Computer Integrated Manufacturing
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Lecture 10
Geometric Modelling (Part 1 of 2)

Welcome back, to this course. This lecture we will focus on Geometric Modelling. Till now, we have covered introduction part, then we understood the need of computers in manufacturing, then we moved to little bit of transformations, CAD; transformations we just went through it.

It is a very big ocean but we try to understand the glimpse of it undergoing the topics of transformation matrix. We studied transformation matrix, we studied homogeneous matrix and then we also start, discussed about concatenation. In this lecture, we will start focusing more on geometric modelling.

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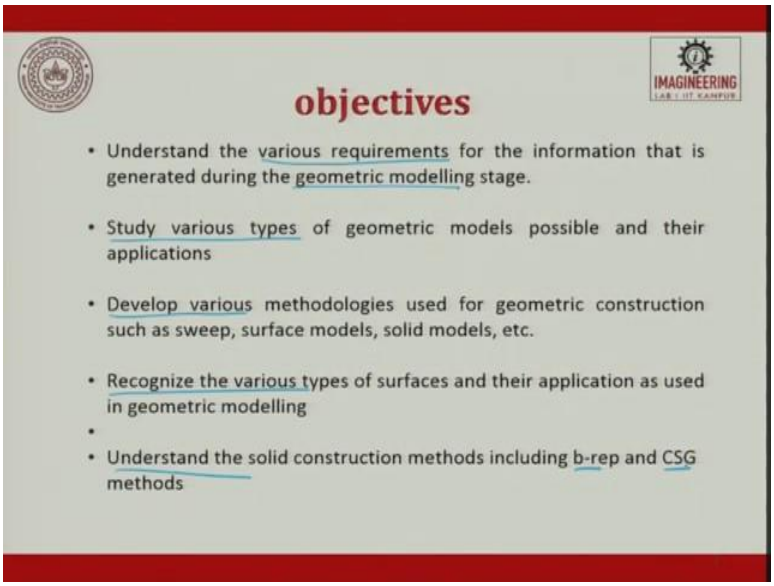

The objective of this lecture, is to basically understand various requirements for the informations that is generated during the geometric modelling. Why is it very important? Because after you make the geometric modelling, then you use for stress analysis, static analysis, kinematic analysis and then you also use it for manufacturing.

So, what are all the informations which you are going to extract. So, that is what we said, for the information that is generated during the geometric modelling stage. Then, we will try to study the various types of geometric modelling, then develop various methodologies, then recognize the various types of surfaces, and finally we will try to understand the solid construction method including b-rep and constructive solid geometry.

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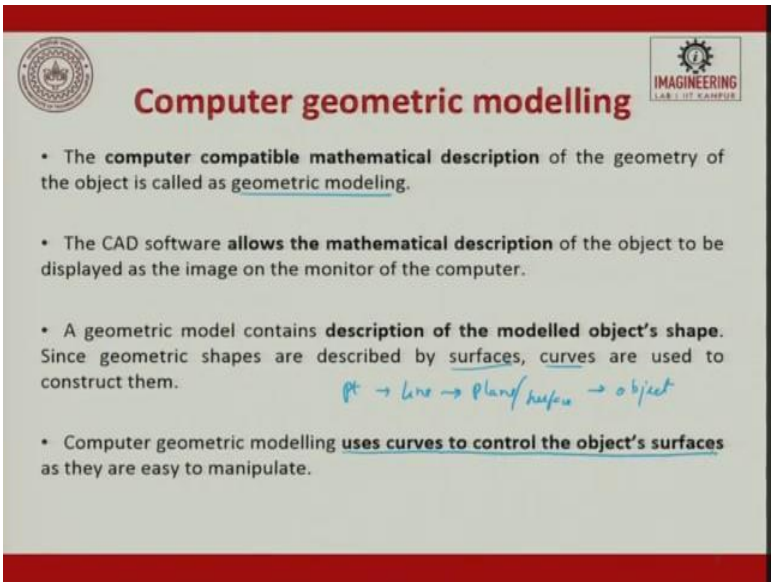

Computer is compatible with mathematical description of the geometry of the objects is called as geometric modelling. So, this tries to explain to us what is geometric modelling. The computer compatible mathematical description of the geometry of the object is called as geometric modelling. The CAD software allows the mathematical description of the object to be displayed as an image on the monitor. The geometric model contains description of the modelling object's shape. Since geometric shapes are described by surfaces, curves and are used to construct them.

So, it is like this point, point to line, line to plane, plane is nothing but surface, surface, several surfaces leading to an object. So, that is what. A geometric model contains description of the modeled object's shape. Since geometric shapes are described by surfaces, and curves are used to construct them. Computer geometric modelling uses curves to control the object surface as they are easy to manipulate, use curves to control the object's surface.

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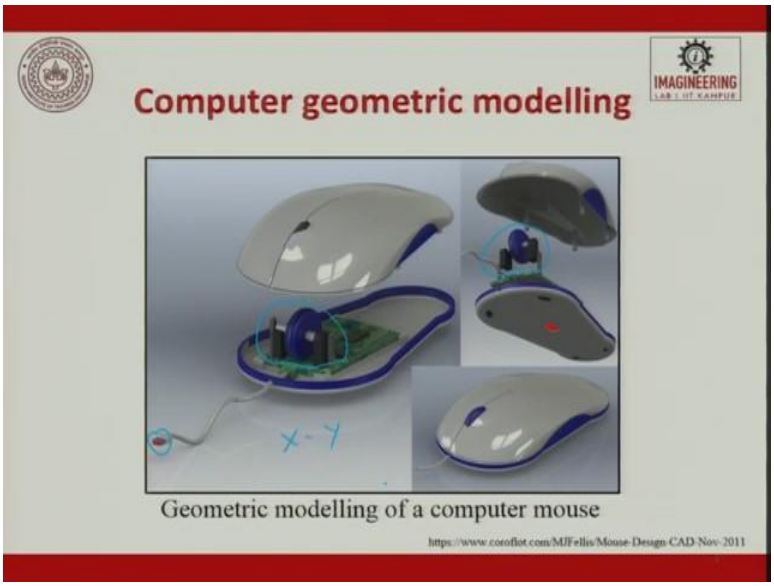

This is a computer geometric model for the mouse. So, you can see here, they have brought in the curved profile. They have given a glossy look. Here are the wheels which is used for scrolling, so this is the scrolling wheel. And then it has, this is the USB which is attached to the port of a computer. So here, you will have to move in the X and Y direction. So, all these things are read in the scroller and as and when you move, there is a lead screw which is, there is a linear scale which measures the X and Y moment on the screen and which is compatible with the mouse moment.

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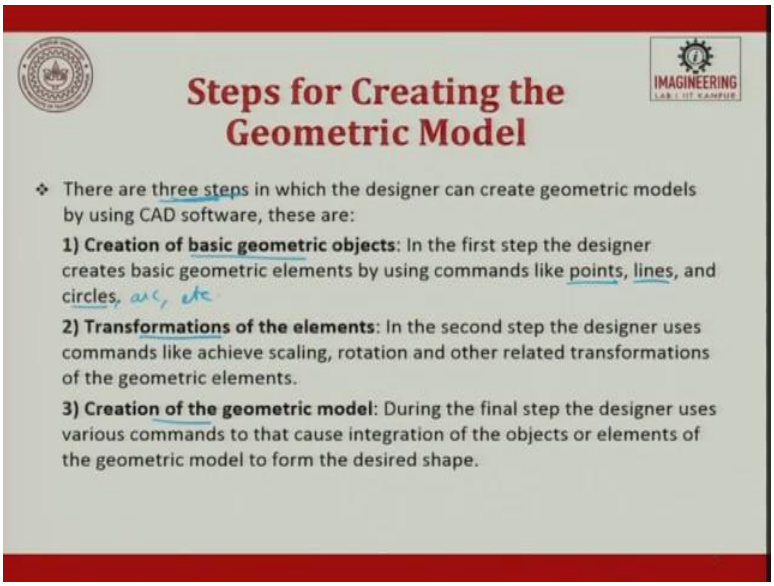

So, the steps involved in creating a geometric model, there are 3 steps in which the designer can create geometric models. Creation of basic geometric objects, in the first step, the designer creates basic geometric elements using commands like points, lines, circles, arcs, etc. This is a creation of a basic geometric model.

The next one is transformation of the elements. In the second step, the designer uses commands like archive scaling, rotation and other related transformation of the geometric elements. The point number 3 is creation of geometric models.

During the final step, the designer uses various commands to that causes integration of the objects or elements of the geometric model to form the desired shape. So, first is basic geometric creation, next is transformation which we studied in the previous lecture, then finally, creation of a geometric model. So, these 3 are the steps a designer follows in creating a model.

