Turbulent Combustion: Theory and Modelling
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Lecture-40
Turbulence (contd...)

Okay, welcome back and let's continue the discussion on the modeling part. So, we are looking at the difference model like Grandsman model in the 2 equations.

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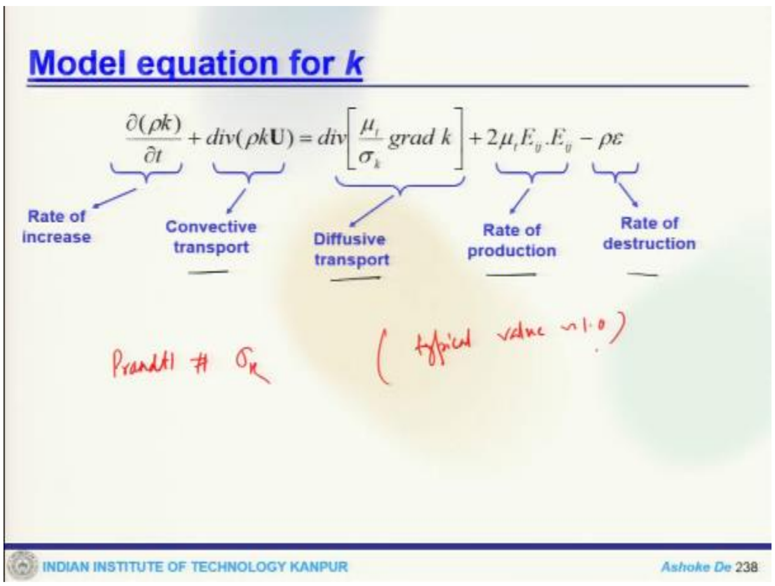

And this is what we looked at the turbulent kinetic energy term where you can see this model equation for the k. Now when you look at that this is for unsteady equation for the k where if you look this is the term which is rate of increase. This is the convective transport, this is diffusive transport, this is production and rate of destruction. So, here the Prandtl number k which connects the diffusivity of k to the eddy viscosity typical value 1 is used. So, that is the typical value is 1 which is used.

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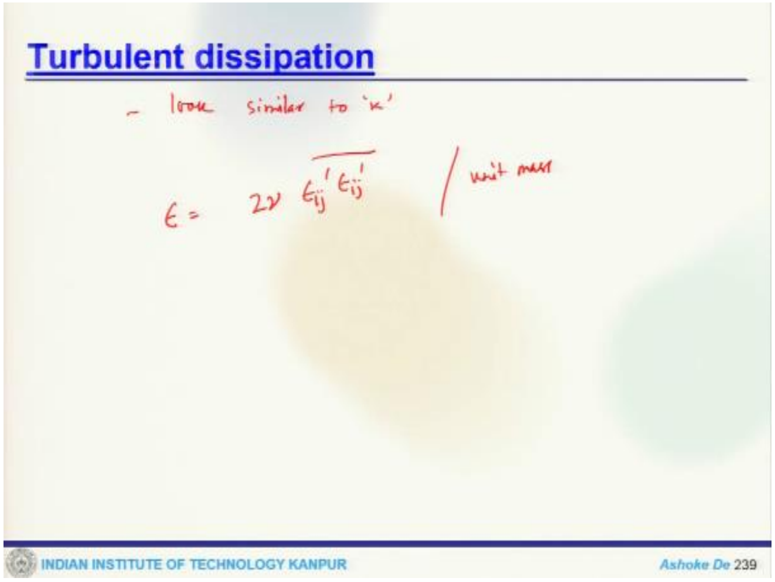

Now, we can look at the turbulent dissipation. This will look similar to k equation, but one has to note that k equation mainly contains the prime quantity indicating that the changes in k are mainly governed by turbulent interactions and also but in the viscous dissipation n there are terms which will be coming from the component due to viscous stresses in like that, so we have to define the dissipation as $2\nu\overline{\varepsilon_{ij}'\varepsilon_{ij}'}$. So, this is for unit mass because this is quite important.

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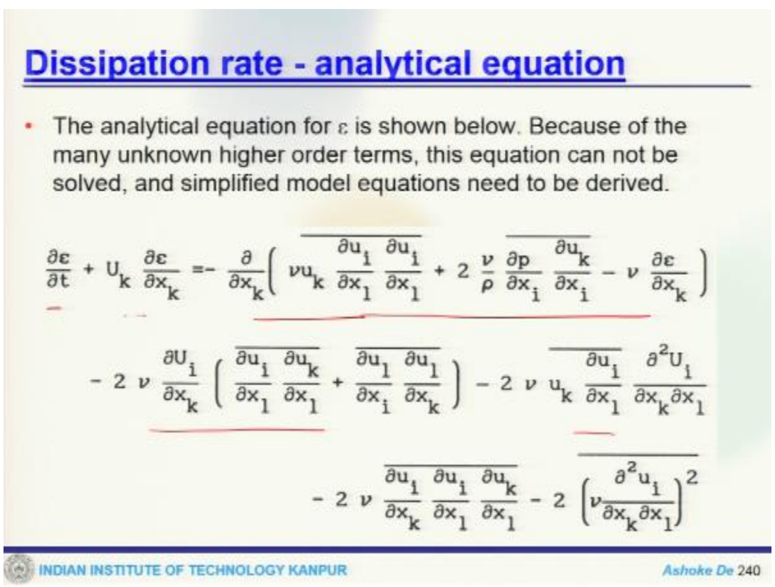

Now if you look at the complete expression for the epsilon equation, this is the analytical equation for the epsilon. So, if you look at that, this is quite complicated or rather apparently looks quite complicated and it cannot be solved that easily. So, this is your unsteady, this is convection. This is the term you have production these so many terms are involved here. Now that is why the simple equations which are used to solve for.

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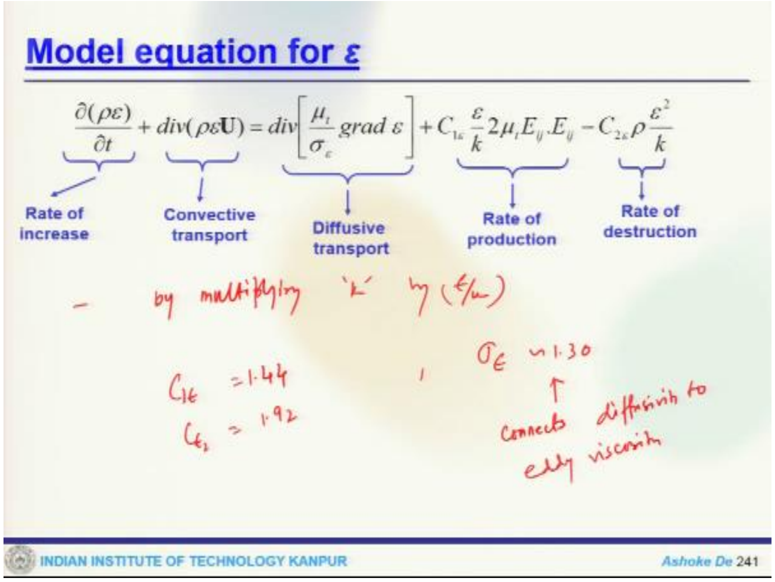

So the model equation which is usually solved this is the unsteady term, this is the convective transport, this is your diffusive transport, production and destruction. So, this is the simplified version. So, this is how we derive. We derive by multiplying k equation by epsilon by k. So, this is a very commonly used epsilon equation where this coefficient $C_{\varepsilon 1}$ is 1.44 is commonly used $C_{\varepsilon 2}$ is 1.92 and σ_ε typically used 1.3, which is again, the number Prandtl number connects the diffusivity to the eddy viscosity this connects diffusivity to eddy viscosity, okay.

