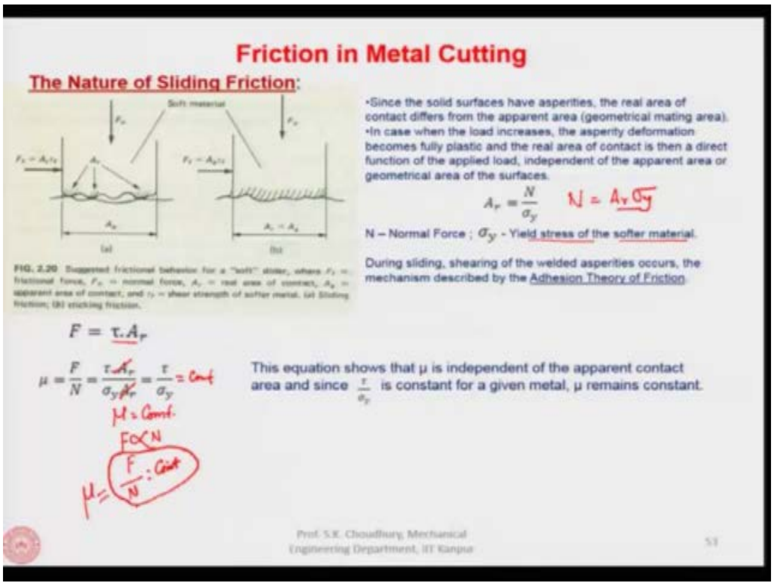
Machining Science - Part I
Prof. Sounak Kumar Choudhury
Department of Mechanical Engineering
Indian Institute of Technology Kanpur

Lecture – 10

Hello and welcome to the 10th lecture of the Machining Science course. In our last session we were discussing the sliding friction and we said that when the two solid bodies meet each other, normally they meet on the asperities. Since, there are asperities on the surfaces and solid surfaces never come with absolutely smooth surface if you look under the microscope, therefore, the real area of contact, which is the area of those asperities on which they are resting differs from the apparent area. Apparent area is the same as the geometrical area of meeting.

(Refer Slide Time: 01:09)



Now, as shown in the above figure, the two surfaces are meeting on the asperities. Initially when the normal force is not very high these asperities will be deformed elastically, because of the normal force acting. As the normal force is increasing, then these asperities will be deformed plastically. Since the solid surfaces have asperities, the real area of contact or the area of the asperities on which the two surfaces are meeting, differs from the apparent area. Apparent area is the geometrical meeting area that depends on the size of the parts. In case when the load increases, the asperity deformation becomes fully plastic, then the real area of contact is a direct function of the applied load.

So, when the force is increasing, then we can write that the real area of contact is equal to $\frac{N}{\sigma_y}$. σ_y is the yield stress of the softer material (as the area is equal to force by stress). Here it is normal stress. Now, during the sliding when they start sliding on each other, the welded asperities will be broken. The mechanism is then described by the adhesion theory of friction, which gives rise to a formula of the friction force, which is determined by the product of τ, shear stress and the real area of contact between the surfaces.

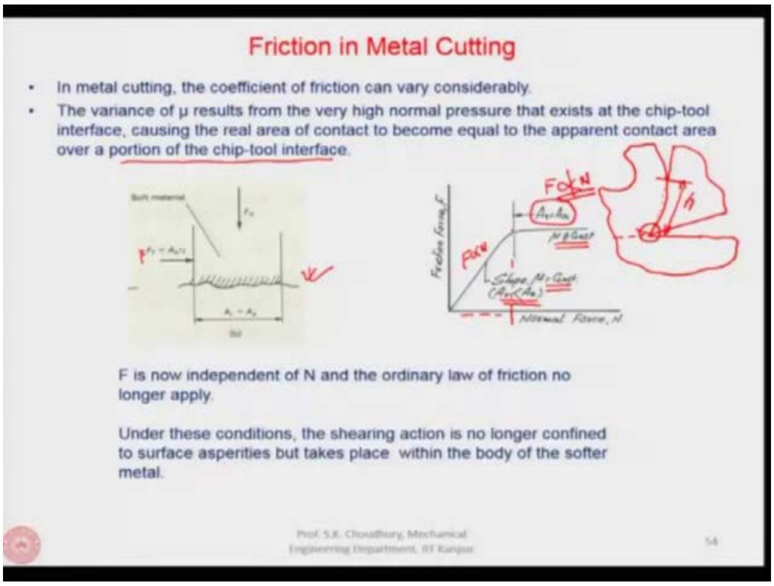
We have the normal force equal to $A_r \sigma_y$ and the friction force equal to $\tau \times A_r$. In both cases we are considering the real area of contact. μ, which is coefficient of friction is equal to the friction force upon normal force. The friction force is given as the $\tau \times A_r$ and we can get normal force is equal to $A_r \sigma_y$, where σ_y as I said is the yield stress of the softer material.

