Computer Integrated Manufacturing
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Lecture 08
Computer Graphics (Part 3 of 4)

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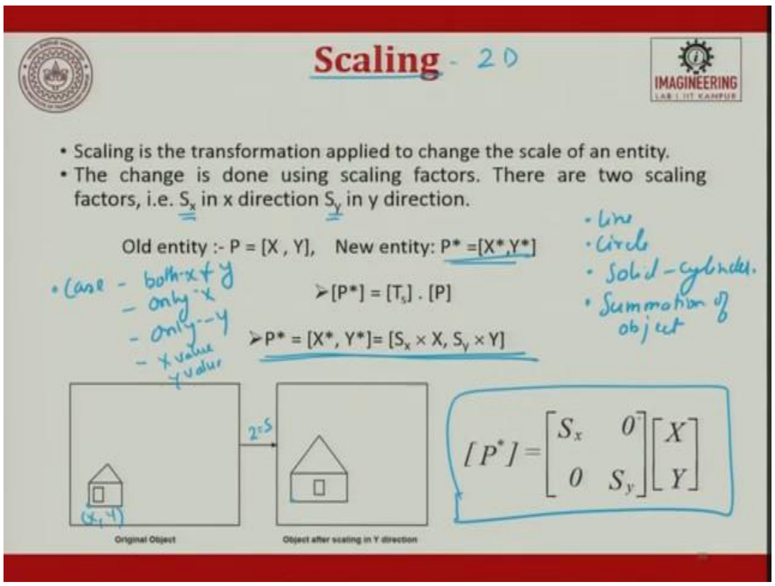

So, the next one we will see is scaling. Scaling is nothing but you have matrices, you multiply or you divide, so you get to the answer. So, instead of division you reduce the number, so you go less than one or you go more than one. Scaling is a transformation applied to change the scale of an entity. The entity can be a line, it can be a circle, it can even be a solid-cylinder, whatever it is. It can be a summation of several objects, of several objects put together it forms.

So, there are 2 scaling factors, one is scaling in x direction and scaling in y direction. In olden days, if you see when you try to do, use a certain software applications, they use to always tries to freeze the scaling both the directions uniform. Today, you have been given a freedom of choosing only one direction scaling and the other direction no scaling. So, it will be skewed object, but maybe if the requirements are there.

So, what are the cases? The cases in scaling can be both, x and y scaling, only x scaling and only y scaling. So, 3 cases are possible. So here, we are all focused only towards 2D, scaling in 2D, transformation 2D, just for simplicity in understanding.

So, the old coordinates, point is nothing but this is the old coordinates, I am choosing this point, which is (X, Y) and then I try to do scaling, maybe 2 times. So, S is equal to 2 times, and now you see that the entire thing has transformed into a new point P^*. Where,

P* =[X*, Y*]

So, this is how it is, so P^* is nothing but (X^*, Y^*) which is nothing but scaling of X multiplication, scaling of Y multiplication. When I said only X, only Y, both X and Y you can also have one more case, X giving a value and Y giving a value. That is also possible. Are you clear? First, both same value and the last one what I am talking about, X give 10 times, Y give 2 times, possible.

If your requirement is there, you can do that also. So, P^* is nothing but (X^*, Y^*), you get that from scaling in X direction multiplied by X comma scaling in Y direction multiplied by Y, you try to get the answer. So now, we are converting this into a matrices form, given by,

$$[P^*] = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

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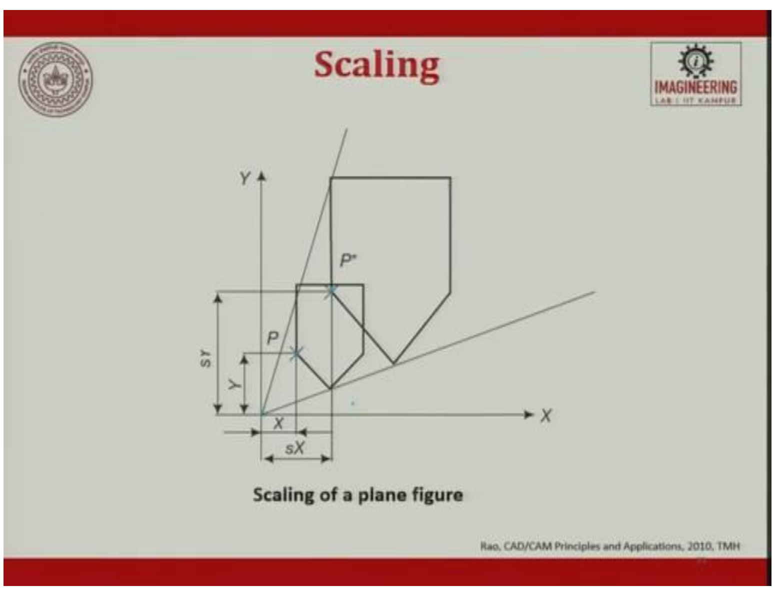

So, if you look at it, this is what it is I was talking to you about with respect to this point. So, you have taken point P, now point P is scaled in X direction and Y direction maybe once or twice. So, you see here, the point P is now offsetted and now you try to get a larger image.

