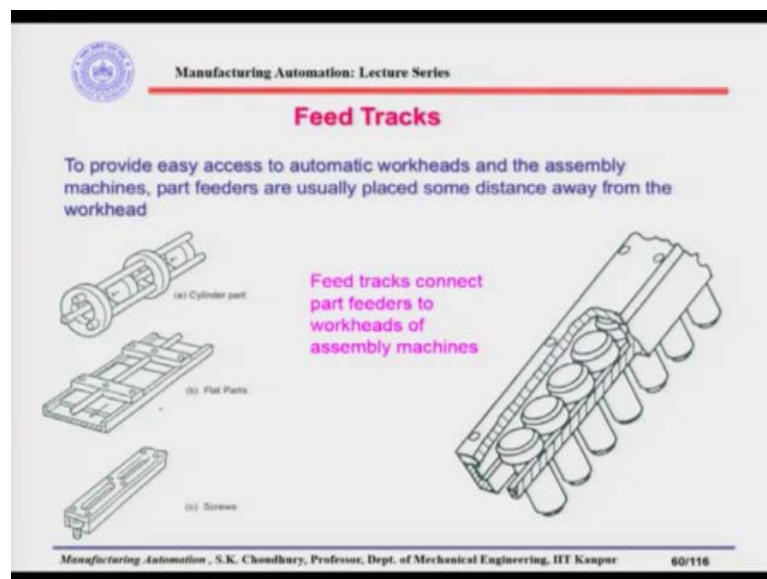


**Manufacturing Automation**  
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**Lecture – 15**  
**Feed Tracks and their analysis**

Welcome back. So, let me remind you that we were discussing in earlier class the feed tracks. And we said that feed tracks are normally connect the bowl feeder with the assembly machine. This is because the bowl feeders cannot be located very close to the machine assembly machine and therefore, there has to be a track. Now while designing the part feeders we said that the part feeders should be designed in such a way. So, that inside the part feeder the parts which are going in a right orientation should not be reoriented and lose their orientation I mean.

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Now, we have shown some of the part feeders for a feed tracks. So, these kind of feed tracks for example, you have seen that this is for cylindrical part, for flat parts, for parts headed parts which are hanging in the slot.

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(a) Horizontal delivery of parts

For the vertical design, the time of delivery,  $t_p$  will be given by the time taken for a part to fall a distance equals to its own length:

$$t_p = \left( \frac{2L}{g} \right)^{0.5}$$

$L$  – part length,  $g$  – acceleration due to gravity =  $9.81 \text{ m/s}^2$

(b) Vertical delivery of parts

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On then we said that the part feeders sorry feed tracks are normally of two types; they can be of the vertical delivery of the parts, where the parts are fed with the gravity and in that case that the time taken for a part to fall will be equal to  $2L$  by  $g$  root over because this is the acceleration due to gravity that we are using to deliver the parts.

Now, in the horizontal delivery of parts section will be horizontal. For example, here this section and this section these are horizontal; however, this part is actually not horizontal, but these are called the horizontal delivery of parts.

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### Analysis of Horizontal Delivery Feed Track

This portion consists of a horizontal section AB of length  $L_1$ , preceded by a curved portion BC of constant radius  $R$ , which, in turn, is preceded by a straight portion inclined at an angle  $\alpha$  to the horizontal.

It is assumed in the following analysis that a certain fixed number of parts are maintained in the track above the delivery point.

