

Machining Science - Part I
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Lecture – 11

Hello and welcome to the 11th lecture of the Machining Science course. So, let me remind you that in our earlier session we were discussing the Zorev's model of friction.

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Friction in Metal Cutting

At the point $X = (l_f - l_{st})$ the normal stress is given by (τ_{st}/μ) . Further, from the equation (1) it is given by:

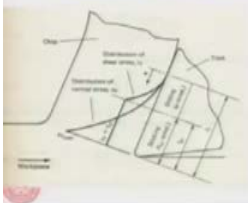
$$\sigma_f = \sigma_{fmax} \left(\frac{X}{l_f} \right)^y \quad \dots [1]$$

Therefore, $\mu \sigma_{fmax} \left(\frac{l_f - l_{st}}{l_f} \right)^y = \tau_{st} \quad \dots (3)$ $[F = \tau_{st} \cdot a_w \cdot l_{st} + \frac{\mu \sigma_{fmax} a_w (l_f - l_{st})^{1+y}}{l_f^y (1+y)}] \quad \dots [2]$

Substituting Eq. (3) into Eq. (2), the expression of F can be simplified as:

$$F = \tau_{st} a_w l_{st} + \frac{\tau_{st} a_w (l_f - l_{st})}{1+y}$$

The mean coefficient of friction on the tool face can now be expressed as :



$\tan \lambda = \frac{F}{N} = \frac{\tau_{st}}{\sigma_{fmax}} \left(1 + y \frac{l_{st}}{l_f} \right) \quad \dots (4)$

λ - friction angle

$$[N = a_w \int_0^{l_f} \sigma_{fmax} \left(\frac{X}{l_f} \right)^y dx = \frac{\sigma_{fmax} a_w l_f}{(1+y)}]$$

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And what we said is that here is the distribution of the normal shear stress along the chip tool contact length which is l_f and this is how the distribution of the shear stress looks like.

Now, the shear stress remains constant in the sticking zone because the normal pressure is very high and when the normal pressure decreases in the sliding zone, the μ remains constant. So, towards the point where the chip loses contact with the tool, this is decreasing, that is, the shear stress. Now, we integrate this curve from 0 to l_f ; X value is 0 at this point and X value is l_f at this point.

So, in that case we can actually find out the normal force - that is what we did here and then we have integrated this area under the shear stress and we found out the shear force. Then we have simplified the formula of the F and initially by integrating we got this,

we have simplified by substituting equation 3; in this equation and we got this. So, then the F by; we got the tan of the friction angle and therefore, the friction angle can be found out as the arc tan of this.

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Friction in Metal Cutting

The mean normal stress on the tool face is given by: $\sigma_{fav} = \frac{N}{a_w l_f} = \left(\frac{\sigma_{fmax}}{1 + \gamma} \right)$

Therefore, $\sigma_{fmax} = (1 + \gamma) \sigma_{fav}$

Substituting for σ_{fmax} in Eq. (4) gives: $\lambda = \arctan \left(\frac{\tau_{st}}{\sigma_{fav}} \left(\frac{1 + \gamma}{1 + \gamma} \right) \left(\frac{l_{st}}{l_f} \right) \right)$

In experimental works it is found that the term $\frac{\tau_{st} (1 + \gamma) (l_{st} / l_f)}{1 + \gamma}$ remains sufficiently constant for a given material over a wide range of unlubricated cutting condition, and therefore the expression becomes:

$\lambda = \arctan \frac{K}{\sigma_{fav}}$

This equation shows that the mean angle of friction is mainly dependent on the mean normal stress on the tool face. This explains the following fact: as working normal rake increases, the component of the resultant tool force normal to the tool face will decrease and therefore, the mean normal stress will decrease and the friction angle will increase.

$\mu = \frac{F_c}{N} = \tan \lambda$

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Next, we said that the mean normal stress on the tool face is given this way. We found out the mean normal stress and from there we can find out the $\sigma_{f_{max}}$ which is $(1 + \gamma) \sigma_{f_{av}}$. So, finally we get that the friction angle is this and it was said that this factor remains sufficiently constant in practice (see the slide above) and therefore, this can be assumed to be equal to $\arctan \frac{K}{\sigma_{f_{av}}}$.