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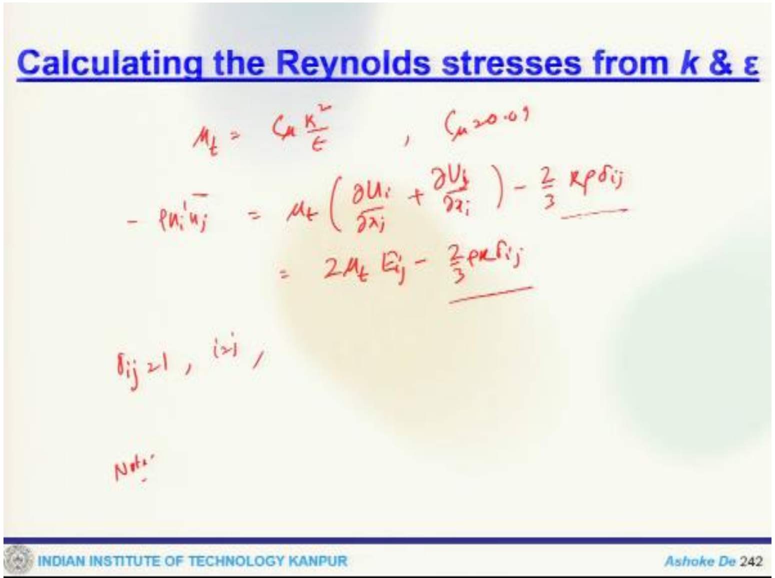

Now we can calculate the Reynolds stresses from k and epsilon. So, our eddy viscosity is approximated like

$$\mu_t = C_\mu \frac{k^2}{\epsilon}$$

where C_μ is point 0.9 and the Reynolds stress is

$$\mu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} k\rho\delta_{ij}$$

which is nothing but

$$2\mu_t E_{ij} - \frac{2}{3} k\rho\delta_{ij}$$

But one has to note that that k Epsilon model leads to all normal stresses being equal which is usually an inaccurate.

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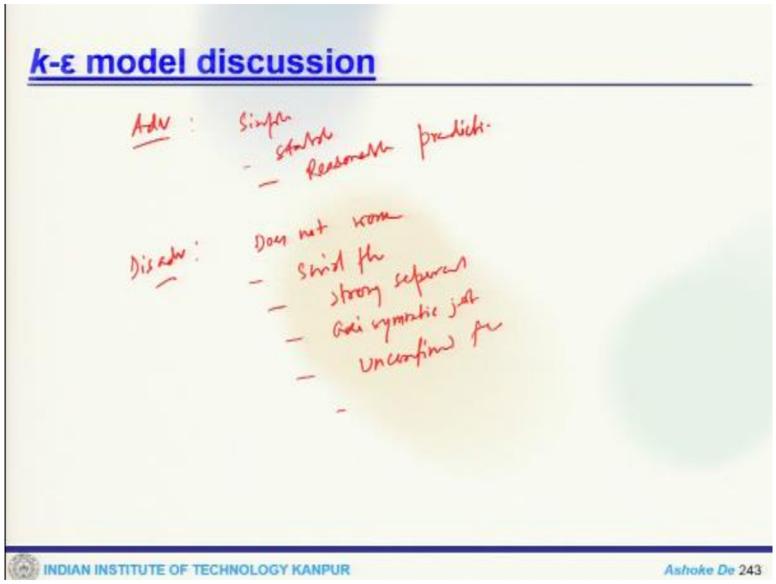

Now there are advantages there are disadvantages. So, the advantages simple to implement let us do stable calculations and converge relatively easily. Stable calculation then reasonably good prediction reasonable predictions, but if you look at the disadvantages this is absolutely provide you poor prediction for swirling and rotating flow, flows with strong separation axis symmetric Jets. So, does not work essentially for swirl flow strong separation axis symmetric Jets unconfined flow some unconfined flows fully developed with but it is quite valid for fully turbulent flow and simplistic Epsilon equation.

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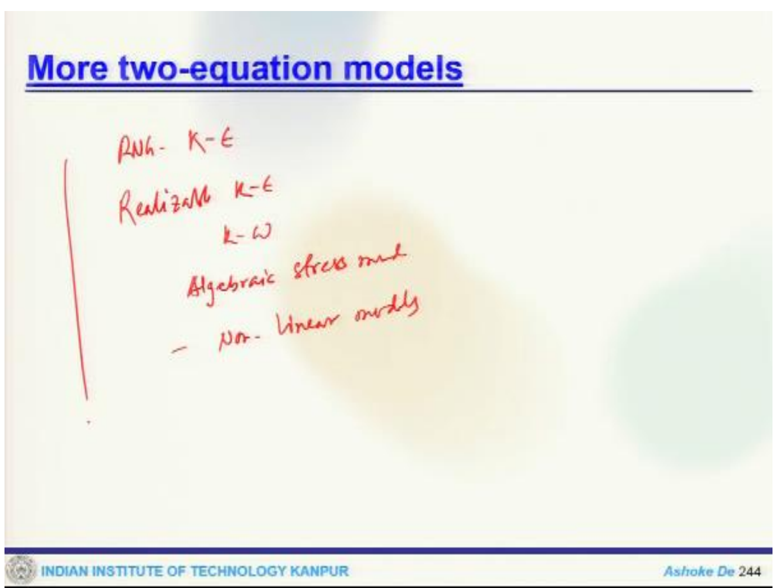

So there are other brands of epsilon models, which are k epsilon RNG k Epsilon. We have realizable k epsilon, we have k Omega Model. We have algebraic stress model and also nonlinear model. So, these are some of the other models also comes under the banner of 2 equation models.

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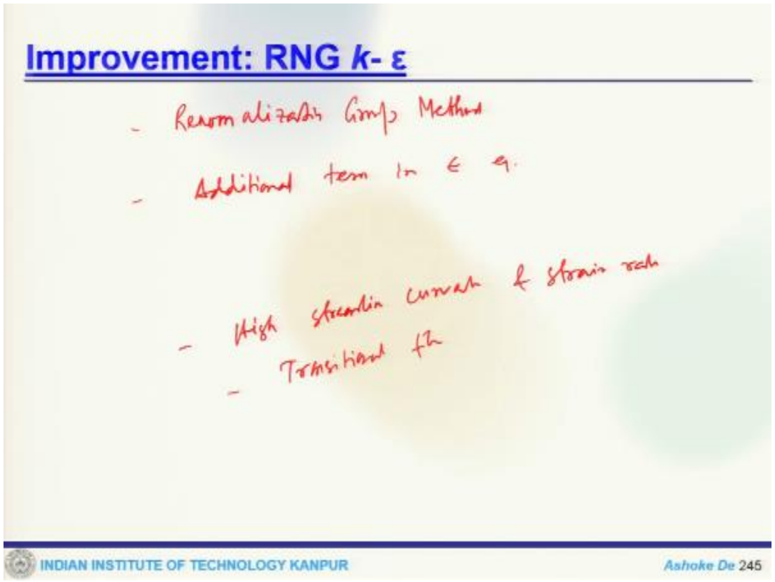

Now, what is the improvement of RNG k Epsilon so there is a k and Epsilon equations are derived from the application of rigorous statistical technique, which is called renormalization? Group Method to instantaneous Navier Stokes equation. So, the equation looks similar to k-epsilon equation. But it includes some additional term in epsilon equation for interaction of turbulence and dissipation means here.

So it takes into account the effect of swirl. So, analytical formula, so it provides you some improved prediction for high streamline curvature and strain rate some transitional flows and well heat and mass transfer.

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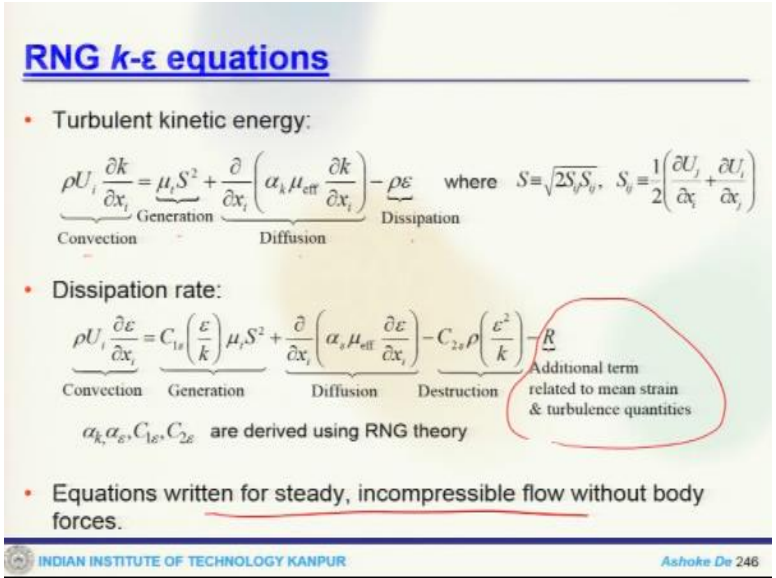

But it still does not predict the spreading of round jet correctly. So, if you look at this RNG k-Epsilon based equations, so this is my convection term generation term. So, this is again for steady incompressible flow. This is diffusion and this is the Epsilon equation where additional term related to mean strain and turbulence quantities. So, this is what comes into the picture when you compared to standard k-Epsilon.

