Turbulent Combustion: Theory and Modelling
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Lecture-59
Combustion in 2 Phase Flows (Contd...)

Welcome back. So, let us continue the discussion on 2 phase flow or combustion in 2 phase media. So, we started looking at the 2 phase media and how a single droplet combustion can be estimated, where we can actually estimate the evaporation rate and time.

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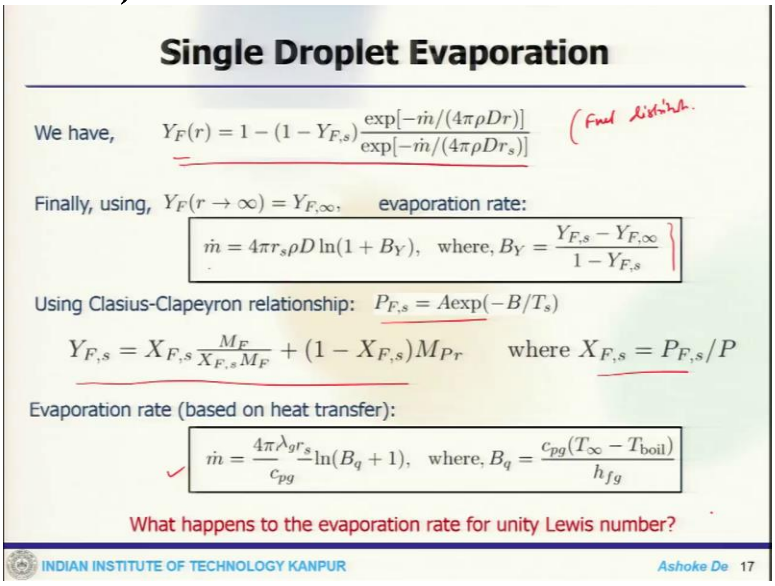

Single Droplet Evaporation

We have, $Y_F(r) = 1 - (1 - Y_{F,s})\frac{\exp[-\dot{m}/(4\pi\rho D r)]}{\exp[-\dot{m}/(4\pi\rho D r_s)]}$ (Fuel distribn.

Finally, using, $Y_F(r \to \infty) = Y_{F,\infty}$, evaporation rate:

$$\boxed{\dot{m} = 4\pi r_s \rho D \ln(1 + B_Y), \quad \text{where, } B_Y = \frac{Y_{F,s} - Y_{F,\infty}}{1 - Y_{F,s}}}$$

Using Clasius-Clapeyron relationship: $P_{F,s} = A\exp(-B/T_s)$

$$Y_{F,s} = X_{F,s}\frac{M_F}{X_{F,s}M_F} + (1 - X_{F,s})M_{Pr} \qquad \text{where } X_{F,s} = P_{F,s}/P$$

Evaporation rate (based on heat transfer):

$$\checkmark\ \boxed{\dot{m} = \frac{4\pi\lambda_g r_s}{c_{pg}}\ln(B_q + 1), \quad \text{where, } B_q = \frac{c_{pg}(T_\infty - T_{\text{boil}})}{h_{fg}}}$$

What happens to the evaporation rate for unity Lewis number?

So, we will continue the discussion. So, this is where we actually stopped.

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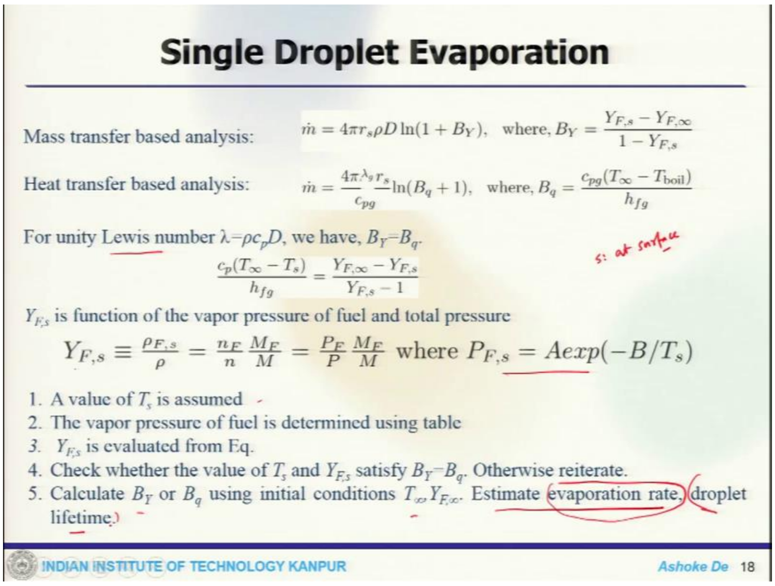

Single Droplet Evaporation

Mass transfer based analysis: $\dot{m} = 4\pi r_s \rho D \ln(1 + B_Y)$, where, $B_Y = \frac{Y_{F,s} - Y_{F,\infty}}{1 - Y_{F,s}}$

Heat transfer based analysis: $\dot{m} = \frac{4\pi\lambda_g r_s}{c_{pg}}\ln(B_q + 1)$, where, $B_q = \frac{c_{pg}(T_\infty - T_{\text{boil}})}{h_{fg}}$

For unity Lewis number $\lambda = \rho c_p D$, we have, $B_Y = B_q$.

s: at surface

$$\frac{c_p(T_\infty - T_s)}{h_{fg}} = \frac{Y_{F,\infty} - Y_{F,s}}{Y_{F,s} - 1}$$

$Y_{F,s}$ is function of the vapor pressure of fuel and total pressure

$$Y_{F,s} \equiv \frac{\rho_{F,s}}{\rho} = \frac{n_F}{n}\frac{M_F}{M} = \frac{P_F}{P}\frac{M_F}{M} \text{ where } P_{F,s} = A exp(-B/T_s)$$

1. A value of T_s is assumed
2. The vapor pressure of fuel is determined using table
3. $Y_{F,s}$ is evaluated from Eq.
4. Check whether the value of T_s and $Y_{F,s}$ satisfy $B_Y = B_q$. Otherwise reiterate.
5. Calculate B_Y or B_q using initial conditions $T_\infty, Y_{F,\infty}$. Estimate evaporation rate, droplet lifetime.

So, this is what we got from the mass transfer based analysis and the second one is the heat transfer based analysis. So, for now, the assumption here is that, if the Lewis number is infinity, then we can write this balance and then finally we can find out the fuel at the surface. So, that is 's' stands for this at surface. Now, how we estimate that, we can see this T_s is assumed.

The vapour pressure of the fuel is also determined using some table then $Y_{F,s}$ is evaluated from the equation and we can check whether T_s and $Y_{F,s}$ satisfy this. Otherwise, we will go for iterative. And then finally, we calculate B_Y and B_q using the initial condition of T_∞ and $Y_{F,\infty}$. And then we can estimate the evaporation rate, droplet lifetime. So, these are the 2 important parameter that we wanted to calculate.

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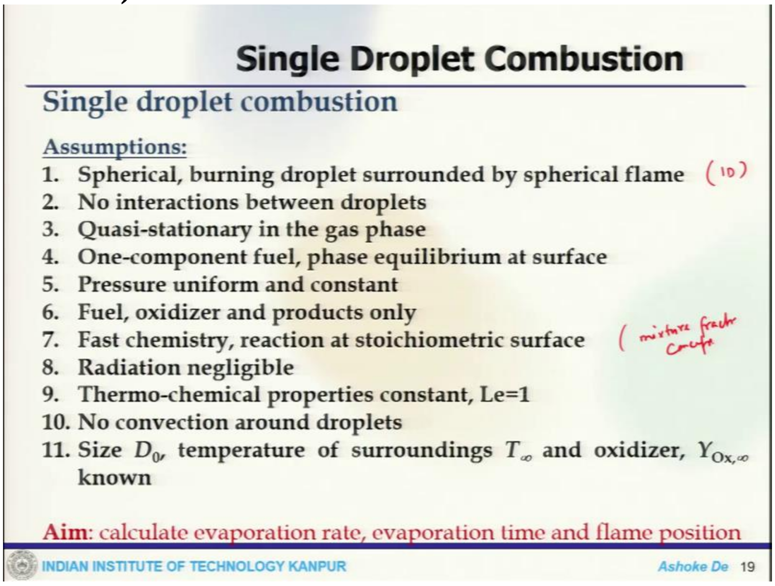

Single Droplet Combustion

Single droplet combustion

Assumptions:

1. Spherical, burning droplet surrounded by spherical flame (1D)
2. No interactions between droplets
3. Quasi-stationary in the gas phase
4. One-component fuel, phase equilibrium at surface
5. Pressure uniform and constant
6. Fuel, oxidizer and products only
7. Fast chemistry, reaction at stoichiometric surface (mixture fraction concept
8. Radiation negligible
9. Thermo-chemical properties constant, Le=1
10. No convection around droplets
11. Size D_0, temperature of surroundings T_∞ and oxidizer, $Y_{Ox,\infty}$ known

Aim: calculate evaporation rate, evaporation time and flame position

Now, when you go to the single droplet combustion, there are also certain assumptions. So, one is that the first assumption is that it is spherical, burning droplet surrounded by spherical flame. Which means essentially we are assuming itself or simplifying to a 1-D problem, where $\dot{m}$ is a constant and function of radius. Now, second one is that no interaction between droplets. That means, it does not take into account droplet collision and also sudden change in droplet size and number partially premixed combustion.

Therefore, diameter only function of $\dot{m}$ and time, It is an Quasi steady stationary in the gas phase which clearly means that steady in the gaseous phase and we can use those assumption. Then one-component fuel, phase equilibrium at the surface. So, which means no mixing inside perfusion diffusion to the droplet surface if one know the temperature. Like Y_g.

Pressure is uniform and constant which simplifies the analysis. Fuel, oxidizer and products only. So that also is simplification. Then fast chemistry, reaction at stoichiometric surface which allows to use the concept killers approach like mixture fraction concept and then radiation is negligible. So, which also simplify the energy equation. Thermo-chemical properties are constant at Lewis number 0. No convection around the droplets. So, that mean all velocity is 0 and the size D_0, temperature of the surroundings T_∞.

So, the whole idea is to calculate the evaporation rate, evaporation time and flame position. The approach here actually extends the development done for this spherically symmetric droplet evaporating in quiescent environment, as we have discussed earlier. So, I have written the assumption of stagnant environment and spherical symmetry in the initial development, but subsequently we can show how the spherically symmetric results can be adjusted to take into account the environment of burning caused by convection either all these things.

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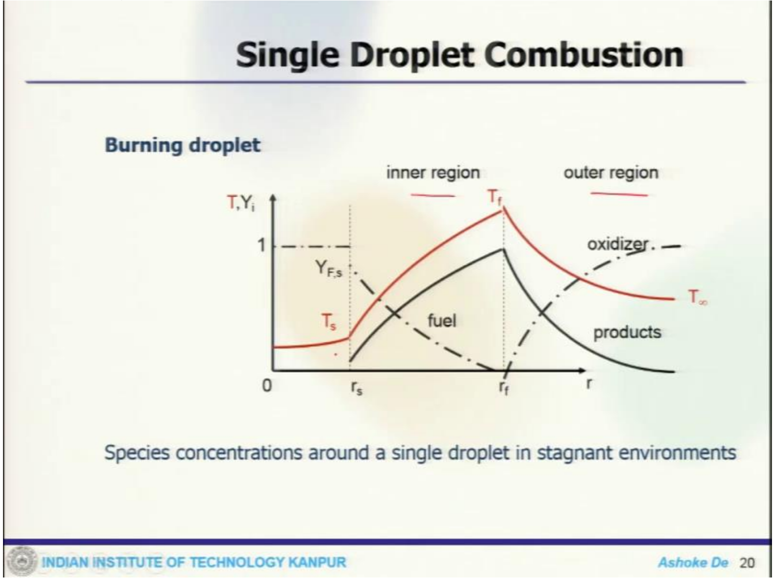

Single Droplet Combustion

Burning droplet

Species concentrations around a single droplet in stagnant environments

So that is essentially gives you this is how the inner region, this is the outer region from where oxidizer comes, this is fuel, temperature and the product and this is the surface from where it goes. So, this is the liquid zone, which is between the inner zone and the outer zone.