The ratio of F and N will be $\frac{\tau \times A_r}{\sigma_y \times A_r}$; A_r getting cancelled, and the ratio is equal to $\frac{\tau}{\sigma_y}$. This equation shows that μ is independent of the apparent area of contact, this is very important. The coefficient of friction when the two bodies slide on each other, the coefficient of friction does not depend on the apparent area of contact.

In fact, there is no area of contact mentioned here. The ratio of $\frac{\tau}{\sigma_y}$ reasonably remains constant for a wide range of material. Therefore, for a given metal μ remains constant; that means, F is proportional to N or $\frac{F}{N}$ is constant, which is μ.

This is what happens in the case of the natural or a normal sliding friction, when the two bodies are moving on each other and the normal pressure is not very high so that the apparent area of contact and the real area of contact does not become the same. It does not become the same because asperities although they get plastically deformed, they do not get plastically deformed to that extent that the real area of contact becomes equal to the apparent area of contact or geometrical area of contact.

(Refer Slide Time: 06:10)



What happens in metal cutting ? In metal cutting the coefficient of friction can vary considerably. Here we have seen, that in sliding friction, $\frac{F}{N}$, which is μ, is constant, because $\frac{F}{N}$ is actually the ratio of the shear stress and the normal stress, which for a given metal remains constant.

Therefore, μ remains constant and in the case of metal cutting, the coefficient of friction can vary considerably. It varies because the normal pressure applied by the tool on to the work piece is very high. Particularly it is maximum when the tool tip is in contact with the work piece and along the chip tool contact length it gets decreased.

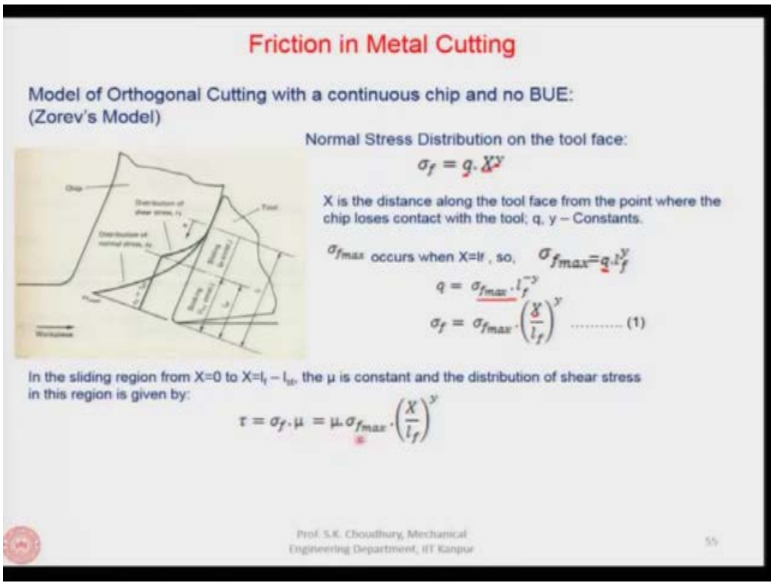
Therefore, this variance of μ results from the very high normal pressure, that exists at the chip tool interface, when the chip is moving along the rake surface of the tool, causing the real area of contact to become equal to the apparent or the geometrical area of contact over a portion of the chip tool interface. Referring to the above figure, consider the workpiece, tool and the chip. In the chip tool contact length, we said that the coefficient of friction varies considerably. Let us say h be the chip tool contact length. At the tip of the tool, the pressure is maximum because at this point the real area of contact becomes almost equal to the apparent area of contact or geometrical area of contact. As it goes towards the point where the chip loses contact with the tool, then it decreases.

Therefore, over a portion of the chip tool interface the normal pressure remains sufficiently high so that the real area of contact becomes equal to the apparent area of contact. Because the pressure is so high that the asperities get plastically deformed. The asperities have been maximally deformed and the real area of contact is almost equal to the apparent area of contact.