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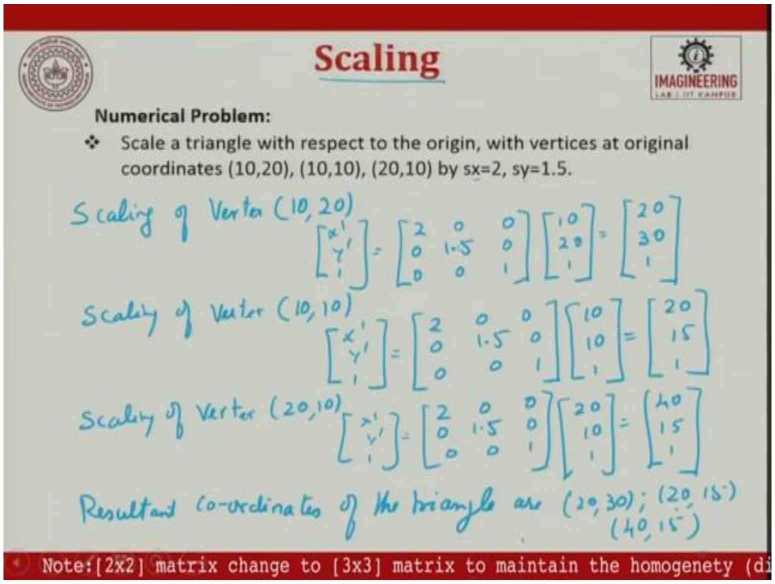

So, let us try to do one numerical in terms of scaling, so that you can understand and appreciate. So, let us take an example of a same triangle. Scale a triangle with respect to the origin, with vertex at original coordinates (10, 20), the same triangle, (10, 20), (10, 10), and (20, 10) is scaled in X by 2 times and Y by 1.5. If you know to do this, then you can know to do only X, only Y, both X and Y, and X and Y same value.

Let us start solving the problem. So, scaling of vertex (10, 20), X^1, Y^1, 1. So, you are trying to take the scaling that is 2, 0, 0, 0, 1.5 times, 0, 0, 0, 1. If you want to do only in X, this value becomes 0; only in Y, this value becomes 0; both same values for whatever case I have said, multiplied by 10 comma 20 comma 1. So, you will try to get the answer 20, 30 and 1.

Now, let us try to do scaling of vertex (10, 10), X^1, Y^1, 1. The new point is nothing but 2, 0, 0, 0, 1.5 times, 0, 0, 0, 1. I multiply it with 10, 10 and 1, so I approximately get the answer as 20, 15 and 1. Now let us try to do scaling of vertex, 20 comma 10. So, X dash, Y dash, 1 is equal to 2 times 0, 0, 0, 1.5, 0, which is multiplied by 20, 10, 1, you try to get as 40, 15 and 1.

So now, the resultant coordinates of the triangle are (20, 30), (20, 15), (40, 15). So, till now we have seen 2 types of transformation, one is moving translation, the other one is scaling.

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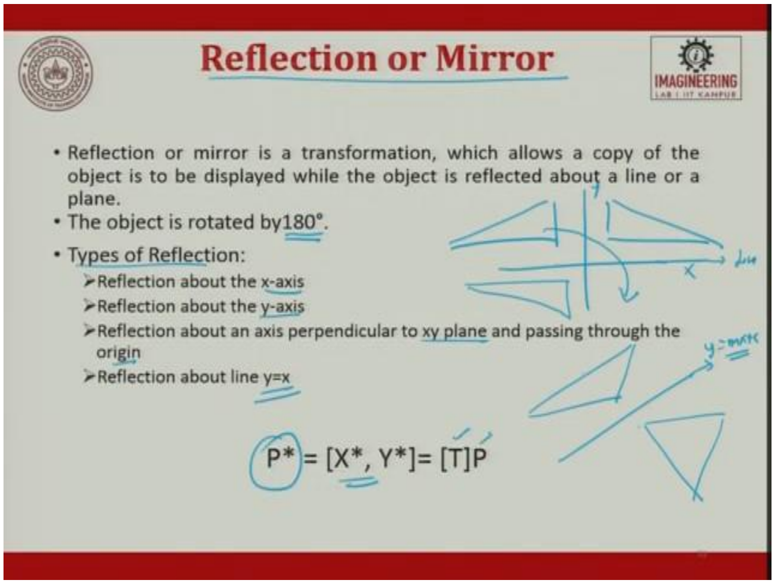

Now let us look into reflection or mirroring. Reflection or mirror is a transformation which allows a copy of the object is to be displayed, while the object is reflected about a line or a plane. So, as I dealt early, it is done about a line. So, this line is Y, X, so you can do it above Y, you can do above X, you can do it about this way, you can then you can do it across, you can do it above, this is an axis you can also do about a line.

For example, this is a line, I have an object like this, so you can have a similar object like this. So, it can be on a line of the axis, it can be on a line where you define a line, it is possible. All these things are possible. It is only you define about a line, the line need not be at the origin. Please understand. So, you can define any line and say about this line, do a mirroring. So that’s what we have written here.

The reflection or mirror is a transformation, which allows a copy of the object to be displayed, while the object is reflected about a line or a plane. The object is rotated by 180 degrees. Later, we will see how do you rotate about certain angles, but here it is a trivial one, so we are about 180 degrees. Then what are the different types of reflection you can have, as I told you, you can have about x, you can have about y. The reflection about an axis perpendicular to xy plane you can have, passing through the origin.

You can have y equal to x, about a reflection about the line also you can do. So, what it does finally is P^* is nothing but X^* and Y^* which are the points. So how do you do it? You just try to do a transformation matrix, multiply with the points whatever is given.