This equation shows that the mean angle of friction is mainly dependent on the mean normal stress on the tool $\sigma_{f_{av}}$. This explains the following fact that as working normal rake increases; working normal rake is α_N , the component of the resultant tool force normal to the tool face will decrease. If you remember, we said in the diagram of the cutting force and the rake angle that as the rake angle increases, the cutting force decreases.

So, as the α is decreasing F_c is increasing and vice versa. So, this is the same thing we are saying here - as working normal rake increases towards this, the component of the resultant tool force normal to the tool face will decrease, because of this, as we have

explained in this curve. And therefore, the main normal stress will decrease and if it is decreasing then the friction angle will increase.

So, this is the conclusion that can be made from the Zorev's model and this is very important when the working normal rake variation is considered and how to select the working normal rake.

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Our next topic will be the practical machining operations and here in this slide, you can see various machining operations, practical machining operations like turning, milling, drilling, shaping, planing, broaching, sawing - these are shown here.

Now, in turning for example, you can get the cylindrical surface or groove or the threaded part or the threaded surface. In milling, normally the flat surfaces are obtained or the grooves or the contours, very complicated contours, when the CNC milling is used. In drilling, normally the holes are drilled - either it is a blind hole or it is a through hole; in case of sawing it is a parting off - that is we are parting off a part into two parts.

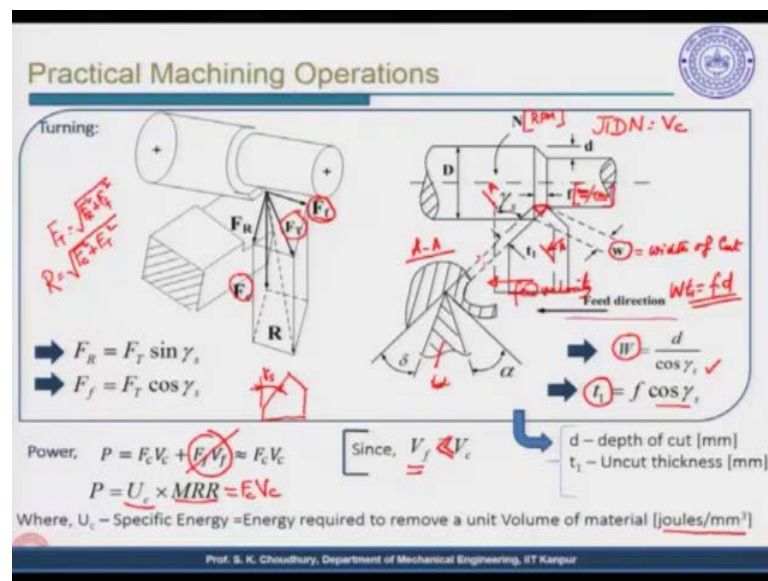
In broaching, these are the internal slots or external slots; this can be machined at a very high accuracy and the shaping and planing, as you know that in case of shaping, the tool moves to and fro - reciprocates and the work piece is given the feed movement, perpendicular to the movement of the tool. And in case of planing, it is the other way

round, that is the workpiece moves and the tool is given feed perpendicular to the movement of the workpiece.

Shaping is used when the work piece is smaller and planing is used when the workpiece is very large. Depending on that in case of shaping the quick return mechanism is used because when the tool is moving forward, it actually cuts, removes the material and since it does not remove any material while going back, so, going back is normally at a higher speed than the forward stroke or the forward movement.

Now, in case of planing the same thing happens - there is a quick return mechanism, but that quick return mechanism is different than the shaping. As many of you might be knowing that in case of planing, there is a cross belt and loose and the tight pulleys system used and in case of shaping it is the Whitworth quick return mechanism which is used.

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Well, let us see the turning operation. The workpiece is shown here in the schematic diagram; it is a pictorial view and earlier also we have discussed that this is the F_f which is the Friction force. Friction force is parallel to the workpiece axis and it is normally given in the direction of the feed.

