Turbulent Combustion: Theory and Modelling
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Lecture - 58
Combustion in 2 Phase Flows

Welcome back. So, let us continue the discussion on our turbulent combustion topics. So, now, we have looked at our different combustion models, they are theoretical description pertaining to their computational application and along with their different advantages and disadvantages. So, we started with the gaseous combustion where we have looked at the simple models and then we looked at the statistical models also and then some models which are very specific to the premixed case and then finally, non-premixed case.

And now, we move to the discussion on the multi phase or two phase system, because so for our discussion which are restricted to the single phase system like and gaseous combustion and when you move to the 2 phase system, this will give you a different kind of approach and different kind of issues.

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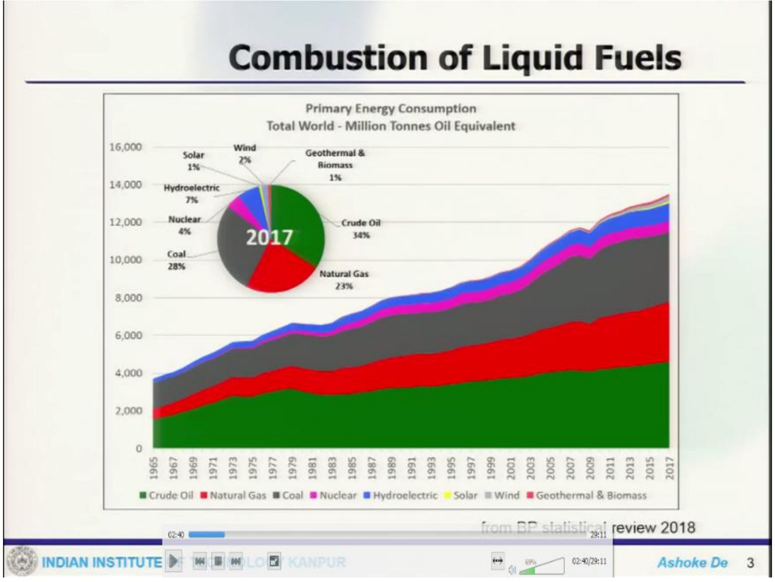

Combustion of Liquid Fuels

from BP statistical review 2018

Now, in 2 phase system, so that means, we are talking about multi phase you have obviously, it is not only gas you have some liquid and then gas, but one please note here that even in your 2 phase system, the combustion always takes place in the gaseous situation, but when we inject the liquid, what it does the liquid has to first go into droplet and the droplet to small space and then finally it evaporates.

So, finally, the reaction takes place in gaseous phase. So that means there is in process or there is a time which is required to go through these different stages of the liquid. So, it is not like that unless it is super critical condition, where actually liquid can just like your solid combustion where your solid actually react to oxidizer quickly. Because due to paralysis process solid immediately goes to gaseous state. But in a regular situation, the liquid first needs to be evaporated to gaseous state or needs to move to the gaseous state then the combustion takes place.

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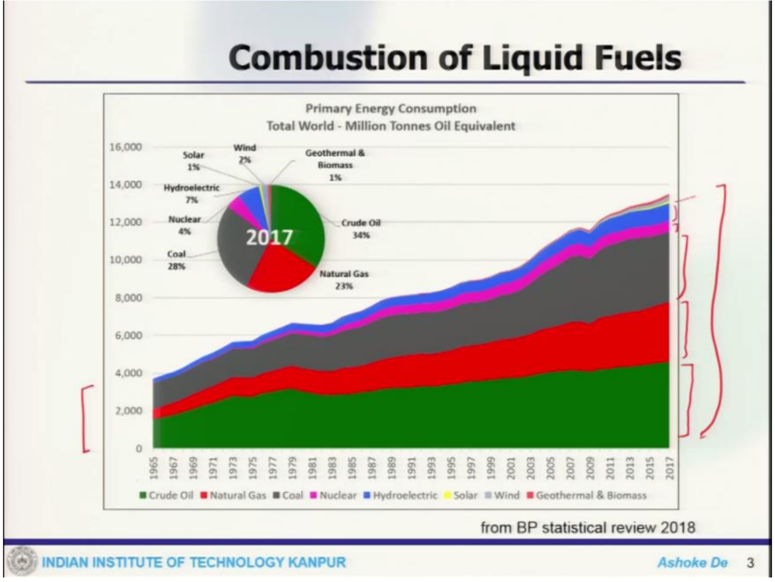

Combustion of Liquid Fuels

from BP statistical review 2018

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Now, before going this is an BP statistical review of 2018 you can see the energy consumption situation across the world what is the right now, and the database is collected from 1965 to 2017 and the green one this is where actually you use the crude oil that means that all these oil burning and all these things from there that the level up. Then this is natural gas, this is coal a small portion is nuclear, there is a small strip of solar, this is hydro, then wind, geo thermal, so, that is becoming smaller and smaller like that.

But interestingly, you can see the energy consumption which has gone up from this level to this and now even today, this is going in an increasing order that is why, we have to keep on thinking, how one can actually come up with the different sources to compensate or to fulfil the demand of the energy of the whole world. Now, the thing is that if you use natural resources, they are also getting and getting exhausted.

So we have to come up with some alternative resources, and that is why we got biomass, wind, hydro these are becoming quite popular, but the conversion rate of the energy production rate is not that high, which can provide really sustainable and meet the demand of that thing. So, this is the 2018 review.

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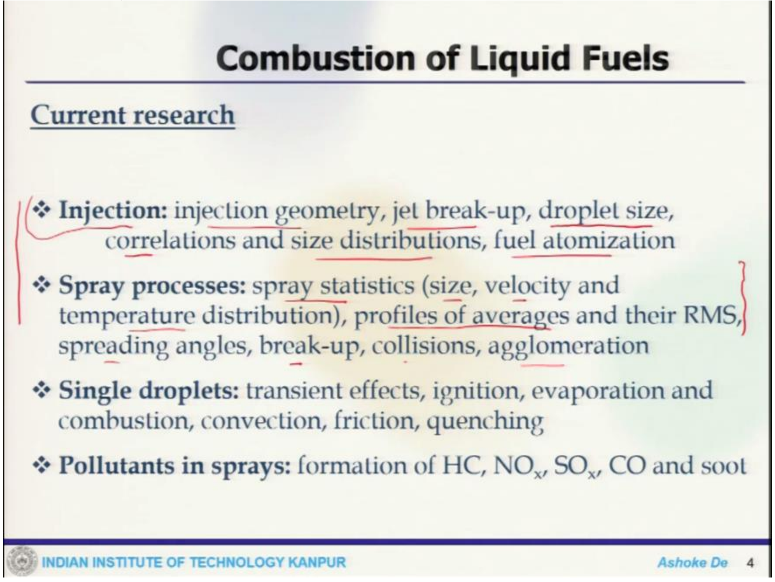

Combustion of Liquid Fuels

Current research

- **Injection:** injection geometry, jet break-up, droplet size, correlations and size distributions, fuel atomization
- **Spray processes:** spray statistics (size, velocity and temperature distribution), profiles of averages and their RMS, spreading angles, break-up, collisions, agglomeration
- **Single droplets:** transient effects, ignition, evaporation and combustion, convection, friction, quenching
- **Pollutants in sprays:** formation of HC, NO_x, SO_x, CO and soot

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Now, what is the situation of the current state of the art of the research in the liquid fuel? So, people may work on the injection. So, there were you look at the injection geometry like how you inject through which you inject that means injected geometry, they are dimensions, then people all looking at the breakup that means when you are injecting the liquid fuel, it has to break up into small droplets or these things.