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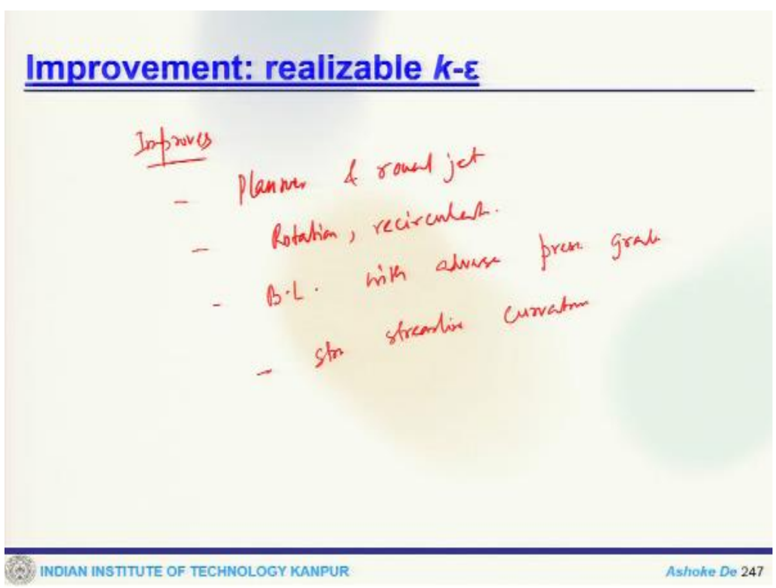

Now, there is a another group where the is realizable k Epsilon. This is improved for this actually a improvement. So, this improves performance for flow involved in planner and round jet, rotation and recirculation, so boundary layer undergo strong boundary layer with adverse pressure gradient and also for strong streamline curvature. So, these are the improvements which actually there for the realizable k Epsilon model.

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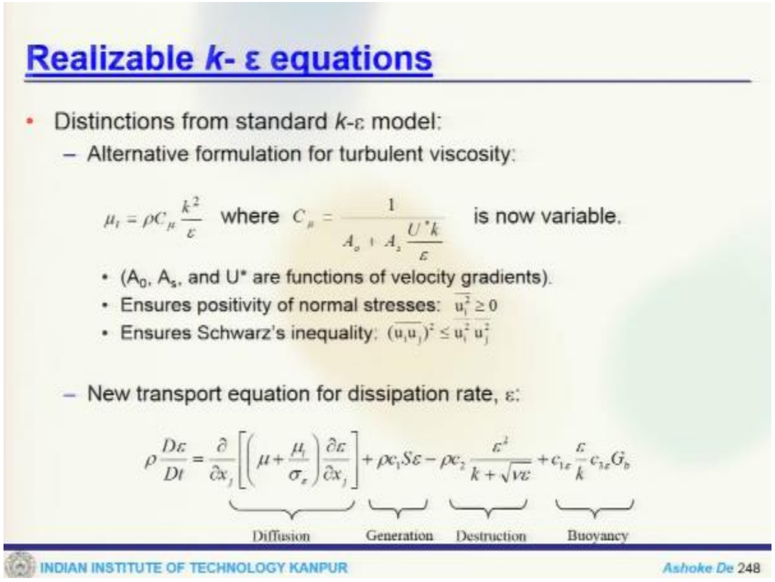

So some of the form the standard k-Epsilon model this deviate and the one of the major deviation is that the estimation of the turbulence eddy viscosity, which is μ_t is calculated and C_μ looking like that here it ensures positivity of the normal stresses that is number 1 number 2 also, it ensures the squares inequality. So, the dissipation rate equation has typical diffusion generation.

This is the destruction term is modified and also the effect of buoyancy. So, if you see there is a quite a bit of improvement which takes place here for the realizable k epsilon model.

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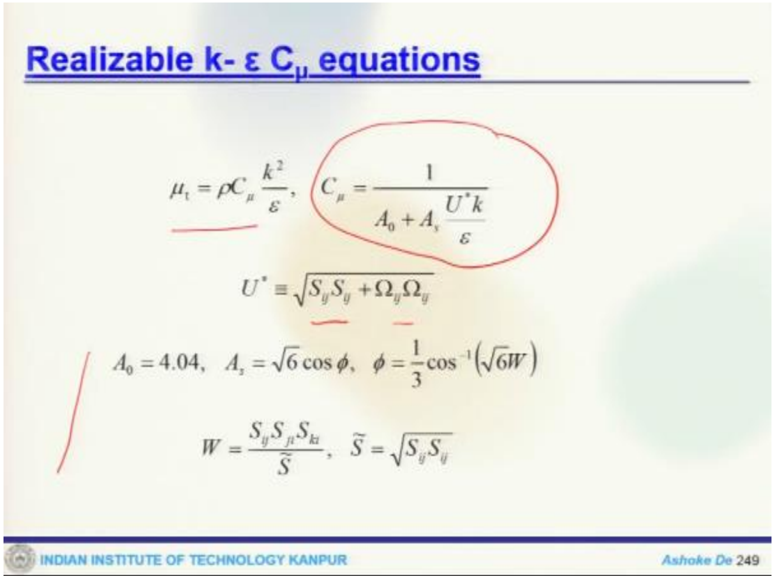

Now here the model constants are estimated like μ_t is estimated with a model constant C_μ and $\frac{k^2}{\varepsilon}$ where C_μ takes care of this effect. It is not constant anymore. And where you start takes in the both the strain and the rotation part. This is rest of the parametric constant.

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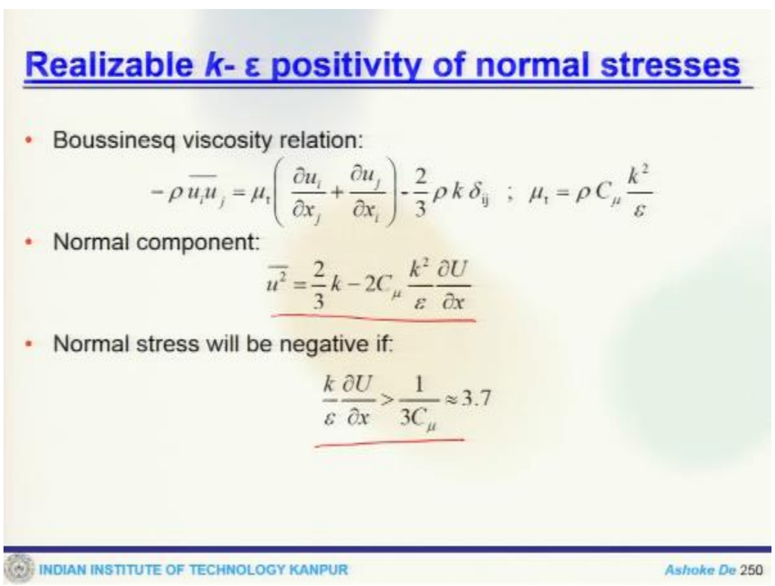

Now if you look at the Boussinesq hypothesis in this, so the normal components are estimated like that in the realization k Epsilon case and this would be negative if this is greater than so these are some small.

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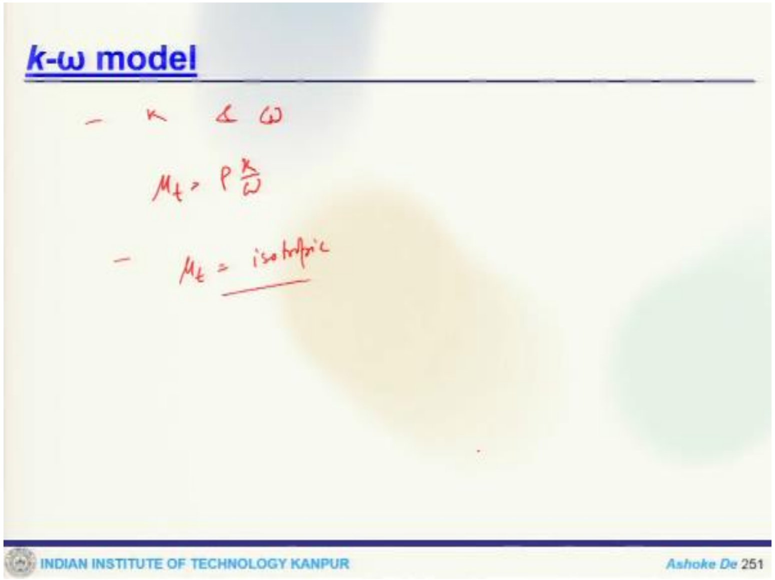

Now, the other set of model is the k Omega Model. So, this is another 2 equation model. So, this Omega is the turbulence frequency. So, this solves for 2 equations one is the k and another equation for Omega and here the μ_t is calculated as $\frac{\rho_k}{\omega}$. So, it is behavior is similar to standard k-Epsilon model, but it has some drawback that here this μ_t assumption is isotropic which is a huge drawback of this particular model for the application in large scale problem.