So, the direction of the feed is shown here, this is the feed direction and along that the F_f which is the feed force; F_R is the radial force; radial force is perpendicular to the

axis of the workpiece and F_R and F_f both are located at the horizontal surface, their resultant will be the F_T ; this we have already discussed. So, the F_T is becoming $\sqrt{F_R^2 + F_f^2}$, F_R once again is the radial force and F_f is the feed force.

Now, if you draw the parallelepiped like this, then, vertically located is the cutting force F_C and the resultant force, which is $\sqrt{F_C^2 + F_T^2}$, will be located from the back point that is the tool point to the front point of the parallelepiped. So, the resultant force is becoming $\sqrt{F_C^2 + F_T^2}$. So, these are the four components basically that is F_R , F_T , F_f and the F_C and main two components are the F_C and the F_T .

So, I also said earlier that when cutting forces are measured, they are normally the F_C and the F_T - these two components are measured. Now, if you see the view like this as it is shown here, this is the tool and it started machining; so, this much depth of cut has been machined. And let us say the tool position at a time is here and after one revolution of the workpiece the linear movement of the tool is called the feed. The revolution of the workpiece is N; which is given in RPM, that is the revolution per minute. After one revolution of the workpiece the tool has moved from here to here.

So, this distance the movement of the tool or advancement of the tool per revolution of the workpiece is given as a feed. Therefore, the feed unit is millimeter per revolution; how much millimeter that the tool has moved with one revolution of the workpiece. This is the width of cut. And if you now draw these two lines; extend these two lines or if you take a section like this, let us say A A section, that A A section will look like this (see the slide above).

Here the distance between these two lines will be given as the t_1 ; which is the uncut thickness. And if you see the A A, section this is how you will see the workpiece and this is the tool. So, this is the rake face because over the rake face the chip moves and this angle is the rake angle which defines the inclination of the rake face. And this is the flank face which makes an angle with the already machined work surface, known as the flank angle.

So, this is the rake angle and this is the flank angle which we have discussed also earlier. Now, in the force components the F_R can be found out from F_T and the $\sin \gamma_s$; γ_s is the side cutting edge angle.

So, γ_s is here. From this figure particularly you can find out that the F_R is equal to F_T and the \sin of this angle, that is the side cutting edge angle. Similarly, the F_f that is the feed force component - this will be equal to thrust force into the $\cos \gamma_s$; γ_s again is the side cutting edge angle. And in the geometric parameters; width of cut will be equal to the $\frac{d}{\cos \gamma_s}$; that you can find out from this small triangle. And from a very small triangle

like this here, this triangle will define the value of the t_1 ; t_1 is from this point to this point. This is equal to – feed f is this; this is the movement of the tool per revolution of the workpiece multiplied by \cos of the side cutting edge angle. So, these are the geometrical parameters. Wt_1 as you understand is equal to $d \times f$ because the $\cos \gamma_s$ is getting cancelled. So, W and the t_1 is equal to f and the d , that is what we get from the geometrical relationships.

Now the power consumption during the machining, during turning in this case can be given as $F_c V_c$. F_c is the cutting force component which is here into the V_c ; $V_c = \pi dN$; this is the cutting velocity plus $F_f V_f$; F_f is the feed force, feed forces here into the V_f which is the feed velocity; this is the feed and the velocity is given as a feed velocity.

Now, normally the V_f is much less than the V_c , i.e. the feed velocity is much less than the cutting velocity; so, in here the F_f into V_f can be ignored and the power can be written approximately as a product of $F_c V_c$; that is the cutting force and the cutting speed. Power can also be given as the U_c , this is the specific energy and the Material Removal Rate.

Now, the specific energy in case of turning is the energy which is spent to remove unit volume of material and this is given in Joules per millimeter cube as it is shown here. So, finally we can say that the power P can be written as either $F_c \times V_c$ or $U_c \times MRR$; these are the same thing $U_c \times MRR = F_c \times V_c$.

