

Manufacturing Automation
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Lecture – 09
Centreboard Hopper Feeder and its analysis

Welcome back and let me remind you that last time we were discussing the reciprocating tube hopper feeder, after we have discussed the vibratory bowl feeder.

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Reciprocating Tube Hopper Feeder

Level of parts
hopper
Parts
Reciprocating delivery tube

Side of reciprocating tube
Part
Hopper wall
F1
F2
N1
N2
W

$$F_1 + W + F_2 \cos \phi = N_2 \sin \phi \quad (4.1)$$

$$N_1 = N_2 \cos \phi + F_2 \sin \phi \quad (4.2)$$

$$= N_2 (\cos \phi + \frac{F_2}{N_2} \sin \phi)$$

$$= N_2 (\cos \phi + \mu \sin \phi)$$

$$F_1 (1 + \cos \phi) D/2 + W(D/2) \cos \phi = N_2 (D/2) \sin \phi \quad (4.3)$$

F1 is the force driving the tube down
F2 is the friction force

where ϕ is the hopper wall angle, and D the diameter of the part.
 Eliminating W from Equation 4.1 and Equation 4.3 gives, after rearrangement,

$$N_2 = \frac{N_1}{\cos \phi + \mu \sin \phi}$$

$$N_1 \sin \phi = F_1 (1 + \cos \phi) + \cos \phi \times (N_2 \sin \phi - F_2 \cos \phi - F_1) \quad (4.4)$$

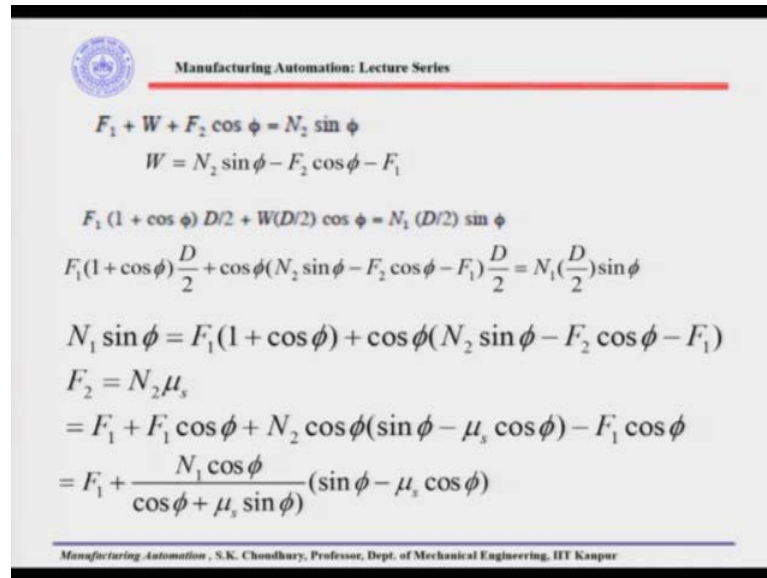
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Now, the reciprocating tube hopper feeder is also a feeder which is used for small engineering parts. And here in this design, I will remind you that through the mass of the parts which is located in the bowl, the tube reciprocates and the tube has a hollow inside and the hollow silhouette of the tube is accordingly made as per the part shape; when the tube reciprocates, through the hollow part of the tube the parts can come to the outlet.

So, here in this design. As I said, that there are certain aspects which have to be considered. One of the basic aspects is that the part should not jam between the side of the reciprocating tube and the hopper wall. So, for that, I have already discussed it, that if we have the part jamming as shown in the figure, we have three equations that we can find out. This is the force balance equation along the vertical direction, this is along the horizontal direction and this equation we have drawn by taking the moments about the

point A. So, the value of W , we can take from this first equation and put it in the third equation which is here.

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$$F_1 + W + F_2 \cos \phi = N_2 \sin \phi$$

$$W = N_2 \sin \phi - F_2 \cos \phi - F_1$$

$$F_1 (1 + \cos \phi) \frac{D}{2} + W(D/2) \cos \phi = N_1 (D/2) \sin \phi$$

$$F_1 (1 + \cos \phi) \frac{D}{2} + \cos \phi (N_2 \sin \phi - F_2 \cos \phi - F_1) \frac{D}{2} = N_1 \left(\frac{D}{2} \right) \sin \phi$$

$$N_1 \sin \phi = F_1 (1 + \cos \phi) + \cos \phi (N_2 \sin \phi - F_2 \cos \phi - F_1)$$

$$F_2 = N_2 \mu_s$$

$$= F_1 + F_1 \cos \phi + N_2 \cos \phi (\sin \phi - \mu_s \cos \phi) - F_1 \cos \phi$$

$$= F_1 + \frac{N_1 \cos \phi}{\cos \phi + \mu_s \sin \phi} (\sin \phi - \mu_s \cos \phi)$$