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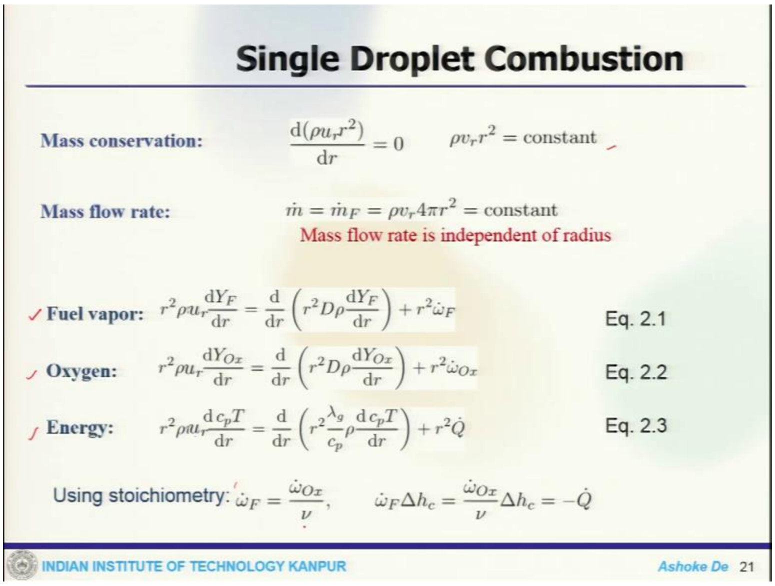

Single Droplet Combustion

Mass conservation: $\frac{\mathrm{d}(\rho u_r r^2)}{\mathrm{d}r} = 0 \qquad \rho v_r r^2 = \text{constant}$

Mass flow rate: $\dot{m} = \dot{m}_F = \rho v_r 4\pi r^2 = \text{constant}$

Mass flow rate is independent of radius

✓ **Fuel vapor:** $r^2 \rho u_r \frac{\mathrm{d}Y_F}{\mathrm{d}r} = \frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 D\rho \frac{\mathrm{d}Y_F}{\mathrm{d}r}\right) + r^2 \dot{\omega}_F$ Eq. 2.1

✓ **Oxygen:** $r^2 \rho u_r \frac{\mathrm{d}Y_{Ox}}{\mathrm{d}r} = \frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 D\rho \frac{\mathrm{d}Y_{Ox}}{\mathrm{d}r}\right) + r^2 \dot{\omega}_{Ox}$ Eq. 2.2

✓ **Energy:** $r^2 \rho u_r \frac{\mathrm{d}\,c_p T}{\mathrm{d}r} = \frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 \frac{\lambda_g}{c_p} \rho \frac{\mathrm{d}\,c_p T}{\mathrm{d}r}\right) + r^2 \dot{Q}$ Eq. 2.3

Using stoichiometry: $\dot{\omega}_F = \frac{\dot{\omega}_{Ox}}{\nu}, \qquad \dot{\omega}_F \Delta h_c = \frac{\dot{\omega}_{Ox}}{\nu} \Delta h_c = -\dot{Q}$

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So, we can write the mass conservation equation and then the mass flow rate would be this. So, one can write fuel mass transfer equation in terms of fuel mass fraction Y_F, oxidizer mass fraction equation which is Y_{Ox}, energy equation and then we can use the stoichiometric. So,

$$\dot{\omega}_F = \frac{\dot{\omega}_{Ox}}{\nu}$$

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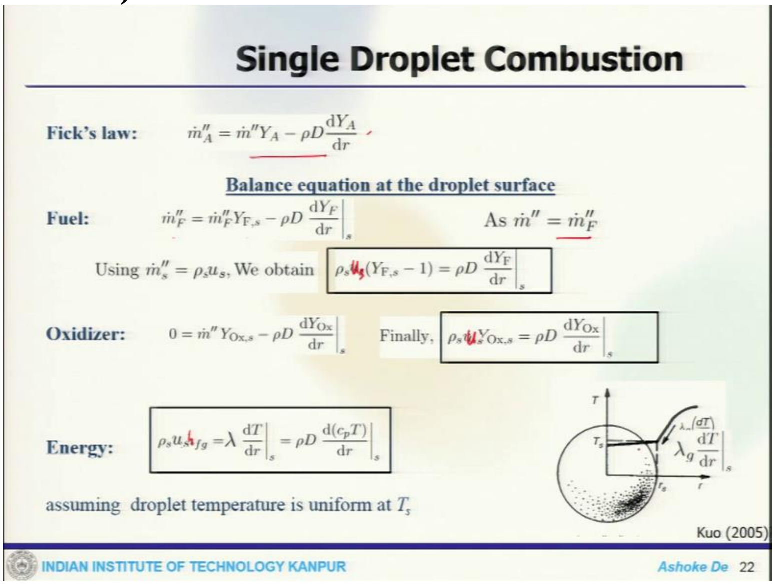

Single Droplet Combustion

Fick's law: $\dot{m}''_A = \dot{m}'' Y_A - \rho D \frac{\mathrm{d}Y_A}{\mathrm{d}r}$

Balance equation at the droplet surface

Fuel: $\dot{m}''_F = \dot{m}''_F Y_{F,s} - \rho D \left.\frac{\mathrm{d}Y_F}{\mathrm{d}r}\right|_s$ As $\dot{m}'' = \dot{m}''_F$

Using $\dot{m}''_s = \rho_s u_s$, We obtain $\boxed{\rho_s u_s (Y_{F,s} - 1) = \rho D \left.\frac{\mathrm{d}Y_F}{\mathrm{d}r}\right|_s}$

Oxidizer: $0 = \dot{m}'' Y_{Ox,s} - \rho D \left.\frac{\mathrm{d}Y_{Ox}}{\mathrm{d}r}\right|_s$ Finally, $\boxed{\rho_s u_s Y_{Ox,s} = \rho D \left.\frac{\mathrm{d}Y_{Ox}}{\mathrm{d}r}\right|_s}$

Energy: $\boxed{\rho_s u_s h_{fg} = \lambda \left.\frac{\mathrm{d}T}{\mathrm{d}r}\right|_s = \rho D \left.\frac{\mathrm{d}(c_p T)}{\mathrm{d}r}\right|_s}$

assuming droplet temperature is uniform at T_s

Kuo (2005)

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Now, you also use the Fick's law of diffusion. So, which is like this and then we put it back in the equation system. So, what we get:

$$\dot{m}''_A = \dot{m}'' Y_A - \rho D \frac{dY_A}{dr}$$

where this is like this and using this what we get that $\rho_s u_s$ and this and here also finally, $\rho_s u_s$ this is also $u_s \rho D$ and energy equation becomes this is h_{fg} and if you look at the surface this is the droplet surface through which the heat transfer comes in. So, the droplet temperature here is uniform.

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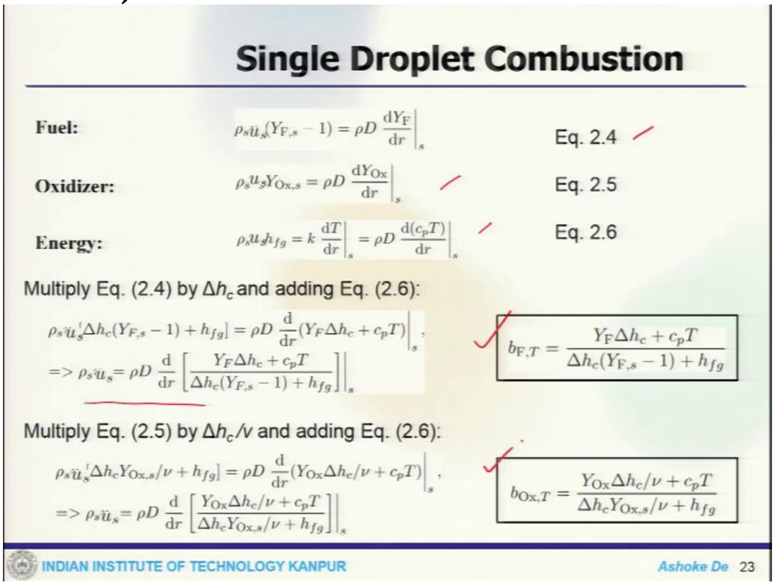

Single Droplet Combustion

Fuel: $\rho_s u_s (Y_{F,s} - 1) = \rho D \left.\frac{dY_F}{dr}\right|_s$ Eq. 2.4 ✓

Oxidizer: $\rho_s u_s Y_{Ox,s} = \rho D \left.\frac{dY_{Ox}}{dr}\right|_s$ ✓ Eq. 2.5

Energy: $\rho_s u_s h_{fg} = k \left.\frac{dT}{dr}\right|_s = \rho D \left.\frac{d(c_p T)}{dr}\right|_s$ ✓ Eq. 2.6

Multiply Eq. (2.4) by Δh_c and adding Eq. (2.6):

$$\rho_s u_s [\Delta h_c (Y_{F,s} - 1) + h_{fg}] = \rho D \left.\frac{d}{dr}(Y_F \Delta h_c + c_p T)\right|_s ,$$

$$\Rightarrow \underline{\rho_s u_s} = \rho D \left.\frac{d}{dr}\left[\frac{Y_F \Delta h_c + c_p T}{\Delta h_c (Y_{F,s} - 1) + h_{fg}}\right]\right|_s$$

✓ $$\boxed{b_{F,T} = \frac{Y_F \Delta h_c + c_p T}{\Delta h_c (Y_{F,s} - 1) + h_{fg}}}$$

Multiply Eq. (2.5) by $\Delta h_c/\nu$ and adding Eq. (2.6):

$$\rho_s u_s [\Delta h_c Y_{Ox,s}/\nu + h_{fg}] = \rho D \left.\frac{d}{dr}(Y_{Ox} \Delta h_c/\nu + c_p T)\right|_s ,$$

$$\Rightarrow \rho_s u_s = \rho D \left.\frac{d}{dr}\left[\frac{Y_{Ox} \Delta h_c/\nu + c_p T}{\Delta h_c Y_{Ox,s}/\nu + h_{fg}}\right]\right|_s$$

✓ $$\boxed{b_{Ox,T} = \frac{Y_{Ox} \Delta h_c/\nu + c_p T}{\Delta h_c Y_{Ox,s}/\nu + h_{fg}}}$$

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So, we write the fuel equation, oxidizer equation and energy equation then multiply this 1 by Δh_c and adding to this, we get this from we calculate $b_{F,T}$. Similarly, we will do some algebra. We can also get $b_{Ox,T}$. So, this one can get. Again going back to this. So, for the $b_{Ox,T}$ we can get.

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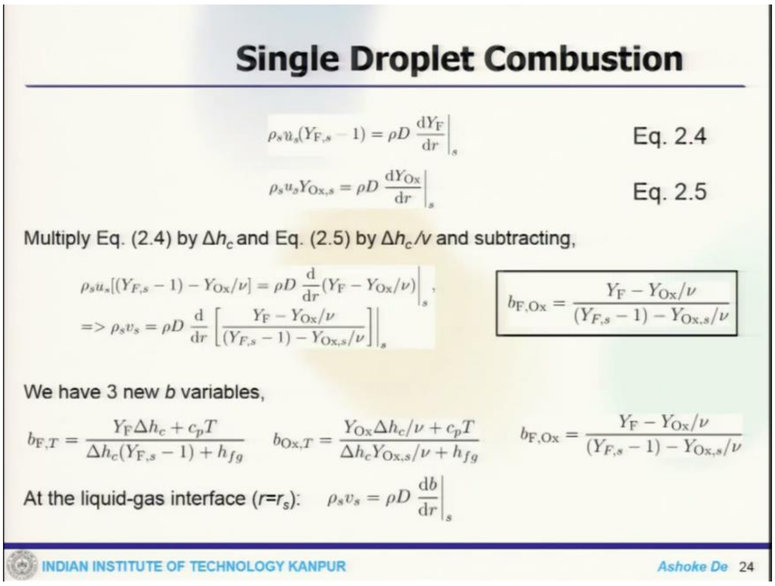

Single Droplet Combustion

$\rho_s u_s (Y_{F,s} - 1) = \rho D \left.\frac{dY_F}{dr}\right|_s$ Eq. 2.4

$\rho_s u_s Y_{Ox,s} = \rho D \left.\frac{dY_{Ox}}{dr}\right|_s$ Eq. 2.5

Multiply Eq. (2.4) by Δh_c and Eq. (2.5) by $\Delta h_c/\nu$ and subtracting,

$$\rho_s u_s [(Y_{F,s} - 1) - Y_{Ox}/\nu] = \rho D \left.\frac{d}{dr}(Y_F - Y_{Ox}/\nu)\right|_s ,$$

$$\Rightarrow \rho_s v_s = \rho D \left.\frac{d}{dr}\left[\frac{Y_F - Y_{Ox}/\nu}{(Y_{F,s} - 1) - Y_{Ox,s}/\nu}\right]\right|_s$$

$$\boxed{b_{F,Ox} = \frac{Y_F - Y_{Ox}/\nu}{(Y_{F,s} - 1) - Y_{Ox,s}/\nu}}$$

We have 3 new *b* variables,

$$b_{F,T} = \frac{Y_F \Delta h_c + c_p T}{\Delta h_c (Y_{F,s} - 1) + h_{fg}} \quad b_{Ox,T} = \frac{Y_{Ox} \Delta h_c/\nu + c_p T}{\Delta h_c Y_{Ox,s}/\nu + h_{fg}} \quad b_{F,Ox} = \frac{Y_F - Y_{Ox}/\nu}{(Y_{F,s} - 1) - Y_{Ox,s}/\nu}$$

At the liquid-gas interface ($r=r_s$): $\rho_s v_s = \rho D \left.\frac{db}{dr}\right|_s$