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Practical Machining Operations

$\Rightarrow U_c = U_o (t_1)^{-0.4}$; U_o – Specific energy to remove 1 mm of t_1 .
→ Specific energy constant

$\Rightarrow MRR = \frac{f d \pi D N}{60}$ [mm³/sec]
 Material Removal Rate (area) (velocity)

\Rightarrow Number of revolution/pass = $\left(\frac{L}{f}\right)$ ✓

\Rightarrow Time/pass = $\left(\frac{L}{f N}\right)$

\Rightarrow Total Time; T = $\left(\frac{L}{f N}\right)^n$

n – number of passes
 L – cylinder length [mm]
 N – spindle speed [rpm]
 f – feed [mm/rev]

Diagram: A cylinder of length L is shown with a horizontal line representing the center line. Above the cylinder, the parameters N and f are indicated.

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Now, the U_c , that is this specific energy, is expressed by an empirical formula. Empirical formula is the formula that is obtained by experimental and analysis or the experimental results. So, the U_c can be expressed as $U_o(t_1)^{-0.4}$; t_1 is the uncut thickness and U_o is the specific energy to remove 1 millimeter of the t_1 . So, sometimes it is called the specific energy constant; this is also called the specific energy constant or coefficient. Because it is the energy that is spent to remove 1 millimeter of the t_1 ; now the MRR, Material Removal Rate is normally given as a product of area and volume.

So, area here in case of the turning is the feed into the depth of cut and the area and the velocity is the $\pi D N$ and we are dividing by 60 to make it as a mm^3/s . So, material removal rate is given in the unit of mm^3/s and U_c is the joules per millimeter cube (J/mm^3).

Number of revolutions per pass: suppose, we have a part which is being turned. Let us say this is the center line and here we have the length of L and this is being turned with the RPM of N , with the feed of f . So, then the number of revolutions per pass - pass you understand that in one go some part of the material is removed and that is called one pass. There may be several passes for getting the final depth of cut removed. So, number of revolutions per pass will be dependent on the L and the f ; that is the feed that we are using.

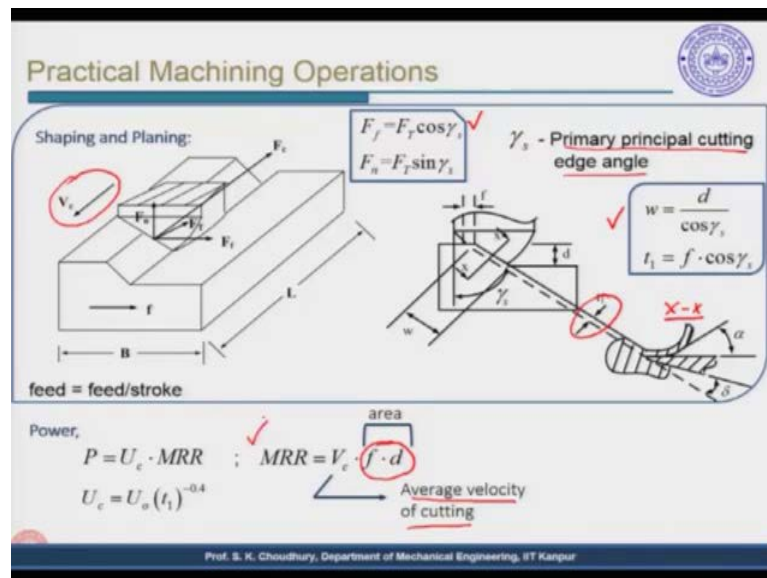
So, it will be given by $\frac{L}{f}$; therefore, the time that is taken in one pass will be given by

$\frac{L}{f}$ number of revolutions into the total number of revolutions that you are using N .

Therefore, the total time that is taken for machining this depending on the number of passes that will be equal to $\frac{L}{fN} \cdot n$ which is the time taken per each pass into the number of passes; number of passes can be 1, number of passes can be several; it can be 2, 3 depending on how the material is being removed what kind of material and what kind of tool and so on.

Small n is the number of passes, L as we said the cylinder length, spindle speed is N and the feed in millimeter per revolution is f .