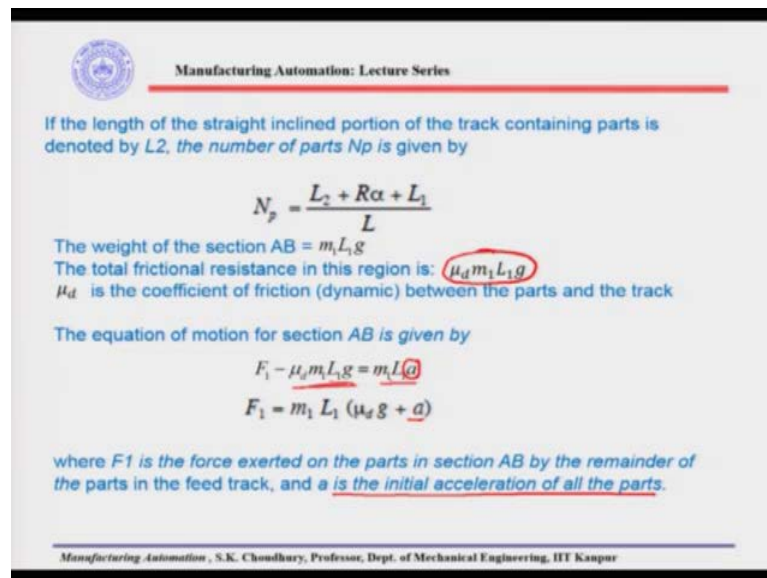
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Now, let us have an analysis of the horizontal delivery feed tracks and let us see how the horizontal delivery feed tracks are designed. Let us take an example that this is the feed track that we have and here we have the inclined portion; inclined straight portion we have the curved portion and then there is a straight portion.

So, AB this is the straight horizontal portion of the feed track, this is the curved BC is the curved portion of the feed track and C D is the straight inclined portion of the feed track. So, in here what happens that this inclined one is inclined to this by an angle let us say alpha, this angle is alpha. And this track is let us assume that this track is filled with the parts. And after this the parts are coming from the feeder from this side and it will go through the feed track and it will be delivered to the work head at this after A, this is the escapement of the parts.

Now, it is assumed in the following analysis that a certain fixed number of parts are maintained in the track above the delivery point. So, there let us assume that this feed track is filled up with the parts.

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If the length of the straight inclined portion of the track containing parts is denoted by  $L_2$ , the number of parts  $N_p$  is given by

$$N_p = \frac{L_2 + R\alpha + L_1}{L}$$

The weight of the section AB =  $m_1 L_1 g$   
The total frictional resistance in this region is:  $\mu_d m_1 L_1 g$   
 $\mu_d$  is the coefficient of friction (dynamic) between the parts and the track

The equation of motion for section AB is given by

$$F_1 - \mu_d m_1 L_1 g = m_1 L_1 a$$

$$F_1 = m_1 L_1 (\mu_d g + a)$$

where  $F_1$  is the force exerted on the parts in section AB by the remainder of the parts in the feed track, and  $a$  is the initial acceleration of all the parts.

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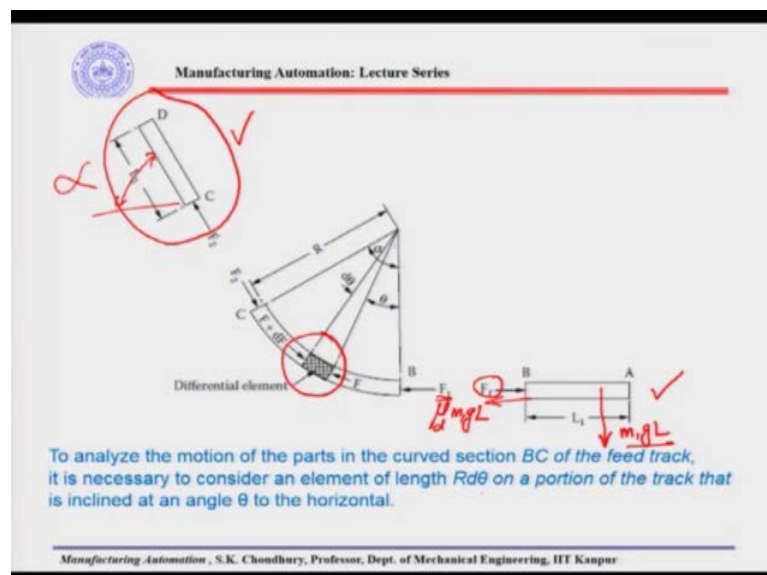
Now, if the length of the straight inclined portion of the track containing parts is denoted by  $L_2$  this is the  $L_2$  and the length of the straight path is  $L_1$ , and the curved portion has a radius of  $R$  and this angle is alpha that is starting of the curve portion and the end of the curve portion.