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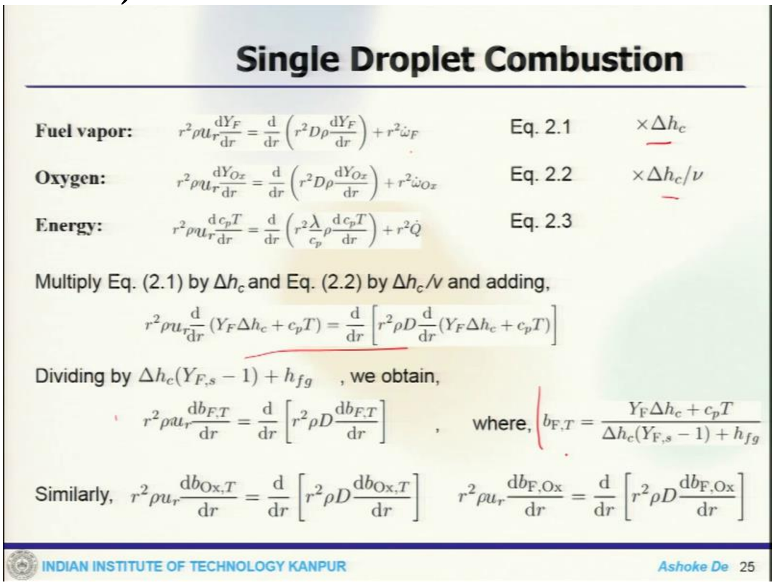

Single Droplet Combustion

Fuel vapor: $r^2\rho u_r\frac{dY_F}{dr} = \frac{d}{dr}\left(r^2 D\rho\frac{dY_F}{dr}\right) + r^2\dot{\omega}_F$ Eq. 2.1 $\times\Delta h_c$

Oxygen: $r^2\rho u_r\frac{dY_{Ox}}{dr} = \frac{d}{dr}\left(r^2 D\rho\frac{dY_{Ox}}{dr}\right) + r^2\dot{\omega}_{Ox}$ Eq. 2.2 $\times\Delta h_c/\nu$

Energy: $r^2\rho u_r\frac{d c_p T}{dr} = \frac{d}{dr}\left(r^2\frac{\lambda}{c_p}\rho\frac{d c_p T}{dr}\right) + r^2\dot{Q}$ Eq. 2.3

Multiply Eq. (2.1) by Δh_c and Eq. (2.2) by $\Delta h_c/v$ and adding,

$$r^2\rho u_r\frac{d}{dr}(Y_F\Delta h_c + c_pT) = \frac{d}{dr}\left[r^2\rho D\frac{d}{dr}(Y_F\Delta h_c + c_pT)\right]$$

Dividing by $\Delta h_c(Y_{F,s}-1)+h_{fg}$, we obtain,

$$r^2\rho u_r\frac{db_{F,T}}{dr} = \frac{d}{dr}\left[r^2\rho D\frac{db_{F,T}}{dr}\right] , \quad \text{where, } b_{F,T} = \frac{Y_F\Delta h_c + c_pT}{\Delta h_c(Y_{F,s}-1)+h_{fg}}$$

Similarly, $r^2\rho u_r\frac{db_{Ox,T}}{dr} = \frac{d}{dr}\left[r^2\rho D\frac{db_{Ox,T}}{dr}\right]$ $r^2\rho u_r\frac{db_{F,Ox}}{dr} = \frac{d}{dr}\left[r^2\rho D\frac{db_{F,Ox}}{dr}\right]$

So, these are the calculations one can do and so, it just like fuel vapour multiply these oxygen doing these and then you do the algebra. So, you get these where $b_{F,T}$ you obtain like that.

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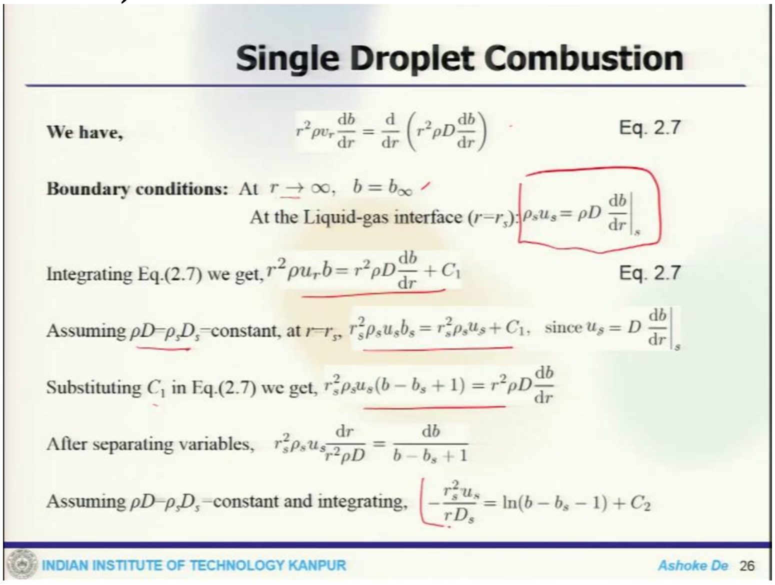

Single Droplet Combustion

We have, $r^2\rho v_r\frac{db}{dr} = \frac{d}{dr}\left(r^2\rho D\frac{db}{dr}\right)$ Eq. 2.7

Boundary conditions: At $r\to\infty$, $b=b_\infty$

At the Liquid-gas interface ($r=r_s$): $\boxed{\rho_s u_s = \rho D\left.\frac{db}{dr}\right|_s}$

Integrating Eq.(2.7) we get, $r^2\rho u_r b = r^2\rho D\frac{db}{dr} + C_1$ Eq. 2.7

Assuming $\rho D=\rho_s D_s=$constant, at $r=r_s$, $r_s^2\rho_s u_s b_s = r_s^2\rho_s u_s + C_1$, since $u_s = D\left.\frac{db}{dr}\right|_s$

Substituting C_1 in Eq.(2.7) we get, $r_s^2\rho_s u_s(b-b_s+1) = r^2\rho D\frac{db}{dr}$

After separating variables, $r_s^2\rho_s u_s\frac{dr}{r^2\rho D} = \frac{db}{b-b_s+1}$

Assuming $\rho D=\rho_s D_s=$constant and integrating, $-\frac{r_s^2 u_s}{rD_s} = \ln(b-b_s-1)+C_2$

So, this is a simple algebra one can carry out and get it and now if the boundary condition if we put r tends to infinity, b would be b_∞. So, this will give me this at the interface. Now, we integrate this, first integration constant. Assuming ρD is this. Then we can get. Then substitute C in this equation and we get this 1 and separation of variable give us this and then finally, after integration.

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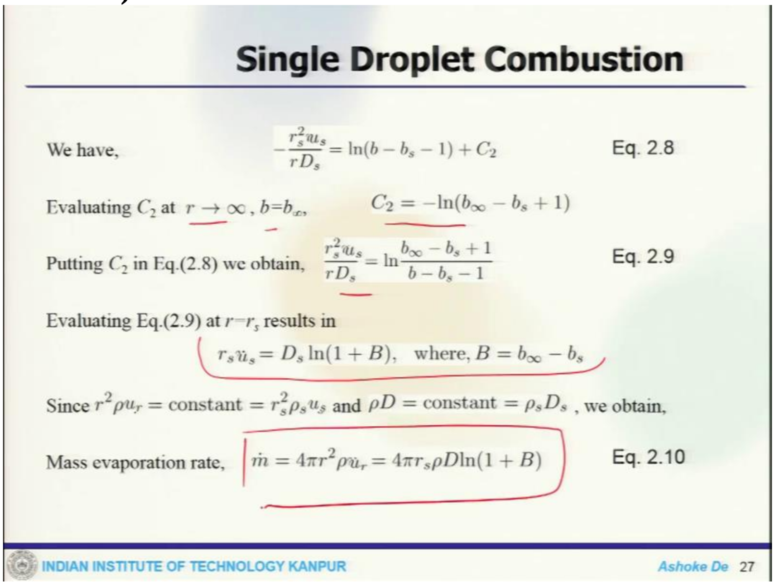

Single Droplet Combustion

We have, $-\frac{r_s^2 u_s}{r D_s} = \ln(b - b_s - 1) + C_2$ Eq. 2.8

Evaluating C_2 at $r \rightarrow \infty$, $b=b_\infty$, $C_2 = -\ln(b_\infty - b_s + 1)$

Putting C_2 in Eq.(2.8) we obtain, $\frac{r_s^2 u_s}{r D_s} = \ln\frac{b_\infty - b_s + 1}{b - b_s - 1}$ Eq. 2.9

Evaluating Eq.(2.9) at $r=r_s$ results in

$$r_s \dot{u}_s = D_s \ln(1 + B), \quad \text{where, } B = b_\infty - b_s$$

Since $r^2 \rho u_r = \text{constant} = r_s^2 \rho_s u_s$ and $\rho D = \text{constant} = \rho_s D_s$, we obtain,

Mass evaporation rate, $\dot{m} = 4\pi r^2 \rho \dot{u}_r = 4\pi r_s \rho D \ln(1 + B)$ Eq. 2.10

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Then the second boundary condition r tends to infinity, b is b_∞. So, we will get C$_2$ we put it back this is what we get. And then finally, this is what it is. So, these is an evaporation rate:

$$\dot{m} = 4\pi r^2 \rho u_r$$

This is how it is estimated.

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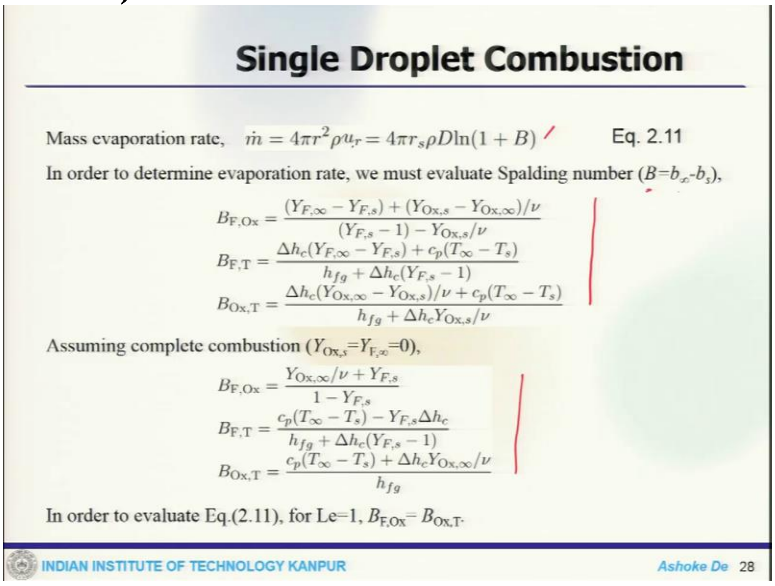

Single Droplet Combustion

Mass evaporation rate, $\dot{m} = 4\pi r^2 \rho u_r = 4\pi r_s \rho D \ln(1 + B)$ Eq. 2.11

In order to determine evaporation rate, we must evaluate Spalding number ($B=b_\infty-b_s$),

$$B_{F,Ox} = \frac{(Y_{F,\infty} - Y_{F,s}) + (Y_{Ox,s} - Y_{Ox,\infty})/\nu}{(Y_{F,s} - 1) - Y_{Ox,s}/\nu}$$

$$B_{F,T} = \frac{\Delta h_c (Y_{F,\infty} - Y_{F,s}) + c_p (T_\infty - T_s)}{h_{fg} + \Delta h_c (Y_{F,s} - 1)}$$

$$B_{Ox,T} = \frac{\Delta h_c (Y_{Ox,\infty} - Y_{Ox,s})/\nu + c_p (T_\infty - T_s)}{h_{fg} + \Delta h_c Y_{Ox,s}/\nu}$$

Assuming complete combustion ($Y_{Ox,s}=Y_{F,\infty}=0$),

$$B_{F,Ox} = \frac{Y_{Ox,\infty}/\nu + Y_{F,s}}{1 - Y_{F,s}}$$

$$B_{F,T} = \frac{c_p (T_\infty - T_s) - Y_{F,s} \Delta h_c}{h_{fg} + \Delta h_c (Y_{F,s} - 1)}$$

$$B_{Ox,T} = \frac{c_p (T_\infty - T_s) + \Delta h_c Y_{Ox,\infty}/\nu}{h_{fg}}$$

In order to evaluate Eq.(2.11), for Le=1, $B_{F,Ox} = B_{Ox,T}$.

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Now, the mass conservation or mass evaporation rate also estimated like this and in order to determine the evaporation rate we calculate the spalding number $b_\infty - b_s$ using this information and this. So, these are standard calculations if you look at any textbook you can find this.