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So, W from the first equation is this; so putting W here, in the third equation, we are getting this expression and since $F_2 = N_2 \mu_s$, so we are getting this. And then we are rearranging putting the value of N_2 . this you can find out from here : N_1 we can write as

$$N_1 = N_2 \left(\cos \phi + \frac{F_2}{N_2} \sin \phi \right).$$

So, we are getting the N_2 common and $\frac{F_2}{N_2}$ is nothing but the μ_s . If you see that F_2 is

the friction force; so $\frac{F_2}{N_2}$ is the μ_s , which is the coefficient of friction between the hopper

wall and the part. So, this will be $N_1 = N_2 (\cos \phi + \mu_s \sin \phi)$. So, from this equation:

$$N_2 = \frac{N_1}{\cos \phi + \mu_s \sin \phi}.$$

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$$F_1 + W + F_2 \cos \phi = N_2 \sin \phi$$

$$W = N_2 \sin \phi - F_2 \cos \phi - F_1$$

$$F_1 (1 + \cos \phi) \frac{D}{2} + W(D/2) \cos \phi = N_1 (D/2) \sin \phi$$

$$F_1 (1 + \cos \phi) \frac{D}{2} + \cos \phi (N_2 \sin \phi - F_2 \cos \phi - F_1) \frac{D}{2} = N_1 \left(\frac{D}{2}\right) \sin \phi$$

$$N_1 \sin \phi = F_1 (1 + \cos \phi) + \cos \phi (N_2 \sin \phi - F_2 \cos \phi - F_1)$$

$$F_2 = N_2 \mu_s$$

$$= F_1 + F_1 \cos \phi + N_2 \cos \phi (\sin \phi - \mu_s \cos \phi) - F_1 \cos \phi$$

$$= F_1 + \frac{N_1 \cos \phi}{\cos \phi + \mu_s \sin \phi} (\sin \phi - \mu_s \cos \phi)$$

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So, we are using here this value of N_2 and getting the value as this.

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$$\text{since, } N_2 = \frac{N_1}{\cos \phi + \mu_s \sin \phi}$$

$$N_1 \sin \phi (\cos \phi + \mu_s \sin \phi) = F_1 (\cos \phi + \mu_s \sin \phi) + N_1 \cos \phi (\sin \phi - \mu_s \cos \phi)$$

$$\boxed{\frac{F_1}{N_1} = \frac{\mu_s}{\cos \phi + \mu_s \sin \phi}}$$

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So, finally what we are getting is that $\frac{F_1}{N_1} = \frac{\mu_s}{\cos \phi + \mu_s \sin \phi}$. So, this condition has to be satisfied so that the part is at the verge of the jamming.

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Reciprocating Tube Hopper Feeder

The maximum value of F_2 is given by $\mu_s N_2$ (where μ_s is the coefficient of static friction) and, thus, writing $F_2 = \mu_s N_2$ in Equation 4.2 and Equation 4.4 and eliminating N_2 give

$$\frac{F_1}{N_1} = \frac{\mu_s}{\cos \phi + \mu_s \sin \phi} \quad (4.5)$$

For the tube to slide, $F_1/N_1 > \mu_s$, and, therefore, from Equation 4.5,

$$\frac{1}{\cos \phi + \mu_s \sin \phi} > \mu_s \quad (4.6)$$

$\phi = 0^\circ \Rightarrow \mu_s < 0.577$

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Now, the maximum value of F_2 is given by $F_2 = N_2 \mu_s$ where μ_s is the coefficient of static friction. And thus writing $F_2 = N_2 \mu_s$ in equation 4.2, and eliminating N_2 ,

what we are getting is: $\frac{F_1}{N_1} = \frac{\mu_s}{\cos \phi + \mu_s \sin \phi}$. Now for the tube to slide what happens

that $\frac{F_1}{N_1}$ has to be more than the coefficient of friction, μ_s . Therefore, from equation 4.5

we can say that for the tube to slide we have to have $\frac{1}{\cos \phi + \mu_s \sin \phi} > 1$.

