

Manufacturing Automation
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Lecture – 07
Analysis of Vibratory Bowl Feeder

Welcome back. So, I will like to remind you that last time what we discussed is the vibratory bowl feeder and some aspects of the bowl feeder design. We said that how the bowl feeder works. Now, for the design, basic thing is to design the track inclination angle and the vibration angle; these are the most important things. So, we said that the vibration angle is decided by the suspension springs and how the suspension springs hold the bowl in the bowl feeder.

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Manufacturing Automation: Lecture Series

Mechanics of Vibratory Conveying

For sliding up the track to occur:

$$m_p a_0 \omega^2 \cos \psi > m_p g \sin \theta + F \quad (1)$$

Where,

$$F = \mu_s N = \mu_s (m_p g \cos \theta - m_p a_0 \omega^2 \sin \psi) \quad (2)$$

μ_s - Coefficient of static friction between the part and the track.

Combining (1) and (2), we get:

$$m_p a_0 \omega^2 \cos \psi > (m_p g \cos \theta - m_p a_0 \omega^2 \sin \psi) \mu_s + m_p g \sin \theta$$

Or, $a_0 \omega^2 (\cos \psi + \mu_s \sin \psi) > g (\sin \theta + \mu_s \cos \theta)$ *a_0 - amplitude of vibration*

Or, $\frac{a_0 \omega^2}{g} > \frac{\mu_s \cos \theta + \sin \theta}{\cos \psi + \mu_s \sin \psi}$

Similarly, it can be shown that for backward sliding to occur during the vibration cycle:

$$\frac{a_0 \omega^2}{g} > \frac{\mu_s \cos \theta - \sin \theta}{\cos \psi - \mu_s \sin \psi}$$

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So, for sliding up the track to occur as we said, we have this force diagram. So, in this force diagram as you remember, we said that this is the total inertia force which is $m_p a_0 \omega^2$, m_p is the mass of the part, a_0 is the amplitude of vibration and the ω is the angular frequency. So, this total inertia force can be resolved into the two components one component is along the track which is $m_p a_0 \omega^2 \cos \Psi$; Ψ is the angle between the total inertia force and the line of the track.

So, in here along the N, that is the normal force, which is acting on the part, this will be similarly $m_p a_0 \omega^2 \sin \Psi$. Similarly, we have the mass of the part which is acting here as $m_p g$; g is the gravity and this $m_p g$ will be resolved into two components along the N component is the $m_p g \cos \theta$ and along the F, it is $m_p g \sin \theta$.

So, in this case for sliding up the track, $m_p a_0 \omega^2 \cos \Psi$ component has to be more, because the part has to slide up the track, than the forces acting on the other side and forces acting on the other side will be F and the $m_p g \sin \theta$. Therefore, $m_p a_0 \omega^2 \cos \Psi > F + m_p g \sin \theta$ that is one aspect of it. Now, the F, which is the friction force because of the normal force acting on the part, this is $N \mu_s$ we said, N is the normal force and the μ_s is the static coefficient of friction between the part and the track.

So, therefore, this N we can substitute by the $m_p g \cos \theta$, this component, minus this component which is $m_p a_0 \omega^2 \sin \Psi$, that is what we have written here. So, N is substituted by this and therefore, the F becomes this much. So, this equation, equation 1, becomes that if we put the value of F as in this case the equation 1 can be converted into this form. Now, from here if we simplify, $\frac{a_0 \omega^2}{g} > \frac{\mu_s \cos \theta + \sin \theta}{\cos \Psi + \mu_s \sin \Psi}$, then this will be the condition.

So, this is the condition for the part to slide up the track. Similarly if you resolve the components in the way when the part is going down the track, you will find out that the condition will be $\frac{a_0 \omega^2}{g} > \frac{\mu_s \cos \theta - \sin \theta}{\cos \Psi - \mu_s \sin \Psi}$. This will be very similar to the earlier one, only you can see that the sign is the minus. So, this will be the condition for the for the part to move down the track, that means, on the other side.