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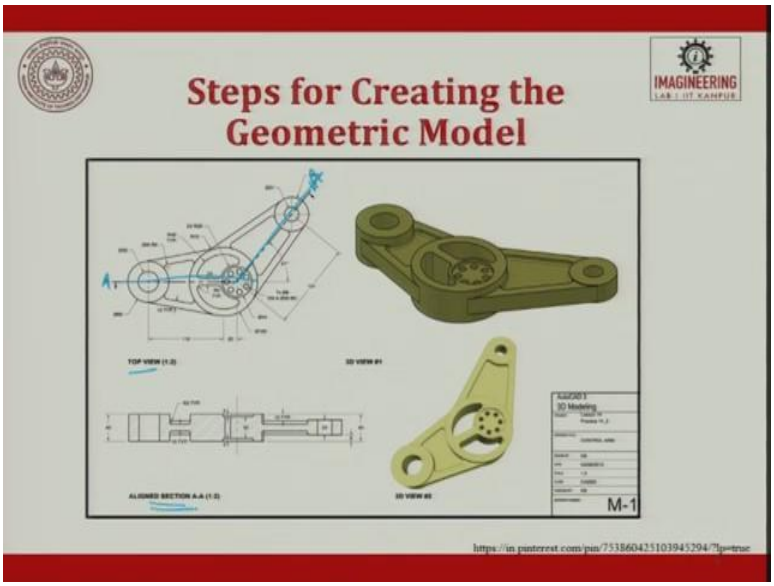

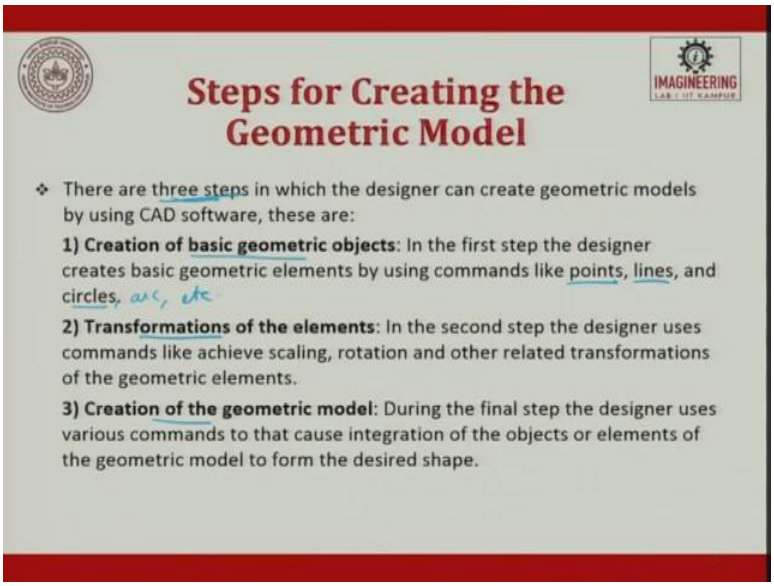

So, whatever I have described till now, you can see here. This is a 3D view generated and for this 3D view it is the top view for the object is drawn here and then you will have a aligned section which is done AA for the object. Where is the section done, the section is done here, this is the section for the object. So, you have drawn, how is the aligned section AA.

So, this is, this clearly states you start from a basic geometry, do the transformation of the elements and then create the geometric model.

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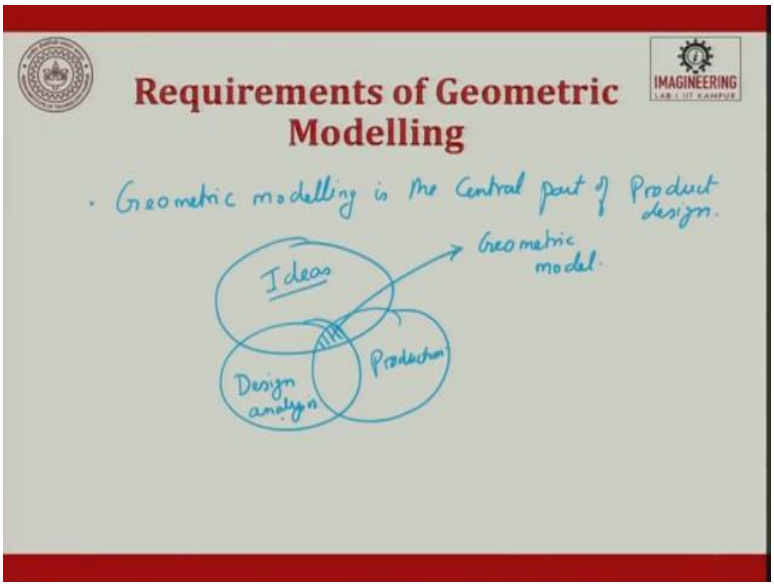

When we try to put the requirements for the geometric model, we will see geometric modelling is the central part of product design. So even the product design, you have ideas, you have design analysis, and you have production. So, geometric model comes here, which is a A union B union C, so it is nothing but ideas, design analysis and production. For all these things, geometric model is very important.

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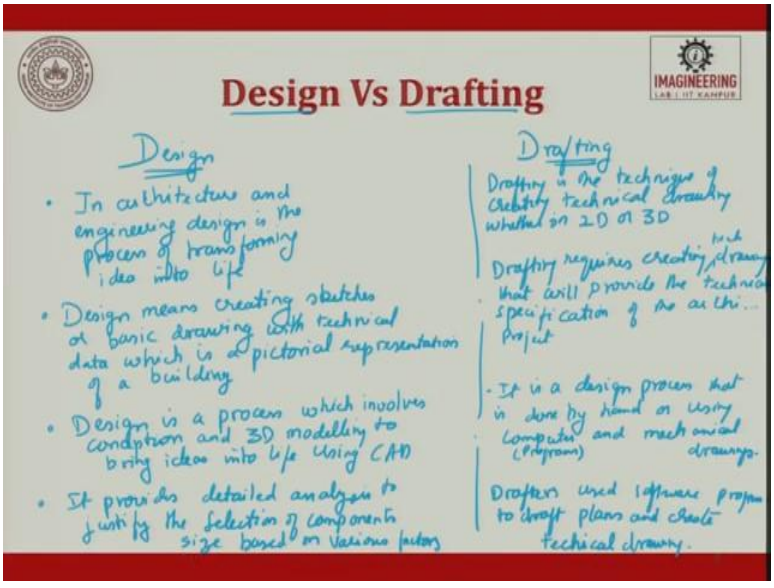

When we see what is the difference between design and drafting, so let us first list down all in terms of design and then let us compare with drafting. In design, in architecture and engineering, design is the process of transforming idea into life. The second point is, design means creating sketch or basic drawing with technical data which is a pictorial representation of a building.

Design is a process which involves conception and 3D modelling to bring ideas into life using computers, or I would put it as CAD. The last point is, it provides detail analysis to justify the selection of components size based on various factors.

When you draw a counter part for it, drafting is the technique, technique of creating technical drawing, whether in 2D or 3D. Next, drafting requires creating drawing, creating technical drawing that will provide the technical specification of the architectural project. The third point is, it is a design process that is done by hand or using computer, mechanical drawing. Computer, here I would say computer, in the sense I am talking about programs, mechanical drawings.

The last one is, drafters use software program to draft plans and create technical drawings. This tries to clearly tell you the difference between, what is design or design versus drafting.

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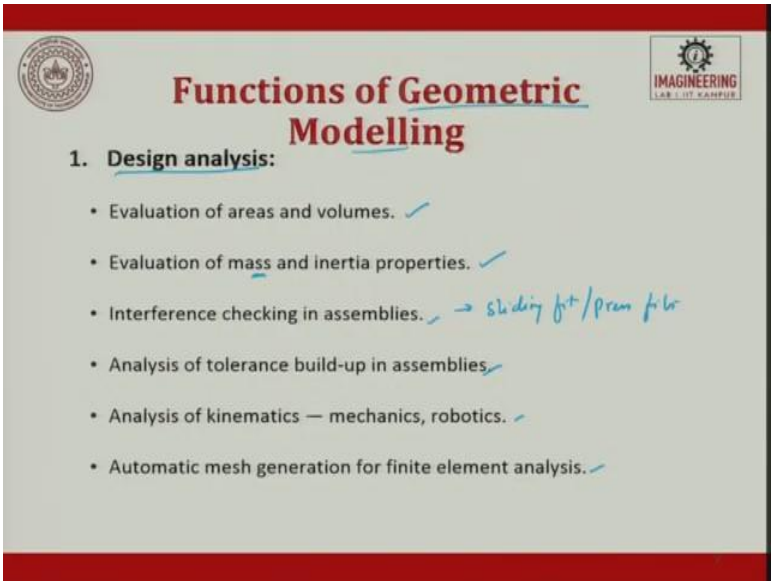

When we try to look into the functions of geometric model, it helps in design analysis in evaluating area, volume, evaluation of mars, mass, inertial property, in interference checking in the assembly. So, when we talk about sliding fit, sliding fit, press fit, so all these things, so interference checking in assemblies, then analysis of tolerance built in assembly, then analysis of kinematic like mechanisms and robotics, automatic mesh generation for finite element analysis. These are some of the design analysis where geometric model are used or functions of geometric modelling is used.

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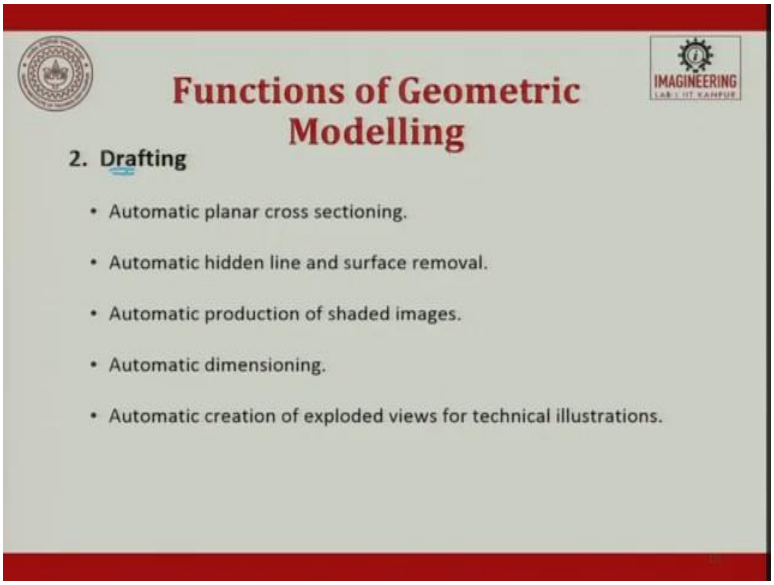

When we talk about in drafting, it is used in automatic planar cross sectioning creator, automatic hidden lines and surface removal, automatic production of shaded images, automatic dimensioning, automatic creation of exploded views for technical illustrations. So, these are some of the drafting help, drafting functions a geometric model does.