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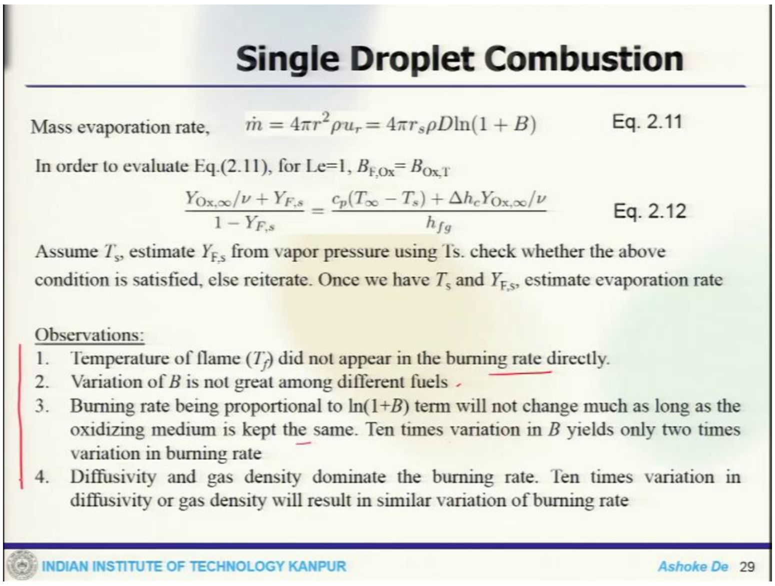

Single Droplet Combustion

Mass evaporation rate, $\dot{m} = 4\pi r^2 \rho u_r = 4\pi r_s \rho D \ln(1+B)$ Eq. 2.11

In order to evaluate Eq.(2.11), for Le=1, $B_{F,Ox} = B_{Ox,T}$

$$\frac{Y_{Ox,\infty}/\nu + Y_{F,s}}{1 - Y_{F,s}} = \frac{c_p(T_\infty - T_s) + \Delta h_c Y_{Ox,\infty}/\nu}{h_{fg}} \quad \text{Eq. 2.12}$$

Assume T_s, estimate $Y_{F,s}$ from vapor pressure using Ts. check whether the above condition is satisfied, else reiterate. Once we have T_s and $Y_{F,s}$, estimate evaporation rate

Observations:

1. Temperature of flame (T_f) did not appear in the burning rate directly.
2. Variation of B is not great among different fuels
3. Burning rate being proportional to ln(1+B) term will not change much as long as the oxidizing medium is kept the same. Ten times variation in B yields only two times variation in burning rate
4. Diffusivity and gas density dominate the burning rate. Ten times variation in diffusivity or gas density will result in similar variation of burning rate

So, more we can go into this what we observe here. The temperature of the flame did not appear in the burning rate directly. So, that is one of the observation that one can have. Secondly, the variation of B is not great among the different fuels that is second thing. Then the burning rate being proportional to the ln(1 + β) will not change much as long as the oxidizing medium is kept the same.

So 10 times variation in B yields only 2 times variation in the burning rate, and, the diffusivity and gas density dominate the burning rate.

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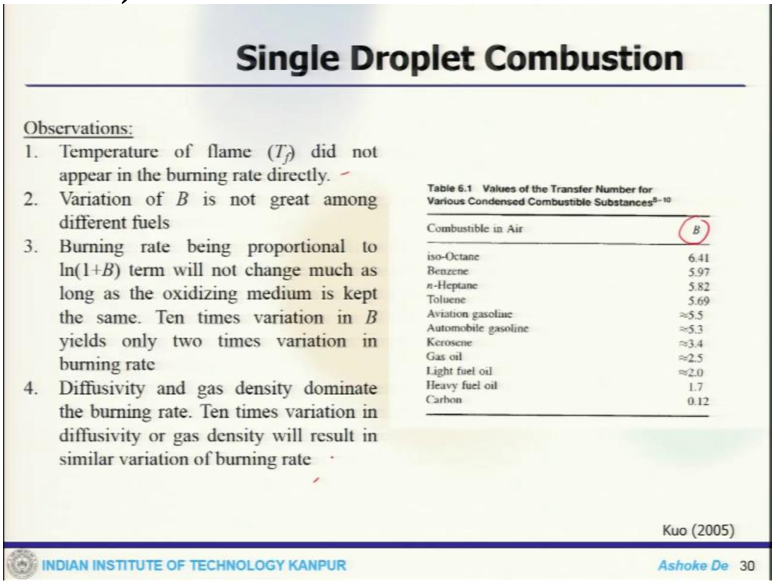

Single Droplet Combustion

Observations:

1. Temperature of flame (T_f) did not appear in the burning rate directly.
2. Variation of B is not great among different fuels
3. Burning rate being proportional to ln(1+B) term will not change much as long as the oxidizing medium is kept the same. Ten times variation in B yields only two times variation in burning rate
4. Diffusivity and gas density dominate the burning rate. Ten times variation in diffusivity or gas density will result in similar variation of burning rate

Table 6.1 Values of the Transfer Number for Various Condensed Combustible Substances[8-10]

Combustible in Air	B
iso-Octane	6.41
Benzene	5.97
n-Heptane	5.82
Toluene	5.69
Aviation gasoline	≈5.5
Automobile gasoline	≈5.3
Kerosene	≈3.4
Gas oil	≈2.5
Light fuel oil	≈2.0
Heavy fuel oil	1.7
Carbon	0.12

Kuo (2005)

So, these are the observation which allows and now these are the table which shows that the different transfer number for different fuels. So, this is the B or the Spalding number. So temperature did not appear in the burning rate variation of B is not great, burning rate being proportional to ln(1 + β) will not change much as the medium is the same. And the diffusivity and the gas density dominate the burning rate and there is a 10 times variance in the diffusivity. So, that is what one get.

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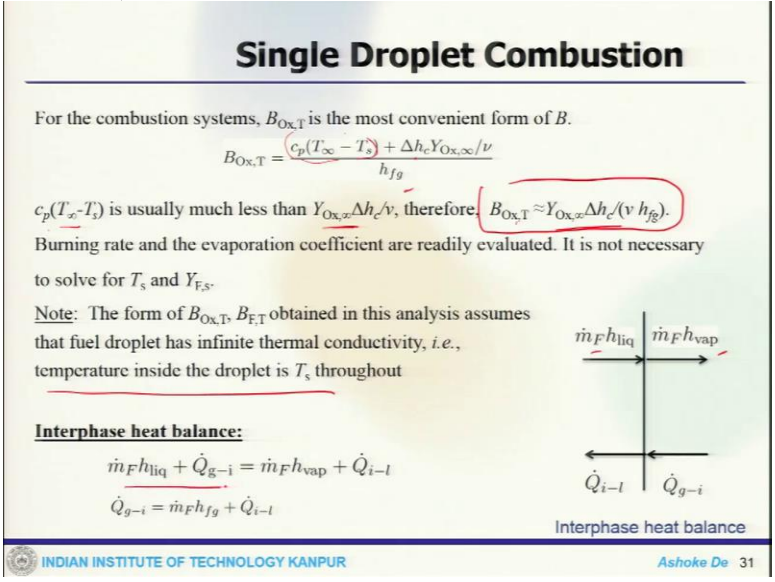

Single Droplet Combustion

For the combustion systems, $B_{\mathrm{Ox,T}}$ is the most convenient form of B.

$$B_{\mathrm{Ox,T}} = \frac{c_p(T_\infty - T_s) + \Delta h_c Y_{\mathrm{Ox},\infty}/\nu}{h_{fg}}$$

$c_p(T_\infty - T_s)$ is usually much less than $Y_{\mathrm{Ox},\infty}\Delta h_c/\nu$, therefore $\boxed{B_{\mathrm{Ox,T}} \approx Y_{\mathrm{Ox},\infty}\Delta h_c/(\nu h_{fg})}$.

Burning rate and the evaporation coefficient are readily evaluated. It is not necessary to solve for T_s and $Y_{F,s}$.

Note: The form of $B_{\mathrm{Ox,T}}$, $B_{\mathrm{F,T}}$ obtained in this analysis assumes that fuel droplet has infinite thermal conductivity, *i.e.*, temperature inside the droplet is T_s throughout

Interphase heat balance:

$$\dot{m}_F h_{\mathrm{liq}} + \dot{Q}_{\mathrm{g-i}} = \dot{m}_F h_{\mathrm{vap}} + \dot{Q}_{i-l}$$

$$\dot{Q}_{g-i} = \dot{m}_F h_{fg} + \dot{Q}_{i-l}$$

Interphase heat balance

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Now from the combustion system $B_{\mathrm{Ox,T}}$ is the most convenient form of B. So, one can write that there is a change in the resulting term. This particular term is less than the standard. So, basically the first term in the numerator is not that high compared to the second term. So, one can actually approximate this one like this. This is a fairly simple assumption and good enough assumption.

Now burning rate and the evaporation coefficient are readily evaluated. So it is not necessary. Now here one can note the form of $B_{\mathrm{Ox,T}}$ or $B_{\mathrm{F,T}}$ obtained in the analysis, assume that the fuel droplet has infinite thermal conductivity, that is temperature inside the droplet is throughout that is maintained as T_s. So, that is one of the assumption. Now at the interface if you do the heat balance, this is what coming out of the liquid. This goes as a vapour goes in. So, if we write down the heat balance this is what we get.

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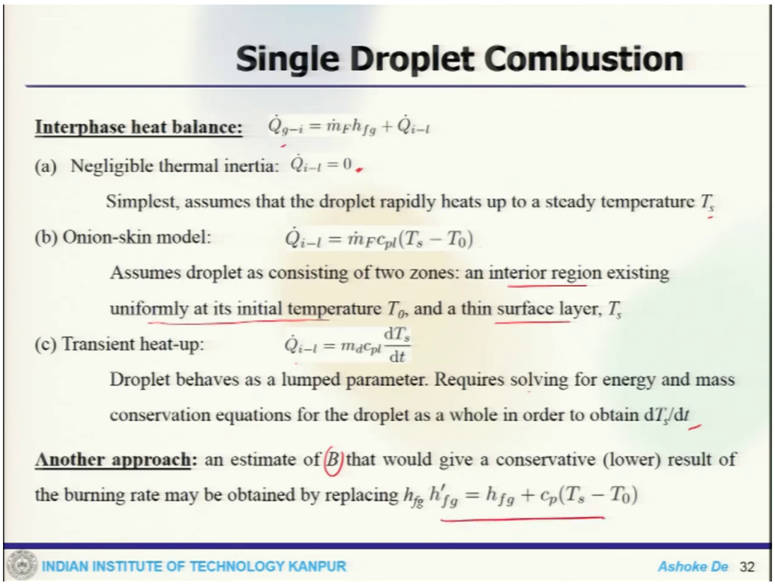

Single Droplet Combustion

Interphase heat balance: $\dot{Q}_{g-i} = \dot{m}_F h_{fg} + \dot{Q}_{i-l}$

(a) Negligible thermal inertia: $\dot{Q}_{i-l} = 0$

Simplest, assumes that the droplet rapidly heats up to a steady temperature T_s

(b) Onion-skin model: $\dot{Q}_{i-l} = \dot{m}_F c_{pl}(T_s - T_0)$

Assumes droplet as consisting of two zones: an interior region existing uniformly at its initial temperature T_0, and a thin surface layer, T_s

(c) Transient heat-up: $\dot{Q}_{i-l} = m_d c_{pl} \frac{dT_s}{dt}$

Droplet behaves as a lumped parameter. Requires solving for energy and mass conservation equations for the droplet as a whole in order to obtain dT_s/dt

Another approach: an estimate of B that would give a conservative (lower) result of the burning rate may be obtained by replacing h_{fg} $h'_{fg} = h_{fg} + c_p(T_s - T_0)$

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Now, interface one can write:

$$\dot{Q}_{g-i} = \dot{m}_F h_{fg} + \dot{Q}_{i-l}$$

So the negligible thermal inertia simplest assume that the droplet rapidly heats up to temperature T_s. And there is an onion skin model which says that:

$$\dot{Q}_{i-l} = \dot{m}_F c_{pl}(T_s - T_0)$$

Here assumption is that the droplet as consisting of 2 zones an inner region existing uniformly at its initial temperature T_0 and a thin surface layer T_s. Then one can have transient heat up where it can be.

So, droplet behaves as a lumped parameter. Requires solving for energy and mass conservation equation for the droplet as a whole in order to obtain $\frac{dT_s}{dt}$. Now, alternative approach is that, one can estimate the B and that would give you a conservative results of the burning rate obtained by using this equation.