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Mechanics of Vibratory Conveying

The operating conditions of vibratory conveyor may be expressed in terms of dimensionless Normal Track Acceleration, A_n/g_n , where,

$$A_n = a_n \omega^2 = a_0 \omega^2 \sin \psi \quad \text{Normal Track Acceleration}$$

$$g_n = g \cos \theta \quad \text{Normal Acceleration due to gravity}$$

g – Acceleration due to gravity = 9.81 m/sec²

So, $\frac{A_n}{g_n} = \frac{a_0 \omega^2 \sin \psi}{g \cos \theta} \dots (3)$ Substituting (3) in earlier equations,

For Forward Sliding, $\frac{A_n}{g_n} > \frac{\mu_s + \tan \theta}{\cot \psi + \mu_s}$

And, For Backward Sliding, $\frac{A_n}{g_n} > \frac{\mu_s - \tan \theta}{\cot \psi - \mu_s}$

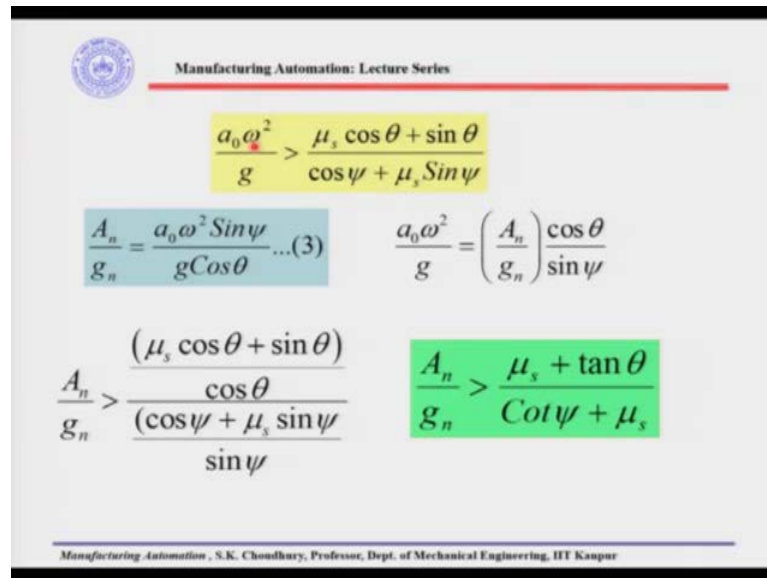
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Then we said that the operating condition of the vibratory conveyor may be expressed in terms of the normal track acceleration and the normal track acceleration we said as $a_0 \omega^2 \sin \Psi$; ω is the angular frequency and a_0 is the amplitude of vibration.

So, this is the normal track acceleration and the normal acceleration due to gravity will be the $g \cos \theta$. Therefore, the $\frac{A_n}{g_n} = \frac{a_0 \omega^2 \sin \Psi}{g \cos \theta}$. Now, if we substitute this in the earlier

equations then we will be getting, $\frac{a_0 \omega^2}{g} = \left(\frac{A_n}{g_n} \right) \frac{\cos \theta}{\sin \Psi}$

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$$\frac{a_0 \omega^2}{g} > \frac{\mu_s \cos \theta + \sin \theta}{\cos \psi + \mu_s \sin \psi}$$

$$\frac{A_n}{g_n} = \frac{a_0 \omega^2 \sin \psi}{g \cos \theta} \dots (3)$$

$$\frac{a_0 \omega^2}{g} = \left(\frac{A_n}{g_n} \right) \frac{\cos \theta}{\sin \psi}$$

$$\frac{A_n}{g_n} > \frac{(\mu_s \cos \theta + \sin \theta)}{(\cos \psi + \mu_s \sin \psi) \sin \psi}$$

$$\frac{A_n}{g_n} > \frac{\mu_s + \tan \theta}{\cot \psi + \mu_s}$$

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So, how it is getting? Let me explain it to you. We have this equation

$$\frac{a_0 \omega^2}{g} > \frac{\mu_s \cos \theta + \sin \theta}{\cos \psi + \mu_s \sin \psi}$$

this is the condition for the part to move up and we have

$$\frac{A_n}{g_n} = \frac{a_0 \omega^2 \sin \psi}{g \cos \theta}; \frac{A_n}{g_n}$$

we got from here. Now, if we substitute this from here, we find

out that, $\frac{a_0 \omega^2}{g} = \left(\frac{A_n}{g_n} \right) \frac{\cos \theta}{\sin \psi}$.