So, in that case the number of parts  $N_p$ ; total number of parts  $N_p$  in the whole sector on the whole of the feed track, entire feed track from here to here. This will be equal to  $L_2$  plus  $R \sin \alpha$  plus  $L_1$  divided by capital  $L$ ;  $L_2$  is here this is the length plus  $R \sin \alpha$  here plus  $L_1$  divided by the length of the part like this. Now the weight of the section AB here, this is equal to  $m_1 L_1$  and the  $g$ . Now  $m_1$  is the mass of the feed track with the parts ok. So, that is  $m_1$  let say and  $L_1$  is the length of the straight portion of the feed track into  $g$ ;  $g$  is the acceleration due to gravity. The total frictional resistance in this region will be  $\mu_d m_1 L_1$  and the  $g$ .

So, what exactly that you understand that suppose we have this portion this is AB. So, this straight AB portion let say this is A and B this will have the  $m_1 g$  and this way that is what we are showing here that  $m_1 L_1$  and the  $g$  because this is  $L_1$  so  $L$ , and here it will be  $\mu_d m_1 g L$  that is what we are showing it here  $M_1 L_1$  and the  $g$  into the  $\mu_d$ .

So, the  $\mu_d$  is the coefficient of friction this is the dynamic coefficient of friction between the parts and the track alright. The equation of motion for section AB therefore, will be given as  $F_1$  minus  $\mu_d m_1 L_1 g$  is equal to  $m_1 L_1$  and the acceleration this is  $a$  is the initial acceleration of all the parts.

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Let me going ahead show you this that here in this AB, we have this  $F_1$  if it is a free body diagram, free body diagram of the entire feed track; so AB which is the straight part of length  $L_1$ , this has then  $F_1$  here and  $F_1$  reaction here. So, therefore, if we have

this  $m_1 g$  and the  $L$  so, then this force which is going against that, it will be  $\mu_d m_1 g$  and the  $L$ . So, therefore, the equation will be  $F_1$  and opposite direction is  $\mu_d m_1 g$  and the  $L$ ;  $\mu_d m_1 g L$  and the  $g$  this is equal to acceleration, initial acceleration  $m_1$  into  $L$ .

So, this is the Newton's law that we are writing that  $F_1$  minus  $\mu_d m_1 g L$  is equal to  $m_1 L$  and the acceleration. So, from here we can get it rearranging that  $F_1$  this is equal to  $m_1 L$  is common and this is  $\mu_d g$  plus acceleration, initial acceleration. So, what we have seen is that in the straight path, in the straight path we have this equation of motion that can be actually written.

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Similarly, the column of parts in the straight inclined portion  $CD$  of the feed track is partly restrained by a force  $F_2$ , given by

$$F_2 - m_1 L_2 g \sin \alpha + m_1 L_2 \mu_d g \cos \alpha = m_1 L_2 a$$

$$F_2 = m_1 L_2 (g \sin \alpha - \mu_d g \cos \alpha + a)$$

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Similarly, we can find out that how this inclined section equation can be made. So, here this is the inclined section. Inclined section is this one ok; here we have the  $F_2$  and  $F_2$  here if it was  $F_1$ ; now what is  $F_1$ ? Let say,  $F_1$  is the force by the remaining of the parts to the parts which are located at the  $AB$  ok, and the reaction will be  $F_1$  here. So, similarly  $F_2$  is the force imparted on the parts located here by the parts located at the set sector  $C D$ . So, therefore, what we can find out from here is; that if it is  $L_2$  as we said the distance the length is  $L_2$ . So, this is  $F_2$  which is force reaction force coming here.