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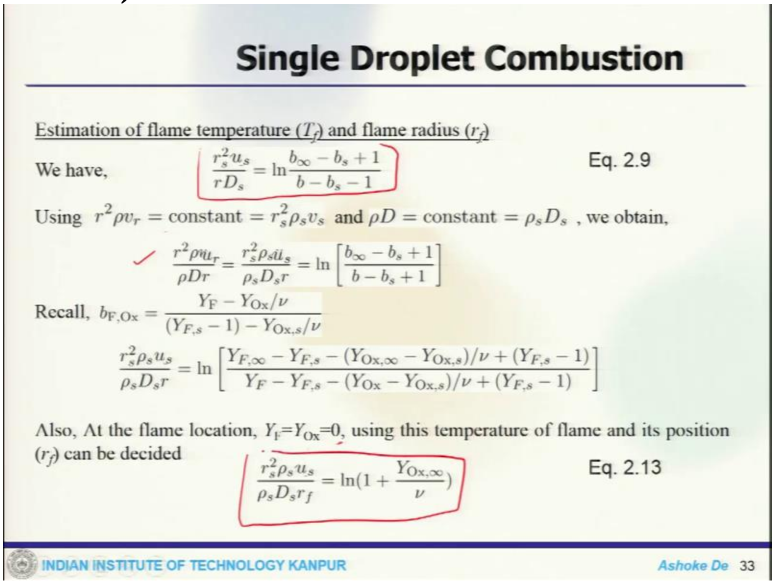

Single Droplet Combustion

Estimation of flame temperature (T_f) and flame radius (r_f)

We have, $$\boxed{\frac{r_s^2 u_s}{r D_s} = \ln \frac{b_\infty - b_s + 1}{b - b_s - 1}}$$ Eq. 2.9

Using $r^2 \rho v_r = \text{constant} = r_s^2 \rho_s v_s$ and $\rho D = \text{constant} = \rho_s D_s$, we obtain,

$$✓ \quad \frac{r^2 \rho u_r}{\rho D r} = \frac{r_s^2 \rho_s u_s}{\rho_s D_s r} = \ln \left[\frac{b_\infty - b_s + 1}{b - b_s + 1} \right]$$

Recall, $b_{\mathrm{F,Ox}} = \dfrac{Y_{\mathrm{F}} - Y_{\mathrm{Ox}}/\nu}{(Y_{F,s} - 1) - Y_{\mathrm{Ox},s}/\nu}$

$$\frac{r_s^2 \rho_s u_s}{\rho_s D_s r} = \ln \left[\frac{Y_{F,\infty} - Y_{F,s} - (Y_{\mathrm{Ox},\infty} - Y_{\mathrm{Ox},s})/\nu + (Y_{F,s} - 1)}{Y_F - Y_{F,s} - (Y_{\mathrm{Ox}} - Y_{\mathrm{Ox},s})/\nu + (Y_{F,s} - 1)} \right]$$

Also, At the flame location, $Y_{\mathrm{F}}=Y_{\mathrm{Ox}}=0$, using this temperature of flame and its position (r_f) can be decided

$$\boxed{\frac{r_s^2 \rho_s u_s}{\rho_s D_s r_f} = \ln(1 + \frac{Y_{\mathrm{Ox},\infty}}{\nu})}$$ Eq. 2.13

Now, the estimation of the flame temperature and the flame radius T$_f$ and r$_f$. So, we have $r_s^2 u_s$ by this already we have obtained. Now, using $r^2 \rho v_r$ here which is constant equals to $r_s^2 \rho_s v_s$. That is the essential at the surface and the so ρD is also constant. So, using this information in this equation, we get this. And then we recall the b$_f$ value and that once we put. Now at the flame location, fuel and oxygen would be 0. Using this, the temperature of the flame and the r$_f$ can be obtained.

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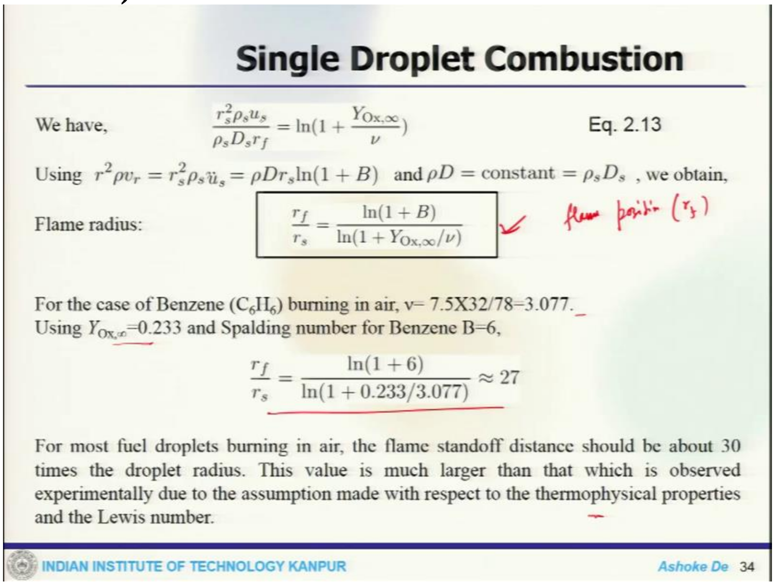

Single Droplet Combustion

We have, $$\frac{r_s^2 \rho_s u_s}{\rho_s D_s r_f} = \ln(1 + \frac{Y_{\mathrm{Ox},\infty}}{\nu})$$ Eq. 2.13

Using $r^2 \rho v_r = r_s^2 \rho_s \ddot{u}_s = \rho D r_s \ln(1 + B)$ and $\rho D = \text{constant} = \rho_s D_s$, we obtain,

Flame radius: $$\boxed{\frac{r_f}{r_s} = \frac{\ln(1 + B)}{\ln(1 + Y_{\mathrm{Ox},\infty}/\nu)}} ✓$$ flame position (r_f)

For the case of Benzene (C_6H_6) burning in air, v= 7.5X32/78=3.077.
Using $Y_{\mathrm{Ox},\infty}$=0.233 and Spalding number for Benzene B=6,

$$\frac{r_f}{r_s} = \frac{\ln(1 + 6)}{\ln(1 + 0.233/3.077)} \approx 27$$

For most fuel droplets burning in air, the flame standoff distance should be about 30 times the droplet radius. This value is much larger than that which is observed experimentally due to the assumption made with respect to the thermophysical properties and the Lewis number.

So, this boils down to this and we use this equation. So, the ratio of r$_f$ by r$_s$ is ln(1 + B) by this. So, that allow you to calculate the flame position r$_f$. Now, for the case of benzene just an example ν is this using these Spalding number 6, we can calculate that this ratio to be 27. Now, for most of the fuel droplets burning in air, the flame standoff distance would be about 30 times the droplet radius.

This value is much larger than which is observed experimentally due to assumption made with respect to the thermo physical properties and the Lewis number that means the equal diffusivity unit Lewis number. So, these are the assumption made these things to happen like that.

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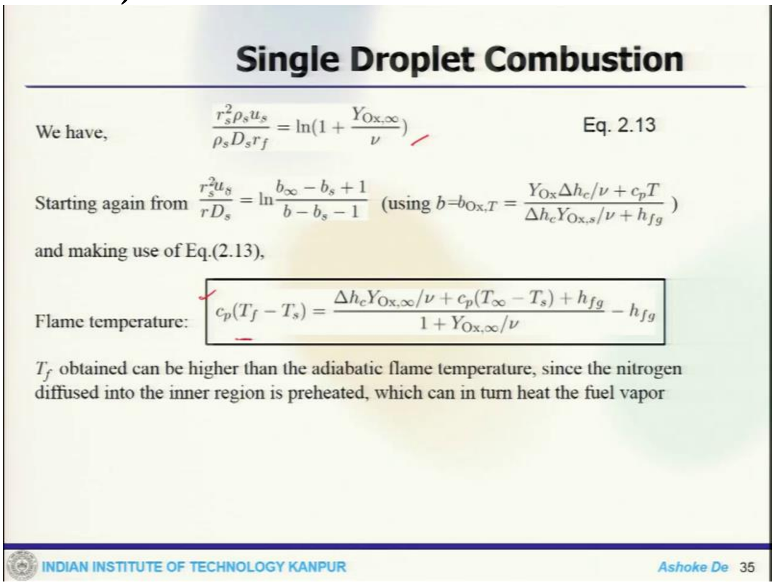

Single Droplet Combustion

We have, $\frac{r_s^2 \rho_s u_s}{\rho_s D_s r_f} = \ln(1 + \frac{Y_{\mathrm{Ox},\infty}}{\nu})$ Eq. 2.13

Starting again from $\frac{r_s^2 u_s}{r D_s} = \ln \frac{b_\infty - b_s + 1}{b - b_s - 1}$ (using $b = b_{\mathrm{Ox},T} = \frac{Y_{\mathrm{Ox}} \Delta h_c / \nu + c_p T}{\Delta h_c Y_{\mathrm{Ox},s} / \nu + h_{fg}}$)

and making use of Eq.(2.13),

Flame temperature: $\boxed{c_p(T_f - T_s) = \frac{\Delta h_c Y_{\mathrm{Ox},\infty}/\nu + c_p(T_\infty - T_s) + h_{fg}}{1 + Y_{\mathrm{Ox},\infty}/\nu} - h_{fg}}$

T_f obtained can be higher than the adiabatic flame temperature, since the nitrogen diffused into the inner region is preheated, which can in turn heat the fuel vapor

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Now, we have these. Now, again we are starting from there. So, using these we can find out the flame temperature. So, T_f obtained can be higher than the adiabatic flame temperature, since the nitrogen diffused in this.

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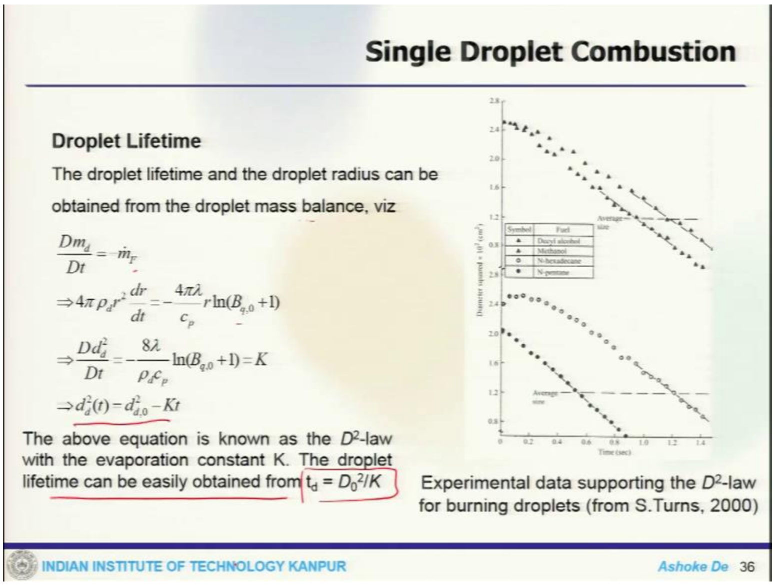

Single Droplet Combustion

Droplet Lifetime

The droplet lifetime and the droplet radius can be obtained from the droplet mass balance, viz

$$\frac{Dm_d}{Dt} = -\dot{m}_F$$

$$\Rightarrow 4\pi \rho_d r^2 \frac{dr}{dt} = -\frac{4\pi\lambda}{c_p} r \ln(B_{q,0} + 1)$$

$$\Rightarrow \frac{Dd_d^2}{Dt} = -\frac{8\lambda}{\rho_d c_p} \ln(B_{q,0} + 1) = K$$

$$\Rightarrow d_d^2(t) = d_{d,0}^2 - Kt$$

The above equation is known as the D^2-law with the evaporation constant K. The droplet lifetime can be easily obtained from $\boxed{t_d = D_0^2/K}$

Experimental data supporting the D^2-law for burning droplets (from S.Turns, 2000)

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And then the droplet lifetime, we have already seen the droplet lifetime in the radius can be obtained from the mass balance. So,

$$\frac{Dm_d}{Dt} = -\dot{m}_F$$

Which is $4\pi\lambda_D$ divided by this and once we integrate we get this. So, this is the well-known D^2 law and you can see four different fuels if you plot that how it varies actually. So, the droplet lifetime can be easily obtained that t. So, that is the lifetime of the droplet.

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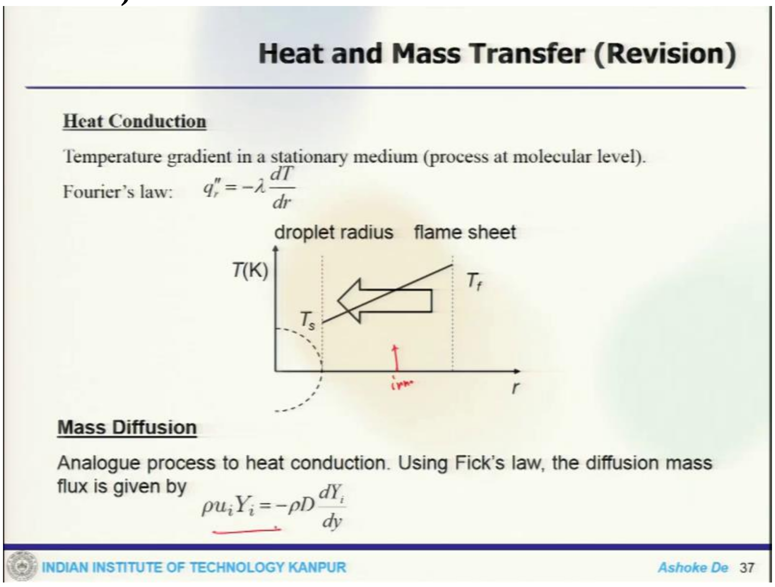

Heat and Mass Transfer (Revision)

Heat Conduction

Temperature gradient in a stationary medium (process at molecular level).

Fourier's law: $q_r'' = -\lambda \frac{dT}{dr}$

Mass Diffusion

Analogue process to heat conduction. Using Fick's law, the diffusion mass flux is given by $\rho u_i Y_i = -\rho D \frac{dY_i}{dy}$

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Now, we look at the heat and mass transfer balance, so, the heat conduction which takes place. So this is our droplet radius, this is the flame sheet. So, in between this is the inner zone. Now, there is a heat transfer which comes in then the mass diffusion analogue process to heat conduction and using Fick's law, we can write this.