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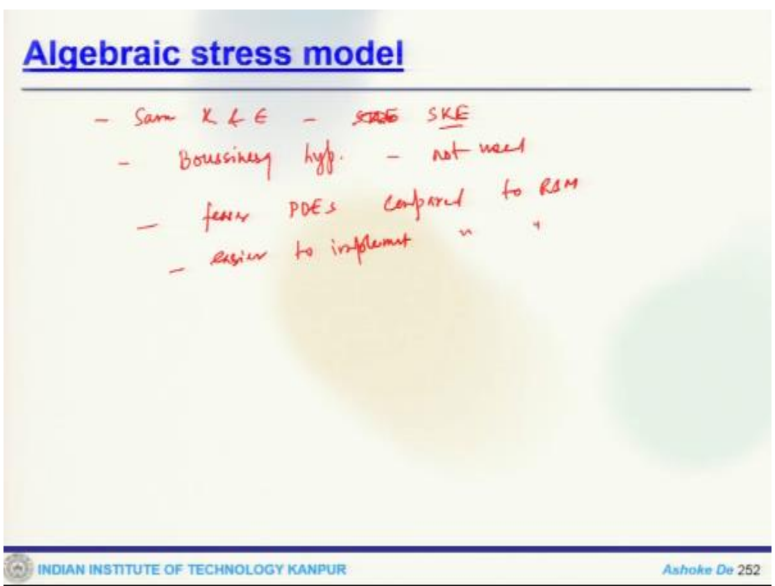

Now, the other set of model is the Algebraic state stress model. This is same k and Epsilon equations, which are solved with standard k Epsilon model or a standard k Epsilon model, but Boussinesq hypothesis not used here. The full Reynolds stress equations are first derived and then some simplifying assumptions are made to allow derivation of algebraic equations for the Reynolds stress. So, has fewer PDEs compared to RSM and also it is quite easier to implement compared to RSM.

So, this algebraic equation and are not very stable. However, computer time is significantly more than the standard k-Epsilon, k Omega modal so this was used in old days, but once the RSM model is available.

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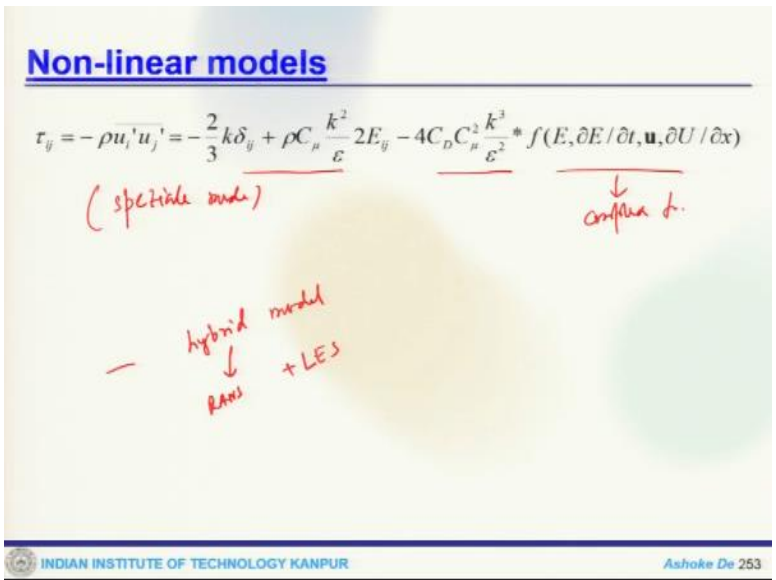

So this is not very often use now. Then another set of correction is the nonlinear corrections, which is standard k-Epsilon model is extended by nonlinear corrections where your Reynolds stress term. This is an example of spatial model where this f is a complex function this is a complex function of deformation denser velocity field and gradient and the rate of change of deformation standard.

So the standard k-Epsilon model here reduces to a spatial case of this model and low Reynolds of deformation model. So, these are relatively new because these nonlinear corrections which take into consideration but then using this nonlinear corrections, there is an another advancement which has been taken place is the development of hybrid models, which uses both RANS and LES.

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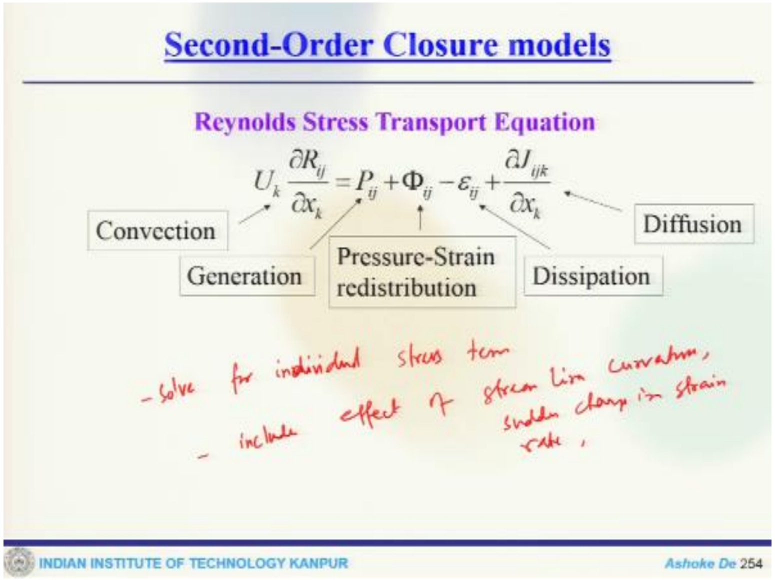

Now, this is a Reynolds stress transport equation. This is where we have already looked at it now you solve for individual stress term. So, it becomes quite computationally expensive. And this is also complex in nature, but it includes the effect of streamline curvature. So, it includes effect of streamline curvature sudden changes in strain rate secondary motions and all these can be taken into consideration.

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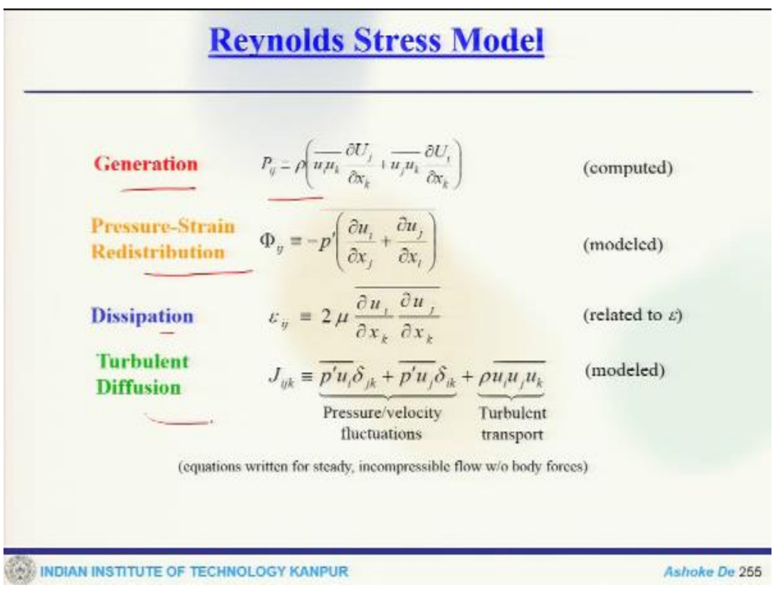

So these are the term which are actually associated with that. This is a generation term. The turbulence generations are given like that. And this is pressure strain term this is dissipation and the diffusion term

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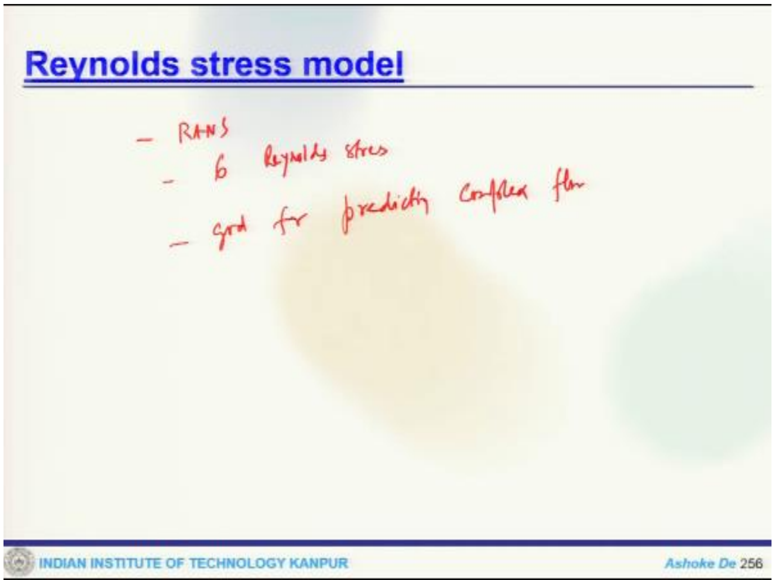

So this is again, we are looking at the now this close the this is based on RANS averaging you have six Reynolds stress equation which we solve so these transport equations are derived and these are solved, so result equation so that is why no Boussinesq hypothesis is required. This is good for predicting complex flows. So, it accounts for the stimulant curvatures. Well rotation high standards. So, this can be used in combustor calculation, cyclone force, rotating flow passages, secondary flows within separation.