Now, we will come back to this and we will put this value here in this equation, $\frac{a_0 \omega^2}{g}$,

we are substituting by this and we are getting that $\frac{A_n}{g_n}$ has to be more than this

component. Therefore, $\frac{A_n}{g_n}$ is the one that we said is the dimensionless normal track

acceleration and this is the component by which we are defining the performance of the

bowl feeder. So, $\frac{A_n}{g_n}$ has to be more than $\frac{\mu_s + \tan \theta}{\cot \psi + \mu_s}$, this is for the part to go up the

track.

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Mechanics of Vibratory Conveying

For values of $\mu_s = 0.8$, $\theta = 3$ degree and $\psi = 30$ degree,
An/gn must be > 0.34 for forward sliding, and
An/gn > 0.8 for backward sliding.

The limiting condition for forward conveying to occur is given by:

$$\frac{\mu_s + \tan \theta}{\cot \psi + \mu_s} = \frac{\mu_s - \tan \theta}{\cot \psi - \mu_s} \text{ or, } \mu_s^2 = \tan \theta + \cot \psi$$

So, the limiting condition is ; $\tan \psi > \frac{\tan \theta}{\mu_s}$

when θ is small, $\tan \psi > \frac{\theta}{\mu_s}$

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Now, we have given an example that for $\mu = 0.8$ and the inclination angle is 3° and the vibration angle is 30° . So, you will be getting that this is the equation and the limiting condition for the forward conveying to occur, we are rearranging this and we are getting this. So, finally, what we are saying is that for small track inclination angle, the $\tan \Psi$ has to be more than the track inclination angle divided by static coefficient of friction, $\tan \Psi > \frac{\theta}{\mu_s}$ and Ψ is the vibration angle.

So, if we know this equation, if we know this inequality, in that case we can find out what should be the track inclination angle, for example, given μ_s and the vibration angle, what should be the vibration angle if we have a track inclination angle and the static coefficient of friction. This is the advantage of deriving this equation, as we call this as the mechanics of vibratory conveying.

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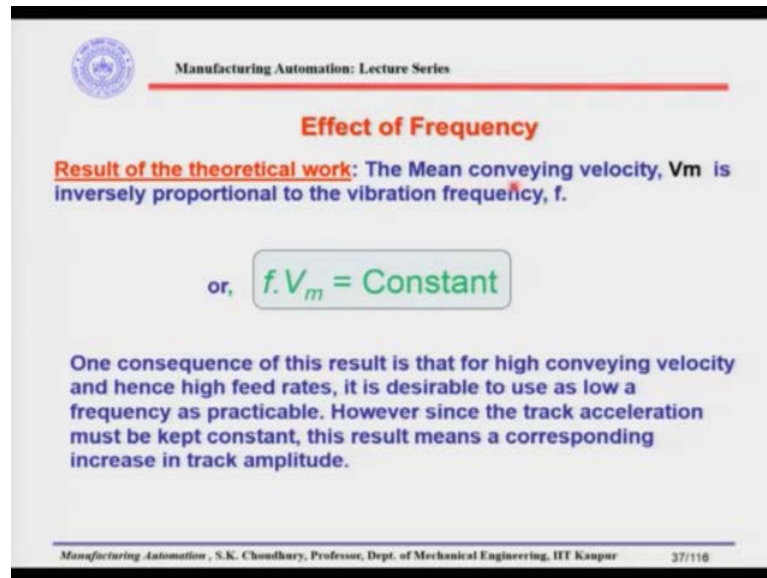
The slide is titled "Mechanics of Vibratory Conveying" and is part of a "Manufacturing Automation: Lecture Series". It explains that for sufficiently large vibration amplitudes, a part will leave the track and 'hop' forward during each cycle. The condition for this is when the normal reaction, N , between the part and the track becomes zero. From the figure, the equation $N = m_p g \cos \theta - m_p a_0 \omega^2 \sin \psi = 0$ is shown. So, for the part to leave the track, the condition $\frac{a_0 \omega^2}{g} > \frac{\cos \theta}{\sin \psi}$, or $\frac{A_n}{g_n} > 1$ is derived. The footer mentions "Manufacturing Automation, S.K. Choudhury, Professor, Dept. of Mechanical Engineering, IIT Kanpur" and the slide number "36/116".

Now, here for sufficiently large vibration amplitudes, the part will leave the track and hop forward during each cycle; what we are saying is that, it is the part which is going along the track. For sufficient conveying velocity, we have to have the part to leave the track, part has to leave the track and hop like a frog. For that we require large vibration amplitudes. Now, the condition for this to occur; that means, when the part will leave, the normal force N has to be equal to 0. When the normal force is 0, the part is ready to leave, it is not leaving yet, it is ready to leave.