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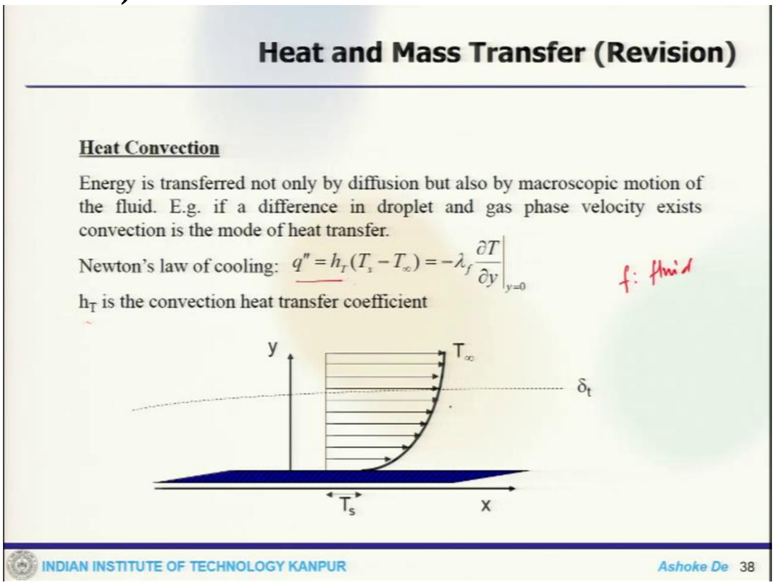

Heat and Mass Transfer (Revision)

Heat Convection

Energy is transferred not only by diffusion but also by macroscopic motion of the fluid. E.g. if a difference in droplet and gas phase velocity exists convection is the mode of heat transfer.

Newton's law of cooling: $q'' = h_T(T_s - T_\infty) = -\lambda_f \left.\frac{\partial T}{\partial y}\right|_{y=0}$

f: fluid

h_T is the convection heat transfer coefficient

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There is the mass diffusion and then this is the convection. This is applicable since at surface no fluid motion and energy transfer occurs only through conduction. Here the f stands for fluid. So, the energy transfer not only by diffusion also by macroscopic motion of the fluid. The difference in droplet and gas space velocity exists. Convection is mode of heat transfer. So, this would be heat flux would be $h_T(T_s - T_\infty)$ which is like that.

Next is the convection heat transfer coefficient. And this is the thermal boundary layer and the temperature profile how and the T_s is the surface temperature.

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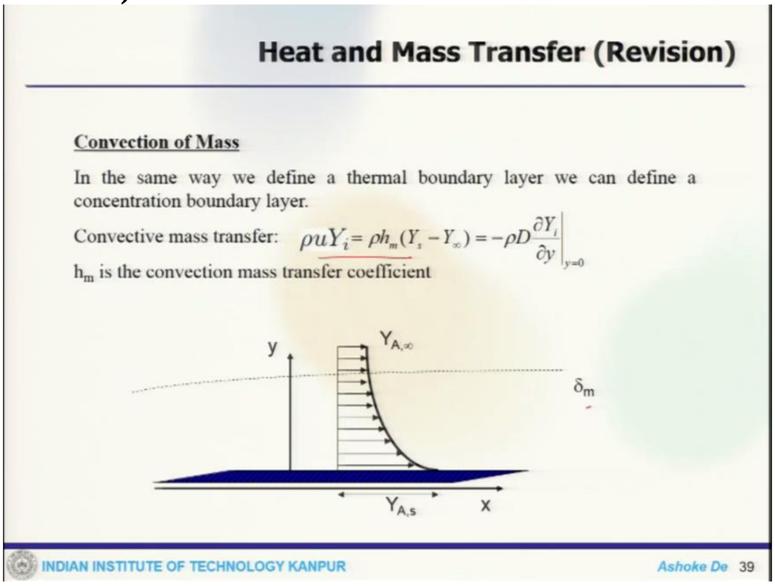

Heat and Mass Transfer (Revision)

Convection of Mass

In the same way we define a thermal boundary layer we can define a concentration boundary layer.

Convective mass transfer: $\rho u Y_i = \rho h_m (Y_s - Y_\infty) = -\rho D \left.\frac{\partial Y_i}{\partial y}\right|_{y=0}$

h_m is the convection mass transfer coefficient

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Similarly, the convection of mass which is there. There will be boundary layer thickness and the diffusion coefficient one can find out.

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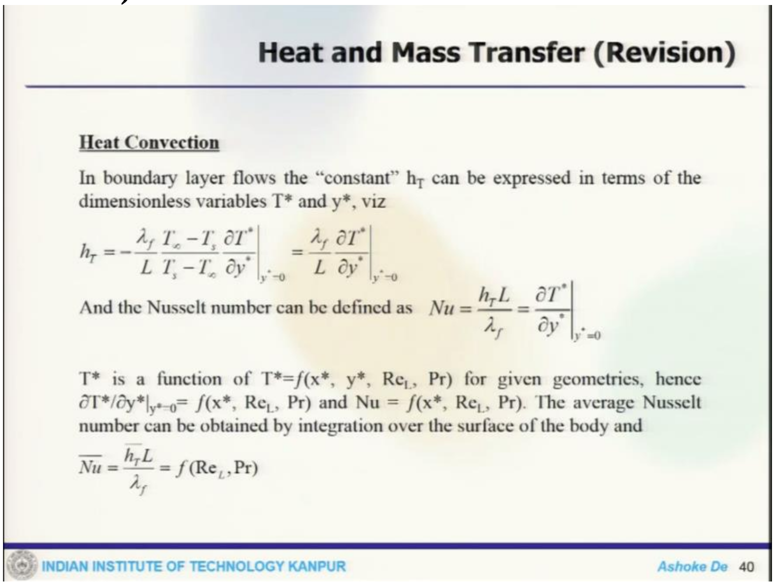

Heat and Mass Transfer (Revision)

Heat Convection

In boundary layer flows the "constant" h_T can be expressed in terms of the dimensionless variables T* and y*, viz

$$h_T = -\frac{\lambda_f}{L}\frac{T_\infty - T_s}{T_s - T_\infty}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = \frac{\lambda_f}{L}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0}$$

And the Nusselt number can be defined as $Nu = \frac{h_T L}{\lambda_f} = \frac{\partial T^*}{\partial y^*}\Big|_{y^*=0}$

T* is a function of $T^* = f(x^*, y^*, Re_L, Pr)$ for given geometries, hence $\partial T^*/\partial y^*|_{y^*=0} = f(x^*, Re_L, Pr)$ and $Nu = f(x^*, Re_L, Pr)$. The average Nusselt number can be obtained by integration over the surface of the body and

$$\overline{Nu} = \frac{\overline{h_T} L}{\lambda_f} = f(\text{Re}_L, \text{Pr})$$

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Now, in boundary layer flows constant heat transfer h_T that can be expressed in terms of dimensionless variable like this. So, these come from the boundary layer equations where the non-dimensional is done and one can get those things. So, the non-dimensional number Nusselt number which is at the boundaries surface $\frac{\partial T}{\partial y}$ is 0 or hysteresis function of this for given geometry hence this derivative would be also function of like this.

And the Nusselt number and function of x*, Reynolds number and Prandtl number. So, the average Nusselt number can be obtained by integration over the surface of the body.

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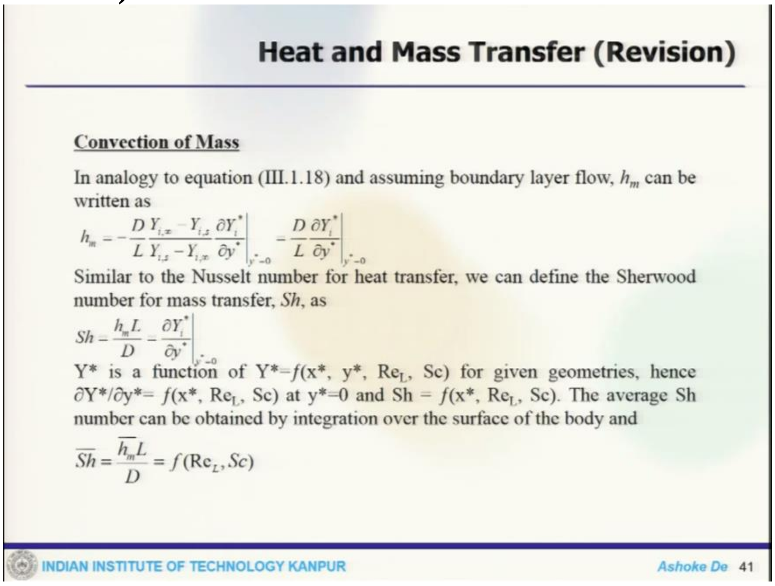

Heat and Mass Transfer (Revision)

Convection of Mass

In analogy to equation (III.1.18) and assuming boundary layer flow, h_m can be written as

$$h_m = -\frac{D}{L}\frac{Y_{i,\infty} - Y_{i,s}}{Y_{i,s} - Y_{i,\infty}}\frac{\partial Y_i^*}{\partial y^*}\bigg|_{y^*=0} = \frac{D}{L}\frac{\partial Y_i^*}{\partial y^*}\bigg|_{y^*=0}$$

Similar to the Nusselt number for heat transfer, we can define the Sherwood number for mass transfer, *Sh*, as

$$Sh = \frac{h_m L}{D} = \frac{\partial Y_i^*}{\partial y^*}\bigg|_{y^*=0}$$

Y* is a function of $Y^* = f(x^*, y^*, Re_L, Sc)$ for given geometries, hence $\partial Y^*/\partial y^* = f(x^*, Re_L, Sc)$ at $y^* = 0$ and $Sh = f(x^*, Re_L, Sc)$. The average Sh number can be obtained by integration over the surface of the body and

$$\overline{Sh} = \frac{\overline{h_m} L}{D} = f(\text{Re}_L, Sc)$$

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Now analogy. So, if boundary layer flow if you assume that you can find out the h_m and then you can find out the Sherwood number and this. So, essentially it means that the convective heat transfer is only function of Nusselt number and Sherwood number for a given geometry. So, it should be a similar for a variety of fluids. Length scale is not only dependent on this Reynolds number, Prandtl number and Schmidt number parameters.

So, Nusselt number you can see it is a function of also I mean by analogy we can see the Nusselt number would be also function of x, Re_L, Sherwood number is also function of x*, Re_L, Sc So, like that.

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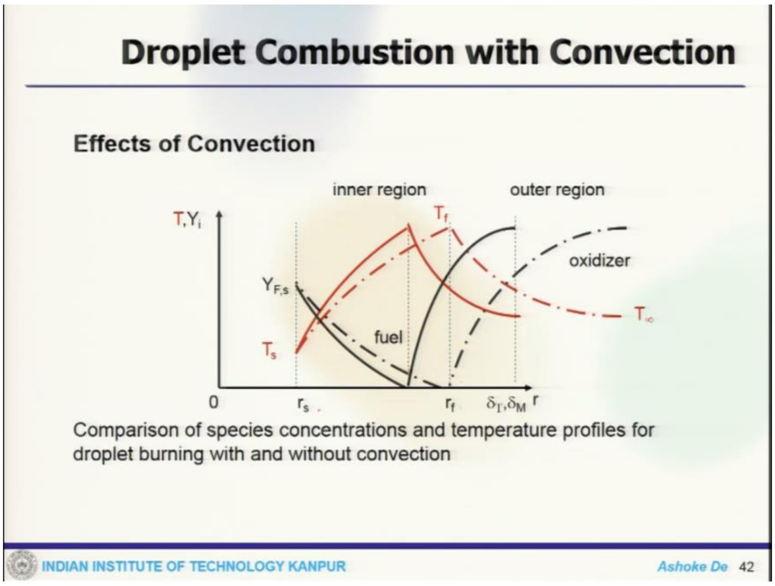

Droplet Combustion with Convection

Effects of Convection

Comparison of species concentrations and temperature profiles for droplet burning with and without convection

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Now, we can see the effect of combustion against the boundary layer of type width so, this is the surface outer region. This is flame surface or flame length. This is how oxidizer varying, fuel varying, this is temperature and all these things. So, this you can see when there is a boundary layer, the profile actually changes and that change takes place because of this boundary layer both and all these things.

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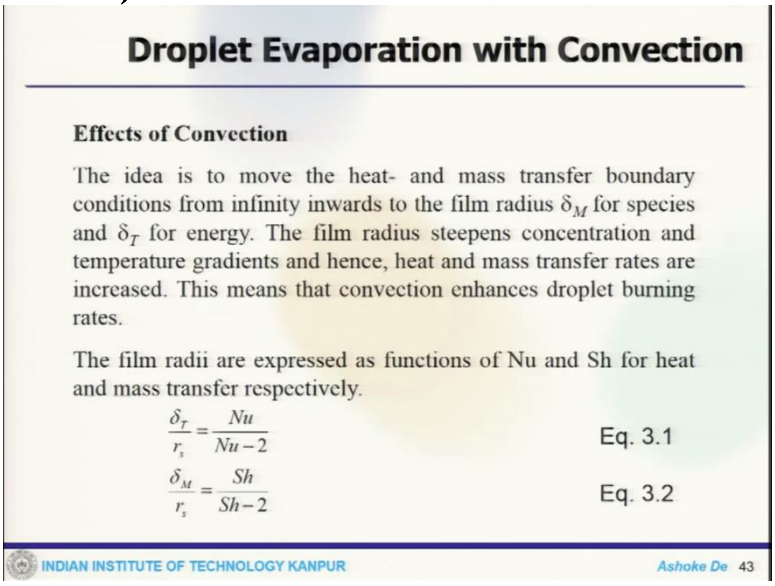

Droplet Evaporation with Convection

Effects of Convection

The idea is to move the heat- and mass transfer boundary conditions from infinity inwards to the film radius δ_M for species and δ_T for energy. The film radius steepens concentration and temperature gradients and hence, heat and mass transfer rates are increased. This means that convection enhances droplet burning rates.