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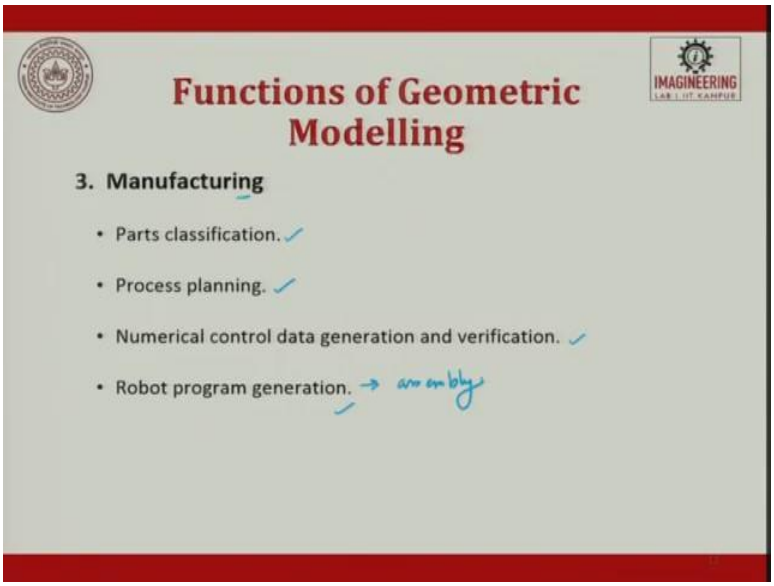

When we talk about manufacturing, it tries to help us in part classification, process planning, numerical control data generation and verification, and robotic program generation, this is basically for assembly operations.

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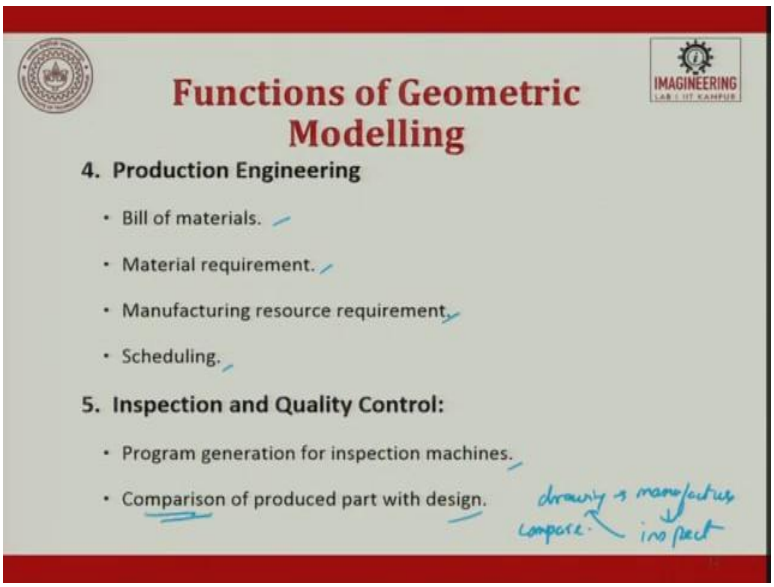

In production engineering, it is used for bill of materials; material requirement, manufacturing resource and requirement, and scheduling. In inspection and quality control, for program generation for inspection machines, and comparison of production parts with

design. So you draw, you manufacture, you inspect, and compare. So, this is what we are saying, comparison of the produced part with the design.

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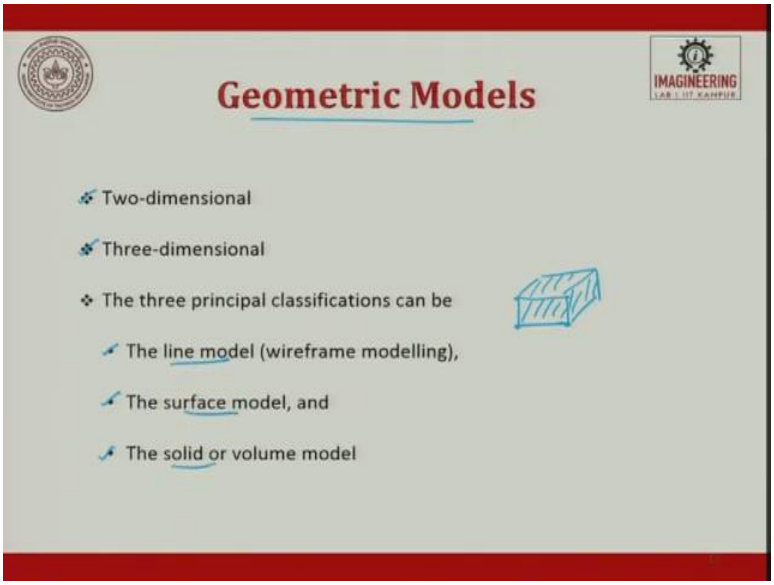

So, the geometric modelling can be 2D, 3D and in 3D, in 3 principal classifications can be line model, surface model, and solid model. So, it can be line model. This, I started with a line and I generated several lines and formed a closed polygon. Now, this closed polygon is called as a surface. Now, several of these surfaces put together forms a solid or a volume. So, you will have line method. These are the 3 principals, surface model and solid model, or volume model.

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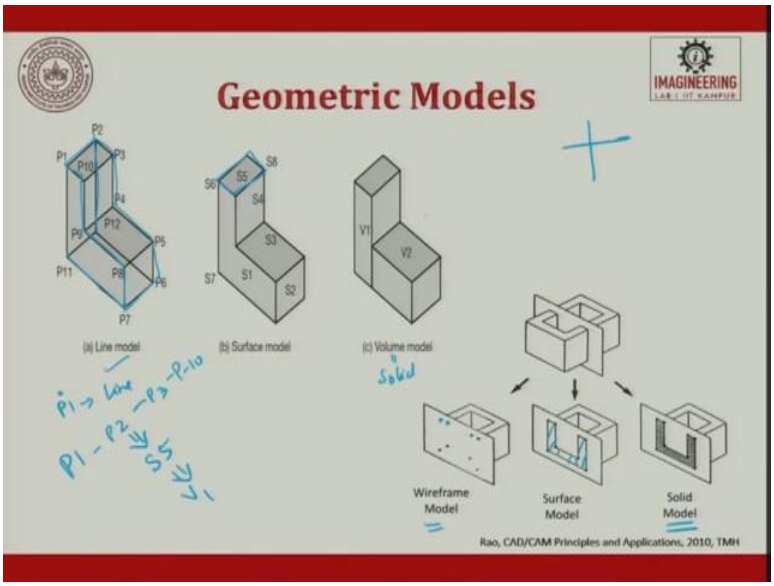

This is a line model. Line model is point, you express in points, points and then you express in lines. So, you join several points to lines. So, for example, here, we start with P1, P2, P3, P4, 5, 6, 7, 8, 9, 10, 11, so it is 10, and then I have 11, and then I have a 12. So, this is a line model. So, then several of these lines, for example, P1, P2, P3, P10, all these fellow joined together, these points and lines joined together to form a surface called as S5, and the S5 is part of V1. So, this is a line model, this is a surface model, and this is a volume model.

Wherever you want to do a drag analysis, wherever you want to find out the aerodynamic flow, what is the response of the aerodynamic flow, we will go for surface modelling. Wherever we wanted to calculate the mass, volume, the static analysis, dynamic analysis, we go for the solid model or surface model, solid model or surface model.

So, when we try to take an object, when you try to draw and when we try to take a cross section of it in a wireframe model, what you will see is only 4 points, because a line when you section it, you see a point. And then, when it is a surface, you see a set of lines joined together to form a cross section surface. And when you try to look, this is only the outer portion so you get the surface. And when it, when the internal surface is also colored, it becomes a solid model.

So, a cross section you do of a cuboid, so you will try to see in wireframe dots. Here in surface you see lines, and the lines will be covered by material property for solid modelling.