Let F be the friction force. Then friction force is equal to $A_a\tau_f$ where τ_f is the shear stress and A_a is apparent area of contact. Here the real area of contact is equal to apparent area of contact. Draw the curve between normal force N versus friction force F. This can be explained in the following way that as the normal force is increasing then up to a certain point the real area of contact is not equal to the apparent area of contact. In this region, the rules of sliding friction apply. μ remains sufficiently constant, because the real area of contact is still less than the apparent area of contact. Now, as the normal force increases, after a certain point the normal force becomes so high that real area of contact is becoming equal to apparent area of contact like it is shown in the above figure representing contacting surfaces. And, the normal rules of the sliding friction do not apply there, then the μ does not remain constant or F is not proportional to N anymore. Initially, up to a certain point, F is proportional to N and the μ remains constant. Beyond that point, when the normal force is very high or normal pressure is very high, μ does not remain constant and the curve become horizontal. So, there F does not depend on the N or F is not proportional to N. F is now independent of N and the ordinary law of friction no longer apply under these conditions; the shearing action is no longer confined to those asperities and it happens inside the softer material.

So far we have seen in sliding friction that the normal force is not sufficiently high. Although, the asperities get plastically deformed, but when the shearing happens those asperities or welded asperities get sheared off. In the sticking region, the real area of contact A_r and the apparent area of contact A_a are same and the shearing happens not along the asperities, but from the softer material. So, within the body of the softer metal the shearing action will take place.

(Refer Slide Time: 12:39)



This has been explained in a scientific way through a model explained by Professor Zorev. He is a Russian scientist. The model says that if you have the metal cutting process represented by the chip flowing over the rake face of the tool. Let l_f be the chip tool contact length. Within this chip tool contact length, let us say from the tip where the normal force is maximum up to a certain point, the normal force remains high and the shear stress is maximum which is given as τ_{st}; τ_{st} because this zone is called the sticking zone.

Here, the shear stress τ remains constant. It becomes maximum and constant because, the normal force is very high in this region. As it goes beyond the point at the end of sticking region towards the point where the chip loses contact with the tool, then the normal force decreases. And, therefore, the *F* remains proportional to *N*, like in the case of the normal sliding friction and then the μ is constant.

The zone from the end of sticking region up to the point where the chip loses contact with the tool is called the sliding zone and from the point of the tool tip to the point from where the sliding zone starts is called the sticking zone. Professor Zorev's model says, that the normal stress distribution on the tool face is maximum at the tool tip and, as it goes towards the point where the chip loses contact to the tool the stress becomes less.

The axis perpendicular to the tool rake face is represented by stress. Normal stress is maximum at the tool tip and is given as $\sigma_{f\max}$. The curve can be described as the $\sigma_f = qX^y$ where *q* and *y* are constants, X is the distance starting from the point where the chip loses contact with the tool. $X = 0$ at the point where the chip loses contact with the tool . As we move towards the tip of the tool, the length increases along the l_f and at tool tip $X = l_f$. So, X value varies from 0 to l_f. Now, the length of the sliding zone will be $l_f - l_{st}$ if the l_{st} is considered to be the length of the sticking zone, where the τ remains constant and the τ remains maximum.

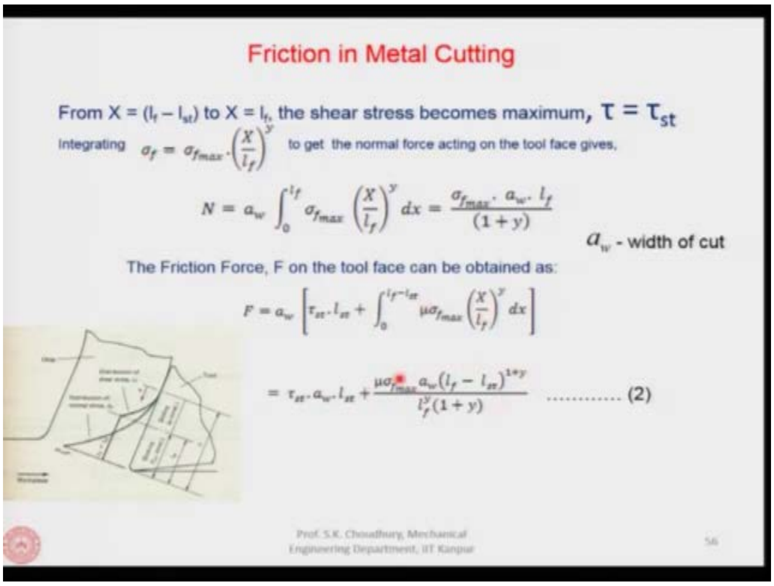
Overall the distribution of the normal stress and the distribution of the shear stress will be as it is shown in the above diagram. Now, *X* is the distance along the tool face from the point where chip loses contact with the tool. And, *q* and *y* are constants. Now, the normal stress, σ_f will be maximum when $X = l_f$. Therefore, $\sigma_{f\max}$ can be determined if we put the value of *X* equal to l_f.