And that is an important phenomena. The reason is that without breaking down into smaller droplet and becoming small spray which will evaporate, reaction would not take place and rather what would happened you end up having a large cell of droplet and then that can reacts suddenly then obviously, the associated phenomena which is of quite interest the droplet size and their correlation, distribution.

So, and then the fuel atomization these are all coming under the injection and the second part which is another interesting is the spray processes that means, which will talk about spray statistics way like size, velocity, temperature distribution like this profiles of averages and their RMS spreading angle, breakup, collision, agglomerations all these are kind of so, you see the immediate change when you move from single phase that like gaseous combustion to the 2 phase liquid combustion.

You start getting this kind of things which are not present in your gaseous case, there I mean itself the gaseous combustion is complicated and there are so many issues and we still kind of research is going on to come up with the best possible model for different regime and different mode of combustion and but still now, as soon as you go to liquid case it impose on it brings in different set of problem, and these are the new things like injection related issues, pre-process spray characteristics related issues. This one has to understand and handle it properly.

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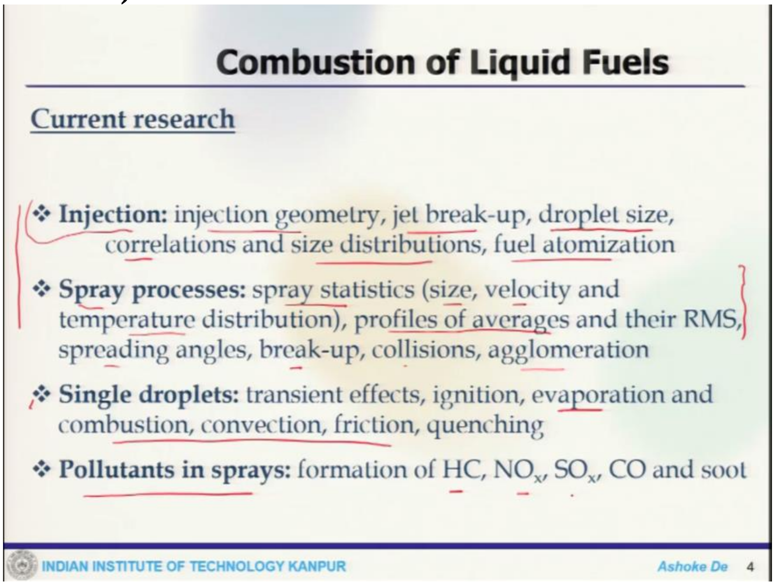

Combustion of Liquid Fuels

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Then if you come down to a single droplet, so, you look at the transient effect, ignition there evaporation, combustion, so in turn, your spray actually contain so many small droplets and that is why we can look at in details the single droplet situation. Now, pollutants in sprays that is another issue which will an obvious problem in the spray calculations where unburned hydrocarbon knocks and all these things should form.

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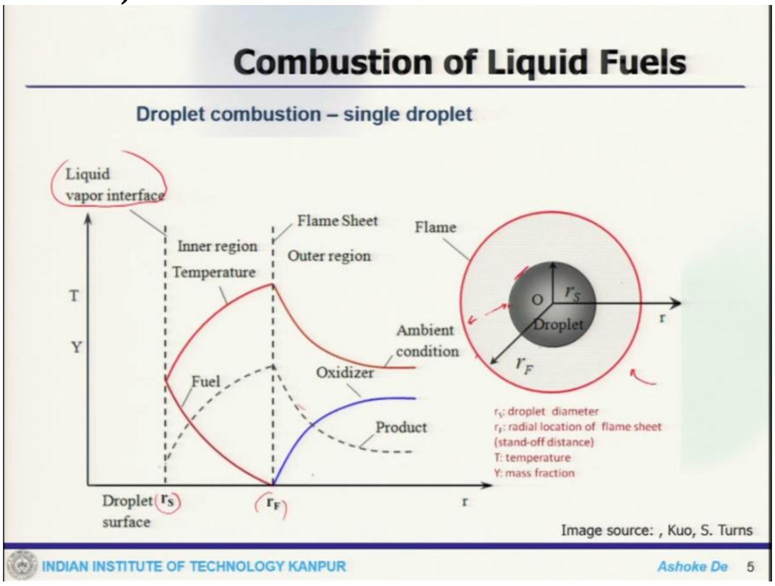

Combustion of Liquid Fuels

Droplet combustion – single droplet

r_S: droplet diameter
r_F: radial location of flame sheet (stand-off distance)
T: temperature
Y: mass fraction

Image source: , Kuo, S. Turns

Now, if you look at a single droplet combustion. So, this is burning at an oxidizing atmosphere. So, here this is the droplet and this is the flame around that droplet, you have the droplets surface. This is an r-coordinate system, this is an r_S that is droplet surface, and this is the flame radius. Now, here if you plot that this is r_S. This is r and this is the flame. Here your fuel is completely consumed and your oxidizer is also completely consumed and then you can see how the product actually goes up and then come down in oxidizer side.

Now, similarly, you can see the temperature here, how and this particular branch between these r_S to r_F. This is an inner region that means, this is between the flame and the droplet and these flame interface you can think about the liquid vapour interface and the r_F or other r_S, one can think about this is the interface r_F is the flame sheet, that means we are completely these gaseous in state.

So, there are now you can see that fuel is evaporated from the so that evaporation takes from this liquid surface and fuel vapour reaches to the flame front and oxygen rather from the surroundings, it moves towards the flame front. So, that is how the thing actually takes place, you have the liquid they are from liquid this moves this side and the oxidizer move this side and that is why you get in flame front on that side.