So, for N is equal to 0, from the figure you can find out the value of N . So, $N = m_p g \cos \theta - m_p a_0 \omega^2 \sin \Psi$ So, when this is equal to 0, it is the limiting condition for the part to leave the track and the part will leave the track when this is more than that.

So, if we do that, if we rearrange this, $\frac{a_0 \omega^2}{g} > \frac{\cos \theta}{\sin \Psi}$ or, $\frac{A_n}{g_n} > 1$. This is an important derivation because, this says that when the dimensionless normal track acceleration, which is $\frac{A_n}{g_n}$, this has to be more than 1, then only the part will leave the track. So, for sufficient conveying velocity, we have to have the normal track acceleration more than 1, this is the condition here. Now, let us discuss the effect of frequency.

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Effect of Frequency

Result of the theoretical work: The Mean conveying velocity, V_m is inversely proportional to the vibration frequency, f .

or, $f \cdot V_m = \text{Constant}$

One consequence of this result is that for high conveying velocity and hence high feed rates, it is desirable to use as low a frequency as practicable. However since the track acceleration must be kept constant, this result means a corresponding increase in track amplitude.

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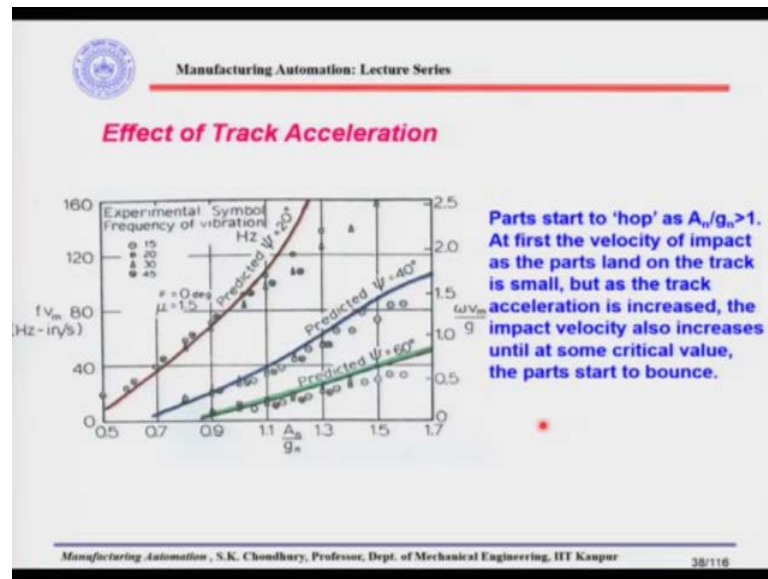
The result of the theoretical work says that the mean conveying velocity V_m is inversely proportional to the vibration frequency f . Now, what do you call it as a mean conveying velocity? It means, this is the speed of the part along the track of the vibratory bowl feeder and the track is inclined which is going spirally along the inside wall of the bowl feeder.

So, we need the large conveying velocity because we need the parts to be transmitted to the exit of the bowl feeder at a faster rate. So, for mean conveying velocity V_m is proportional to frequency. This has been found out theoretically and $fV_m = \text{Const.}$ does mean that for high conveying velocity we need the high feed rates, high conveying velocity means the high feed rates, it is desirable to use as low frequency as possible because $fV_m = \text{Const.}$

So, if you are increasing the V_m so to make them constant f has to be sufficiently low as low as possible. However, since the track acceleration must be kept constant this result means a corresponding increase in that track amplitude. Meaning, we are decreasing the frequency because we want to increase the V_m and fV_m have to remain constant.

Therefore, one option is to increase the track amplitude. This is the conclusion of the effect of frequency that since $fV_m = \text{Const.}$, for higher value of the conveying velocity V_m , we have to use as low frequency as possible and that does mean that we have to increase the track amplitude of vibration.

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Next, let us see the effect of the track acceleration. By track acceleration we mean $\frac{A_n}{g_n}$

which is the normal track acceleration. I would like to remind you that normal track acceleration is the one which decides or defines the performance of the bowl feeder.

Now, for the part to hop, we said that $\frac{A_n}{g_n}$ has to be more than 1. At first, what is

happening is, as you see in the curve, this is actually the experimental curve, and this curve has been taken for an angle of inclination of 0° , i.e., for a flat track and for the static coefficient of friction, let us say, near 1.