So, this is the condition that has to be satisfied. In that case we have two parameters; one parameter is the angle that is the most important parameter and the another parameter is the coefficient of friction. The μ_s will depend actually on the material of the part and the material that you are using for the hopper wall. So, let us say this is fixed because we have selected those.

Therefore, most important part here is this angle which will actually decide whether the part will be jammed between the hopper wall and the side of the reciprocating tube. Otherwise we are saying that whether the tube is able to slide if the ϕ is not properly selected in that case the tube will not be able to slide because the part is jamming here.

So, this is the condition that we have to always see that $\frac{1}{\cos \phi + \mu_s \sin \phi} > 1$.

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Reciprocating Tube Hopper Feeder

The expression indicates that the value of ϕ should be as large as possible to prevent jamming when μ_s is large. However, when $\mu_s < \cot \phi$ the parts cannot slide down the hopper wall. The best compromise is probably given by writing the following two limiting conditions:

$$\mu_s = \cot \phi \quad (4.7)$$
$$\cos \phi + \mu_s \sin \phi = 1 \quad (4.8)$$

Combining Equation 4.7 and Equation 4.8 gives $\phi = 60^\circ$ and, on substitution of this value in expression 4.6, it is found that, to prevent jamming under these conditions, the coefficient for friction μ_s must be less than 0.577. Because this value is greater than that expected in practice, it may be concluded that, with a hopper angle of 45° , the possibility of jamming will generally be avoided if the coefficient of friction is less than 0.414.

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This expression indicates that the value of ϕ should be as large as possible, you can see this from here, to prevent jamming when μ_s is large. However, when μ_s is less than $\cot \phi$, the parts cannot slide down the hopper wall. What we are taking is that this is some absurd value we are saying; that if $\mu_s < \cot \phi$, it is absurd; so this is stuck I mean the parts cannot slide down the hopper wall at all.

So, we are taking that as a condition and said therefore, that the best compromise is probably given by writing the following two limiting conditions as shown in the slide. So, we are saying that since it will not be able to slide, so, if $\mu_s = \cot \phi$ and then here we are saying $\cos \phi + \mu_s \sin \phi = 1$; this will be the limiting condition. So, from this limiting condition, combining equation 4.7 and 4.8, you will find that ϕ is equal to 60° .

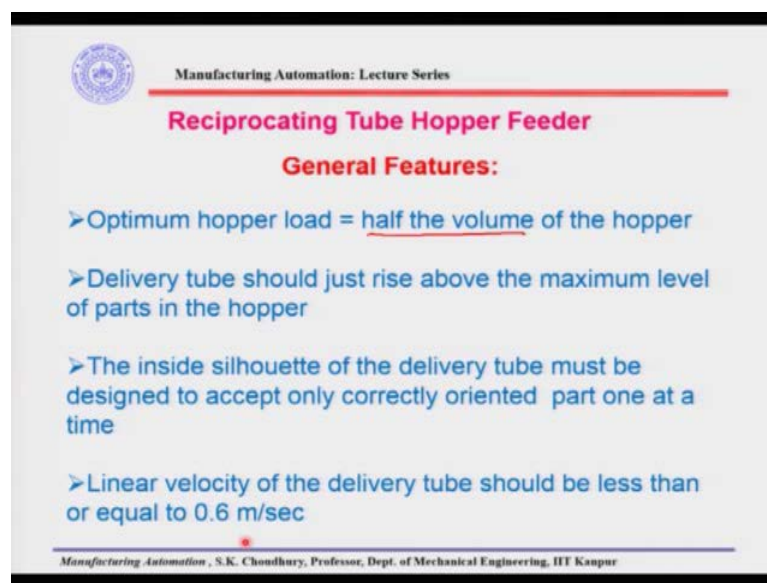
Now on substitution of this value in expression 4.6, it is found that to prevent jamming under these conditions, that is when we have these conditions, the coefficient of friction μ_s must be less than 0.577. Meaning that if we have ϕ equals to 60° , we will find out that the μ_s is equal to somewhere around 0.577. So, this will be less than 0.577 if this condition has to be satisfied that this is more than 1.

In that case what is happening? This value is greater than expected in practice; in the sense that normally in practice the material that we use for the hopper wall and the metal

parts, normally for that combination of material this value is greater than expected in the practice.

So, it may be concluded that with a hopper angle of 45° , instead of 60° for which we are getting μ_s equals to 0.577, with the iteration we can say that we will go for the hopper angle of 45° and for that you can find out from this equation that we are getting the friction less than 0.414, which probably satisfies all the combinations of the material of the part and the hopper wall.