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Now, let us see what happens in case of shaping and planing. Now, in shaping and planing the tool used is almost the similar to the tool that is used in the case of the turning. So, here is the diagram showing shaping or planing, only difference is that in shaping the tool moves, tool reciprocates and in case of planing, the workpiece reciprocates, otherwise there is no difference as it is.

Now, here the forces are the following: the F_c will be acting along the V_c ; F_n will be acting perpendicular to the machine surface and F_f will be acting parallel to the feed

direction, along the feed direction. So, F_n and F_f it will actually give you the F_T . So, F_f can be therefore, found out by multiplying F_T and the $\cos \gamma_s$; γ_s in this case is this angle, alright and it is called the primary principal cutting edge angle. In case of turning, it was the side cutting edge angle, in case of shaping or planing it is called the primary principal cutting edge angle.

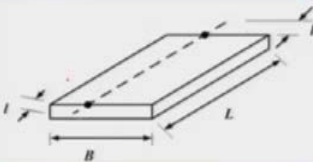
And therefore, the normal F_n can be found out by F_T and the $\sin \gamma_s$, and the geometrical parameters, like in the case of the turning, the width of cut which is here this is found out by $\frac{d}{\cos \gamma_s}$. And the t_1 is the same as we have done in case of turning; that means we are taking a section X X and this is the sectional view X X.

So, this is the t_1 and we have the tool and the workpiece with the rake angle and the flank angle; this is remaining the same as in case of the turning operation as you understand. Now, the power that is utilized, that is consumed during the shaping or planing, this is given by the same way again $U_c \times MRR$. Material Removal Rate, MRR is area into velocity. So, area here is $f \times d$ that is the feed and the depth of cut and the velocity; it is taken as the average velocity of cutting here V_c , this is along the direction of the V_c .

So, we have the geometrical parameters, we have the forces and the power consumption here for the shaping or planing.

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Shaping



l = Approach length
 V = Average velocity of cutting
 f = feed/stroke ✓
 r = Quick return ratio

Time for forward motion = $\frac{L + 2l}{V} = t_f$
 Time for backward motion = $\frac{t_f}{r} = t_b$
 So, total time/stroke = $t_f + t_b = \frac{L + 2l}{V} \left[1 + \frac{1}{r} \right]$
 Total number of strokes to cover the surface = $\frac{B}{f}$
 So, total time/pass = $\frac{B(L + 2l)}{f \cdot V} \left[1 + \frac{1}{r} \right]$ ✓

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Now, let us see how much time it takes for shaping a particular length and the width of a part. Let us say we have a block alright and this block has to be either shaped or it can be produced by planing.

So, in here the length is L ; capital L . The breadth B and here in case of shaping unlike in case of turning, the tool moves very rapidly up to the point where it gets in contact with the work piece. Then the speed of the tool slows down and then from here it again goes with that same speed as it was approaching initially, because when it is approaching it is not cutting the material. So, you understand that the speed could be higher.

So, this is called the approach length; this is small l from here to here the tool initially is located here, it is approaching this point this is called the approach length - this length is the same in here. And this length is the length through which the tool is removing material; cutting process takes place along the length L .

Now, here it is written approach length is small l , V is the average velocity of cutting, f is the feed per stroke. Mind it that the tool will go and come back, for example, in case of shaping; so it goes forward and comes back at a higher speed.

So, this is one stroke, i.e. tool going forward and coming back that time during that stroke, that is going forward and coming back, the workpiece is getting a feed perpendicular to the direction of the tool movement. So, this is called the feed per stroke,

and let us say r is the quick return ratio; depending on how fast you want the tool to come back, in case of shaping I am talking about.

Therefore, time taken for the forward motion will be this L it is covering plus 2 of this approach length it is covering divided by velocity, average velocity of cutting. This will be the time for forward motion. And for the backward motion, it will be t_f this time divided by the quick return ratio, because how fast you want the tool to come back depending on that the value can be determined.