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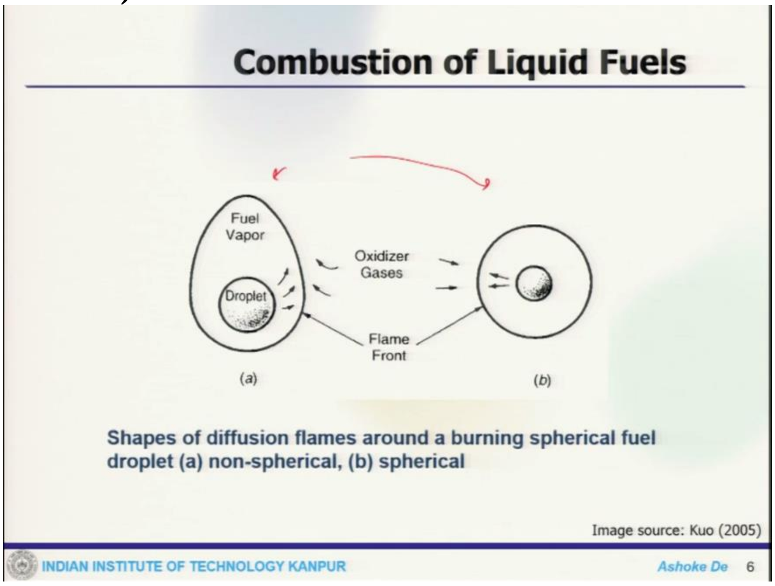

Combustion of Liquid Fuels

Shapes of diffusion flames around a burning spherical fuel droplet (a) non-spherical, (b) spherical

Image source: Kuo (2005)

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Now, this is a kind of one can think about around the droplet what kind of or type of flame we can see. Now, here you can see when there is a droplet burnt either you can have a spherical kind of drop flame shape this is spherical and this non-spherical kind of shape. Now, non-spherical flames are generally caused by the conviction effect due to the relative motion between droplet and surrounding gas space.

So, that means, whatever there is a transfer or heat transfer takes place between that is happens, but when the droplet is very small and the surrounding gases can enter easily the relative velocity between droplet and that gas phase becomes small and then the diffusion flame surrounding the droplet becomes nearly spherical. So, in this case, there is no such change in that relative velocity.

So, I mean though they look as simple as one case spherical case and another case is the non-spherical flame, but there is a reason associated with it. So, this case the convection effects due to relative motion between the droplet and the gas surrounding gas phase is different that is why it happens.

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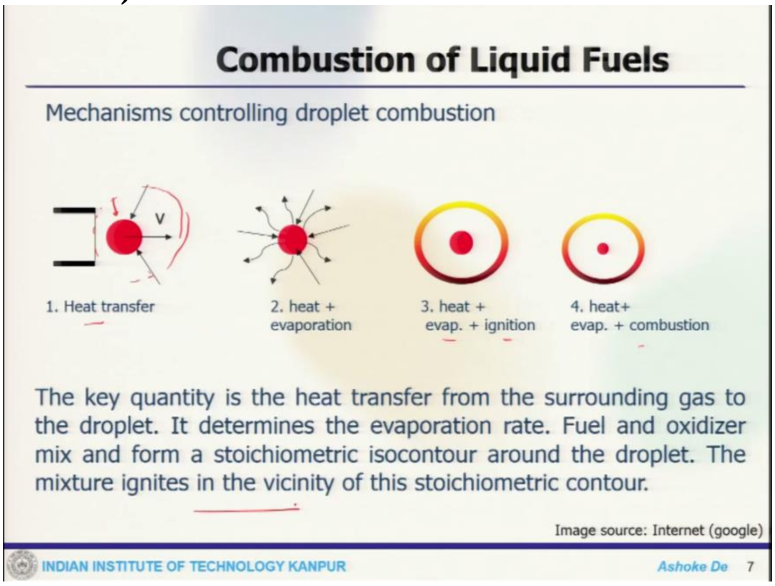

Combustion of Liquid Fuels

Mechanisms controlling droplet combustion

1. Heat transfer
2. heat + evaporation
3. heat + evap. + ignition
4. heat+ evap. + combustion

The key quantity is the heat transfer from the surrounding gas to the droplet. It determines the evaporation rate. Fuel and oxidizer mix and form a stoichiometric isocontour around the droplet. The mixture ignites in the vicinity of this stoichiometric contour.

Image source: Internet (google)

Now, here you can see the mechanism controlling the droplet combustion. So, one of the important things is that here you can see these kind of things happen in your methanol, octane, dodecane then ATA on all these diesel engine. So this is all liquid fuels which actually burn in realistic situation. Now, what happens is that so, this is where the heat transfer effect essentially now, then when the, from the surrounding the heat is transferred to these things.

So, the point one can think about the parameters that controls the rate at which droplet evaporates and burns that means, this is the liquid droplet, it has to actually evaporate and this evaporation will only take place if the heat gets transferred to this droplet from the surroundings and when that happens, it will evaporate like this and then it will go for burning. Now, the rate at which the droplet evaporates and burns is generally considered to be determined by the rate of heat transfer from the flame front to the field surface.

That means, if outside there is a flame front then the flame front as a high temperature. So, amount of heat transfer goes towards the fuel are the liquid fuel that will allow to calculate the evaporation rate and finally, the burning. So, one is the heat transfer then there will be heat plus evaporation, then the heat transfer evaporation plus then ignition and finally the reaction. So the heat quantity here is the heat transfer from the surroundings gas to the droplet. Actually it determines the evaporation rate fuel and oxidizer mix and from a stoichiometric iso-contour around the droplet that mixture ignites in the vicinity of it.

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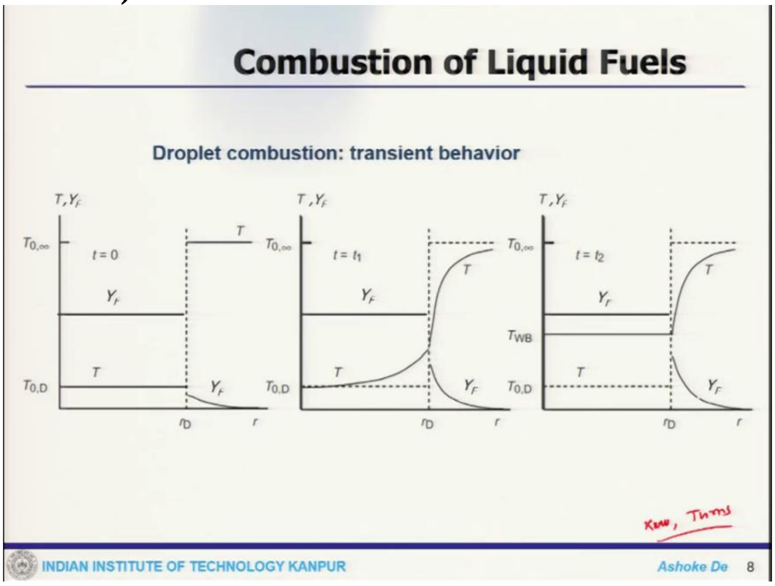

Combustion of Liquid Fuels

Droplet combustion: transient behavior

Krow, Turns

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Now, here you can look at the sort of transient behaviour of the droplet, how it varies, one can look at these either book of Krow or turns. So, this picture is adopted from these books only. So, you can see the change of the temperature from one state to another state.