Now, this has been taken for the different vibration angles, namely 20° , 40° and 60° and when we are increasing the normal track acceleration, as we can see that fV_m which is in the y-axis, that is the conveying velocity, this conveying velocity is increasing. In all these cases, for any value of the vibration angle whether it is 20° or 40° or 60° , with the

increase in the $\frac{A_n}{g_n}$, the conveying velocity increases that is the first conclusion that we can make from the experimental curve which is shown here.

Now, the second conclusion is that when we are increasing, let us say beyond 1.7, these curves are actually saturating, in the sense it is actually not increasing anymore as the increasing order we are seeing for the other values of the normal track acceleration.

Now, what happens here is that when the $\frac{A_n}{g_n}$ is more; that means, the part is hopping more and more.

Now, if it is going beyond 1, it starts hopping and more than 1 up to let us say 1.7, it is hopping more and more. Initially, when it is, let us say, just crossed 1, it is hopping and coming back to the track and the impact velocity is not very high. But beyond 1.7 of the $\frac{A_n}{g_n}$ value, it goes up so much hops high and then impact velocity will be accordingly higher and by landing on the track of the bowl feeder, it actually topples, it becomes erratic.

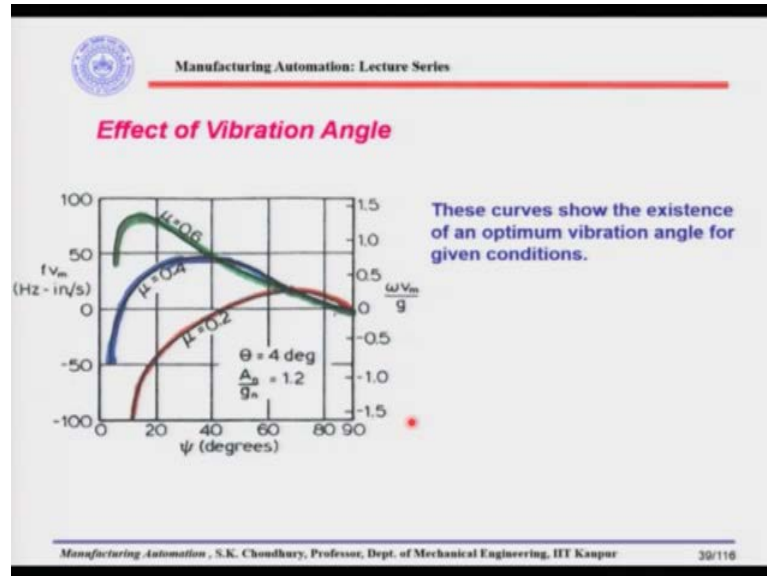
So, the conveying velocity does not increase anymore. This is the conclusion that we have to limit the $\frac{A_n}{g_n}$ to a certain value beyond which the part will be unstable. Here it is

written the part starts to hop as $\frac{A_n}{g_n}$ more than 1, that we have seen earlier. At first, the velocity of impact as the parts landing on the track is small, but as the track acceleration is increased, the impact velocity also increases until at some critical value, the parts start to bounce.

Now, what we say as the critical value, also depends on the mass of the part. It depends on the track, it depends on the material of the part and the track material. So, depending on all those factors this can be somewhere between 1.6, 1.7 up to 2. It has been seen in the practice. So, these are the critical values depending on type of the part, depending on the type of the track, depending on the bowl feeder that you are using overall. This is the effect of track acceleration. Once again summing up, increasing the normal track

acceleration, the conveying velocity increases up to a certain level after that it is unstable.

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Next is the effect of vibration angle. Let us see what happens as the vibration angle is increased. Here x-axis is the vibration angle, y-axis is the conveying velocity fV_m . By the way, conveying velocity is given in Hertz inch per second. So, it is an American system because the data given by the American authors. So, it is inch system, in India of course, it will be centimeter or meter per second.