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Now, for the reciprocating tube hopper feeder the general features are the following: that optimum hopper load, means how many parts you will be putting in the hopper as a mass of parts. So, I will remind you that as we said we have the parts in the hopper; this is the level of parts. There are many of these parts. The idea is that it cannot be too much; it should be normally half the volume of the hopper. If it is more than that, in that case what happens is that the pressure will be on the tube and the tube has to move so that resistance to that movement, because the mass is very high or the parts are large in number; in that case the resistance is higher.

Therefore, optimally it is taken as half the volume of the hopper. Delivery tube should just rise above the maximum level of parts in the hopper. What does it mean? So that the parts can come inside the delivery tube, it is enough for the reciprocating tube to come just above that. I mean it is not necessary to go beyond that because it is absolutely

redundant and as we are moving that more, that means we will spend more energy, we will spend more power. Therefore, it has to just go beyond that level of the parts, just rise beyond the level of the parts.

The inside silhouette of the delivery tube: I have already told you that the tube must be designed to accept only the correctly oriented parts one at a time. Now, I told you earlier when we were discussing the initial part of the lecture that feeders have to feed the parts in the correct orientation. So, there are some reorienting devices which are incorporated inside the bowl.

Now those details we will discuss at a later stage, but at this moment we are saying that if the inside silhouette of the reciprocating tube could be of the same type and the shape of the part and in that orientation which you desire, in that case through the reciprocating tube the parts with the right orientation can be taken. This is what exactly we are saying that this is the inside of the reciprocating tube.

So, inside of the reciprocating tube, the silhouette, the shape will be the same as the shape of the parts. Suppose, the part we want in this way; so inside silhouette of the reciprocating tube should be this way and not this way. So, if the part is coming like this, it will not be accepted because the inside silhouette of the reciprocating tube is made in the opposite way in a different way.

Apart from that, the linear velocity of the delivery tube should be less than or equal to 0.6 m/s. If it is more than that, by inertia the parts may actually not go inside the reciprocating tube. It is not required to be made slower because in that case it will take more time. So, the production rate will decrease. That is why it should be less than or equal to 0.6 m/s; so it is around 0.6 m/s. Another thing probably I should tell you that this inside silhouette of the delivery tube must be designed to accept only correctly oriented part one at a time.

Another advantage of this device is that if the inside silhouette is made exactly according to the shape, size and the accuracy of the part, in that case it can also serve as a kind of an inspection device. Meaning that those parts which are quality wise right, only those parts will go inside the tube. If the tolerance is not right, they will not be able to go inside the silhouette. So, that way many of the feeders actually made that way so that this can be considered as an inspection device as well.

Let us take a numerical example to see how we can implement the theory that we have just now gone through for the reciprocating tube hopper feeder.

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NUMERICAL EXAMPLES:

For a reciprocating tube hopper feeder with a static coefficient of friction between the parts and the hopper wall of 0.3945, determine the hopper angle so that the parts do not jam.

Solution:

Given, $\mu_s = 0.3945$

For parts not to jam, $\frac{1}{\cos\phi + \mu_s \sin\phi} > 1$

or, $\cos\phi + \mu_s \sin\phi < 1$; or, $\mu_s \sin\phi < 1 - \cos\phi$

or, $2\mu_s \sin \frac{\phi}{2} \cos \frac{\phi}{2} < 2 \sin^2 \frac{\phi}{2}$; or, $\tan \frac{\phi}{2} > \mu_s$

or, $\frac{\phi}{2} > \tan^{-1}(\mu_s)$; or, $\phi > 2 \tan^{-1}(\mu_s)$

or, $\phi > 43^\circ$

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For a reciprocating tube hopper feeder with a static coefficient of friction between the parts and the hopper wall of 0.3945, determine the hopper angle so that the parts do not jam. See, this example is taken as a practical example when somebody is designing the reciprocating tube hopper feeder. I will remind you that for designing the reciprocating tube hopper feeder, the most important thing is the hopper angle so that within the silhouette, within the reciprocating tube and the hopper wall the parts do not jam.

So, here suppose the given is the coefficient of friction - it is 0.3945; actually we do not need anything else. For example, we know that we have this condition to satisfy for parts not to jam. So, here if we have the μ_s given we can find out the $\cos\phi$ or the ϕ that is what it is asked; that determines the hopper angle so that the parts do not jam. So, if you take the limiting condition, for example, is equal to 1 or less than 1 the parts are jamming or about to jam.