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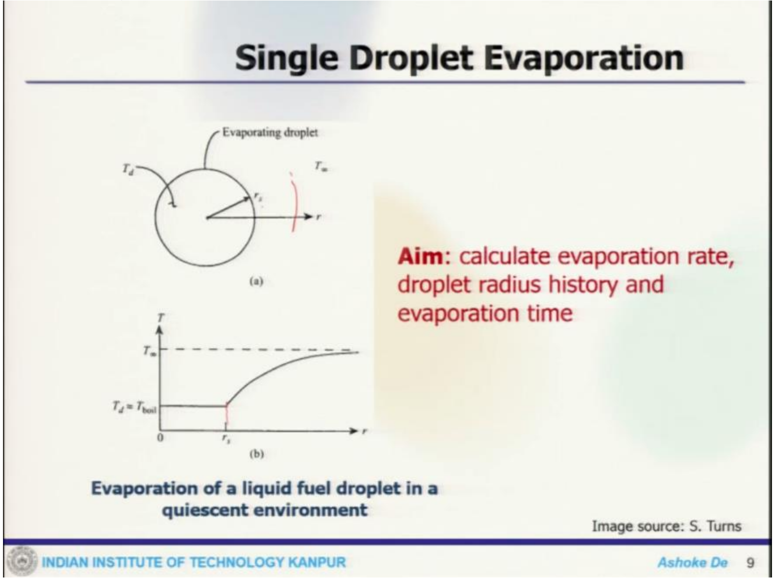

Single Droplet Evaporation

Aim: calculate evaporation rate, droplet radius history and evaporation time

Evaporation of a liquid fuel droplet in a quiescent environment

Image source: S. Turns

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Now, this is again an evaporation of a liquid fuel droplet in the quiescent environment. So, that means the outside environment is come and then how that happens. So, the idea here is to calculate the evaporation rate. So, this is the liquid droplet, this is the droplets surface in the radial direction there will be flame surface, then this is evaporating. So, you can see at the flames are these is the infinity the ambient temperature and this is the boiling temperature well at the droplet surface. This evaporates and then it go due to reaction all these temperature increases, which is along the flame front.

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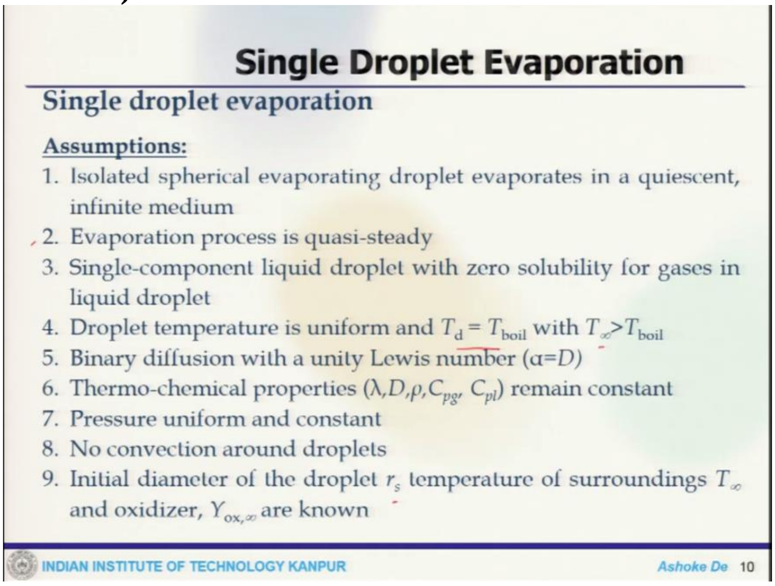

Single Droplet Evaporation

Single droplet evaporation

Assumptions:

1. Isolated spherical evaporating droplet evaporates in a quiescent, infinite medium
2. Evaporation process is quasi-steady
3. Single-component liquid droplet with zero solubility for gases in liquid droplet
4. Droplet temperature is uniform and $T_d = T_{boil}$ with $T_\infty > T_{boil}$
5. Binary diffusion with a unity Lewis number ($\alpha = D$)
6. Thermo-chemical properties ($\lambda, D, \rho, C_{pg}, C_{pl}$) remain constant
7. Pressure uniform and constant
8. No convection around droplets
9. Initial diameter of the droplet r_s temperature of surroundings T_∞ and oxidizer, $Y_{ox,\infty}$ are known

So, if you come to the single droplet evaporation then there are some assumption because already you can see this is quite a bit of complicated phenomena. It is not that easy to handle like our single phase combustion so first assumption that is there it is an isolated spherical evaporating droplet evaporates in quiescent infinite media. Then the second assumption is that evaporation process is quasi-steady that means, at any instant in time the process can be described as if it were in steady state.

So, this particular assumption and actually eliminate the need to deal with the PDF. Now, third the single component liquid droplet with zero solubility for gases in liquid droplet. So, this allows one to use the phase equilibrium at the droplet surface. So, this is a quite useful assumption as you can see. Then the important assumption is that droplet temperature which is uniform and this is like droplet temperature the boiling temperature where the ambient temperature is quite higher than the boiling temperature.

Now, in some of the problems the transient heating up the liquid does not greatly affect the droplet life time and more rigorous calculations show that the droplet surface is only slightly less than the liquid boiling point in combustion environment. So, this particular assumption eliminates the need to solve for the energy equation for the liquid phase.

And importantly, eliminates the need to solve the fuel vapour transport equation in the gas phase. So, that does not require any of these energy equation to be now, binary diffusion with unity Lewis number which allows us to use some sort of a like Shvab-Zeldovich energy equation, then thermo chemical properties remain constant. Although, these properties are greatly vary as we move through the gas phase from the droplet surface to the surroundings, but this constant properties allow a simple closed form solution.

But one has to make very judicious choice of these mean values of these quantities in final analysis, so which will allow you to have a reasonably accurate prediction. Then pressure would be uniform and also constant. No convection around the droplets and initial diameter of the droplet that is r_s. Temperature of the surroundings and oxidizer are known.With all these assumption we can find mass evaporation rate droplet radius history and by writing the gas phase mass energy conservation.

Droplet gas phase interface energy balance and droplet mass energy conservation equation. So, if we write those things, then top of that the gas phase energy equation gives us the temperature distribution within the gas phase. Which in turn allow us to evaluate the conduction heat transfer into the droplet at that surface. So, this is very necessary to evaluate the surface energy balance that yields the evaporation rate. So, once evaporation rate is known, we can easily find the droplet size that is the function of time. Now, we write down the mass conservation equation. So, this is D we are writing in r-coordinate system.