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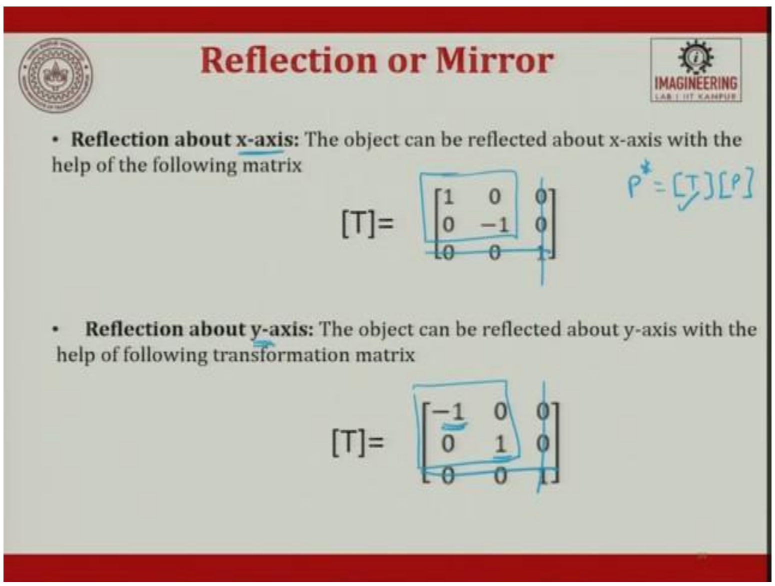

So, for reflection about x, the object can be reflected about X axis with the help of the following matrix. Now, it is very clear P^* is nothing but transformation matrix into P. So now, rather than getting into all troubles, let us start looking into what are all the transformation matrix. So, if the transformation matrix is about x axis,

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

When the transformation is about y axis,

$$[T]=\begin{bmatrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

As far as 2D is concerned, so this always in a transformation matrix, this will become 0, 0, 1 and 0, 0, 1. The transformation major player comes only in this region. So, this is about x and this is about y.

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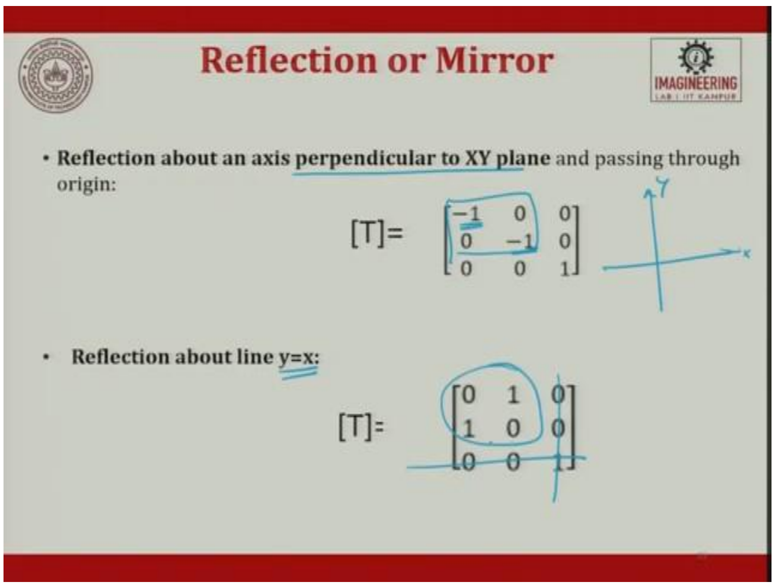

And, if you want to do perpendicular to XY plane, perpendicular to XY plane, this Y, this is X, perpendicular to XY plane, then please note it down both the thing along the diagonal will have a minus symbol. Is it clear?

Then, when we try to do y equal to x, this will be the transformation matrix. So, this is apart, so you try to get this matrix, transformation matrix. So, when we try to multiply, you try to do a mirroring of the image.

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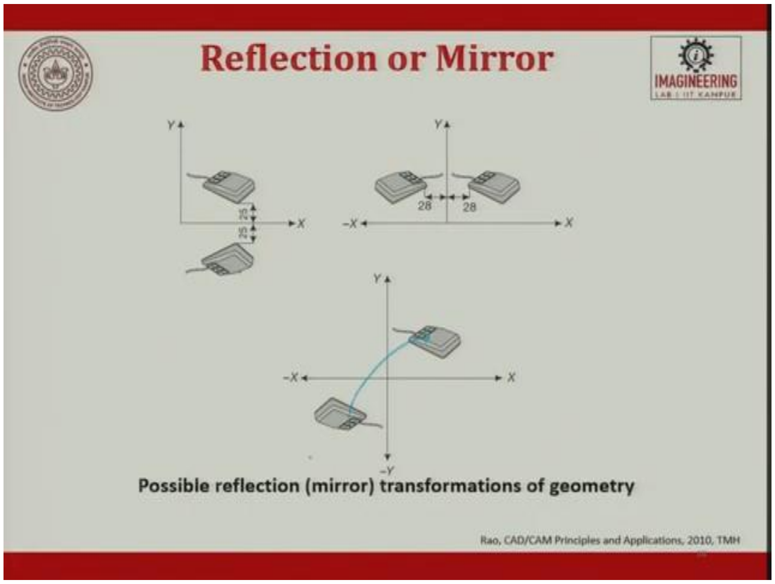

So, this is reflection on the mirror, so you can see 25, 25 is reflected. This is reflection about Y axis, about X axis. This is what I said, this is about this plane, it is not about X and not about Y, but about 2 things.