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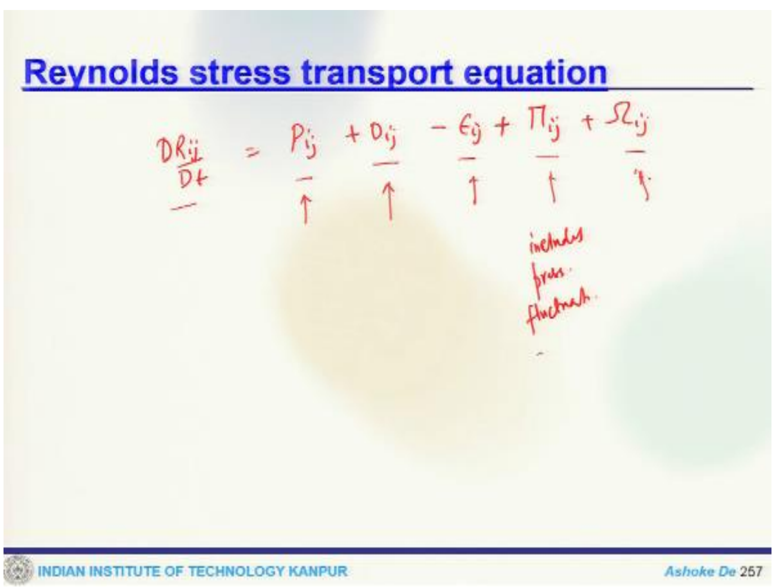

So this is an quite a bit of improvement and the exact equation looks like is that

$$\frac{DR_{ij}}{Dt} = P_{ij} + D_{ij} - \epsilon_{ij} + \Pi_{ij} + \Omega_{ij}$$

where this is the rate of change and this includes rate of change plus transport of average this is production, this is diffusion, this is rate of dissipation. So, this is turbulent pressure strain interaction and this is the rotation. So, these are the components which actually take into account. So, production dissipation this production, diffusion, dissipation, pressures strain and the rotation, so production is retained in his exact form some comment about that.

So, diffusive transport model using a gradient diffusion assumption that is another aspect of it. The dissipation epsilon is this term is related to epsilon and calculated from the standard k epsilon equation. The pressure strain interactions are quite important because this include so this actually includes pressure fluctuations. Due to eddy interacting with each other at different mean velocity.

So the overall effect to make the normal stress more isotropic and to decrease the safe stress. So, it does not change the total turbulent kinetic energy. So, this is one of the term which is very difficult to model and various models are available common is the launder model improved nonlinear. And finally this term is the transport due to rotation.

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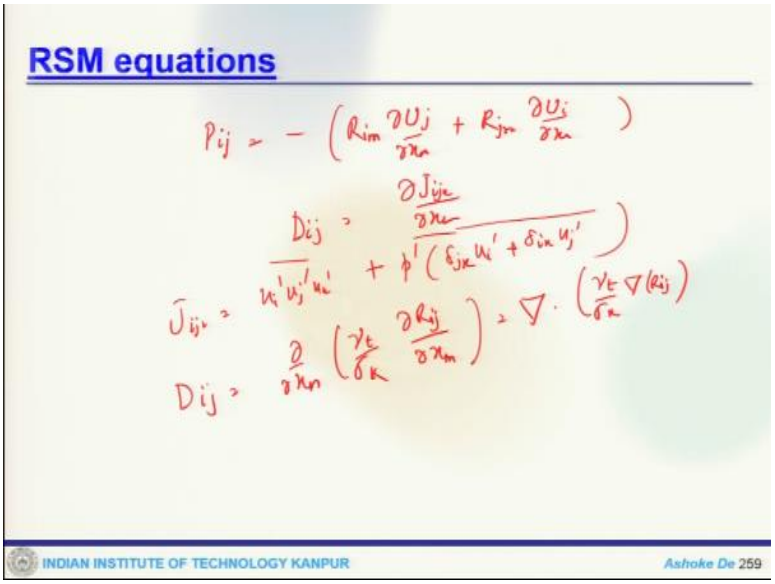

So your production is quite exact as we said which one can write that

$$P_{ij} = -\left(R_{im}\frac{\partial u_j}{\partial x_m} + R_{jm}\frac{\partial u_j}{\partial x_m}\right)$$

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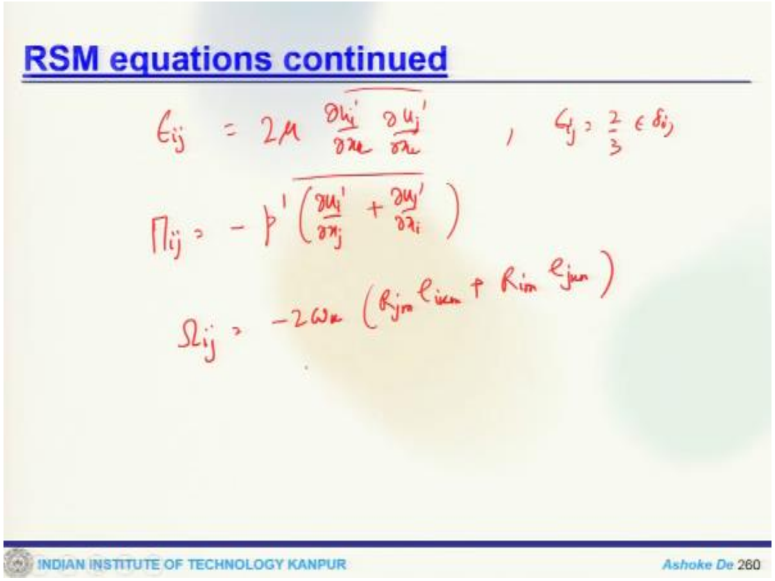

So also the dissipation is the exact so one can approximate that $2\mu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}$ dissipation model uses is $\frac{2}{3}\varepsilon\delta_{ij}$ pressure strain term, which is $-\overline{p'\left(\frac{\partial u_i'}{\partial x_k} + \frac{\partial u_j'}{\partial x_k}\right)}$ and there are models available and rotation term is $-2\omega_k(R_{jm}\rho_{ikm} + R_{im}\rho_{jkm})$ where this is - 1 0 or 1 depending on the situation.

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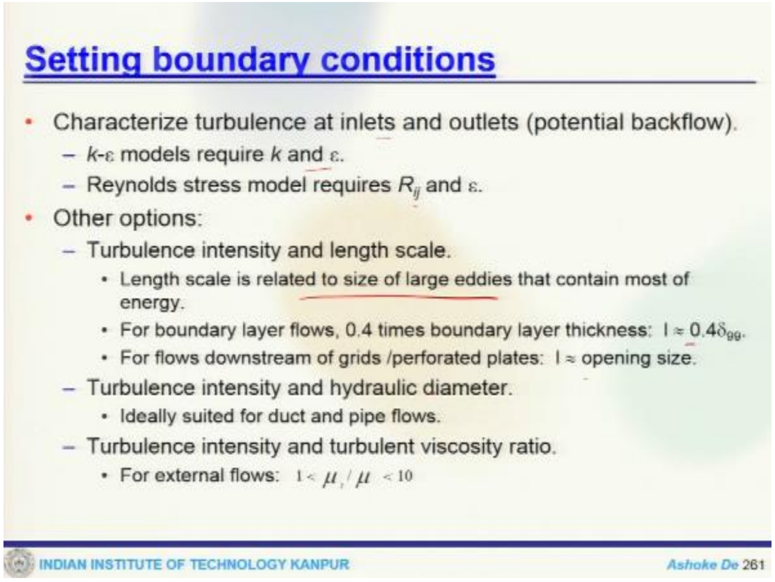

So, one has to set up the boundary condition. So, you need to characterize the turbulence at the inlets and outlets. So, k Epsilon models require k and Epsilon Reynolds stress model requires all the other options turbulence intensity and length scale where the length scale is related to the size of large eddies for boundary layers flows one can define that length scale for other flows, then also turbulence intensity and hydraulic diameter or one can use the viscosity ratio.

