of 39.6^{o} because in a ring structure photoelastic experiment shows that the maximum strains occur at that node, which are located at that 39.4^{o}.

Now, it has been seen that the e 90 at 1, 2, 3, 4 value will be $e_{90} = \pm 1.09\left[\frac{F_q r}{Ebt^2}\right]$; ± 1.09 is the coefficient. And F_q because it will be sensing the F_q, r is the inside radius this is the radius inside means this dotted line what we have shown here, from the centre up to the dotted line it is the r. And e is the Young's modulus of the material of this of this structure, b is the width here as shown and t is the thickness of this wall thickness of this structure.

So, at 90^{o} which will be actually measuring this we have the Wheatstone bridge like this consisting of 1, 2, 3, 4 sensors and this Wheatstone bridge will give you the value which is here which will depend on the $F_q r$ e b and t^2 this is the thin ring elastic theory. At 39.6^0 where the maximum nodes, maximum strain will be occurring, at this node it will be $\pm 2.31\frac{F_p r}{Ebt^2}$.

So, just notice here that at 39.6^0 , 1' 2' 3' and 4'sensors will be sensing the force, that is, it will be sensing the force F_p. So, here it is F_p and along the 90^{o} 1, 2, 3, 4 these sensors will be sensing F_q which is the thrust force or normal force and therefore, at 90^{o} it is $\frac{F_q r}{Ebt^2}$. So, this is the principle by which the forces can be measured by using different kind of structures, be it cantilever beam or a ring structure.

So, here the strains are measured from the Wheatstone bridge and from knowing the value of the r, knowing the value of the Young's modulus of the material, knowing the value of the b and the t which are the physical parameters, we can actually find out the value of the F_q or the F_p. Now, here only disadvantage of the ring type structures is that it will tend to roll when the F_q is applied as it is shown here, from here you can see that when the F_q is applied how this is getting deformed. So, it is trying to roll.

This is the disadvantage that the ring types structures have, but otherwise rings provide a high ratio of sensitivity to stiffness, as I said. And at the same time have adequate stability against the buckling. Because of these properties the ring type structures are used for measuring the forces in practice. Details we will discuss in our next lecture.

Thank you for your attention.