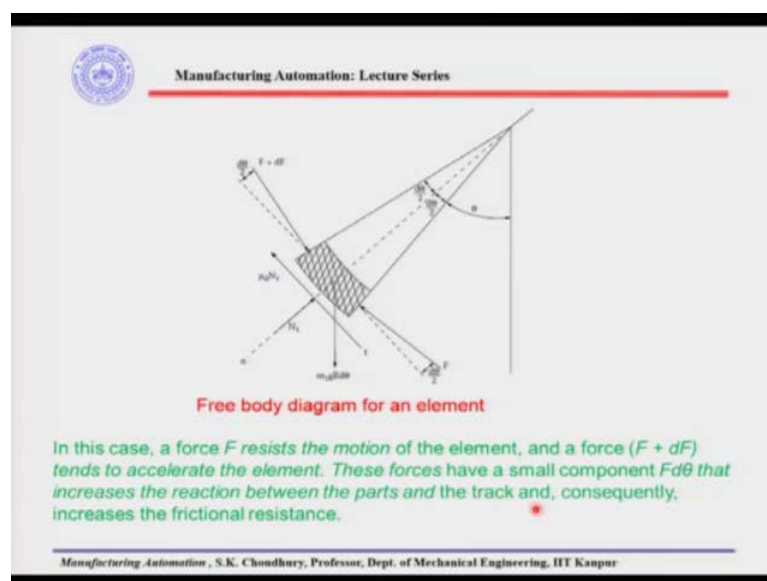
Here it will be  $m_1 L_2$  and the  $g$  like it was for the straight portion like here we are having  $m_1 L_2$  and the  $g$ . So, this force can be resolved into the perpendicular to the axis of the sector and parallel to the axis of this sector. So, here it will be  $m_1 L_2 g$  and the

$\cos \alpha$  this force will be  $m_1 L_2 g$  and the  $\sin \alpha$ . Then there will be a friction force and that will be opposite to this that will be  $m_1 L_2 \mu_d g$  and the  $\cos \alpha$  and  $\mu_d$  is the dynamic coefficient of friction between the part and the feed track. So, therefore, from this diagram we can find out that  $F_2$  that the equation of motion that is  $F_2$  is this opposite direction is  $m_1 L_2 g$  and the  $\sin \alpha$  this one. And then there is one more which is along the  $F_2$  which is  $m_1 L_2 \mu_d$  and the  $\cos \alpha$   $g \cos \alpha$  this one.

So,  $F_2$  and these two forces have the same directions. So, these are in the plus and the opposite direction is  $m_1 L_2 g \sin \alpha$ . So, this one this they will be equal to the  $m_1 L_2$  and the acceleration this can be rearranged by  $F_2$  is equal to  $m_1 L_2$  common  $g \sin \alpha$  minus  $\mu_d$  and the  $g \cos \alpha$  plus  $a$ , because  $m_1 L_2$  is common. So, we have two equations; one is for the straight sector and another is for the straight inclined at an angle with the horizontal by an angle of  $\alpha$ , this is the free body diagram as we said.

So, for these two elements we have the equation of motion and for this element that is the curved portion. To analyse the motion of the part in the curved section BC of the feed track, it is necessary to consider an element of length  $R d\theta$  which is this one on a portion of the track that is inclined at an angle  $\theta$  to the horizontal. So, this is the  $\theta$  angle and here we are taking one small section that is  $R d\theta$  of the length,  $d\theta$  is this angle and  $R$  is the radius of this curved portion. So,  $R$  into  $d\theta$  will be the length.

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Now, here for the section that we have taken that element and in this element let say the forces which are acting we have the  $d\theta$  here. So,  $d\theta$  then we have divided into  $d\theta$  by 2  $d\theta$  by 2 in the middle and here this is the normal force which is acting let say this is  $N_1$ . So, therefore, because of this normal force there is a friction force which is  $\mu dN_1$  and this is the mass of the part acting as  $m_1 g$  and the  $R d\theta$  is the length as we said. And here we will have two forces that is  $F$  and  $F + dF$  on this element small element. So, they will be acting at an angle of  $d\theta$  by 2 as we said here and here it is  $F$  and incremental of that, so  $F + dF$ . Now in this case a force  $F$  resists the motion of the element that is the  $F$  and a force  $F + dF$  tends to accelerate the element because we have selected that element.