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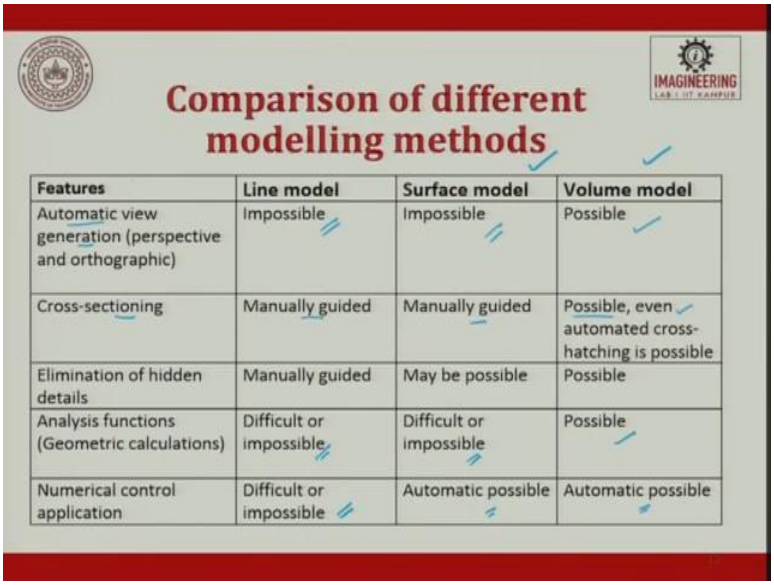
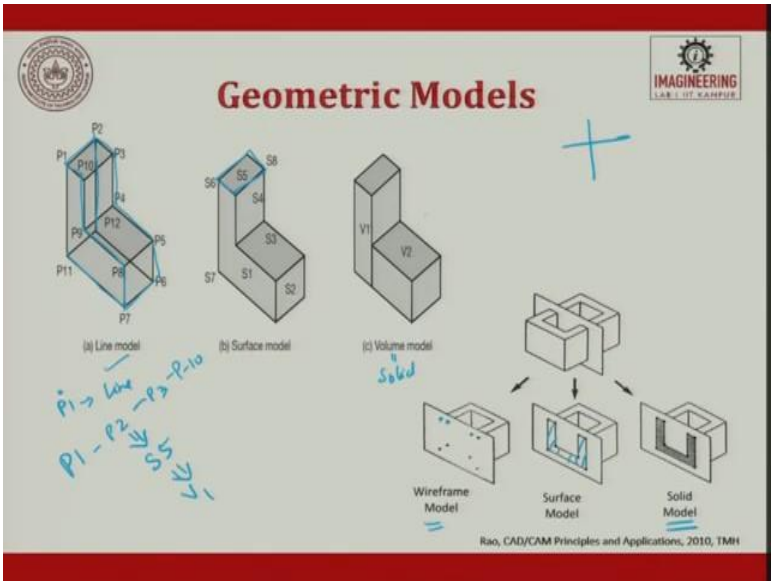


Features	Line model	Surface model	Volume model
Automatic view generation (perspective and orthographic)	Impossible	Impossible	Possible
Cross-sectioning	Manually guided	Manually guided	Possible, even automated cross-hatching is possible
Elimination of hidden details	Manually guided	May be possible	Possible
Analysis functions (Geometric calculations)	Difficult or impossible	Difficult or impossible	Possible
Numerical control application	Difficult or impossible	Automatic possible	Automatic possible



Now let us try to compare various features, with different model which are available. When we want to do automatic view generation, it is impossible to do in line model, surface model also it is impossible to do, but in volume model or solid model it is possible to generate automatic view generation.

When we are trying to do a cross section, it needs manual guidance in line, it needs manual guidance in surface. It is possible even automated cross hatching is possible in volume, so it is more automatic friendly or system generation friendly. Elimination of hidden details, it has to be done inline, manually done. Surface, maybe possible, in volume, it is definitely possible.

You look into it, here the hidden lines is seen, here if it is possible, it can be seen, here it is not at all seen.

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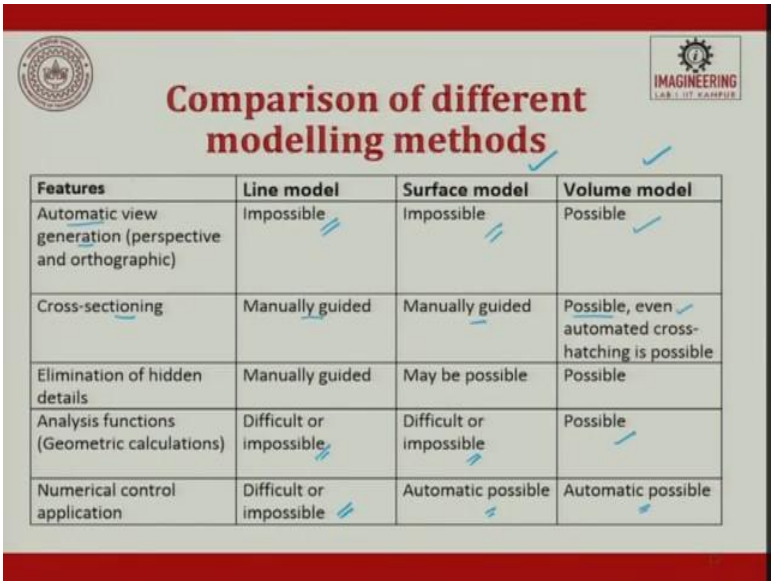

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Then analysis function, difficult or impossible to use, difficult or impossible to use, here it is possible for geometric calculation, mass, volume, inertia, whatever it is. Numerical control application, it is difficult, line model; surface model is possible; and in solid model, it is definitely possible. So, it looks like in reality, these 2 models are going to take a big way as compared to that of line modelling. Then why to study about line modelling?

It occupies a very small space and this is how the CAD initially develop from a line to surface, surface to solid. If you can know this, the way how the technology evolution has happened, then it becomes easy for you. If you have a complex problem, to split the complex problem into lines or surfaces or solid, to get the output whatever you want.

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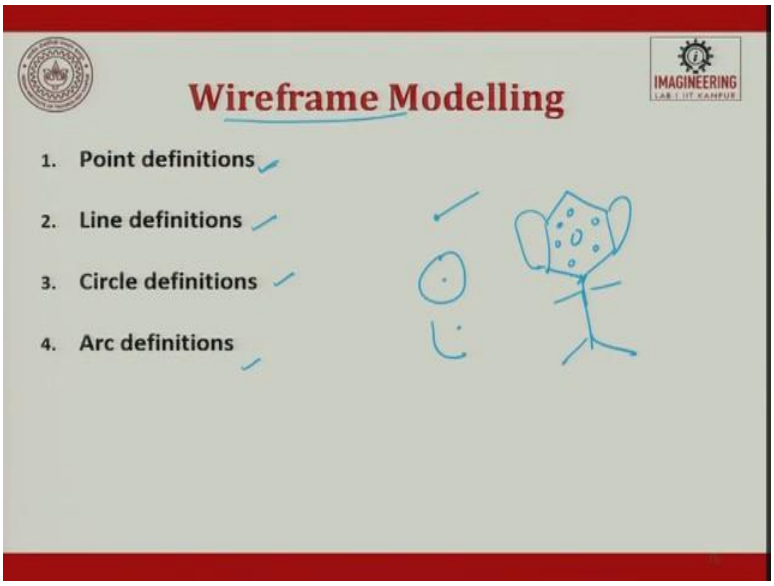

Let us now first look into a line model, which is otherwise called as wireframe model. So here, we will try to define a point, a line, a circle and an arc. As I told you, if I know to define a point, if I know to define a line, if I know to define a circle, if I know to define an arc, then I can split any complex geometry into these lines and then I can try to split this into lines, circles, arcs and then get the data points.

So now let us see how to define a point, line, circle, and arc in wireframe model.

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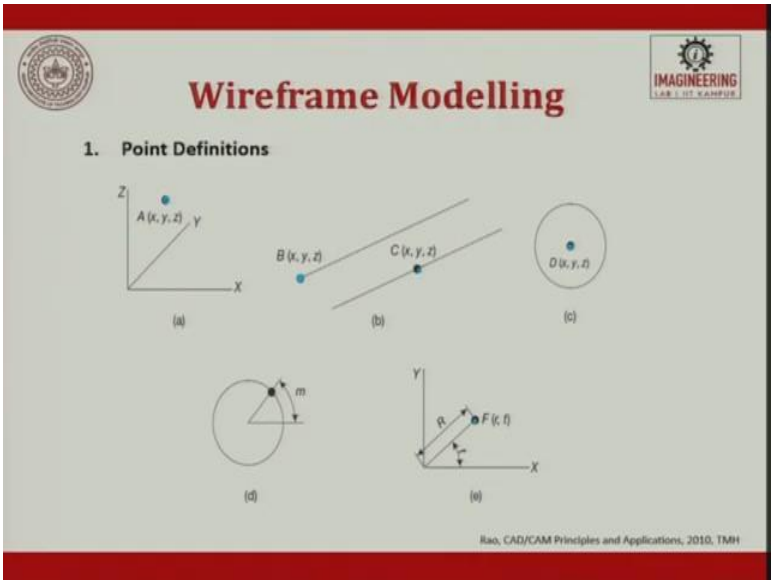

So now let us see, how to define a point. So, a point can be defined in an XY plane, XY, or you can define a point. So, you can define x, y, z and you can try to say parallel to this line, draw a point, possible. You can try to draw a point center of a circle, you can try to draw a point in the circumference of a circle, you can try to draw a point and then you can also try to tell what is its displacement from the origin, and at a what angle it is moved out. So, these are all different ways of defining a point in wireframe model.