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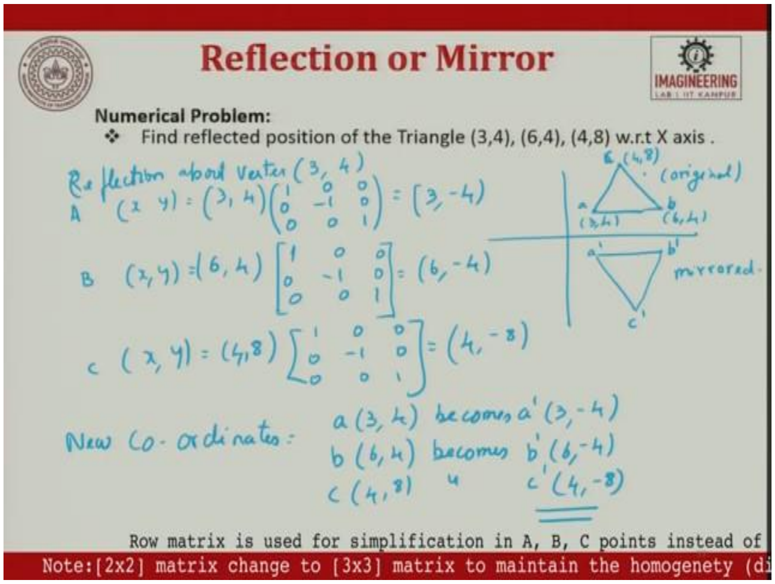

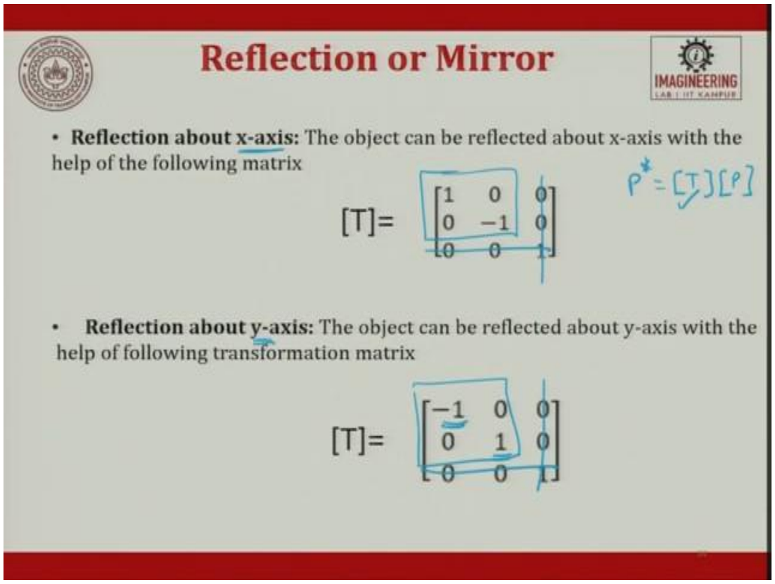

So, let us try to solve a simple problem to have an understanding of the reflection matrix. So, this is the original, so which is (4, 8) and this will be (6, 4) and this is (3, 4). When I do a mirror, it will be something like this. This is the mirrored image, mirrored. So, if I put it as a, b and c, this will be a dash, I put b here and I put c here, b dash and c dash. So, reflection about vertex (3, 4) is, (x, y) equals (3, 4) multiplied to the matrix with values 1, 0, 0, 0, -1, 0, 0, 0, 1. So, what I get is (3, -4)

So then, the next point is going to be (x, y) which is nothing but (6, 4). I do the matrix which is 1, 0, 0, 0, minus 1, 0, 0, 0, 1. So, why is this -1 coming? So what am I trying to do? I am trying to do this transformation about x. So, this is about x, so I get to get the same. So, the answer will be (6, -4).

So the last point, so let us take A point, B point and C point, (x, y) is nothing but (4, 8) multiplied by 1, 0, 0, 0, minus 1, 0, 0, 0, 1. So, the answer is going to be (4, -8). So the new coordinates, new coordinates or the resultant coordinates are going to be, a which was (3, 4) will become a dash which is (3, -4), b which was (6, 4) becomes b dash (6, -4), c which was (4, 8) becomes c dash (4, -8). So, this is the resultant matrix which you will get for this triangle.

So, we have seen the third transformation. First, translation; next, we saw scaling; and the third one we saw, mirroring.

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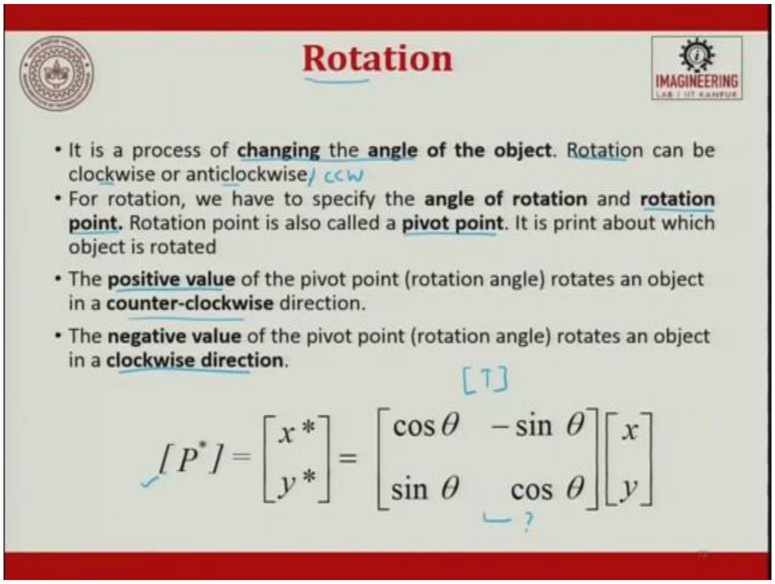

The next transformation in CAD is going to be rotation. If I know to linearly transform an object or rotate an object in 2D space or 3D space, then I can do any type of transformation of the object. Rotation, here when you talk about rotation there are 2 things, one is clockwise rotation, anticlockwise rotation. You have to keep it in mind.