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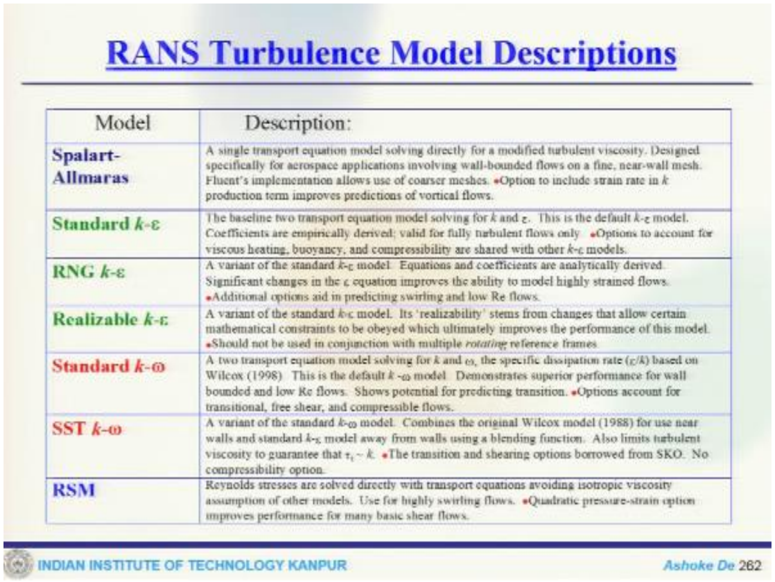

Model	Description:
Spalart-Allmaras	A single transport equation model solving directly for a modified turbulent viscosity. Designed specifically for aerospace applications involving wall-bounded flows on a fine, near-wall mesh. Fluent's implementation allows use of coarser meshes. •Option to include strain rate in k production term improves predictions of vortical flows.
Standard k-ε	The baseline two transport equation model solving for k and ε. This is the default k-ε model. Coefficients are empirically derived; valid for fully turbulent flows only. •Options to account for viscous heating, buoyancy, and compressibility are shared with other k-ε models.
RNG k-ε	A variant of the standard k-ε model. Equations and coefficients are analytically derived. Significant changes in the ε equation improves the ability to model highly strained flows. •Additional options aid in predicting swirling and low Re flows.
Realizable k-ε	A variant of the standard k-ε model. Its 'realizability' stems from changes that allow certain mathematical constraints to be obeyed which ultimately improves the performance of this model. •Should not be used in conjunction with multiple *rotating* reference frames.
Standard k-ω	A two transport equation model solving for k and ω, the specific dissipation rate (ε/k) based on Wilcox (1998). This is the default k-ω model. Demonstrates superior performance for wall bounded and low Re flows. Shows potential for predicting transition. •Options account for transitional, free shear, and compressible flows.
SST k-ω	A variant of the standard k-ω model. Combines the original Wilcox model (1988) for use near walls and standard k-ε model away from walls using a blending function. Also limits turbulent viscosity to guarantee that $\tau_t \sim k$. •The transition and shearing options borrowed from SKO. No compressibility option.
RSM	Reynolds stresses are solved directly with transport equations avoiding isotropic viscosity assumption of other models. Use for highly swirling flows. •Quadratic pressure-strain option improves performance for many basic shear flows.

Now if we put this RANS base models together then one can see this is spalart-Allmaras model standard k-Epsilon model RNG k Epsilon, so these are the improvement standard k-Omega SST k Omega. So, these are the different variable RANS models which are put together.

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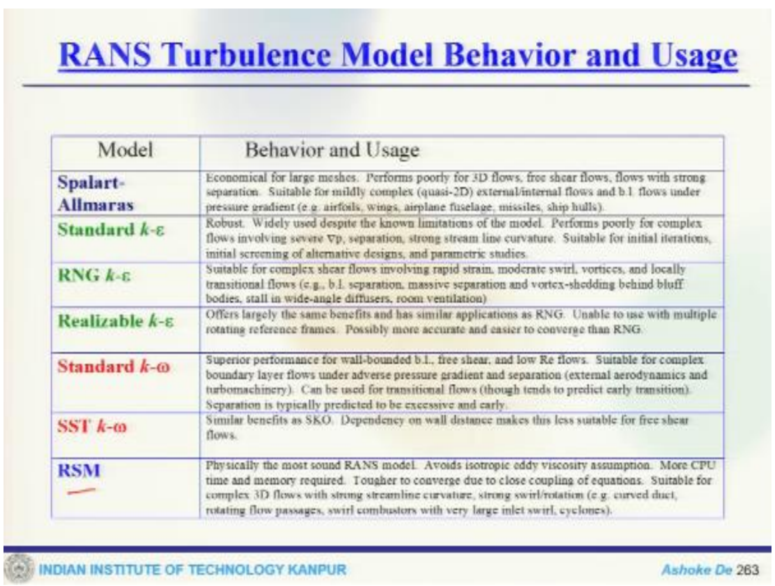

Model	Behavior and Usage
Spalart-Allmaras	Economical for large meshes. Performs poorly for 3D flows, free shear flows, flows with strong separation. Suitable for mildly complex (quasi-2D) external/internal flows and b.l. flows under pressure gradient (e.g. airfoils, wings, airplane fuselage, missiles, ship hulls).
Standard k-ε	Robust. Widely used despite the known limitations of the model. Performs poorly for complex flows involving severe ∇p, separation, strong stream line curvature. Suitable for initial iterations, initial screening of alternative designs, and parametric studies.
RNG k-ε	Suitable for complex shear flows involving rapid strain, moderate swirl, vortices, and locally transitional flows (e.g., b.l. separation, massive separation and vortex-shedding behind bluff bodies, stall in wide-angle diffusers, room ventilation)
Realizable k-ε	Offers largely the same benefits and has similar applications as RNG. Unable to use with multiple rotating reference frames. Possibly more accurate and easier to converge than RNG.
Standard k-ω	Superior performance for wall-bounded b.l., free shear, and low Re flows. Suitable for complex boundary layer flows under adverse pressure gradient and separation (external aerodynamics and turbomachinery). Can be used for transitional flows (though tends to predict early transition). Separation is typically predicted to be excessive and early.
SST k-ω	Similar benefits as SKO. Dependency on wall distance makes this less suitable for free shear flows.
RSM	Physically the most sound RANS model. Avoids isotropic eddy viscosity assumption. More CPU time and memory required. Tougher to converge due to close coupling of equations. Suitable for complex 3D flows with strong streamline curvature, strong swirl/rotation (e.g. curved duct, rotating flow passages, swirl combustors with very large inlet swirl, cyclones).

And what is their behavior one is very simple used for it has certain advantages that we have discussed. So, depending on their advantages and disadvantages one has to use it. This is quite involved. It can take care lot of the other aspect of it.

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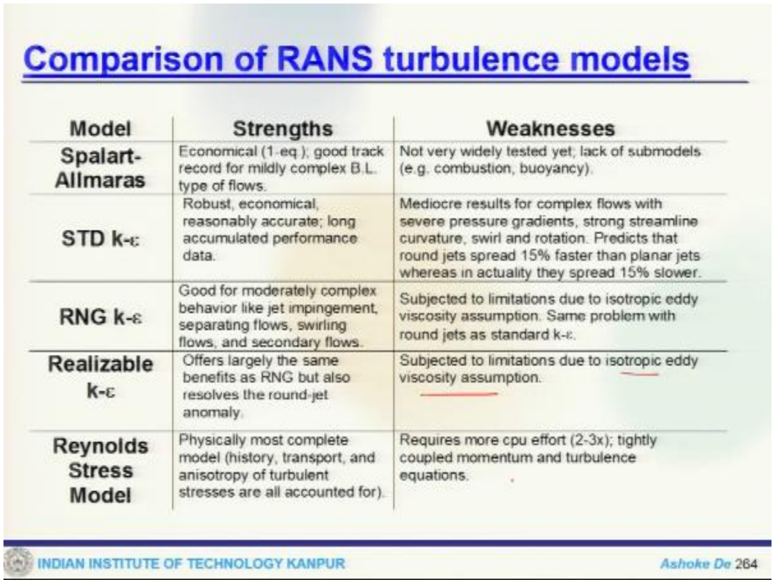

So if we look at the some of the basic strength and weakness of these models, if you look at SA model, it is economical good track record for mildly complex boundary layer flow, but not very widely tested. Standard k-Epsilon robust reasonably accurate long remembered but mediocre results for complex flows. It does not take into account swirl rotation RNG. It is good for moderate behavior like jet impingement separating flows it has also certain limitations. Realizable k Epsilon this is quite good modification to the standard model and it is subject to limitation of the isotropic eddy viscosity assumption.