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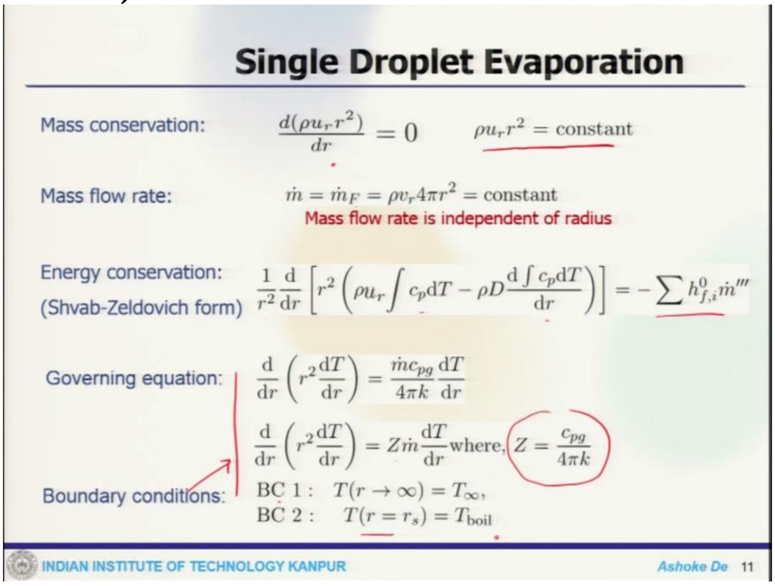

Single Droplet Evaporation

Mass conservation: $\frac{d(\rho u_r r^2)}{dr} = 0$ $\quad \rho u_r r^2 = \text{constant}$

Mass flow rate: $\dot{m} = \dot{m}_F = \rho v_r 4\pi r^2 = \text{constant}$

Mass flow rate is independent of radius

Energy conservation: (Shvab-Zeldovich form) $\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[r^2\left(\rho u_r \int c_p \mathrm{d}T - \rho D \frac{\mathrm{d}\int c_p \mathrm{d}T}{\mathrm{d}r}\right)\right] = -\sum h^0_{f,i}\dot{m}'''$

Governing equation: $\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}T}{\mathrm{d}r}\right) = \frac{\dot{m}c_{pg}}{4\pi k}\frac{\mathrm{d}T}{\mathrm{d}r}$

$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}T}{\mathrm{d}r}\right) = Z\dot{m}\frac{\mathrm{d}T}{\mathrm{d}r}$ where, $Z = \frac{c_{pg}}{4\pi k}$

Boundary conditions: BC 1 : $T(r \to \infty) = T_\infty$,
BC 2 : $T(r = r_s) = T_{\text{boil}}$

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So,

$$\frac{d(\rho u_r r^2)}{dr} = 0$$

which means:

$$\rho u_r r^2 = constant$$

Now mass flow rate if it is independent of radius then massflow rate of the fuel and droplets size would be constant. Now this is an energy conservation equation which is a Shvab-Zeldovich kind of form. We can write:

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\left(\rho u_r\int c_p dT - \rho D\frac{d\int c_p dT}{dr}\right)\right] = -\sum h_{f,i}^0 \dot{m}'''$$

And change in the enthalpy in the epson term. So that is how you write the energy equation. Now the governing equation becomes then this is the set of governing equations:

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = \frac{\dot{m}c_{pg}}{4\pi k}\frac{dT}{dr}$$

So one can write:

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = Z\dot{m}\frac{dT}{dr}$$

Where Z is different like this. So that is a parameter or term which you can define and rewrite the temperature equation and then to get the solution for this you need 2 boundary condition when r goes to infinity it should be the infinity temperature. So, using these 2 boundary conditions, you can find out the solution.

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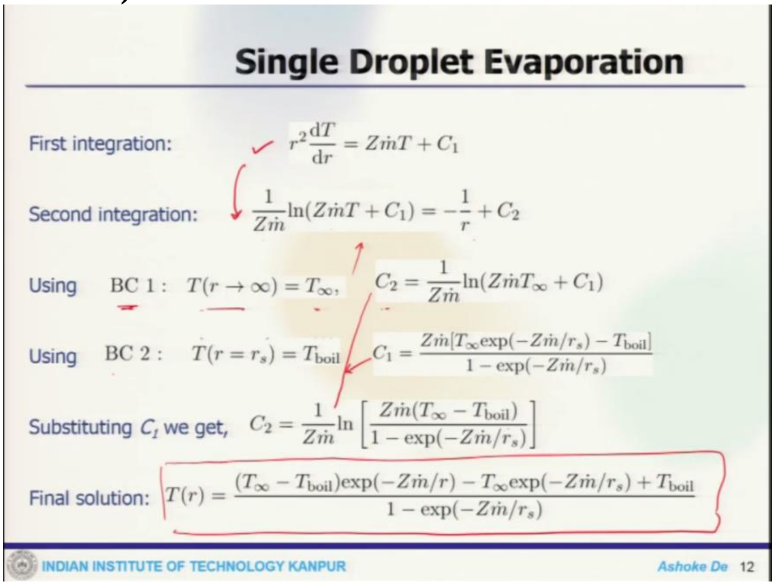

Single Droplet Evaporation

First integration: $r^2\frac{dT}{dr} = Z\dot{m}T + C_1$

Second integration: $\frac{1}{Z\dot{m}}\ln(Z\dot{m}T + C_1) = -\frac{1}{r} + C_2$

Using BC 1 : $T(r \to \infty) = T_\infty$, $C_2 = \frac{1}{Z\dot{m}}\ln(Z\dot{m}T_\infty + C_1)$

Using BC 2 : $T(r = r_s) = T_{boil}$ $C_1 = \frac{Z\dot{m}[T_\infty \exp(-Z\dot{m}/r_s) - T_{boil}]}{1-\exp(-Z\dot{m}/r_s)}$

Substituting C_1 we get, $C_2 = \frac{1}{Z\dot{m}}\ln\left[\frac{Z\dot{m}(T_\infty - T_{boil})}{1-\exp(-Z\dot{m}/r_s)}\right]$

Final solution: $\boxed{T(r) = \frac{(T_\infty - T_{boil})\exp(-Z\dot{m}/r) - T_\infty \exp(-Z\dot{m}/r_s) + T_{boil}}{1-\exp(-Z\dot{m}/r_s)}}$

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So, the first integration will get us:

$$r^2\frac{dT}{dr} = Z\dot{m}T + C_1$$

And the second integration gives us:

$$\frac{1}{Z\dot{m}}ln(Z\dot{m}T + C_1) = -\frac{1}{r} + C_2$$

So, from the previous equation we get this, then from here we get this. Now, the first boundary condition if we apply. When r tends to infinity, it would be T infinity. So, if we put this C_2 becomes so you can use this one here and get C_2 and once we put the second boundary condition at the droplet surface, it will be boiling then C_1 becomes like this.

Now, you substitute the C_1,we get C_2 equals so here you put from C_2 using the value of C_1 we get:

$$C_2 = \frac{1}{Z\dot{m}} ln \left[\frac{Z\dot{m}(T_\infty - T_{boil})}{1 - exp(-Z\dot{m}/r_s)} \right]$$

So the final solution will look like this. So, which you look like $T_\infty - T_{boil}$. That is an expression which you can and this is simple we started with the temperature equation and through integration, you can obtain that.