So then comes the question is there a standard? So, while discussing we will see what is the standard. Rotation transformation it is a process of changing the angle of the object. The rotation can happen clockwise or in anticlockwise or counter-clockwise. For rotation, we have to specify the angle of rotation and rotation point, you are trying to rotate about what?

For example, Earth goes around itself in an axis, Earth goes around the Sun in its path. So, these 2 rotations are different. So the important to note is rotation about what, is important. So, for rotation, we have to specify the angle of rotation and the rotation point. The rotation point is also called as the pivot point. It is print about which object is rotated. So, the positive value of the pivot point, rotation angle rotates an object in a counter-clockwise direction or anticlockwise direction.

So, if it rotates in the counter-clockwise direction, it is taken as positive values. The negative values of the pivot point rotates an object in the clockwise direction. So, this is the transformation matrix for rotation about a point. So, you have a new point, new point you get it in the matrices. So, these are the coordinates. How did you get these coordinates, you multiplied a transformation matrix of rotation with respect to the existing point, you got a new point. Let us now see, how did we get this transformation matrix?

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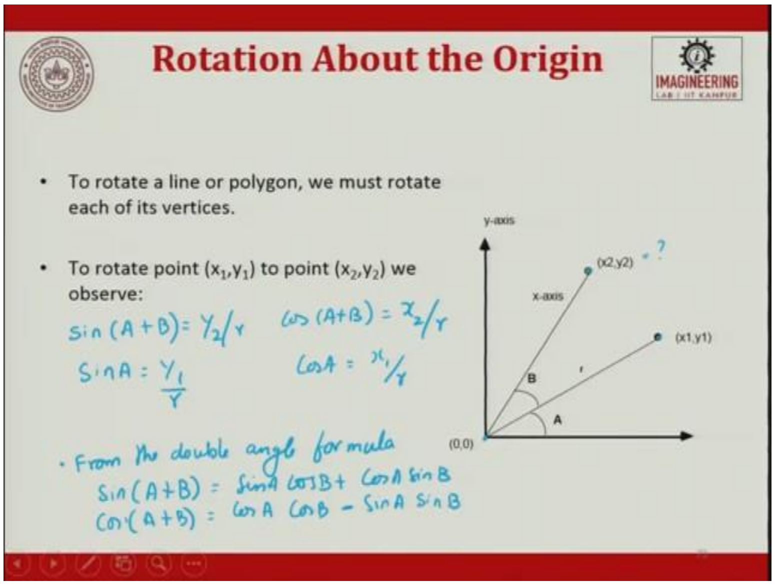

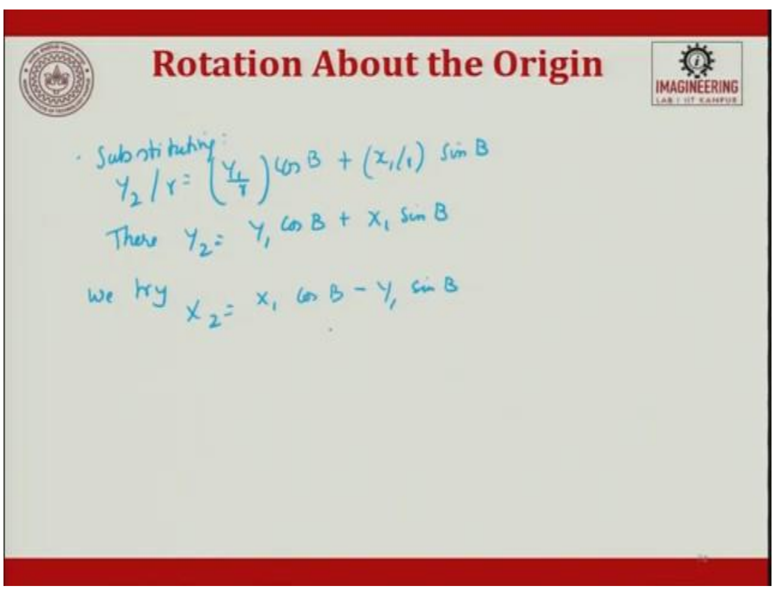

So, there are 2 cases. So, if you rotate about the origin, to rotate a line or a polygon, we must rotate each of its vertices. So here, x1, x2, I have to move from x1 to x2. So, what we do is we try to move x1 directly to x2 or we try to take x1, y1 back to the origin, rotate it and then transform it.

Here there are multiple steps, we will do that later. What is that multiple steps and all, but here as of now you try to understand that this point about the origin is rotated at some angle beta or B to (x2, y2). Now, find out the new coordinates. This is what we are trying to understand. For doing this, we need to find out the transformation matrix.

So, to rotate point (x1, y1) to point (x2, y2), we do the following mathematical observations. We will try to take the 2 formulas, sin (A + B) is nothing but y2/r and cos (A + B) is nothing but x2/r, sin A is nothing but y1/ r and cos A is nothing but x1/r.