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It is more complex model but requires more CPU time, so it is tightly coupled with the momentum. So, one another important aspect when you look at the boundary layers are all banded flow. You need the near wall treatment. So, this is a special treatment required to mean profile is a standard wall function non-equilibrium wall functions or 2 layer zonal wall function.

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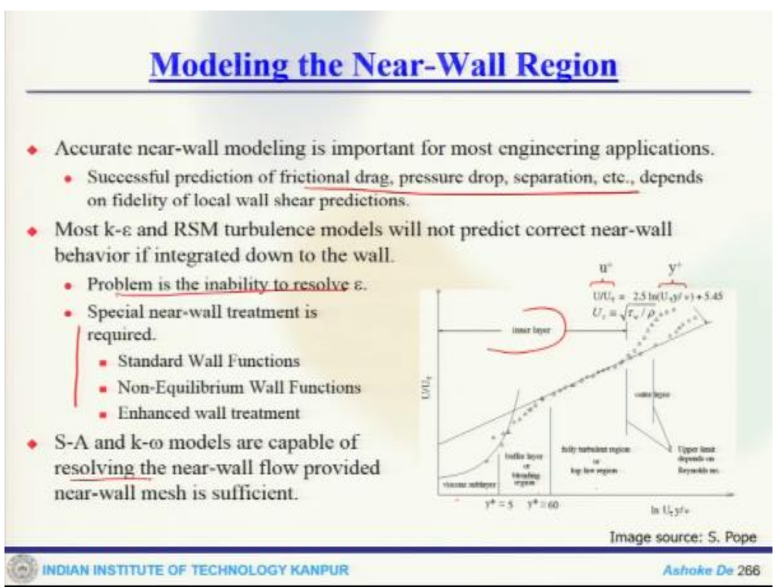

So if you look at the wall profile, which we have discussed, this is our viscous sub layer in the boundary layer a buffer layer. This is fully turbulent zones or this rather can be totally inner layer and this is the outer layer. So, you need near wall modeling for engineer application because of the drag pressure drop separation all these important aspect which require so most of the problem in the inability to resolve the Epsilon.

So we require some treatment either Standard wall treatment so S-A and K-Omega models are capable of resolving the near all flow provided near all mess is sufficient. That means if you resolve the mess.

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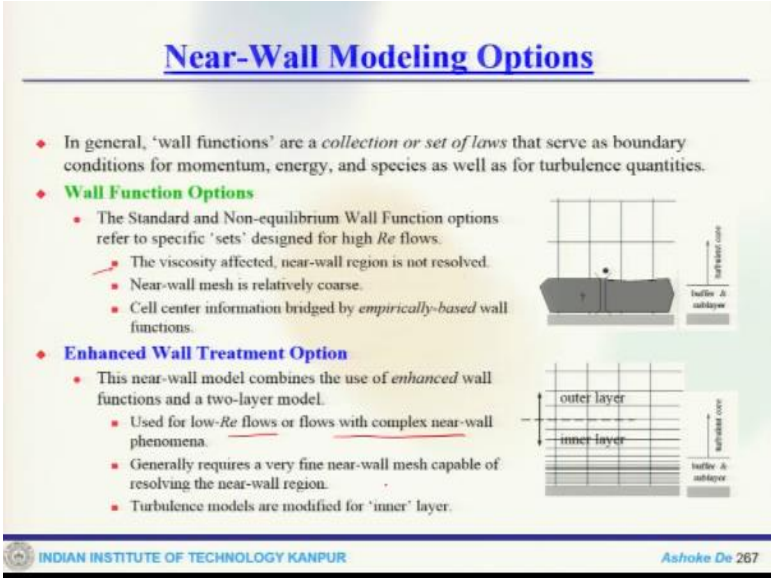

So that means if you look at this is the buffer layer and this is zone and this is the inner layer and outer layer. So, it depending on the mesh resolution one can so the wall function options the standard and non-equilibrium malfunctions, which are sets designed for high Reynolds number flow. So, there is a viscosity affected near all reason is not resolved. So, near mess resolved is relatively coarse and empirically based model.

And if you go by essentially enhanced wall function situation, then it is used for low Reynolds number flows or flows with complex near all phenomena. This is requires find near mess capability and modified at the inner layer.

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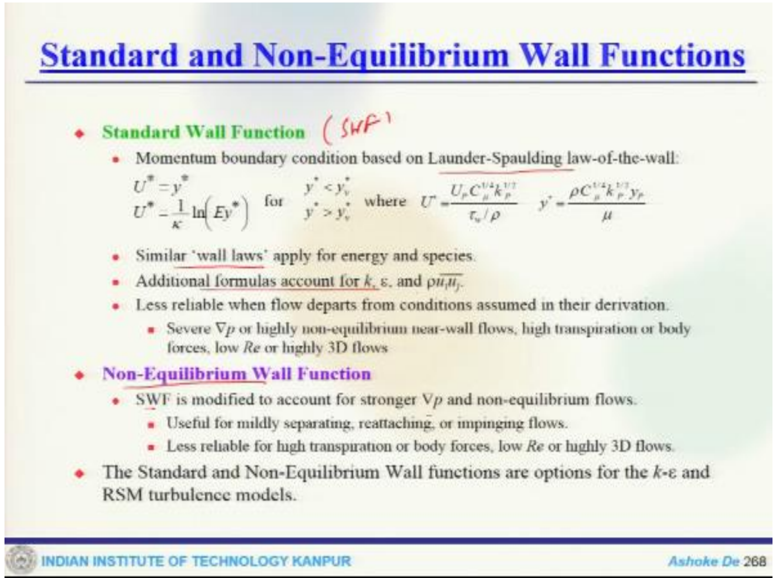

So these are the some of the standard wall function which is quite often used and by using Launder Spaulding law that is U star Y star U star is this depending of the Y star definition. So, this is similar to wall laws addition formulas account for k Epsilon and Reynolds stress. Less reliable when the flow departs from conditions assumed in their derivation so that severe delta p. p is non equilibrium wall function. So, this is modified Standard wall function is modified. This is SWF.

For stronger delta p and nonlinear flows. This is also less reliable for high transportation or body forces. So, these are the options one can use for k Epsilon and RSM based turbulence model.

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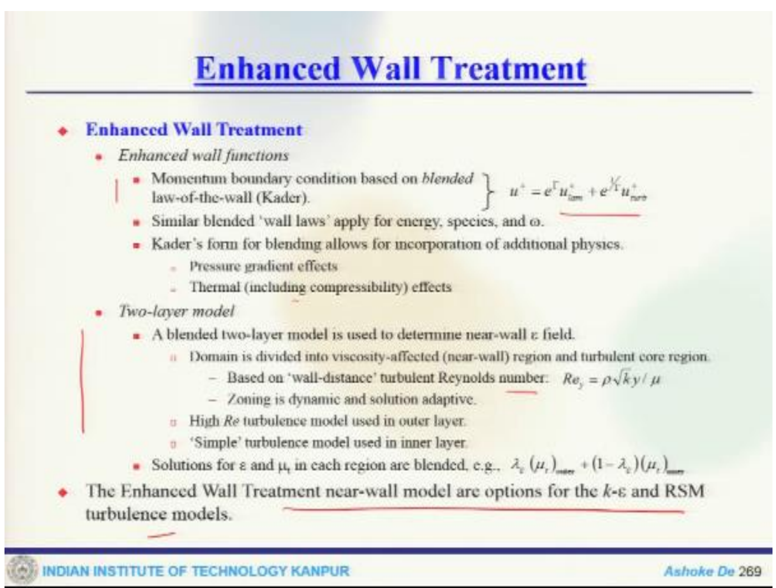

Now this is Enhanced Wall treatment. So, where your momentum boundary layer is Blended similar blended wall laws and then pressure gradient affect thermal effect. These are what is taken care of and if you go for 2 layer model where this blended 2 layer models is used to determine the near Epsilon field. So, the domain is divided into the near wall region and the turbulent core region where while distance and so this has high Reynolds turbulence model.

So the enhance wall treatment near wall model are options again for k Epsilon and RSM turbulence model where one can use it.