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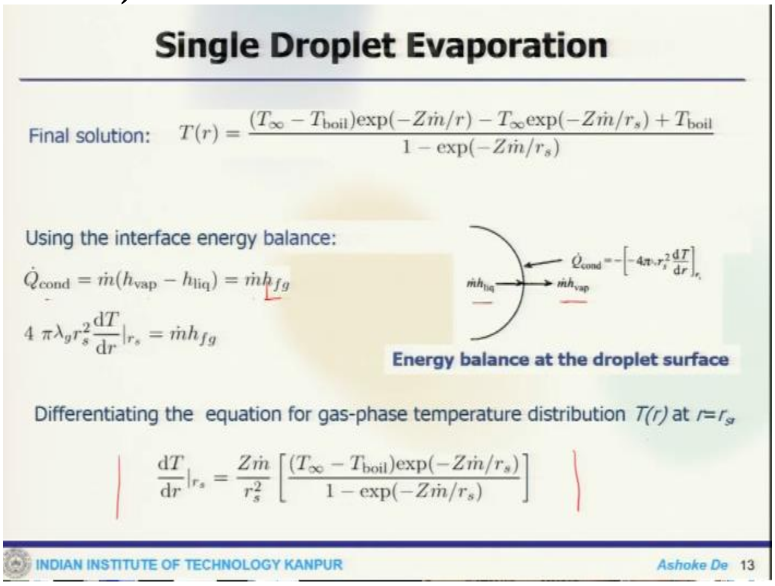

Single Droplet Evaporation

Final solution: $T(r) = \frac{(T_\infty - T_{\mathrm{boil}})\exp(-Z\dot{m}/r) - T_\infty \exp(-Z\dot{m}/r_s) + T_{\mathrm{boil}}}{1 - \exp(-Z\dot{m}/r_s)}$

Using the interface energy balance:

$$\dot{Q}_{\mathrm{cond}} = \dot{m}(h_{\mathrm{vap}} - h_{\mathrm{liq}}) = \dot{m}h_{fg}$$

$$4\,\pi\lambda_g r_s^2 \frac{\mathrm{d}T}{\mathrm{d}r}|_{r_s} = \dot{m}h_{fg}$$

Energy balance at the droplet surface

Differentiating the equation for gas-phase temperature distribution $T(r)$ at $r = r_s$

$$\frac{\mathrm{d}T}{\mathrm{d}r}|_{r_s} = \frac{Z\dot{m}}{r_s^2}\left[\frac{(T_\infty - T_{\mathrm{boil}})\exp(-Z\dot{m}/r_s)}{1 - \exp(-Z\dot{m}/r_s)}\right]$$

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Now, that is the final solution. Now, we go to the next level of balance where at the droplets interface is the energy balance at the droplet surface. So, from flame that heat will come out and then the droplets surface will evaporate or the liquid evaporate and this is the liquid which is going in that means, $\dot{m}h_{liq}$ which is coming in that will be the energy this is coming in. So, the conduction energy would be $\dot{m}(h_{vap} - h_{liq})$. Okay?

So, at this surface the energy balance will get us that which is $\dot{m}h_{fg}$. Now, the rate at which happens, so if we write:

$$4\pi\lambda_g r_s^2 \frac{dT}{dr}\Big]_{r_s} = \dot{m}h_{fg}$$

So differentiating the equation of the gas phase temperature dr at r_s, we get $\frac{dT}{dr}\Big]_{r_s}$ equals to this. Now, this is what we get from that this the $\frac{dT}{dr}\Big]_{r_s}$. That is our gas phase calculation. So, we are differentiating that gas phase equation at r equals to r_s and we get this.

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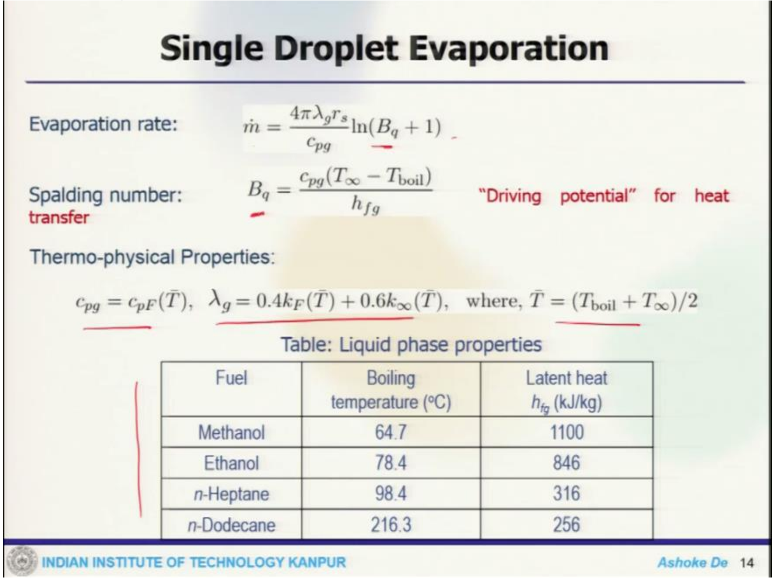

Single Droplet Evaporation

Evaporation rate: $\dot{m} = \frac{4\pi\lambda_g r_s}{c_{pg}}\ln(B_q + 1)$

Spalding number: $B_q = \frac{c_{pg}(T_\infty - T_{\text{boil}})}{h_{fg}}$ "Driving potential" for heat transfer

Thermo-physical Properties:

$c_{pg} = c_{pF}(\bar{T})$, $\lambda_g = 0.4k_F(\bar{T}) + 0.6k_\infty(\bar{T})$, where, $\bar{T} = (T_{\text{boil}} + T_\infty)/2$

Table: Liquid phase properties

Fuel	Boiling temperature (°C)	Latent heat h_{fg} (kJ/kg)
Methanol	64.7	1100
Ethanol	78.4	846
n-Heptane	98.4	316
n-Dodecane	216.3	256

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So now,we can find out the evaporation rate which will be:

$$\dot{m} = \frac{4\pi\lambda_g r_s}{c_{pg}} ln\left(B_q + 1\right)$$

And B_q is known as the Spalding number. So, this expression from the Spalding number is derived based on the heat transfer consideration alone.

Other definition of Spalding number is possible and they are functional form will depend on that assumption that is made. Then also one can clearly see that evaporation rate is dependent on the heat transfers through the Spalding number. This is what it is. And it depends on the gas phase properties such as conductivity, specific heat, for which we need to use suitable average values between droplets surface and ambient.

Now, for different fuel it has different volume temperature different latent heat h_{fg} and Spalding number and evaporation rate will change with multiple fuel which is quite obvious, because these thermo physical properties are they are which are used in this particular equation and different fuel as the different. And also B_q this factor has a different definition if a spherically symmetric flame is assumed to surround the droplet. So, this will be discussed later on when we will be talking about the burning droplet. So, now, if you look at this, it complete the analysis of the gas phase, knowing the evaporation rate will allow us right to mass balance which can be used to calculate the droplet diameter.