The film radii are expressed as functions of Nu and Sh for heat and mass transfer respectively.

$$\frac{\delta_T}{r_s} = \frac{Nu}{Nu-2} \quad \text{Eq. 3.1}$$

$$\frac{\delta_M}{r_s} = \frac{Sh}{Sh-2} \quad \text{Eq. 3.2}$$

Idea have moved to heat and mass transfer boundary conditions from flame radius δ_M for species and for energy flame radius stiffens the concentration and temperature gradient and heat and mass transfer rate are increased. So, the $\frac{\delta_T}{r_s}$ is expressed as Nusselt number function and the δ_M is a function of Sherwood number.

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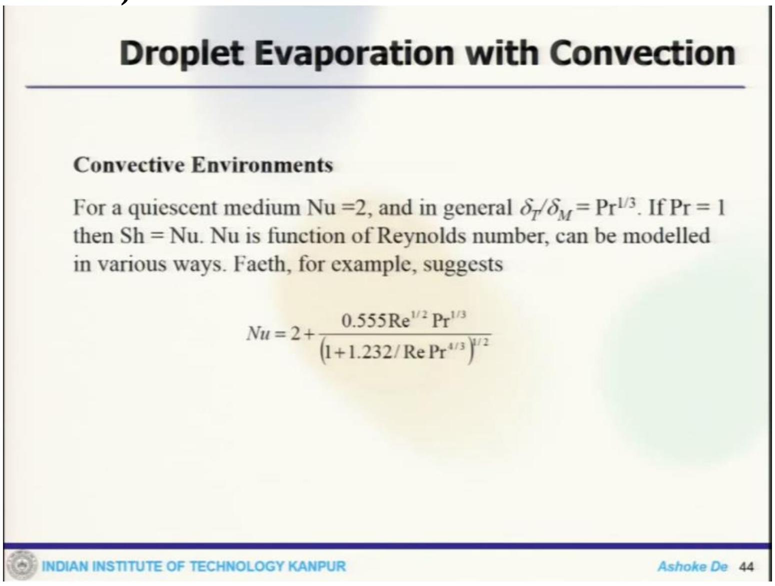

Droplet Evaporation with Convection

Convective Environments

For a quiescent medium Nu =2, and in general $\delta_T/\delta_M = \mathrm{Pr}^{1/3}$. If Pr = 1 then Sh = Nu. Nu is function of Reynolds number, can be modelled in various ways. Faeth, for example, suggests

$$Nu = 2 + \frac{0.555\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{\left(1+1.232/\mathrm{Re}\,\mathrm{Pr}^{4/3}\right)^{1/2}}$$

Now, if you have a convective environments for a quiescent medium the Nusselt number equals to 2 and in general ${}^{\delta_T}/_{\delta_M}$ equals to Prandtl number to power one third. If Prandtl number is one then Sherwood number equals to Nusselt number. So, this would be estimated. So, there are different correlations which actually exist and one may look at that thing how.

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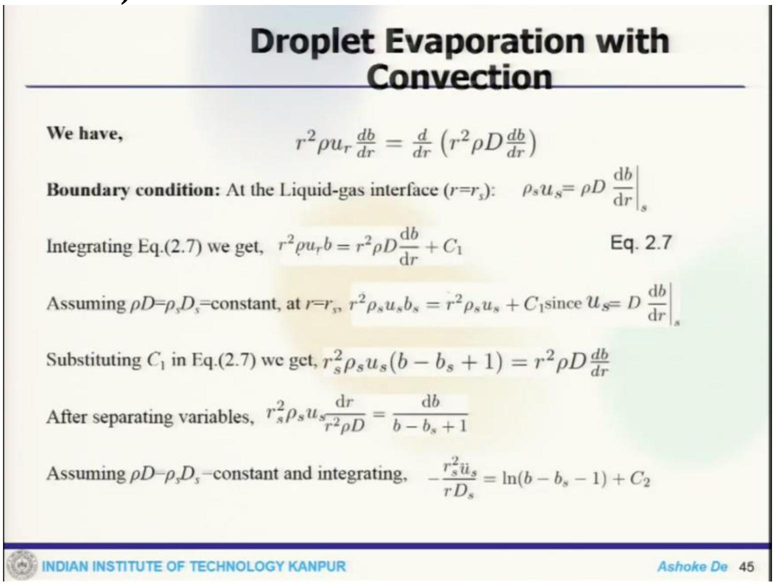

Droplet Evaporation with Convection

We have, $r^2\rho u_r \frac{db}{dr} = \frac{d}{dr}\left(r^2\rho D\frac{db}{dr}\right)$

Boundary condition: At the Liquid-gas interface ($r=r_s$): $\rho_s u_s = \rho D \left.\frac{db}{dr}\right|_s$

Integrating Eq.(2.7) we get, $r^2\rho u_r b = r^2\rho D\frac{db}{dr} + C_1$ Eq. 2.7

Assuming $\rho D=\rho_s D_s$=constant, at $r=r_s$, $r^2\rho_s u_s b_s = r^2\rho_s u_s + C_1$since $u_s = D\left.\frac{db}{dr}\right|_s$

Substituting C_1 in Eq.(2.7) we get, $r_s^2\rho_s u_s(b-b_s+1) = r^2\rho D\frac{db}{dr}$

After separating variables, $r_s^2\rho_s u_s\frac{dr}{r^2\rho D} = \frac{db}{b-b_s+1}$

Assuming $\rho D=\rho_s D_s$=constant and integrating, $-\frac{r_s^2\ddot{u}_s}{rD_s} = \ln(b-b_s-1)+C_2$

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Now, again we can come back and look at the evaporation with convection. So, we have:

$$r^2\rho u_r\frac{db}{dr} = \frac{d}{dr}\left(r^2\rho D\frac{db}{dr}\right)$$

The boundary condition at the liquid gas interface that is r equals r_s now, we have $\rho_s u_s$ because to this for integrating this one will get C_1 assuming they are always D_S this at r_s we get this then get this C_1 putting back to get this separate the variable and finally, after integration and get the C_2.

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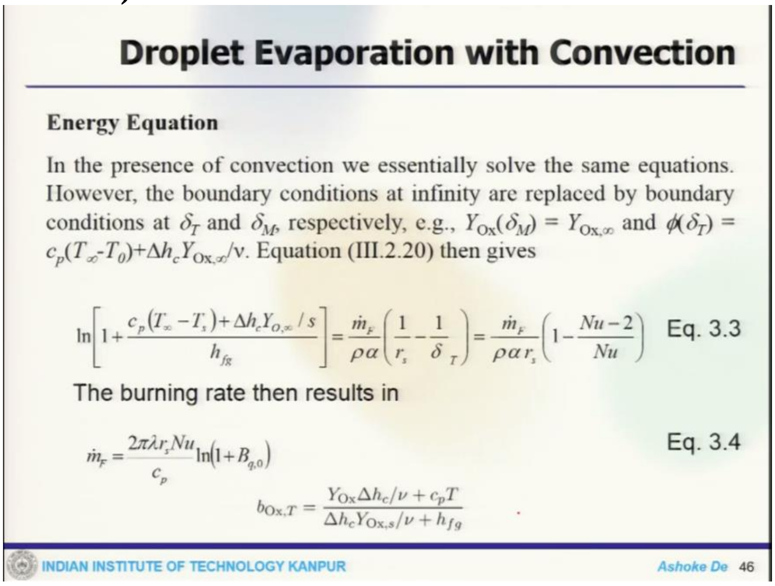

Droplet Evaporation with Convection

Energy Equation

In the presence of convection we essentially solve the same equations. However, the boundary conditions at infinity are replaced by boundary conditions at δ_T and δ_M, respectively, e.g., $Y_{Ox}(\delta_M) = Y_{Ox,\infty}$ and $\phi(\delta_T) = c_p(T_\infty - T_0)+\Delta h_c Y_{Ox,\infty}/\nu$. Equation (III.2.20) then gives

$$\ln\left[1+\frac{c_p(T_\infty-T_s)+\Delta h_c Y_{O,\infty}/s}{h_{fg}}\right] = \frac{\dot{m}_F}{\rho\alpha}\left(\frac{1}{r_s}-\frac{1}{\delta_T}\right) = \frac{\dot{m}_F}{\rho\alpha r_s}\left(1-\frac{Nu-2}{Nu}\right) \quad \text{Eq. 3.3}$$

The burning rate then results in

$$\dot{m}_F = \frac{2\pi\lambda r_s Nu}{c_p}\ln\left(1+B_{q,0}\right) \quad \text{Eq. 3.4}$$

$$b_{Ox,T} = \frac{Y_{Ox}\Delta h_c/\nu + c_p T}{\Delta h_c Y_{Ox,s}/\nu + h_{fg}}$$

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Now, in the presence of convection the energy equation however, the boundary condition at the infinity are replaced by the boundary condition at δ_T and δ_M. So, we have already seen solving the energy equation we can find out now due to the convection now, instead of r tends to infinity use at that thermal boundary layer. So, oxygen profile at the mass boundary layer thickness would be this.

So, this one will get me back these one side put it this. So, the burning rate then results in:

$$\dot{m}_F = \frac{2\pi\lambda r_s Nu}{c_p}\ln(1 + B_{q,0})$$

One can note here this is not the only way to do these things. So, one can also do this by starting from the other equations or integrating the different equation and then apply the boundary condition, we can get this variable 2.

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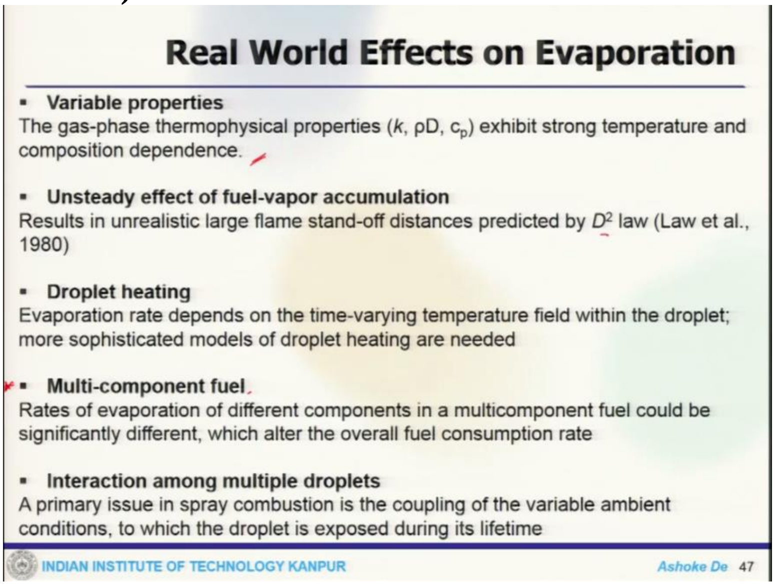

Real World Effects on Evaporation

- **Variable properties**
  The gas-phase thermophysical properties (k, ρD, c_p) exhibit strong temperature and composition dependence.
- **Unsteady effect of fuel-vapor accumulation**
  Results in unrealistic large flame stand-off distances predicted by D^2 law (Law et al., 1980)
- **Droplet heating**
  Evaporation rate depends on the time-varying temperature field within the droplet; more sophisticated models of droplet heating are needed
- **Multi-component fuel**
  Rates of evaporation of different components in a multicomponent fuel could be significantly different, which alter the overall fuel consumption rate
- **Interaction among multiple droplets**
  A primary issue in spray combustion is the coupling of the variable ambient conditions, to which the droplet is exposed during its lifetime

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Now, we can see the effects. So, one of this variable parameters the gas-phase thermophysical properties exhibit strong temperature and composition dependence. So, this is an important parameter and thermophysical properties, which actually control this droplet in combustion, then there will be unsteady effect of fuel-vapour accumulation. So, this is an unrealistic large flame stand-off distances predicted by D^2 law droplet heating.

Now, the evaporation rate depends on time bind temperature field within the droplet. So, we required more sophisticated model to capture that. Then it is a multicomponent fuel which is very important. So, if you look at these liquid fuels, they are not whether it is taking kerosene or petrol, diesel or something, they are really a multi component fuel there would be aromatic ring there would be other rings so the rate of evaporation of different component in a multi component fuel could be significantly different which actually alter the fuel consumption rate. So, this is very important property because in the practical system that we burn, we burn these fuels which are having actually multi component ingredients and that changes the whole properties that we estimate from a single component fuel. Now, the interaction among multi component droplet is primary issue in spray combustion.

The variable ambient conditions to which the droplet is exposed during its lifetime. Then, another very specific case which may occur in our space application, or the semi cryo or cryogenic application, where there is supercritical droplet combustion. Here the pressure is higher than the critical pressure. So, the latent heat of the vaporization becomes 0 and the evaporation rate becomes infinity.