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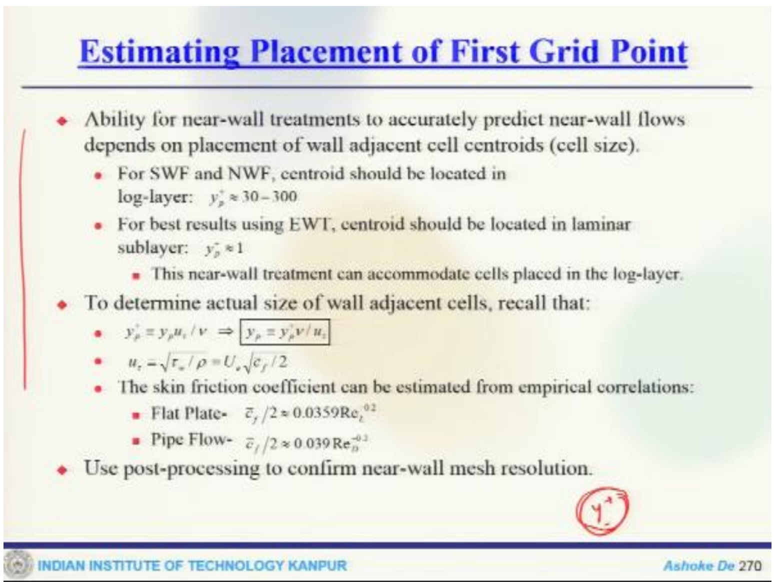

So any of this CFD packages or the commercial packages that is available that has this kind of wall treatment that one can use this is estimating the first grid point because once you use the wall treatment, then you have to estimate your first grid point or the size of the grid point so that you estimate so the details you can find it any turbulence book or any turbulence modeling book that what would be the value of y^+. So, this is a critical parameter. That one has to look at it

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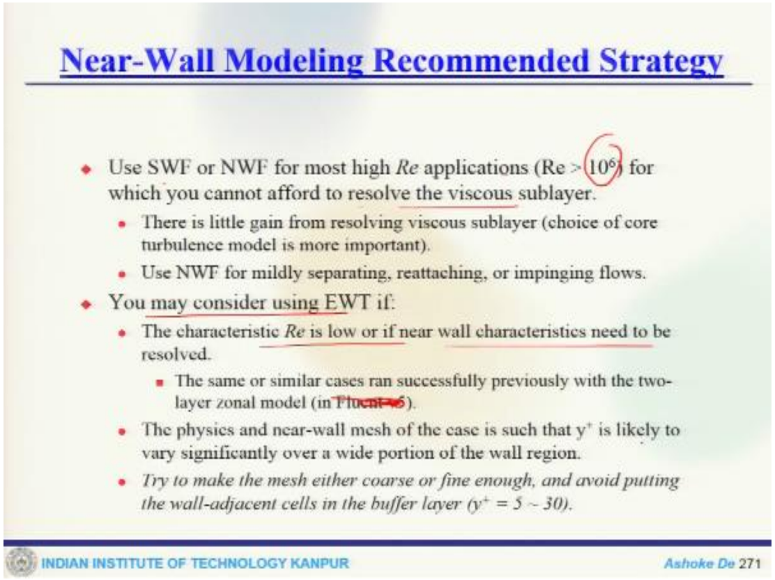

When they now if you go by the sum of this recommendation, so one can use the Standard wall function or near non equilibrium wall function for most high Reynolds number applications, which cannot afford to resolve the viscous sub layer that because you know, somebody so high we cannot resolve it there is little gain, resolving viscous sub layer use non equilibrium wall function more than mildly flows, but you may consider using if the characteristics are is low or even are all characters need to be resolved.

So this is some cases one can so the physics are near all mess of these cases such that y^+ is likely very significantly over this.

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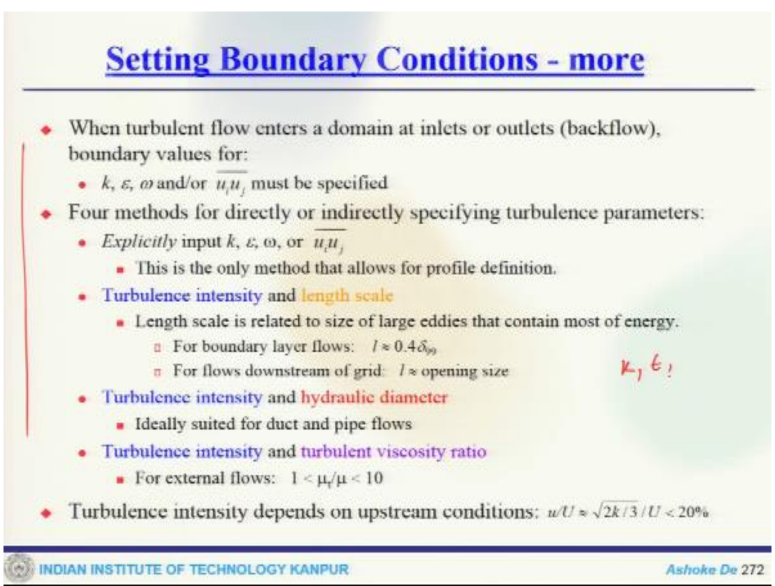

Now then at the same time you need to set the boundary conditions. So, obviously one has to provide boundary condition for k Epsilon and other stuff.

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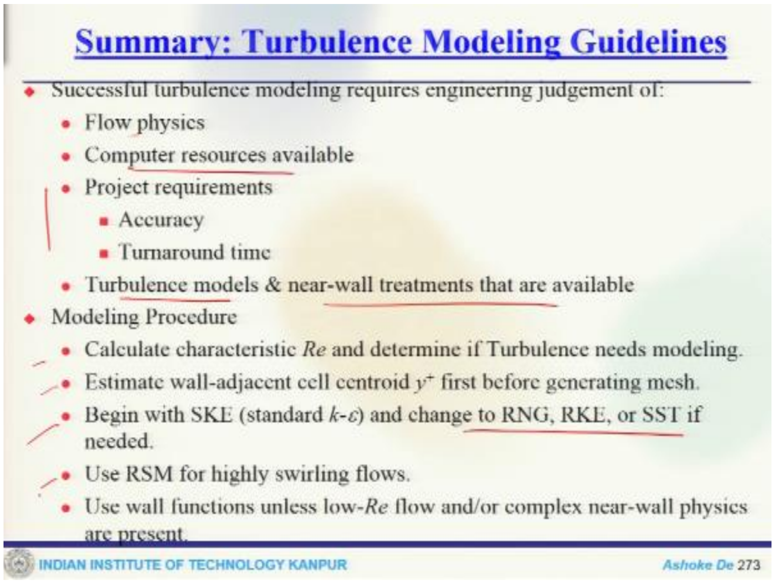

So if you put the summary of these guidelines so successful modeling requires flow physics computer resources available and the requirement like accuracy and turnaround time on so then you can depend on the model and are all treatments available. So, you calculate the characteristics Re estimate wall adjacent determine on the y^+, you can use some standard k-Epsilon then move to other advanced model or use some RSM and use wall function.

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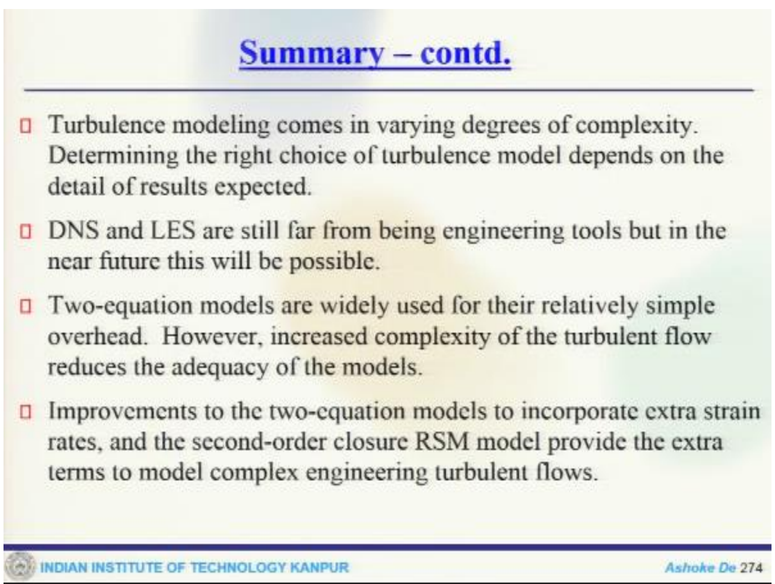

So these are some of the summary which is modeling various degrees of complexity where these are RANS based models are used often for the engineering problems. And if you want to be captured everything then go for DNS or RSM kind of so that is talks about pretty much what kind of modeling which is required for engineering problem or the small scale problem and the issues so then we stop here and start our discussion on the turbulent reacting system in next lecture.