So, $\frac{1}{\cos\phi + \mu_s \sin\phi} > 1$; this is the condition that we have taken. For example, here if you

see in this equation that this is the one that we have derived finally and from here what we said is that these are the limiting conditions. So, in this example we are saying that

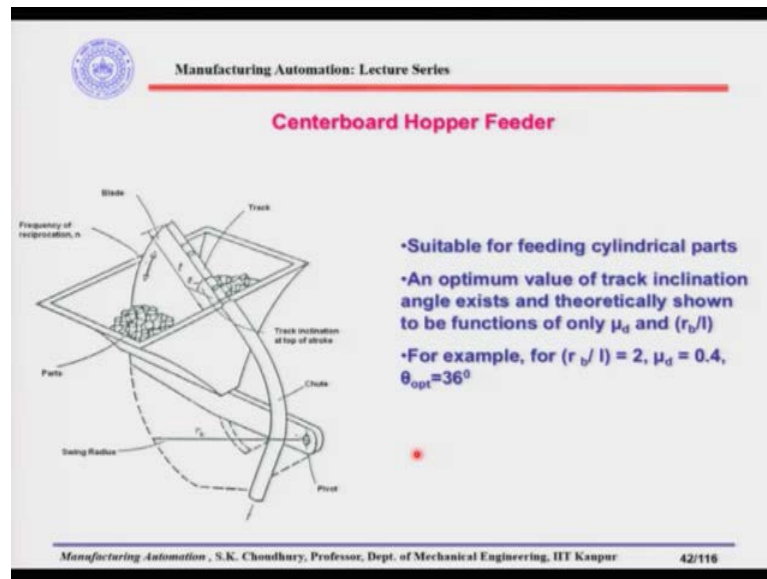
$\frac{1}{\cos \phi + \mu_s \sin \phi} > 1$ for parts not to jam. And from here what we are getting is $\cos \phi + \mu_s \sin \phi < 1$;

Then we are expanding that and we are finding out that $\tan\left(\frac{\phi}{2}\right) > \mu_s$; from here and finally, we are getting that $\phi > 2 \tan^{-1} \mu_s$. Therefore, ϕ is more than 43° because the μ_s given here is 0.3945. So, ultimately what you are getting is that ϕ has to be more than 43° so that the parts do not jam. And here what is given only is the coefficient of friction between the parts and the hopper wall which is 0.3945. Now, in the design problem or when you are designing the reciprocating tube hopper feeder, you are using certain part material of let us say mild steel and you are using a mild steel hopper wall.

So, in the handbook you can find out μ_s between the mild steel and mild steel or any other combination of materials; And from there using this equation, using this inequality you can find out what should be the hopper angle; so it is easy, you know can find out the value of the ϕ . Now, what does it mean that ϕ is more than 43° ? Anything? 90° which is more than 43° ; can we use 90° ? No, it is not like that; it just cannot be less than 43° .

The angle should be more than 43° , judiciously chosen according to the experience that how many parts you will be taking. So, if it is 90° , you know that it will not work. So accordingly from the experience you find out the angle just more than 43° , it should not be less than 43° ; that is the idea of this design criteria.

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Next, we will look into another hopper feeder, this is called the centreboard hopper feeder and from the name centreboard, you can find out from this diagram, that is given in this picture. This is a pictorial view, so there is a bowl and inside the bowl we have the mass of the parts. These are also small engineering parts as in case of the reciprocating tube hopper feeder or the vibratory bowl feeder. Now, through the mass of the parts there is a blade which actually reciprocates; it goes up and comes down and it goes through the mass of the parts.

Here is a cut out for this blade to reciprocate and the mass of the part will be within the feeder. Here is a chute or the delivery chute which is fixed here. So, when the blade will move up and gets aligned to the delivery chute, then all the parts located on the track will be coming down the delivery chute and going for the assembly machines which are at the exit of this. So, the machines will accept the parts one by one, which are coming from the chute.

So, what happens is that this blade has to go up and dwell for some time so that all the parts could slide down the blade. Second thing is that there is a pivot; so this pivot will be attached to this blade that you can see here. The whole thing is actually moving, but the blade is here, this much only and the track which is provided on the blade, i.e., on the track the parts will be nested. So, the blade is only this much, it is connected to this part which is pivoted here.