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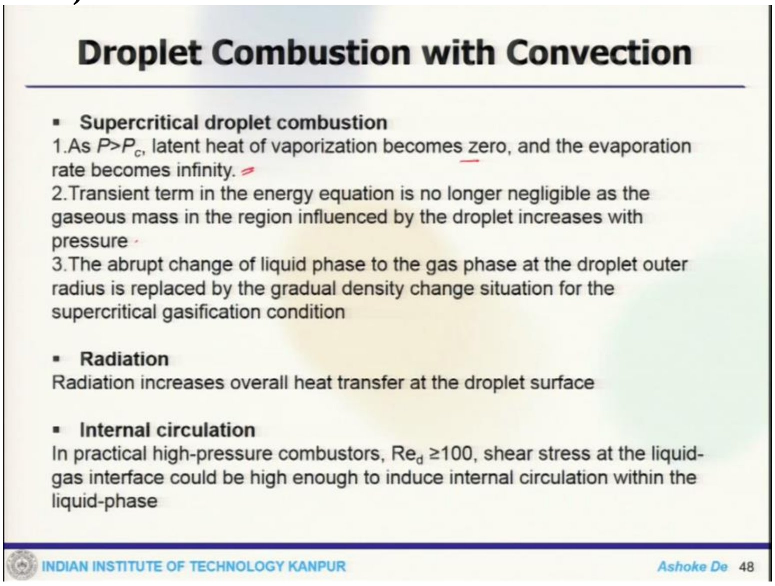

Droplet Combustion with Convection

- **Supercritical droplet combustion**

1.As $P>P_c$, latent heat of vaporization becomes zero, and the evaporation rate becomes infinity.
2.Transient term in the energy equation is no longer negligible as the gaseous mass in the region influenced by the droplet increases with pressure
3.The abrupt change of liquid phase to the gas phase at the droplet outer radius is replaced by the gradual density change situation for the supercritical gasification condition

- **Radiation**

Radiation increases overall heat transfer at the droplet surface

- **Internal circulation**

In practical high-pressure combustors, $Re_d \geq 100$, shear stress at the liquid-gas interface could be high enough to induce internal circulation within the liquid-phase

So, that means, as soon as the liquid comes out it evaporates. Transient term in the energy equation is no longer negligible as the gaseous mass in the region influenced by the droplet increases with pressure then the abrupt change of liquid phase to the gas phase and the droplet outer radius is replaced by the gradual density change.

Then the radiation effect. So, radiation also increases overall heat transfer at the droplet surface because some fuels when they burn actually there is a lot of heat radiation effect that can change those things. And then the internal circulation in practically high pressure combustor shear stress at the liquid gas interface could be high enough to induce internal circulation within the liquid phase.

So, these are the factors one can think about then in realistic situation, they can change the complete evaporation phenomena the bonding rate consumption and all these things, it is not that simple. So, as you can see, as long as you are looking at the gas-phase combustion and when you are coming here in the multiphase combustion, things are quite different, because in the liquid phase there are so many other things that needs to be considered.

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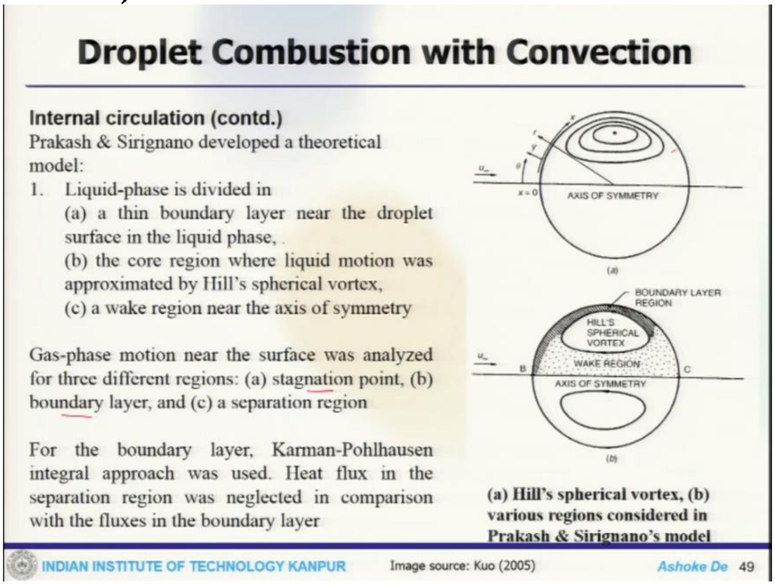

Droplet Combustion with Convection

Internal circulation (contd.)

Prakash & Sirignano developed a theoretical model:

1. Liquid-phase is divided in
   (a) a thin boundary layer near the droplet surface in the liquid phase,
   (b) the core region where liquid motion was approximated by Hill's spherical vortex,
   (c) a wake region near the axis of symmetry

Gas-phase motion near the surface was analyzed for three different regions: (a) stagnation point, (b) boundary layer, and (c) a separation region

For the boundary layer, Karman-Pohlhausen integral approach was used. Heat flux in the separation region was neglected in comparison with the fluxes in the boundary layer

(a) Hill's spherical vortex, (b) various regions considered in Prakash & Sirignano's model

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Now, when you look at this internal circulation this is a Hill's spherical vortex. This one is the Hill's spherical vortex and the different regions which are considered in Prakash and Sirignano's model so this is a boundary layer region. This is the Hill's spherical vortex, this is behind the wake region near the axis of symmetry and all this. So, Prakash and Sirignano they developed a theoretical model or this liquid phase, this is within the droplet.

So, the liquid phase is divided in a thin boundary layer, near the droplet surface in the liquid phase. So, this is what the thin boundary layer then the core region for the liquid motion was approximated by Hill's spherical vortex that means, there is a region where the spherical vortex would be there and the wake region near the axis of symmetry.

Now, when you look at the system, the gas phase analysis or the motion near the surface analysed by three different regions stagnation point boundary layer and the separation region and for boundary layer Karman-Pohlhausen integral approach can be used. So, this is quite important while determining the heat and mass transfer within the droplet. And that is why it also altered the evaporation rate.

So, this is an important phenomena that takes place within the droplet. And there is another thing also within the droplet which actually happens is the flux boiling. So that we will talk a little bit when you go to actually the breakup and all these things, and we will see how that.

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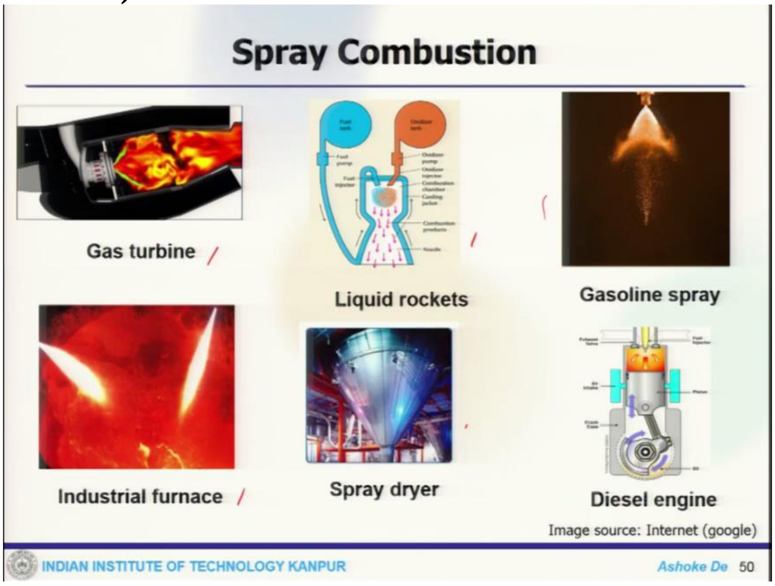

Spray Combustion

Gas turbine

Liquid rockets

Gasoline spray

Industrial furnace

Spray dryer

Diesel engine

Image source: Internet (google)

Now the application point of view, we have already seen this is a gas turbine application industrial furnace, this is your liquid rocket, gasoline spray in the engine, spray dryer, diesel engine. So, these are all using different kind of fuels, which are actually multi component. So, simplified analysis is good enough to give you some idea, but the realistic situation is quite different as this multi component system play an important role.

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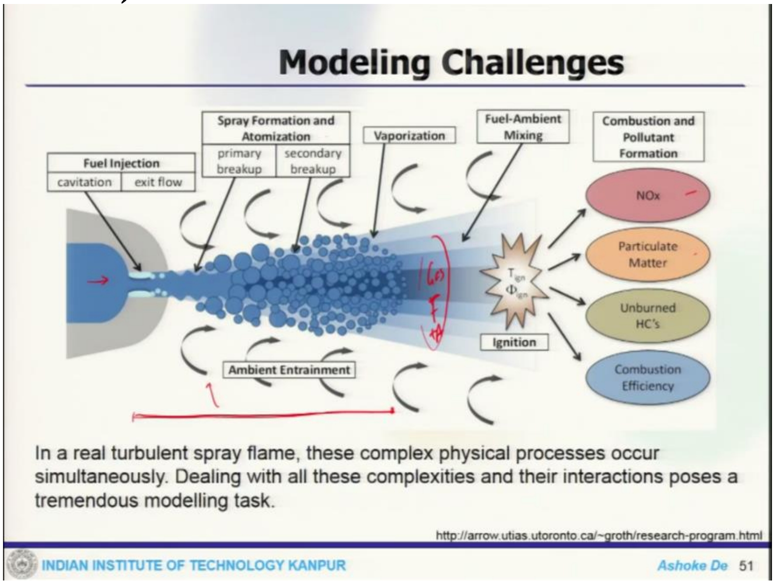

Modeling Challenges

In a real turbulent spray flame, these complex physical processes occur simultaneously. Dealing with all these complexities and their interactions poses a tremendous modelling task.

http://arrow.utias.utoronto.ca/~groth/research-program.html

Now, coming to another important picture when this actual show. So for our discussion was on the single droplet combustion the evaporation looking at the surface now, whole point is that before we get the single component droplet or single droplet, it has to because the liquid is injected like this or the fuel is injected like this as a form of fuel injector that does not produce these single droplets, it goes through a process and the process is first the fuel is injected.

So, there could be exit flow there could be cavitation these are the things which may occur then primary there would be ignition and where there is a primary breakup takes place or the primary atomization. So, this is where the spray formation takes place. And then there would be entrainment from the ambient and then there would be secondary breakup that means from the primary, your sheet break into smaller.

I mean the large size droplet and then this will evaporate and mix I mean rather the secondary breakup and the smaller size droplets are formed and then it evaporates. So, that is why the vaporisation is there, once it evaporates, so, it is becoming the gas phase. So, this is the gas phase, where the fuel and air is mixed. And then once you ignite the ignition takes place and the combustion goes on.

So, from these to these there are so you can see we talk about the single droplet combustion going from the gas phase to heat transfer and all these which is actually in this position, but before that the liquid which is also injected that also goes through different stages. And that is why and once it burns you have a pollutant formation like NOx, particulate matter, unburned hydrocarbon and you can look at these.

So, the physical process occurs very simultaneously. So dealing with all these complexities, so that is why to but in order to advance with current modelling capabilities.

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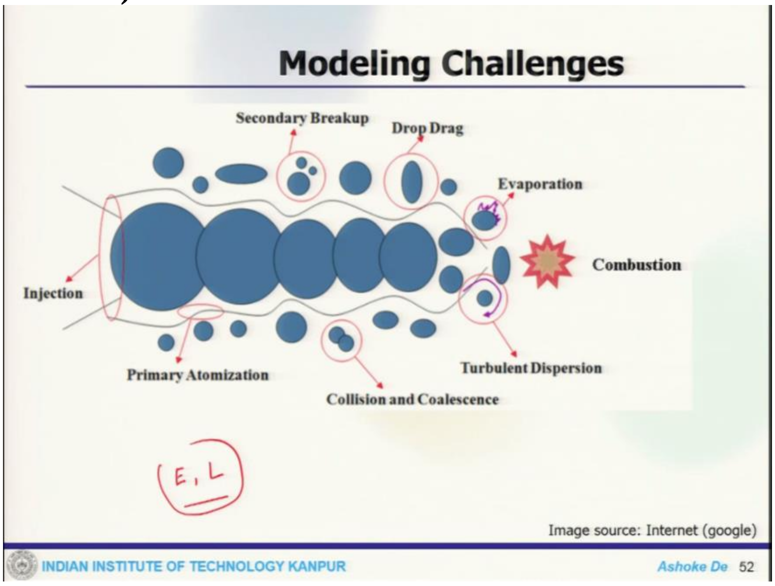

Modeling Challenges

Image source: Internet (google)

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So, it seems reasonable to aim for progress individuals are various like one can look up the breakup, somebody can look up the dispersion, mixing and combustion but, which in other way cannot be viewed in complete isolation. Moreover, one has to consider the advantages and disadvantages of the different modelling approaches like the DNS, LES and simulation based on RANS, property density function kind of approach and all these things.

So, and also important is that you have injection and primary atomization and secondary then drop collisions and all these things dispersion the important is that you have both Eulerian framework and Lagrangian framework, so the coupling between that 2 or completely handling in Lagrangian framework. So these are the challenges that one has to look at it. So we will see how these things are handled from the modelling perspective, and we will stop here today and continue the discussion in the next lecture.