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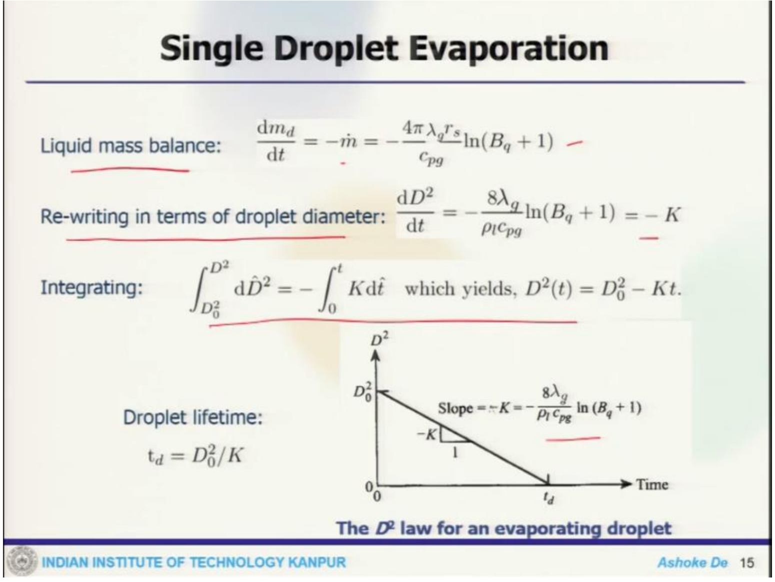

Single Droplet Evaporation

Liquid mass balance: $\frac{dm_d}{dt} = -\dot{m} = -\frac{4\pi\lambda_g r_s}{c_{pg}}\ln(B_q+1)$

Re-writing in terms of droplet diameter: $\frac{dD^2}{dt} = -\frac{8\lambda_g}{\rho_l c_{pg}}\ln(B_q+1) = -K$

Integrating: $\int_{D_0^2}^{D^2} d\hat{D}^2 = -\int_0^t K d\hat{t}$ which yields, $D^2(t) = D_0^2 - Kt.$

Droplet lifetime:

$t_d = D_0^2/K$

The D^2 law for an evaporating droplet

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Now, we write the liquid mass balance equation. So,

$$\frac{dm_d}{dt} = -\dot{m}$$

Then if you rewrite the terms of droplet diameter it will $\frac{dD^2}{dt}$ in like this and once we integrate we get this. So, this is a flame or D^2 law of evaporating droplet and the slope would be minus K which is like this. So, this is now in the liquid phase. So, once we had the gas phase and moving to the liquid phase, this is what we get.

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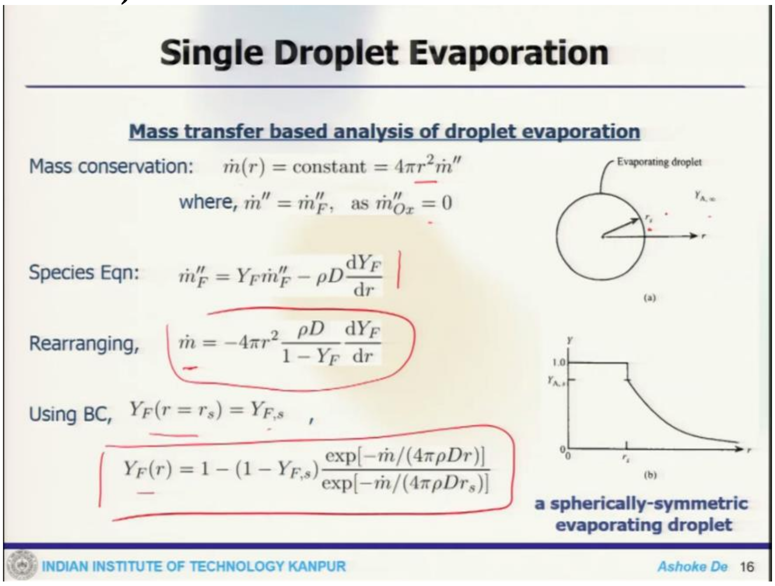

Single Droplet Evaporation

Mass transfer based analysis of droplet evaporation

Mass conservation: $\dot{m}(r) = \text{constant} = 4\pi r^2 \dot{m}''$

where, $\dot{m}'' = \dot{m}''_F$, as $\dot{m}''_{Ox} = 0$

Species Eqn: $\dot{m}''_F = Y_F \dot{m}''_F - \rho D \frac{dY_F}{dr}$

Rearranging, $\dot{m} = -4\pi r^2 \frac{\rho D}{1 - Y_F} \frac{dY_F}{dr}$

Using BC, $Y_F(r = r_s) = Y_{F,s}$,

$Y_F(r) = 1 - (1 - Y_{F,s}) \frac{\exp[-\dot{m}/(4\pi\rho D r)]}{\exp[-\dot{m}/(4\pi\rho D r_s)]}$

a spherically-symmetric evaporating droplet

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Now we can look at the mass transfer based analysis of droplet evaporation. This is the droplet and r_s is the droplet size, this is the environment and the mass conservation $\dot{m}(r)$ would be constant.

$$\dot{m}'' = \dot{m}''_F$$

And oxidizer would be 0. So, the species equation:

$$\dot{m}''_F = Y_F \dot{m}''_F - \rho D \frac{dY_F}{dr}$$

This is along the radian directions how the fuel is changing. So, once we realize that this $\dot{m}$ would be $4\pi r^2$ like this so after here you can like. Now it requires 2 boundary conditions. So at r_s it should be $Y_{F,s}$ and that it means the fuel at the surface and $Y_F(r)$ would looks like this. So, this is an expression that you get how the fuel vary from the liquid surface to the other side.

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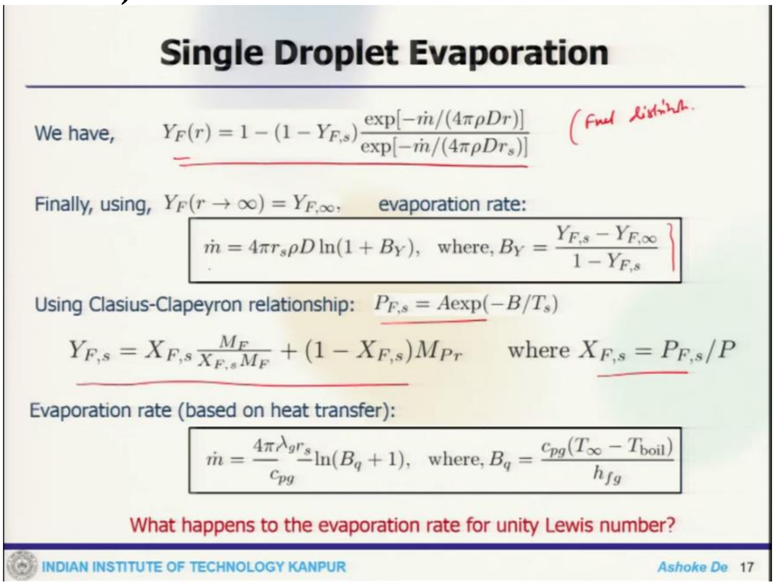

Single Droplet Evaporation

We have, $Y_F(r) = 1 - (1 - Y_{F,s}) \frac{\exp[-\dot{m}/(4\pi\rho D r)]}{\exp[-\dot{m}/(4\pi\rho D r_s)]}$ (Fuel distribn.