These forces have a small component  $F d\theta$  that increases the reaction between the parts and the track and, consequently, increases the frictional resistance.

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The equilibrium equation of forces in the radial direction  $n$  is

$$N_1 = m_1 g R \cos \theta d\theta + F d\theta$$

Ignoring  $dF/d\theta$

The equation of motion in the tangential direction  $t$  is

$$dF + m_1 g R \sin \theta d\theta - \mu_d N_1 = m_1 R d\theta$$

rearranging the terms, the following first-order linear differential equation with constant coefficients is obtained:

$$\frac{dF}{d\theta} - \mu_d F = m_1 R (a + \mu_d g \cos \theta - g \sin \theta)$$

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Now, the equilibrium equation of force once again I have drawn this here the same diagram as in here. So, from here you can see the equilibrium equation of forces in the radial direction  $n$ , let say in this direction ok. In here what we have is the  $N_1$  itself  $N_1$  and this is opposed by  $m_1 g R d\theta$  and the  $\cos \theta$  which will be a component of this. Meaning that in here if you say that this components I mean this force will have two components; one is in the direction of the  $n$  and another is in the vertical direction of the

vertical to  $n$ . So, along the direction of  $n$  this will be  $\cos \theta$  and vertical to the  $n$  will be the  $\sin \theta$  which will be coming when we will take it in this direction.


In the direction of  $N_1$  we will have  $N_1$  and this force component will be in this direction ok, this will be  $m_1 g R \sin \theta$  and the  $\cos \theta$ . So, since it is opposite to  $n$ . So, this will be  $m_1 g \cos \theta \sin \theta$  plus  $F \sin \theta$ ;  $F \sin \theta$  is coming from here that is this is the  $F$ , and this is the  $F$  and this is the  $F$  plus  $dF$ . So, if we ignore really  $dF$  and the  $d\theta$  that is a very small value. So, then we will have that  $F \sin \theta$  by 2 plus  $F \sin \theta$  by 2. So, this will be  $F \sin \theta$  that is why we have taken and this direction will be along the direction of the  $m_1 g R \sin \theta \cos \theta$ .

So, therefore, these two sines are the same  $N_1 \sin$  is opposite. So, the equilibrium equation will be this that is  $N_1$  minus  $m_1 g R \sin \theta \cos \theta$  minus  $F \sin \theta$  because of this two. Because here the component will be coming in this direction. The equation of motion in the tangential direction let us say in this direction perpendicular to  $N_1$  this will be the friction force  $\mu d N_1$  that is 1, then we will have the  $F$  cancelling and we have the  $dF$ . So, the  $dF$  this is the first element, then we have the  $m_1 g R \sin \theta$  and here it will be  $\sin \theta$ . So, this will be in the opposite direction to  $\mu d N_1$ . So, therefore, this is plus and this is minus  $\mu d N_1$ , this is equal to  $m_1 R \sin \theta$  and the acceleration;  $a$  is the initial acceleration of parts in the track.

So, this is understood that we are taking the equilibrium equation, force equation in the direction of the  $N_1$  and in the direction that are vertical to the  $N_1$ . Rearranging the terms the following first order linear differential equation with the constant coefficient can be obtained. So, we are rearranging these two, this one and this one and we are finding that this is  $dF$  by  $d\theta$  the minus  $\mu d$  and the  $F$  is equal to  $m_1 R$  acceleration plus  $\mu d g \cos \theta$  minus  $g \sin \theta$ .

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**The general solution to Equation**

$$F = Ae^{\mu_d \theta} + m_1 g R \left[ \frac{(1 - \mu_d^2) \cos \theta + 2\mu_d g \sin \theta}{1 + \mu_d^2} - \frac{a}{\mu_d g} \right]$$

This equation relates the acceleration of the line of parts along the curved and horizontal sections of the track when a force  $F$  is applied to the end of the line located at an angle  $\theta$ .