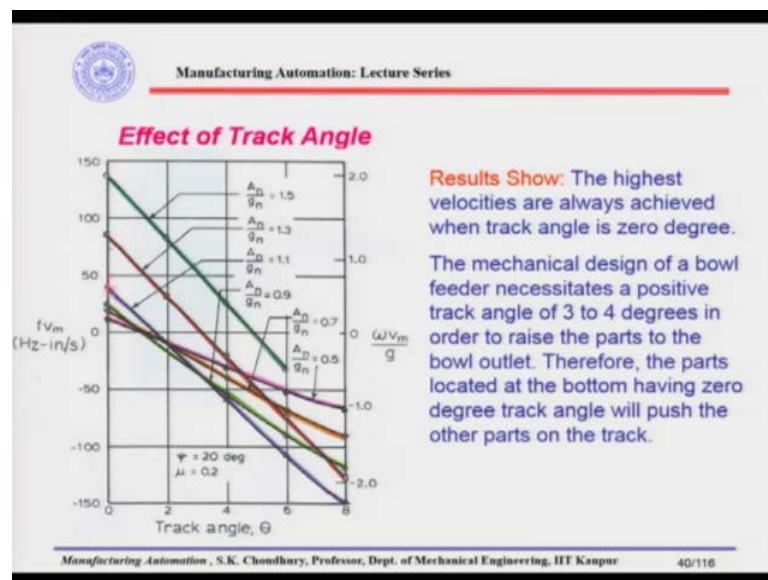
So, this is the frequency f it is in the Hertz and inch per second or centimeter per second and meter per second is the V_m . So, fV_m is in the y-axis and x-axis has the vibration angle. For different values of static coefficient of friction starting from 0.2 to 0.6, you can see that as the vibration angle is increased, the conveying velocity is increasing and after that it is actually decreasing for 0.6, for 0.4 as well as 1.2.

Now, it can be explained by the same phenomena as we have seen earlier. For this case as the vibration angle is increased, fV_m is increasing. With further increase in the vibration angle, $\frac{A_n}{g_n}$ increases and it actually gets into higher impact velocity zone. So, the fV_m , conveying velocity decreases.

These curves also show that there is an optimum vibration angle for a given condition. For example, if we have the μ equals to 0.6, the static coefficient of friction is 0.6. In that case, the maximum conveying velocity can be obtained somewhere around 10° of the vibration angle.

See for example, if we have the 0.4, then this maximum velocity could be somewhere around 30° or a little more. So, this is how we can actually experimentally find out the curves and we can find out what would be the optimum value of the vibration angle, so that we can accordingly design the suspension springs and select the electromagnet for the bowl feeder. Now, these curves are made for a track and inclination angle of 4° and the normal track acceleration as the 1.2.

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Next, let us say effect of track angle. So, this is the track inclination angle in the x-axis and y-axis as always it is the output, so this is the conveying velocity in Hertz inch per second. These are experimental curves taken for the vibration angle of 20° and the static coefficient of friction as 0.2 and for different values of normal track acceleration starting from 0.5, 0.7, 0.9 and so on up to 1.5. We have seen the effect of the track inclination angle; as we are increasing the track inclination angle, what happens to the conveying velocity.

Find one thing here in these curves that this is the 0 conveying velocity, above that there is a positive conveying velocity, below that the conveying velocity is negative; there is

no conveying velocity upwards - it is coming back. So, let us see what happens, for a very small track angle, let us say, somewhere up to this for maximum of maximum of $\frac{A_n}{g_n}$ is equal to 1.5, we have the positive fV_m - positive conveying velocity. But see here

what happens is, one thing you can actually conclude from these curves that for the lower track angle that is 0^0 , the conveying velocity is maximum this is one conclusion that can be drawn from here.

Second conclusion is that as the track angle is increasing to the higher level, let us say maximum of 5 and above, there is no conveying velocity which is positive and all the other conveying velocities will be below 0, that means, backwards, this is negative. Now, from here there is one more thing which is very important that the parts will be going from the below and it will be riding along the track.

So, on the ground level, the track angle is 0 and for the mechanical design of the track, we have to give some angle because it has to be inclined. So, normally it is 3 to 4^0 , that is the track inclination angle. So, what happens is that, at the ground level the conveying velocity will be maximum because here the curve shows that at a 0^0 degree, that is when the track angle is 0, which is flat, the conveying velocity is maximum.

But when the parts will be riding above the ground level, above the flat that is on the inclined track, there the track inclination angle increases and therefore, the parts behind those parts which have gone up, the parts which are located at the ground level at a 0^0 inclination angle, they will actually push the parts above that, you understand what I said? There is a pushing action from the parts which are at the ground level because there the track inclination angle is 0^0 .