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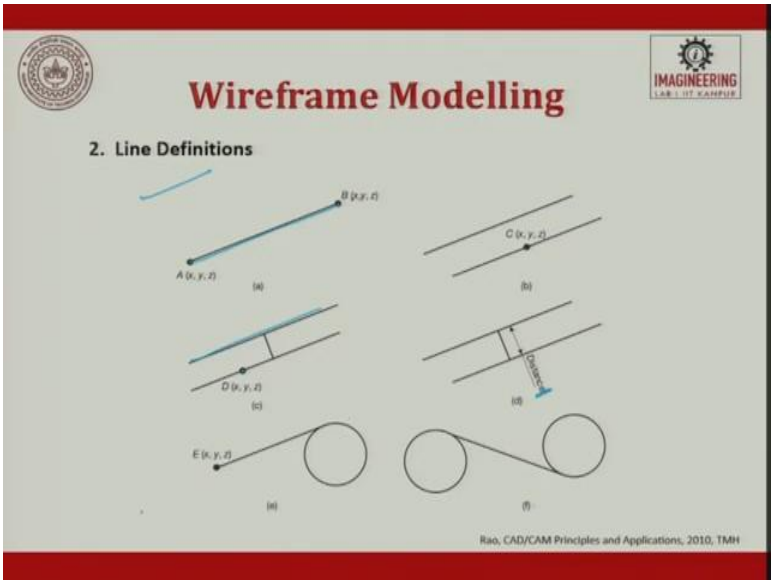

When you try to define a line, 2 points joined together, the shortest distance joined together forms a line. So, you can have 2 points, these 2 points have joined by a line and then you can try to have a point and a line, you draw a parallel line or you can try to have a point and a perpendicular to a line, you can have one more line. You can try to say that line, draw a parallel line and then give to, try to define the offset in distance; or you can try to have a circle and then tangent to the circle becomes a line.

Then you can have 2 circles drawn and tangent to one circle and then meeting to the other circle is also a possible definition for a line. So, a line can be represented by 6 different ways. It can be 2 point joining, it can be a point and a line and a parallel to it, and perpendicular to it, a circle and a tangent to it. These are all different ways of defining line in wireframe model.

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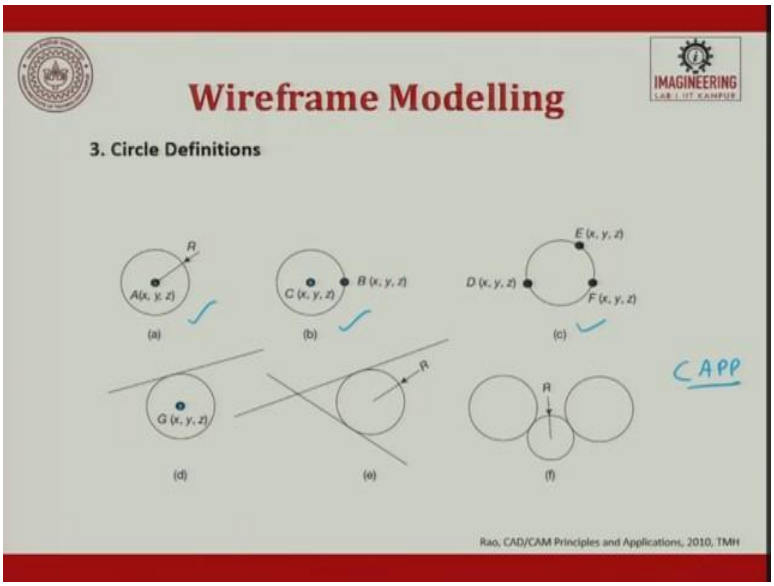

Let us now try to define a circle, a center and a radius; a center and along the radius one point or 3 points on the circumference, or a circle, a center and a center, and a line which pass tangent to the circle or you can try to have 2 lines which are intersecting and then they are trying to draw within these 2 lines. The tangent of these 2 lines form a circle or you can try to define a circle, then between the 2 circles, a point which contacts these 2 and then with that and then keeping a radius you can try to develop.

So, these are the different ways of defining a circle. Generally, we use these 3 things. Other things are used for CAPP, computer aided process planning we will see later. There we do not like to write the geometry in terms of G codes and M codes. We write the entire program in terms of features, so where we use this technique.

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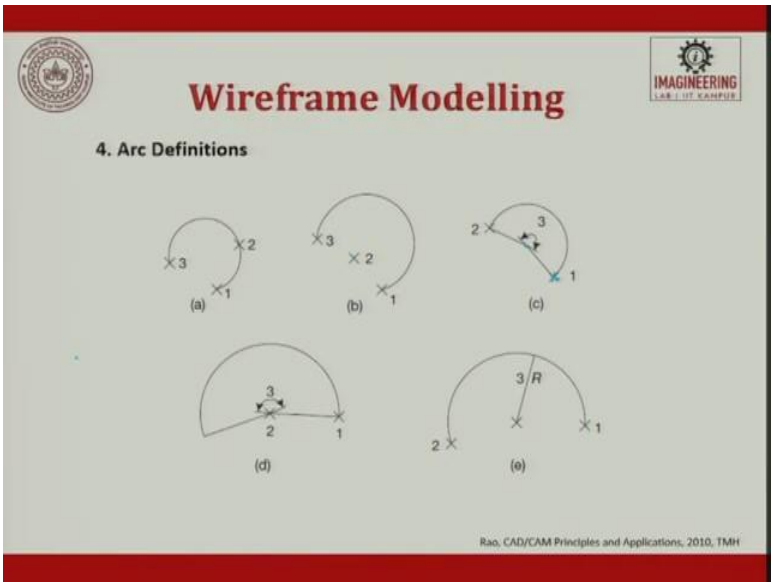

And then when we have to define an arc, 3 points we can define an arc or we can define a center point and 2 points. We can also try to define a center point and you can try to draw a line to it and then define this angle. You can, or define 2 points and then these 2 points meeting and then this angle can do. Then you can also try to do, 2 lines, a line with 2 points where one point is the center and then you can try to define the angle which is there, or you try to define start point, end point, center point and a radius. So this, these are the different ways of defining an arc.

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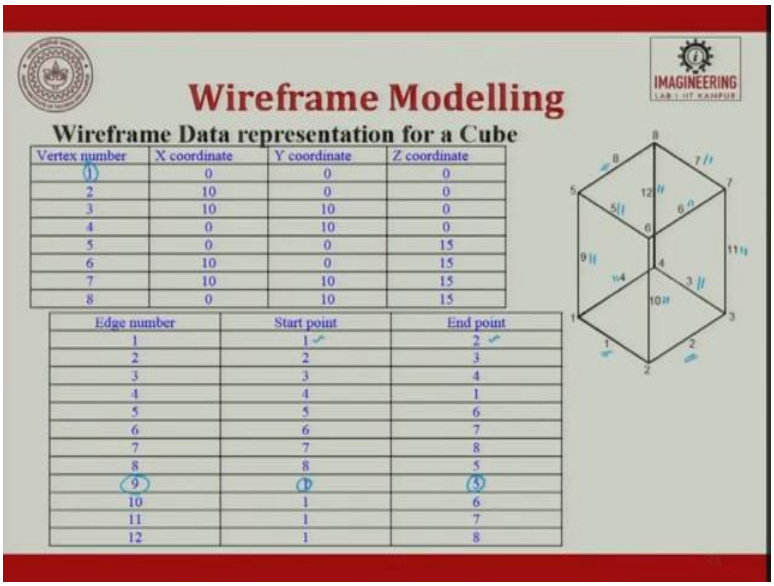

So, in a wireframe model, the, let us see the amount of data which is stored in a wireframe model. So, this is a cuboid, so this cuboid has 1, 2, 3, 4, 5, 6, 7, 8 vertices, so and then these vertices are joined by lines. 1 and 2 is joined in the first line; 2 and 3, second line; 3 and 4, third line; 1 and 4, 4th line; 5 and 6, fifth line; sixth, seventh, eight. Then I am trying to join 5 and 6 with 1 and 2, 9, I will have 10 then 11 and then I will have 12.

So, in wireframe data representation, you will have, these are the vertices, what are each vertices, what are their coordinates, 0, 0, 0. Then you try to define an edge. So, in the edge, what is the starting point and what is their ending point. I am just trying to say how is the data represented in wireframe in your computer? So, you first define the points, then you define the lines and then these are the edges. So, these edges will try to tell what is the starting point and the ending point. So, you get starting point and ending point.

Let us take an example of 9, where is 9 here? 9 is here. So, the starting point will be 1 and the ending point is 5. So, like this the data is stored using wireframe model.

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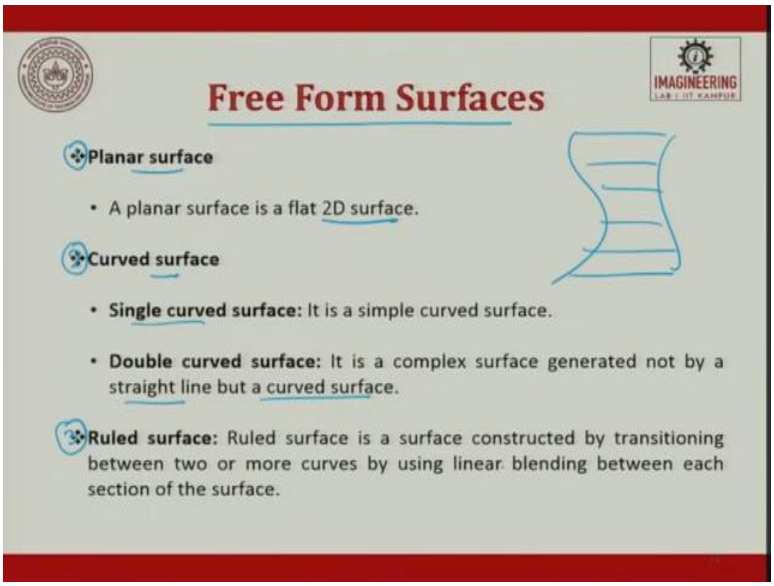

The next one is free form surfaces. All these thing what we studied will now, it is only engineered and geometries which can be defined easily through a mathematical equation. In reality, you have lot of free form surfaces. So, defining these free form surfaces is a challenge. So, in free form surfaces we have planar surface and curved surface. Planar surfaces are flat 2D surfaces which can be mathematically easily defined. In curved surfaces, you have 2 things, one is called as a single curved surface. This is a simple curved surface; and you also have something called as double curved surface.