What is r? r is the linear length of the point which is displayed from the origin. So, from the double angle formula,

$$\sin (A + B) = (\sin A \, . \cos B) + (\cos A \sin B)$$

So this the formula you know, so then

$$\cos (A + B) = (\cos A \, . \cos B) - (\sin A \, . \sin B)$$

Now, when you substituting back into the equation, so what do you get? Substituting in the equation,

$$(y2/r) = [(y1/r). \cos B] + [(x1/r). \sin B]$$

So,

$$y2 = (y1.\cos B) + (x1.\sin B)$$

and,

$$x2 = (x1. \cos B) - (y1. \sin B)$$

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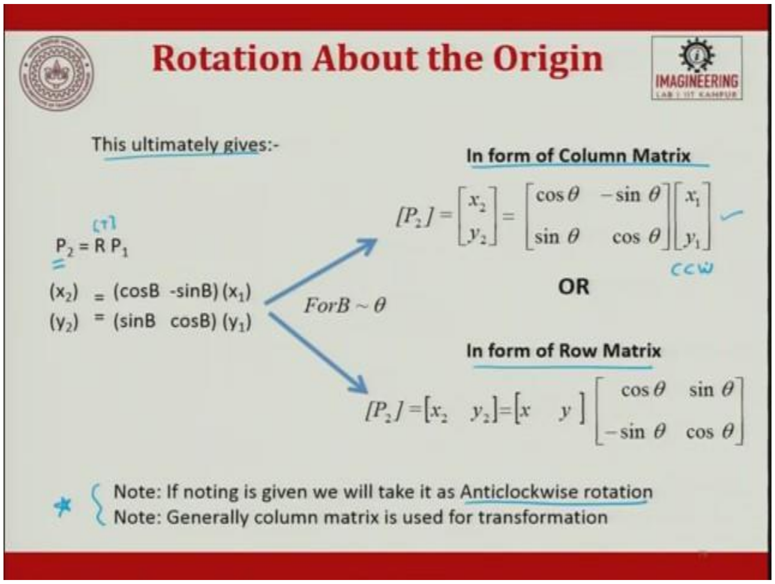

So now what we do is, we try to transform it into a matrices form, so we will ultimately get these two. Whatever transformation we get this comes to, so P2 is nothing but a transformation matrix, this is T or this is rotation. So, transformation matrix into P1 so x2, y2 is nothing but cos B minus sin B x1 and this is y1. So, for B approximately equal to theta, so we can have two things. P2 in the column matrix form,

$$[P_2] = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\ \theta \\ \sin\ \theta & \cos\ \theta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

If you want to write in a row matrix form, then it becomes

$$[P_2] = \begin{bmatrix} x_2 & y_2 \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\ \theta \\ -\sin\ \theta & \cos\ \theta \end{bmatrix}$$

Note, these 2 are very important. Note, if nothing is given, we will take it as anticlockwise rotation. Then the second note is, generally the column matrix is used for transformation. So, this is used and CCW, Counter-Clock Wise rotation matrix is used, if you do not specify anything in particular.

So, in the examination you can expect some small simple matrix calculation, which in turn relates to the CAD.

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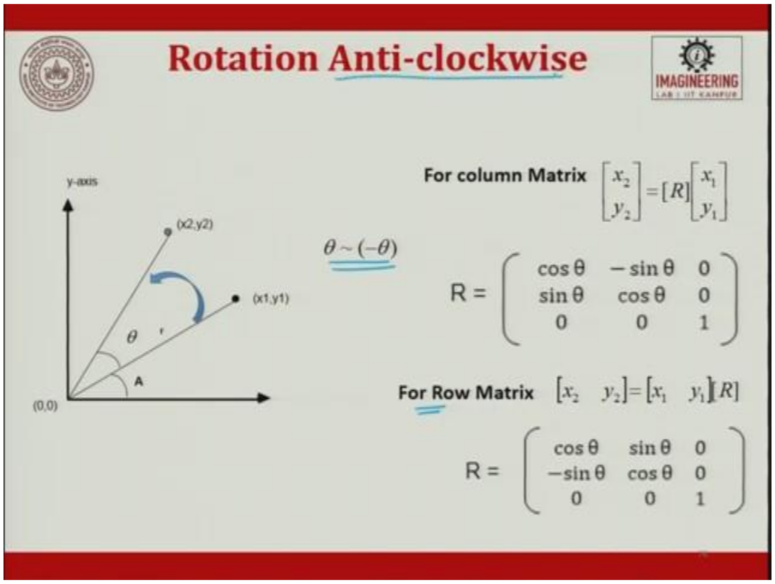

So, if we wanted to express it in the column matrix form, so x2, y2 is nothing but the rotation matrix transformation whatever you do and then it is x1, y1. So, you will write it as this, when it is approximately equal to minus theta so then this becomes like this. So, 2 cross 2 in order to have a proper multiplication becomes 3 cross 3 and when we try to do for row, it becomes like this. So, this is for anticlockwise direction or counter-clockwise direction.