So, it will be $ql_f^{\ y}$. From here we can find out the constant *q*, which will be $\sigma_{f\max} l_f^{-y}$. From here we can find out σ_f by putting the value of *q* into the expression of σ_f. We get, $\sigma_f = \sigma_{f\max}\left(\frac{X}{l_f}\right)^y$.

Now, in the sliding region i.e. from $X = 0$ to $X = l_f - l_{st}$, μ is constant because it is a sliding zone and here the normal force is not very high. Therefore, the real area of contact is not equal to the apparent area of contact and the real area of contact remains less than the apparent area of contact.

Therefore, μ remains constant and the distribution of shear stress in this region can be represented by the normal stress into μ.

(Refer Slide Time: 19:01)



Now, in the other region that is from $X = l_f - l_{st}$ to $X = l_f$, the shear stress becomes maximum. The shear stress is equal to τ_{st} which is the maximum shear stress. The normal force, N is the area under the curve multiplied by the width of cut, a_w. Area under the curve is the integration of $\sigma_f = \sigma_{f\max}\left(\frac{X}{l_f}\right)^y$ from $X = 0$ to $X = l_f$.

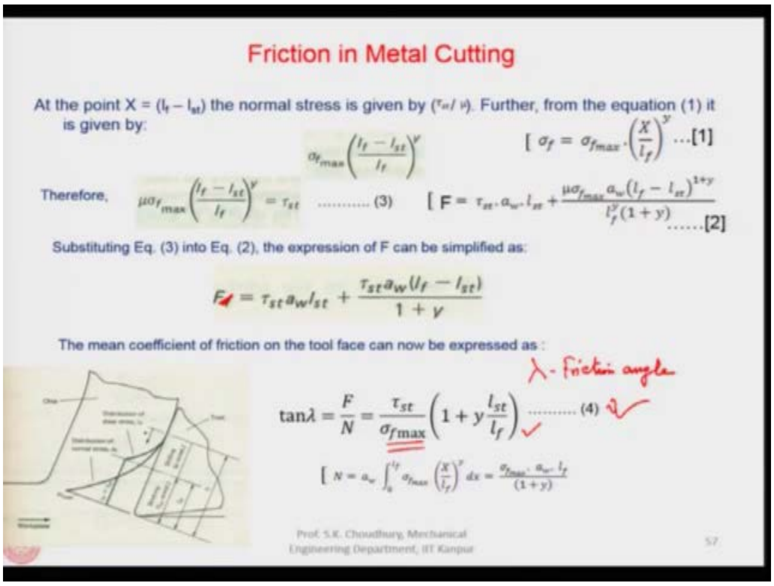
So,

$$N = a_w \int_0^{l_f} \sigma_{f\max}\left(\frac{X}{l_f}\right)^y dx = \frac{\sigma_{f\max}.a_w.l_f}{(1+y)}.$$

Now, the friction force, F is

$$F = a_w\left[\tau_{st} l_{st} + \int_0^{l_f - l_{st}} \mu\sigma_{f\max}\left(\frac{X}{l_f}\right)^y dx\right] = a_w \tau_{st} l_{st} + \frac{\mu\sigma_{f\max}.a_w.(l_f - l_{st})^{1+y}}{l_f^y (1+y)}$$

(Refer Slide Time: 22:05)



Now, at the point $X = l_f - l_{st}$ the normal stress is given by $\frac{\tau_{st}}{\mu}$, because earlier we have seen that normal stress into μ is equal to shear stress. The normal stress can be given as $\frac{\tau_{st}}{\mu}$ at this point, but from other side we can find out that from equation (1),

$$\sigma_f = \sigma_{f\max}\left(\frac{X}{l_f}\right)^y \tag{1}$$

$$F = \tau_{st} a_w l_{st} + \frac{\mu\sigma_{f\max}.a_w.(l_f - l_{st})^{1+y}}{l_f^y(1+y)} \tag{2}$$

If we put the value of $X = l_f - l_{st}$, then $\sigma_f = \sigma_{f\max}\left(\frac{l_f - l_{st}}{l_f}\right)^y$.