Double curved surface is a complex surface generated, not by a straight line but by a curved surface, we will see the example later. So, the other one, so planar surface, the second one was curved surface, the third one was ruled surface. Ruled surface is a surface constructed by transitioning between 2 or more curves by using linear blending between each section of the surface. So, it is something like this. So, ruled surface is a surface constructed by transitioning between 2 curves by using linear blend surface, each section on the surface.

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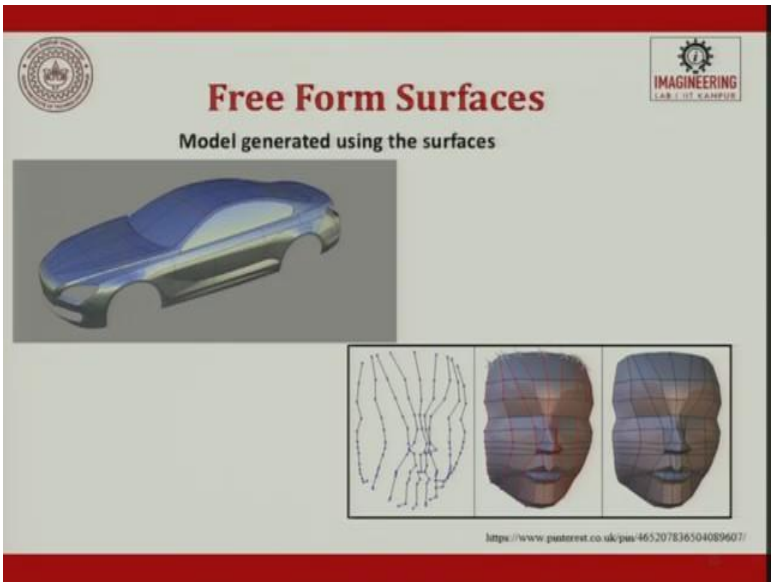

So, this is a model, which is generated for surfaces. So, if you look at real time the objects which are there it is all very complex, it is all free form surfaces. Just by planar, you cannot define. So, you see a combination of several modelling surfaces are available and these are wireframe. So, from wireframe, you can see how these next lines are joined in 2 planes. These are all along a straight line, these are all ruled surfaces joined and now I remove all the points and then I get the face mask. You look at it how complex is it to define a face. And it is practically impossible to define a facemask by a simple equation.

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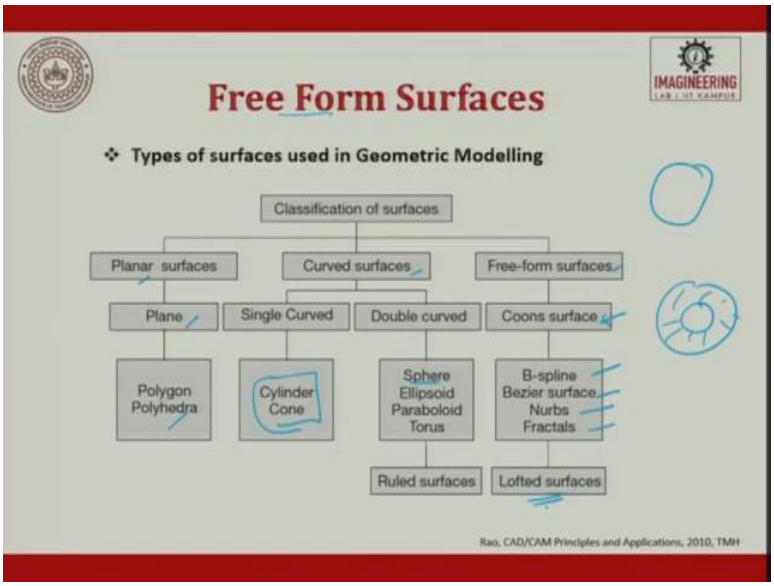

So, when we look at free form surfaces, they are classified as planar surface, curved surface and free form surface. The planar surface is by a plane and you will have polygons and polyhedra, which is easy to, for defining. When we look at the curved surface, you will have single curve, double curve. Single curve, the examples are cylinder and cone; and for double curve, you will have hemisphere, ellipsoid, paraboloid and torus, torus. So, these all come under double curved surface. And in this classification comes the ruled surface.

In the free form, truly free form surface, we have coon surfaces which we will see later. So, under this category, you will have B-spline, Bezier, NURBS and fractal design. And we will try to generate the surface by using lofting techniques.

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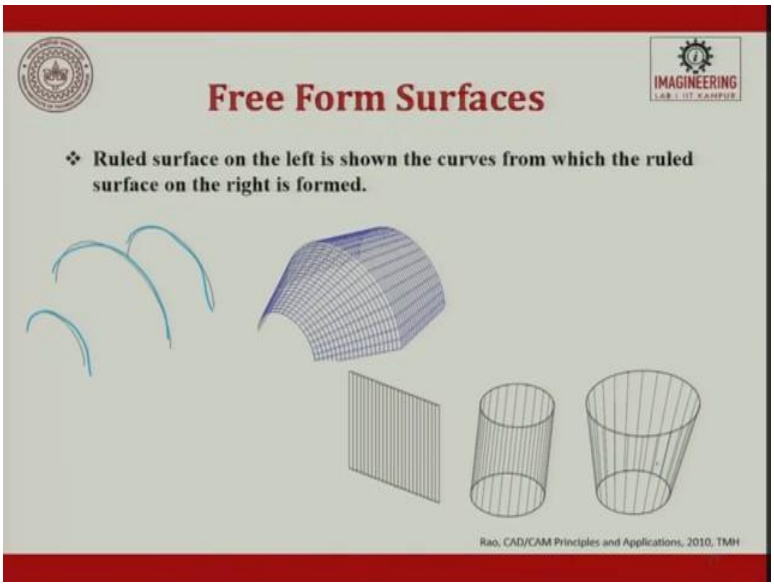

A ruled surface on the left is shown the curves from which the ruled surface on the right is generated. So, this is what is the curved surface. So you will see here, these are the rulings which are made, such that you can try it. So here you defined only the curves. So now the ruled surfaces are done by it. So, if you look at it a line and a line, a rule is done; a circle and a circle, rule is done; a larger diameter, smaller diameter, a frustum of a cone or a tumbler is done.

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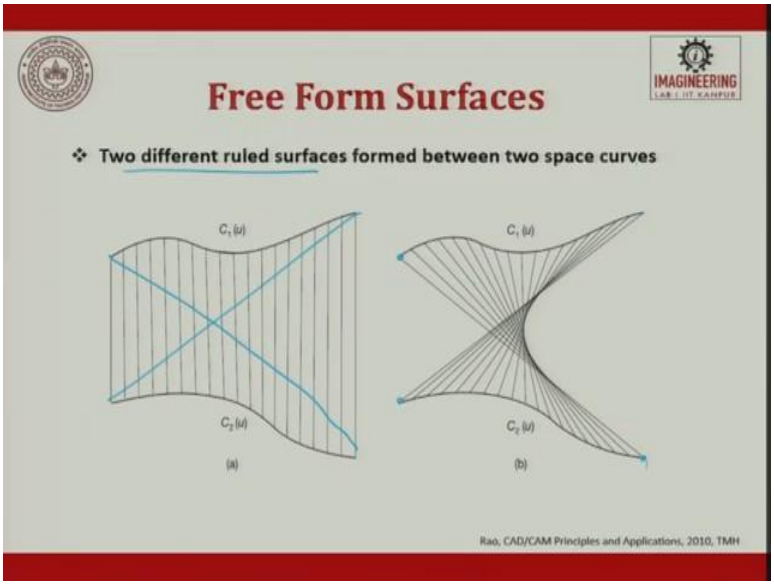

When we use 2 different rule surfaces form 2 different ruled surfaces form between 2 different face shape curves, so you have a curve one C1, C2. If you want to join C1 and C2 start point to start point, end point to end point, you get a rule surface to this. If you want to define start point and then the next surface, these 2 surfaces you will link and these 2 surfaces you link. You see in the space you will have 2 different curves which are getting form, so 2 different rule surfaces formed by between 2 space curves. So, start point end point, start point end point you see different surfaces getting formed.