There are two applicable boundary conditions:  $\theta = 0$ ,  $F = F_1$ , and  $\theta = \alpha$ ,  $F = F_2$ . The first boundary condition gives


$$A = F_1 - m_1 g R \left( \frac{1 - \mu_d^2}{1 + \mu_d^2} - \frac{a}{\mu_d g} \right)$$

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The general solution of this equation, if you take this it will be something like this, alright. So, this is the general solution, this is given in the mathematics, mathematician can find out. So, this equation relates the acceleration of the line of parts along the curved and horizontal sections of the track when a force  $F$  is applied to the end of the line located at an angle of theta.

Now for us this is not so important, what is important is the final equation that we will be getting, the final solution that we will be getting, what are the components that will be the involved in that? So, there are two applicable boundary conditions theta is equal to 0 then  $F$  becomes  $F_1$  ok, you can see this from here theta is equal to 0. So,  $F$  is becoming the  $F_1$  which is coming here; then the another condition is that when the theta is alpha then  $F$  is equal to  $F_2$  this is this; theta is equal to alpha which is the angle here this alpha then this  $F$  is equal to  $F_2$ . So, we are using the boundary condition. The first boundary condition which is theta is equal to 0 where the  $F$  will be equal to  $F_1$ , we can find out this equation alright from here.

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Using this second boundary condition and solving Equation 5.8 for  $a/g$ , we obtain

$$\frac{a}{g} = \frac{(L_2/R)(\sin \alpha - \mu_d \cos \alpha) - (L_1/R)\mu_d e^{\mu_d \alpha}}{(L_2/R) + (L_1/R)e^{\mu_d \alpha} + (e^{\mu_d \alpha} - 1)/\mu_d}$$

This equation shows the relationship between the non-dimensional acceleration  $a/g$  of the entire line of parts and the design parameters  $L_2/R$ ,  $L_1/R$ ,  $\alpha$ , and  $\mu_d$ .

the time  $t_p$  for one part to move into the workhead can be found from the kinematic expression

$$t_p = \left( \frac{2L}{a} \right)^{1/2}$$

Equation can be approximated as

$$\frac{a}{g} = f_1(\mu_d) + f_2\left(\frac{L_1}{R}\right) + f_3(\alpha) + f_4\left(\frac{L_2}{R}\right)$$

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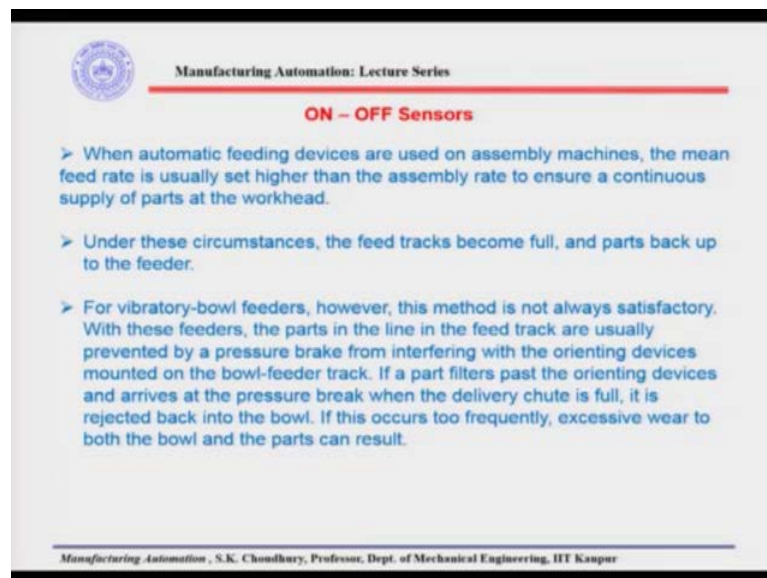
And the second one, using the second boundary condition and solving the equation earlier that was given for  $a$  by  $g$  we can find out is equal to  $L_2/R$  this minus  $L_1/R$   $\mu_d e$  to the power  $\mu_d \alpha$  plus so on so forth. So, as I said that we are actually interested in the final solution the components. These equation shows the relationship between the non dimensional acceleration  $a$  by  $g$  of the entire line of parts and the design parameters  $L_2/R$ ,  $L_1/R$ ,  $\alpha$  and  $\mu_d$  this is more important for us, ok. And by the way this  $a$  by  $g$  we actually said this is  $A_n$  by  $g_n$  this is the same thing please mind it this is equivalent.