Therefore, $\mu \times \sigma_f$ at that point will be equal to τ_{st}.

$$\mu\sigma_{f\max}\left(\frac{l_f - l_{st}}{l_f}\right)^y = \tau_{st} \tag{3}$$

Substituting equation (3) in equation (2), the expression of F will be

$$F = \tau_{st} a_w l_{st} + \frac{\tau_{st}.a_w.(l_f - l_{st})}{(1+y)}$$

Now, we can find out the μ or the tan of the friction angle, λ from the following expression:

$$\tan\lambda = \frac{F}{N}$$

Where,

$$N = a_w \int_0^{l_f} \sigma_{f\,\max} \left(\frac{X}{l_f} \right)^y dx = \frac{\sigma_{f\,\max}.a_w.l_f}{(1+y)}$$

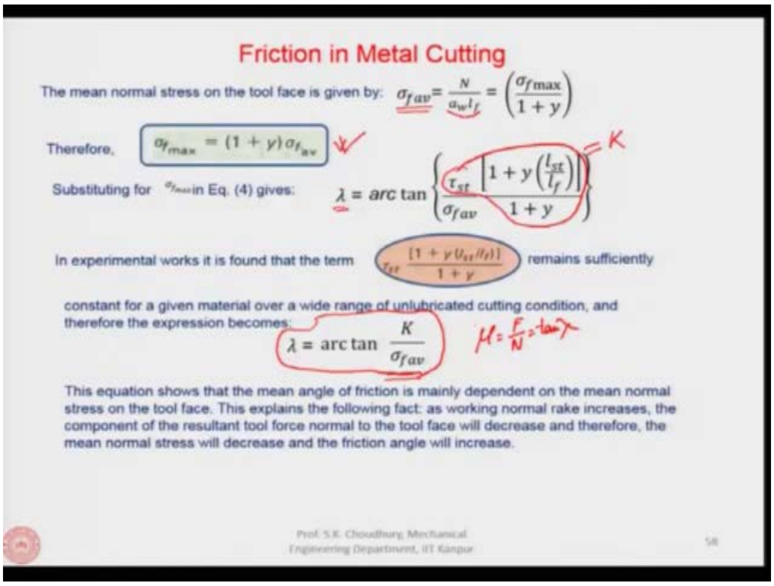
And,

$$F = \tau_{st} a_w l_{st} + \frac{\tau_{st}.a_w.(l_f - l_{st})}{(1+y)}$$

Substituting the equations of F and N into $\tan\lambda = \frac{F}{N}$ we have,

$$\tan\lambda = \frac{\tau_{st}}{\sigma_{f\,\max}} \left(1 + y \frac{l_{st}}{l_f} \right) \qquad (4)$$

(Refer Slide Time: 26:34)



Now, the mean normal stress on the tool phase is given by the normal force divided by the chip tool contact length multiplied by the width a_w, as length into the width is the area. Average value of the normal stress we will get by dividing the normal force and the area for the whole chip contact. The area can be found out by $a_w l_f$.

$$\sigma_{fav} = \frac{N}{a_w l_f} = \left(\frac{\sigma_{f\max}}{1+y}\right)$$

Therefore ,

$$\sigma_{f\max} = \sigma_{fav}(1+y)$$

Now, substituting $\sigma_{f\max}$ in the equation (4), we have

$$\lambda = \arctan\left\{\frac{\tau_{st}\left(1+y\frac{l_{st}}{l_f}\right)}{\sigma_{fav}(1+y)}\right\}$$

Now, in experimental works it has been found that the term $\dfrac{\tau_{st}\left(1+y\dfrac{l_{st}}{l_f}\right)}{(1+y)}$ remains sufficiently constant, for a given material over a wide range of unlubricated cutting condition.

Therefore, the expression becomes

$$\lambda = \arctan\left\{\frac{K}{\sigma_{fav}}\right\}$$

So, this is how we can actually express the friction angle or the coefficient of friction which are the same things.

This has a significance that is the result of Professor Zorev's model on the friction in the metal cutting. And, the consequence is that it means how the mean angle of friction depend on the normal stress.

And, how it varies when normal rake angle changes, which is the working rake angle as we have seen earlier for example, we said that normal rake, orthogonal rake, and the side rake angle they influence the cutting force and the power. And, how we can conclude from this equation that how they vary, I will discuss it in the next class.

Thank you for your attention.