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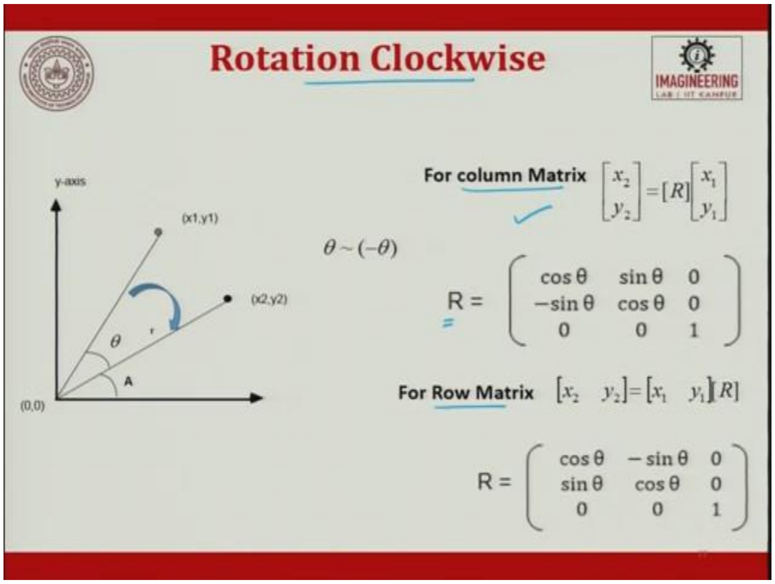

When we have to do the same for clockwise direction, you can see there the theta is approximately equal to minus theta, rotation matrix. Please note down the change in the matrix and in the examination, you should make sure that you choose the proper transformation matrix to get the answer.

So, column matrix and row matrix, we generally use the column matrix and this is how is the rotation matrix used for doing the calculations.

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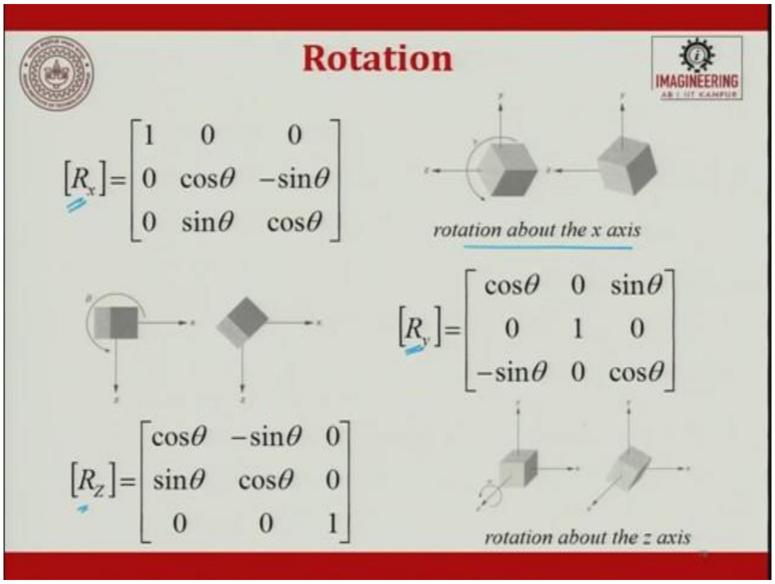

So, this is about rotation about the x axis. You can see the object, this is y and z, and we are rotating about this axis, it is rotation about x axis. So, this is rotation about z axis and this is rotation about y axis. So, you can see the formulas what has derived earlier, so from that derivation we try to see the formulas what gets developed. And why did we move from 2 X 2 to 3 X 3? Because when we try to do it in a 3, in 2 X 2 multiplication, we try to get into homogeneous matrix, homogeneous matrix we get into 3 X 3 form.

So, this is how it is used for rotation about x, rotation about y and rotation about z. The derivation for x is shown. So the next two you can try as home assignment, you will try to get the same answer.

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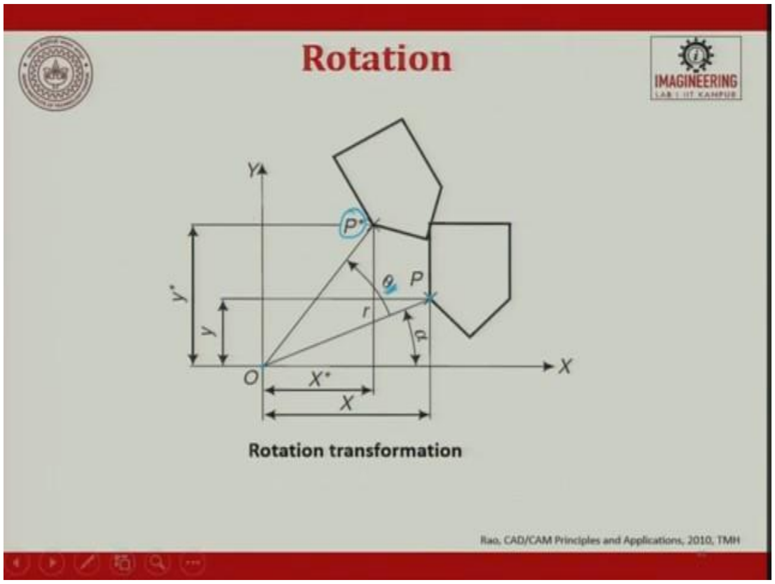

So, if you try, that was a line, now if you try to take it as a triangle and if I have to rotate the P point, P is a point of a triangle wherein which I rotate this point with an angle of theta, it occupies a new point called as P dash. Now, you have to find out what is the angle it has to move with respect to which point. Again, you can put this as a point and rotate the object or the entire object with respect to the world coordinate system and then you rotate by an angle theta.

So, this was the old point, X and Y was the old point. Now because of the theta angle, it becomes X^* and Y^*.