So, the total time taken per stroke will be t_f which is the time for forward motion plus time for backward motion t_b and this will give you $\frac{L+2l}{V} \left[1 + \frac{1}{r} \right]$; Total number of

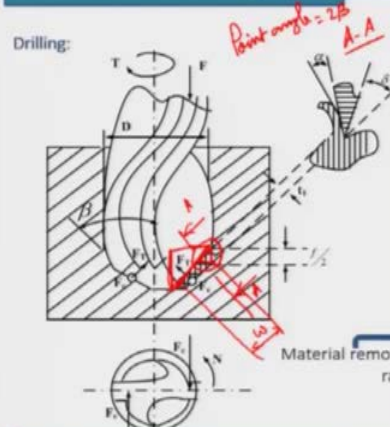
strokes to cover the B will be $\frac{B}{f}$. Therefore, the total time will be equal to

$\frac{B(L+2l)}{f \cdot V} \left[1 + \frac{1}{r} \right]$; this is the total time for removing the required material.

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Practical Machining Operations

Drilling:



Point angle = 2β
A-A

$$t_1 = \frac{f}{2} \sin \beta ; \quad w = \frac{D/2}{\sin \beta}$$

Thrust, $F = 2F_t \sin \beta$

Torque, $T = F_c \cdot \frac{D}{2}$

Power, $P = U_c \cdot MRR$

$$U_c = U_s (t_1)^{-0.4}$$

Material removal rate

$$MRR = \frac{N}{60} \cdot f \cdot \left(\frac{\pi D^2}{4} \right) \text{ [mm}^3/\text{min]}$$

feed velocity area

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Now, in case of drilling, it is different because first of all in case of drilling the forces are different, that is, both the rotation and the feed is given to the tool, that is to the drill and here is a diagram of the drill shown. As you can see that suppose drill initially was at this point and with one movement of the drill, it has penetrated from this point to this point. So, the distance between these two point will be feed by 2; half of the feed because there

are two teeth in comparison to tools used in turning, shaping or planning, here we have two teeth.

So, this f will be distributed amongst these two teeth and the value will be $\frac{f}{2}$ for each tooth. And if you take the cross section here, let us say this is A- A and the A- A will be seen like this. If you see here, this is exactly the same as what we had in case of turning or shaping or milling. This is the rake face of the tool, this is the flank face of the tool and therefore, it is the rake angle and this is the flank angle. This is for one tooth; there will be two teeth for the drill.

Now, here the geometrical parameters will be the $t_1 = \frac{f}{2} \sin \beta$; this is the angle β . So, like in the case of the turning we had the cutting angle, here we have the point angle, point angle is the 2β .

So, the β is half of this point angle and here the $t_1 = \frac{f}{2} \sin \beta$; this you can find out in a similar way as we found out earlier from this small triangle you can find out the value of the t_1 . Now, the width of cut w is here, this is the width and this width you can find out as $\frac{D/2}{\sin \beta}$; this you can find out from a triangle from here.

So, from this triangle you can actually find out the value of the w ; this is related to the diameter of the drill or the diameter of the hole that is being drilled. By the way, when we are drilling a hole normally the hole diameter becomes more; little more than the drill diameter because the drill vibrates. Similarly, we cannot say really that this is the same; it will be slightly different, but let us assume that D is here is the diameter of the hole.

So, this will be $\frac{D/2}{\sin \beta}$; β , once again is half of the point angle. Now, the thrust force and the torque and the power can be measured like this that is the thrust force is $2 \times F_T$; Now F_T will be perpendicular to the teeth towards the axis. F_C will be like this, it is visible in other view and here therefore, it will be as cross and the dot.

So, here you can see that that this is F_c which will be withstood by the torque given to the tool. So, this is the resistance to the torque given. So, it is in another direction as you can see. So, this is the F_c and this is the F_T direction; these values can be found out like the thrust is $2F_T \sin \beta$ and the torque is $F_c \cdot \frac{D}{2}$.

Now, power is taken in a similar fashion as in case of turning and the shaping which is $U_c \times MRR$ or it is $U_o (t_1)^{-0.4}$ which is a an empirical formula. Rest of the things we will discuss in the next lecture.

Thank you for your attention.