Finally, using, $Y_F(r \to \infty) = Y_{F,\infty}$, evaporation rate:

$$\boxed{\dot{m} = 4\pi r_s \rho D \ln(1 + B_Y), \quad \text{where, } B_Y = \frac{Y_{F,s} - Y_{F,\infty}}{1 - Y_{F,s}}}$$

Using Clasius-Clapeyron relationship: $P_{F,s} = A\exp(-B/T_s)$

$$Y_{F,s} = X_{F,s} \frac{M_F}{X_{F,s} M_F} + (1 - X_{F,s}) M_{Pr} \quad \text{where } X_{F,s} = P_{F,s}/P$$

Evaporation rate (based on heat transfer):

$$\boxed{\dot{m} = \frac{4\pi\lambda_g r_s}{c_{pg}} \ln(B_q + 1), \quad \text{where, } B_q = \frac{c_{pg}(T_\infty - T_{\text{boil}})}{h_{fg}}}$$

What happens to the evaporation rate for unity Lewis number?

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Now, we have $Y_F(r)$ this is the distribution of the fuel. So, this is our fuel distribution and finally, we use r tends to infinity it will Y_F to infinity. So the evaporation rate will become:

$$\dot{m} = 4\pi r_s \rho D ln(1 + B_Y)$$

B_Y is given like that. If you use the Clasius Clapeyron relation where $P_{F,s}$ is like this, then the fuel $Y_{F,s}$ can be written where $X_{F,s}$ is like this. So, based on heat transfer, we write $\dot{m}$ as this.

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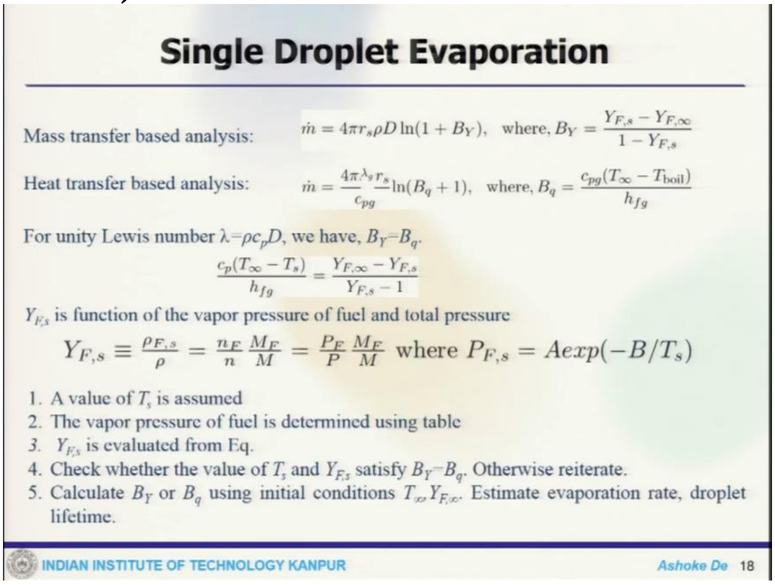

Single Droplet Evaporation

Mass transfer based analysis: $\dot{m} = 4\pi r_s \rho D \ln(1 + B_Y)$, where, $B_Y = \frac{Y_{F,s} - Y_{F,\infty}}{1 - Y_{F,s}}$

Heat transfer based analysis: $\dot{m} = \frac{4\pi\lambda_g r_s}{c_{pg}} \ln(B_q + 1)$, where, $B_q = \frac{c_{pg}(T_\infty - T_{\text{boil}})}{h_{fg}}$

For unity Lewis number $\lambda = \rho c_p D$, we have, $B_Y = B_q$.

$$\frac{c_p(T_\infty - T_s)}{h_{fg}} = \frac{Y_{F,\infty} - Y_{F,s}}{Y_{F,s} - 1}$$

$Y_{F,s}$ is function of the vapor pressure of fuel and total pressure

$$Y_{F,s} \equiv \frac{\rho_{F,s}}{\rho} = \frac{n_F}{n}\frac{M_F}{M} = \frac{P_F}{P}\frac{M_F}{M} \text{ where } P_{F,s} = Aexp(-B/T_s)$$

1. A value of T_s is assumed
2. The vapor pressure of fuel is determined using table
3. $Y_{F,s}$ is evaluated from Eq.
4. Check whether the value of T_s and $Y_{F,s}$ satisfy $B_Y = B_q$. Otherwise reiterate.
5. Calculate B_Y or B_q using initial conditions $T_\infty, Y_{F,\infty}$. Estimate evaporation rate, droplet lifetime.

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So, that requires the effect of your Spalding numbers and these things. Now we can continue the mass transfer based analysis.

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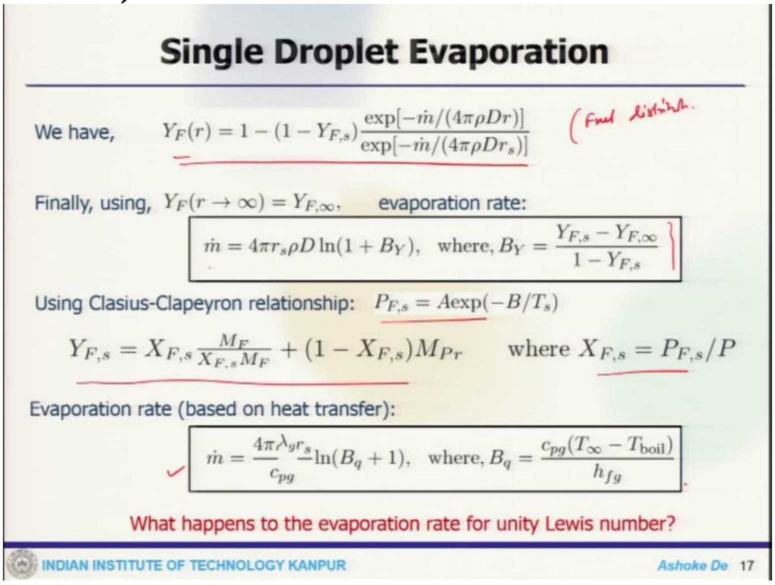

Single Droplet Evaporation

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What happens to the evaporation rate for unity Lewis number?

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So, basically we can look at that when you look at this heat transfer based analysis this is what you get for the evaporation rate. One can question that what happens when the Lewis number is unity. So we will stop here and the mass transfer based analysis will continue in the next lecture. Thank you.