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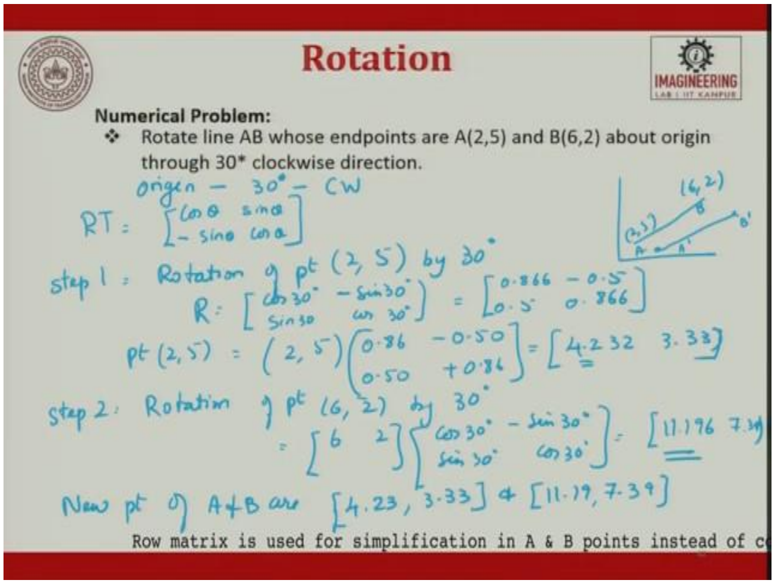

Now, let us try to do a simple numerical and then understand. Rotate a line, a line A, B, whose points are (2, 5) and (6, 2) about the origin by an angle 30 degrees. So, origin is given, so and then 30 degrees. The important thing is what, it has to be clockwise.

So now, let us start writing the rotation transformation matrix. So, I call it this as RT, which is nothing but cos theta, then sin theta, minus sin theta, and cos theta. So, step 1 is nothing but rotate, rotation of point (2, 5) by 30 degrees. So, that is nothing but R is equal to cos 30 minus sin 30, then it becomes sin 30 and cos 30, which is nothing but you get 0.866, it is minus 0.5, then you have 0.5, and then you have 0.866.

Now, when you multiply it with (2, 5), so point is (2, 5), so you have (2, 5) multiplied by 0.86 minus, minus 0.50, 0.50, and plus 0.86, this is nothing but 4.232, 3.33. The same way, if we start doing it for step 2, step 2 is nothing but for the next point. Rotation of point (6, 2) by 30 degrees. So, this will be nothing but (6, 2) into cos 30, minus sin 30, sin 30, and cos 30.

If you substitute all the values, you will finally land up with, you will, if you do all this multiplication, this multiply this multiply, finally what you will get is 11.196 and 7.39. So, AB is the point, so now it will be moved to A dash and B. So, this will be A dash and B dash.

So, this is what happens. So, these are the new points. So, the new points of A and B are (4.23, 3.33) and, (11.19, 7.39). So, this is what happens. So, it is rotated about 30 degrees in the direction of clockwise.

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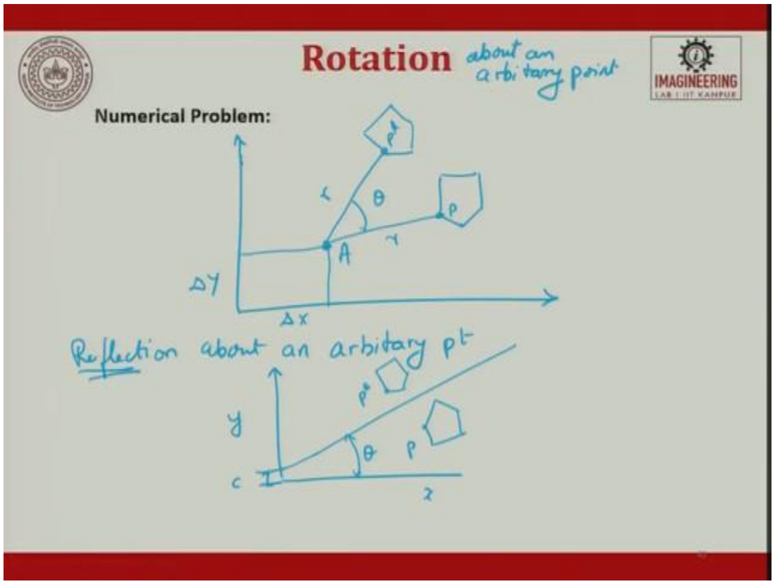

So, if we try to do for arbitrary, so now rotation about an arbitrary point. If I try to do rotation about an arbitrary point. So, for example, if this is the triangle if I have to rotate it this point P about a point A, somewhere in A, so I am defining about this point. It is not about the origin. I am saying that this is delta Y and this is delta X. So, these are the points. So now, from here I give the distance as r, now it is transformed into r so this is theta and the new point is going to be something like this. So, this will be P dash.

So, this is arbitrary about a point we can do rotation. Then we can also do a reflection. So, let me write down reflection about a line, reflection about an arbitrary line. That is also possible. What is reflection? Reflection is 180 degrees, just define that, that is all.

So now if you see there, here you look at it, I have put y, x, I can define a line and then say, this is what is my triangle. So now you see that, about this point if I say, you can start doing it. So, this is the c intercept and this is the theta, whatever it is. So, the point P becomes P^*. So, this is about an arbitrary plane reflection. This is about an arbitrary plane rotation. Thank you very much.