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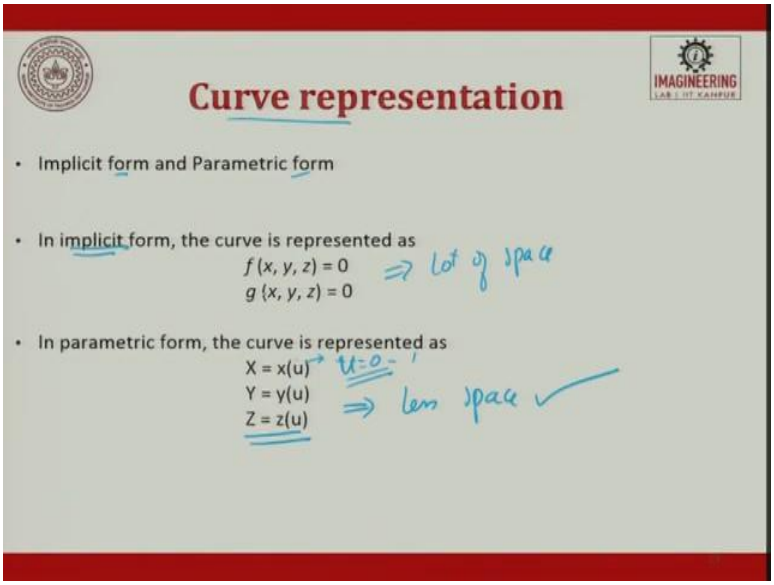

So now, let us see how do we express or represent a curve, because straight line is easy you will do. But now let us see for a curve representation.

So here, you have 2 forms of representation, one is implicit form, the other one is parametric form. Implicit form, the curve is represented as,

$$f(x, y, z) = 0$$

$$g(x, y, z) = 0$$

It is implicit. In parametric form, the curve is represented as,

$$X = x(u)$$

$$Y = y(u)$$

$$Z = z(u)$$

where u takes a value between u is equal to 0 to 1. So, now here what is the advantage, you are representing the entire X, Y, Z in terms of a single variable which is a parameter, where it takes a value 0 to 1. So just by defining this, you can define a curve.

So, this will try to occupy lot of space and this is going to occupy less space. So, in CAD we always prefer less space, so all the CAD softwares will parametric representation rather than implicit form.

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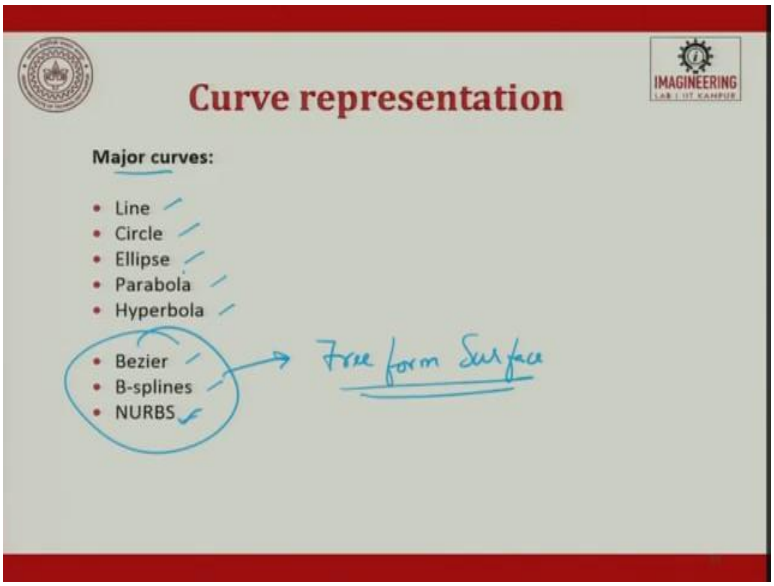

So, the major curves are lines, circles, ellipse, parabola, hyperbola, Bezier, B-spline, NURBS, if you see this, all these fellows form under free form surfaces. Today when you try to do any complex surface, we try to use Bezier curve, B-spline curve, or NURBS. Today we exhaustively use NURBS. Bezier and B-spline were used initially, because today any object you take I said it has lot of free form surfaces.

So, free form surfaces, you will be using Bezier curve, B-spline curve or NURBS curve. But for our academic purpose, we will try to understand also line curves, circle, ellipse, parabola and hyperbola.

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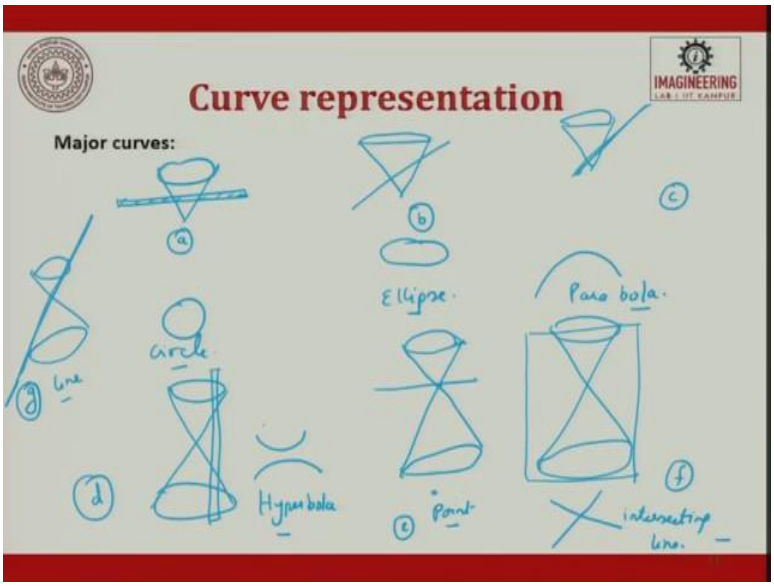

So, let us see some of the curves. If you try to take a section at this point, so what gets generated is a circle. When we try to draw the same at an angle when we try to cut, what gets formed is a ellipse. When we try to form the same cone, when we try to cut at a much steeper angle, it tries to form a parabola. I can even remove this part, so if you are, I can draw a cone and then you try to take.

So, this is all from a triangle. So, now the, we need to have 2 curves. And then if I cut the plane like this, so what I get is a hyperbola. So, if I cut exactly at the center, what I get is a point. I cut here, I get a point. I am just playing with the figure, trying to take and then trying to talk about major curves. When I try to talk about, I draw a circle and a line, so now what I do is I try to take a section exactly here at this point, center. So, what I get is a intersecting line. So, this was a, b, c, d, e, f.

And, I have one thing to do, so suppose if I draw the same object and cut it along the same line here, then I get g as line. So, I know how to create a circle, ellipse, parabola, hyperbola, point, intersecting line and lines. So, these are the major curves with curve representation in it.

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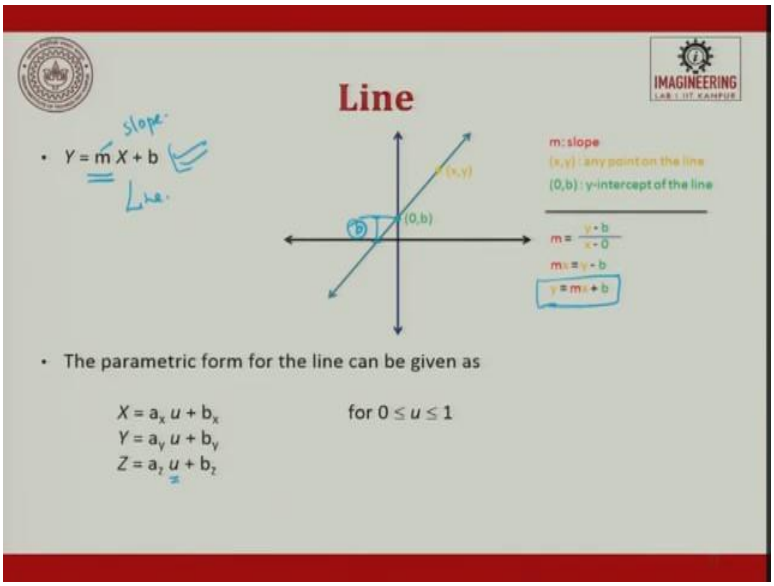

So, when we talk about a line, so we have a line which is not passing through the origin, which is slightly offset. So now, this line can be represented as,

$$Y = mX + b$$

where m is the slope, X and Y are the coordinates, O and b are the y intercepts. This is the y intercept, this is the Y. So, that is mx plus C or b. So, that b is nothing but O, b.

So, if you want to find out the slope,

$$m = \frac{y - b}{x - 0}$$

Now,

$$mX = Y - b$$

This is how you get this equation, y equal to mx plus b for a line.

When I have to express the same thing in parametric form, can be expressed as,

$$X = a_x u + b_x$$

$$Y = a_y u + b_y$$

$$Z = a_z u + b_z$$

So now, u takes the value 0 to 1 is the parametric form of representing a line in X, Y and Z.

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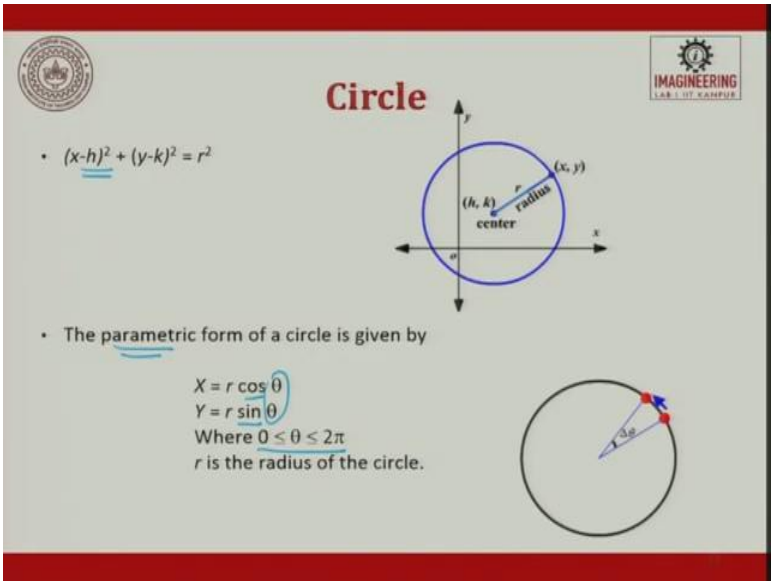

Now, let us try to take a circle. So, a circle, we have a center of a circle, a radius of a circle. Now this, and the center is h comma k and the point on the circumference is x comma y. So, it can be expressed by this form,

$$(x - h)^2 + (y - k)^2 = r^2$$

So, this will be the equation of a circle so that you can try to draw. So, this is the coordinates in x and this the coordinates in y.