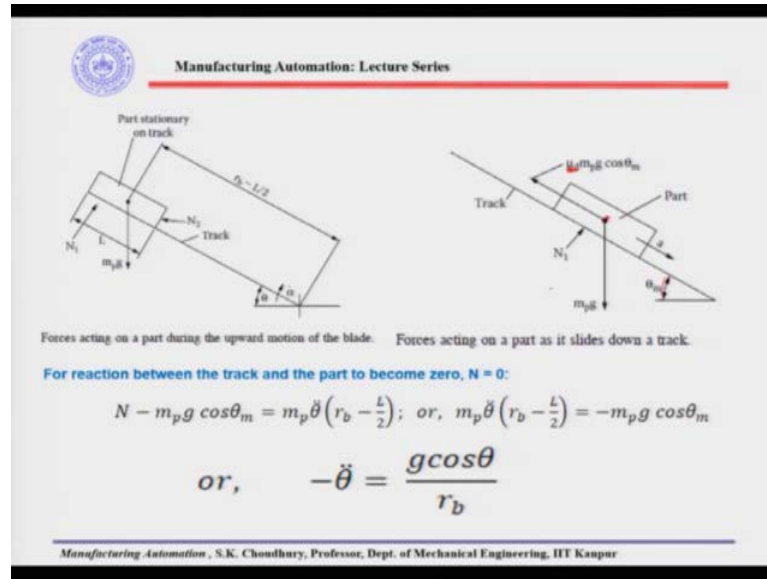
Now, there is one very important aspect that has to be taken care of. The blade which is reciprocating is going up and coming down. If the acceleration is very high, in that case at the top of the blade the parts located on the track will be thrown out. Second thing is that if the inclination is more, in that case the parts will be facilitated to come down the track. So, these are the two aspects: the acceleration should not be very high so that the parts are not thrown out of the track and the inclination angle should not be very low or very high.

Of course, if the track inclination angle is more, in that case it will facilitate the parts to come down; these are the two aspects let us keep in mind. Now, this kind of centreboard hopper feeders are suitable for feeding cylindrical parts mostly. That means, the silhouette of the track or the shape of the track is as per the cylindrical part so that those parts in the right orientation can be nested. An optimum value of track inclination angle exists and theoretically shown to be function of only μ_d and the $\frac{r_b}{l}$ that we will discuss later on.

Now, we are saying about the optimum value of track inclination angle; why it has to be optimum? If the track inclination angle is very high in that case, it will take more time for the reciprocation of the blade. So, more time is less production rate because each time it is reciprocating in one cycle, it is feeding one part to the assembly machine. So, this track inclination angle has to be optimum.

For example, for $\frac{r_b}{l}$ is equal to 2, what is r_b is the swing radius from here to here, this is the radius and l is the length of the track. So, $\frac{r_b}{l}$ is equal to 2 and the dynamic coefficient of friction is equal to 0.4; the optimum angle is around 36° . These are given experimentally; there is a calibration curve which is made and from the calibration curve, we can find out that for the values of $\frac{r_b}{l}$ and the μ_d how much will be the optimum angle; there will be a calibration curve, let us see this.

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Now, these are the two line diagrams given; showing how the forces are acting on the part when the part is at the highest position of the blade and when the part is coming down. Let us say when the part is here forces acting on a part during the upward motion of the blade is like this. Here it will be $m_p g$, that is the mass of the part and here is the normal force which is acting by the track, here is the track and this is the part and here what is shown is that this is the $\left(r_b - \frac{L}{2} \right)$; capital L is the length of the part.

This is, let us say, at the middle of the part. So, it is r_b total minus $\frac{L}{2}$, this much, this is the angular track inclination, and this is the maximum track inclination θ_m ; in fact, here it is θ_m - this is the maximum track inclination. So, when the parts are sliding down the track here a is the linear acceleration of the parts; N_1 is the normal force, this is the free body diagram and the friction force which will be coming here will be $m_p g \cos \theta_m$. Here we can actually resolve these forces; one force will be along the track and one force will be vertical to the track.

So, $m_p g \cos \theta_m$ will be here this is multiplied by the μ_d which is the dynamic coefficient of friction. This will give you the friction force and this is the maximum inclination of

the track. So, we have three forces basically friction force, $m_p g$ is the mass of the part and the N_1 which is the normal force.

Now, we can find out the force balance equation here and the condition for the acceleration. In the next class, we will discuss how the acceleration can be found out and how we can find out the maximum track inclination so that the parts are not thrown out because of that acceleration, and the time taken for the reciprocation of the blade could be optimum.

Thank you.