So, finally, what we are getting is the non dimensional track acceleration and we are getting different kind of parameters here. So, these parameters actually tell us that  $A_n$  by  $g_n$  which is the non dimensional track acceleration, this can be written as a function of  $\mu_d$  function of  $L_1/R$ ;  $L_1$  is the length of the straight path  $R$  is the radius of the curved path, then  $\alpha$  which is the inclination angle of the straight path inclined to the horizontal with an angle  $\alpha$  and  $L_2/R$ .

So, these are the design parameters that is important and then when you have the track if you are designing the track; so that you can find out what will be the  $A_n$  by  $g_n$  that is the normal track acceleration, dimension less normal track acceleration. So, the time  $t_p$  for one part to move into the work head that can be found out from the kinematic expression of  $t_p$  equal to  $2L$  by  $a$  root over. So, this we have seen earlier in the case of the vertical delivery feed track that this is the  $t_p$  that is a time taken. So, here we have the acceleration which can be selected the one that we are getting from here.

So, this is for horizontal delivery feed track which once again I will say that this is not a very big deal that we are just taking the free body diagram of the straight path, of the straight inclined and the curved portion and in each of these we are finding out what should be the equation of motion. And then we are solving them together to get an equation which will give you the general solution of the equation and from here, we are finding out what are the boundary conditions and from the boundary condition we are finding out the  $A_n$  by  $g_n$ ; that is the normal track acceleration. And we see that the normal track acceleration is a function of dynamic coefficient of friction  $L_1$  by  $R$ ,  $\alpha$  and the  $L_2$  by  $R$ . So, this is more important for us to find out that how they are related.

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Let us discuss one thing which is interesting which is called the ON-OFF Sensors in the feed track. See what happens is when automatic feeding devices are used on the automatic assembly machines. The mean feed rate is usually set at a higher rate than the assembly rate to ensure a continuous supply of parts at the work head. So, I told you this thing earlier also, that work heads should not starve because that will then effect the other adjacent work heads.

Mind it that in a flow line as I said I will remind you the work heads are subsequently arranged and the material handling device passing on one sub assembly from one work head to another work head. So, if we cannot feed the part to the at the right time then that machine will suffer, will starve and the other machines will suffer because of that. So,

therefore, not to do that not to you know start the machine we actually put the  $F$  into  $v$   $m$  of the bowl feeder little higher.

Under this circumstances the feed tracks become full, and parts back up to the feeder ok. For vibratory-bowl feeders; however, this method is not always satisfactory. With these feeders, the parts in the line in the feed track are usually prevented by a pressure brake. We have seen the pressure break you remember, this is that the one that prevents the certain parts of certain orientation to go through because the track is narrowed. So, that pressure break from interfering with the orienting devices mounted on the bowl feeder track.

If a part filters past the orienting devices and arrives at the pressure break when the delivery chute is full it is rejected back. So, what we mean is that suppose the feed conveying velocity as we said that it is set little higher. So, because of that the parts which are coming to the assembly machine; the assembly machine is still not ready to take because our feed rate is higher. So, in that case the parts will actually fall to the bottom of the bowl feeder, because the feed track is filled up. So, they it part cannot go to the feed track even.

So, that will create the excessive wear both the bowl and the parts because of the recirculation. So, once again that in that case what happens the pressure break is there; pressure break is narrowing the track and if the part feeder sorry feed track is filled up because the bowl feeder  $F$  into  $v$   $m$  conveying velocity has been set little more than the parts number of parts machine can accept, then the parts will fall on the at the bottom of the bowl. If the part is falling at the bottom of the bowl it will again re-circulate and then it will get into the friction with the other parts, it will create the wear and tear of the parts and of the bowl as well. So, let us see the ON-OFF sensors how it works to prevent this problem? That we will discuss in the next class.

Thank you very much.