If I have to represent the same thing in parametric form, so I can represent as,

$$X = rcos\theta$$

$$Y = rsin\theta$$

where θ varies from 0 to 2 pie and r is the radius. So here, the circle whatever I have explained in the geometric form could be converted into parametric form and stored like this.

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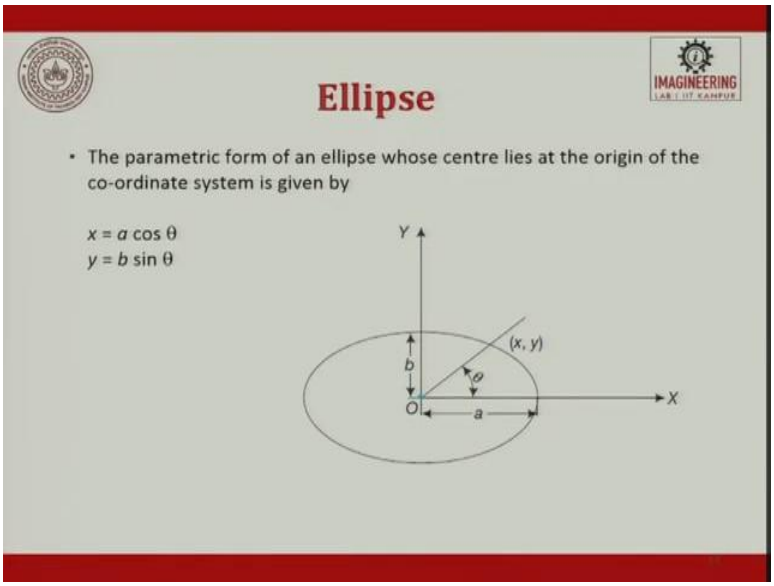

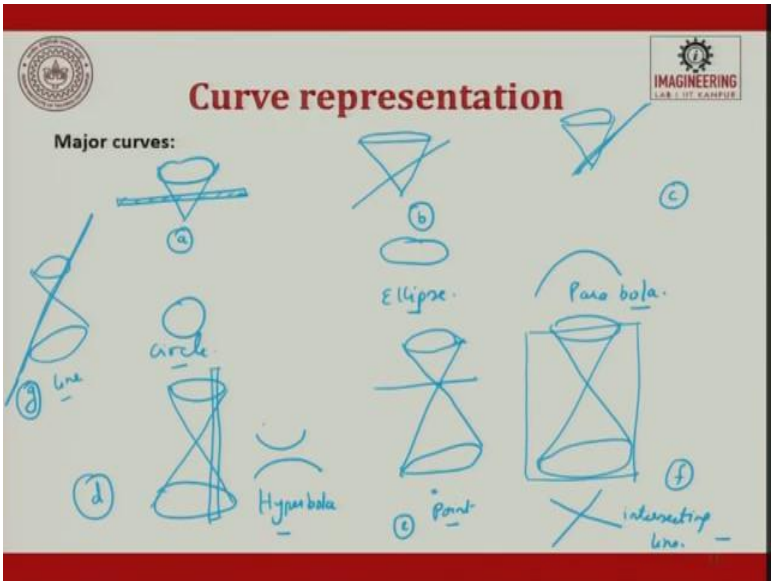

Now, let us see ellipse. So, what I am trying to do is I am going back to all the curves which I have drawn here, line, circle, ellipse, parabola, so I am trying to cover all these things.

So now ellipse, the parametric form of an ellipse whose center lies at the origin of a coordinate system is given by,

$$x = a\cos\theta$$

$$y = b\sin\theta$$

Again, θ varying from 0 to 2 pie. So, this is for an ellipse.

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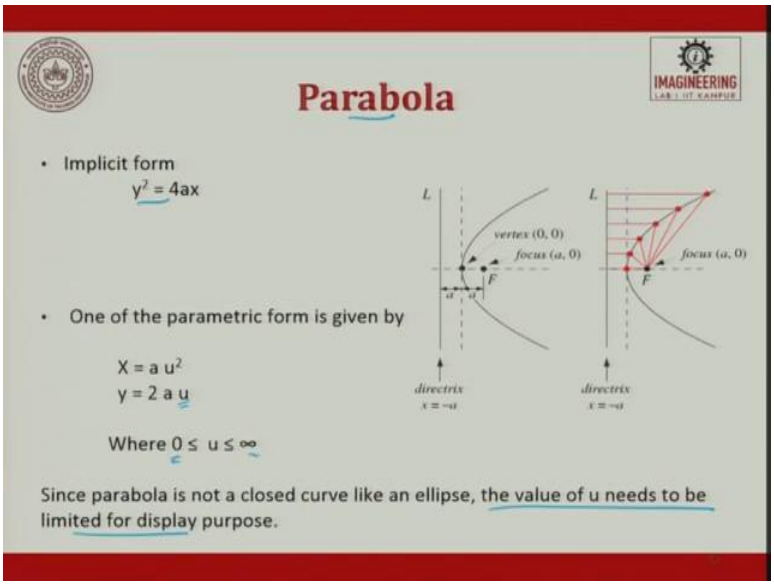

For a implicit form for a parabola,

$$y^2 = 4ax$$

In parametric form,

$$X = au^2$$

$$y = 2au$$

u varies from 0 to infinity, u varies from 0 to infinity. Since, parabola is not a closed curve, like ellipse the value of u needs to be limited for display purpose.

So, this is what is expressed. This is a, this is a and a, this is the F. So, this is how you try to solve the parabola. So, we saw, line, we saw circle, we saw ellipse and we saw parabola, which can be expressed in implicit form as well as parametric form.

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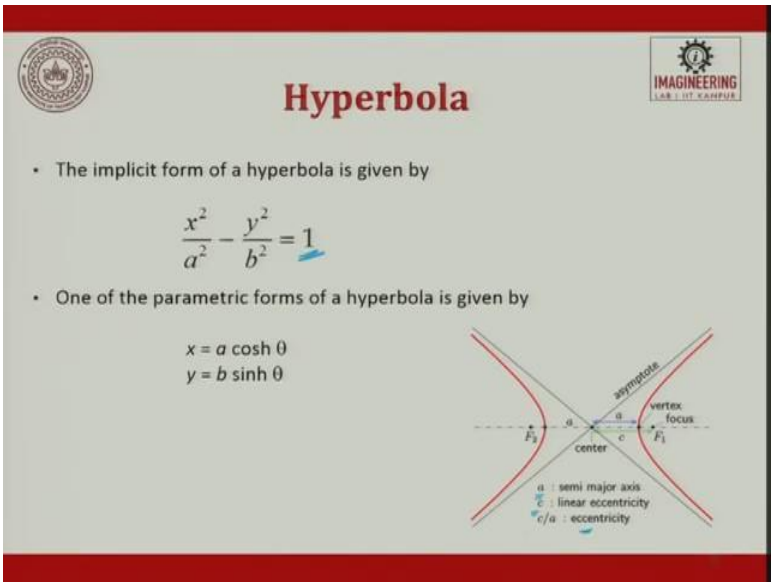

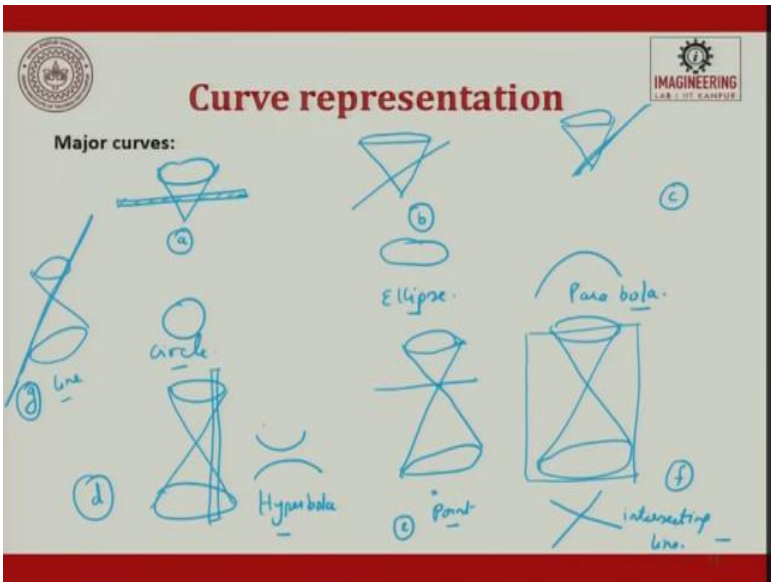

In the implicit form for a hyperbola. So, you go back, we drew the curve hyperbola also. So, this is what we are trying to see here, hyperbola. The implicit form of a hyperbola is given by,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

In parametric form,

$$x = a\cosh\theta$$

$$y = b\sinh\theta$$

So, where a is the semi major axis, c is the linear eccentricity, c by a is giving you the eccentricity. Again, theta here, it varies from 0 to infinity. Thank you.