Those parts will push the parts above and sometimes it may actually jam if the track is filled up with the parts. The parts at the ground level can actually push and it can jam at the exit or in between on the tracks if there are some orienting devices that we will discuss later on.

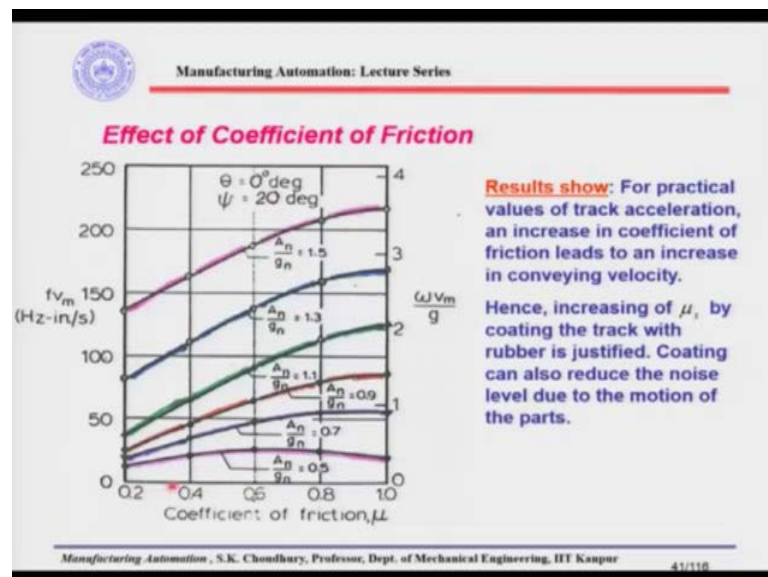
Orienting devices can be jammed by the parts which are pushing, which are at the flat level of the of the track. Here it is, I will read this, the highest velocities are always achieved when the track angle is zero degree, from here it can be seen that at the zero

degree of the track, the maximum fV_m could be achieved. The mechanical design of a bowl feeder necessitates a positive track angle of 3 to 4° because the track is inclined, as I said.

In order to raise the parts to the bowl outlet therefore, the parts located at the bottom having 0° track angle will push the other parts on the track where the angle is more than zero and this may create the jamming at the outlet or jamming at the orienting devices. So, this is a very important phenomenon because we have to find out how these track angles could be designed from this concept point of view. Let us say, what is the effect of the coefficient of friction.

Overall if I do not even look at the experimental curves, we can say that if the coefficient of friction increases between the track and the part, conveying velocity will be higher. That is one conclusion that we can make because it is a rougher surface so there will be less slip, it is not smooth and if the track is very smooth then the parts can slip and the fV_m can decrease. Let us see what happens experimentally.

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Here we have the coefficient of friction in the x-axis, we start from 0.2 and it goes up to 1, static coefficient of friction and in the y-axis, we have that fV_m , conveying velocity in Hertz inch per second and we have taken 0, 50, 100, 150 up to 250 of positive conveying velocity. So, these curves experimentally again have been taken for a 0° of track

inclination angle and 20° of the vibration angle. So, these curves have been taken for different values of the normal track acceleration starting from 0.5, 0.7, 0.9 up to 1.5.

First thing that we can see is that, as the coefficient of friction is increased, the conveying velocity increases. So, this is a normal thing because it is less smooth and it can facilitate the parts to go up if the coefficient of friction is increased. Now, for practical values of track acceleration, an increase in coefficient of friction leads to an increase in the conveying velocity. Hence, increase of static coefficient of friction by coating the track with rubber is justified. Coating can also reduce the noise level due to the motion of the parts.

See as I said the parts are moving, not just sliding over the track mind it, it is actually hopping and landing on the track again, but at the same time it is going forward. So, because of that impact velocity there is a noise and to reduce the noise you can coat the track with rubber, which serves two purposes: one is increasing the coefficient of friction and therefore, the conveying velocity increases, but it is up to a certain level as you can see, the coefficient of friction cannot go beyond 1.

Therefore, up to 1, you can see that the conveying velocity is increasing for all values of the normal track acceleration starting from 0.5 to 1.5. Now, another thing that you have to consider is whether you will be coating the track with the rubber or not that also depends on what kind of material you are using, combination of material you are using for the track and the part. So, depending on that you can actually decide from the experience what kind of coating you will be required, the rest of the things we will discuss in the next class.

Thank you very much.