Turbulent Combustion: Theory and Modeling
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Lecture-39
Turbulence (contd...)

Welcome back, let us continue the discussion on the turbulence and so, we are pretty much looked at all the characteristics scaling and everything. Now the last part, we will look at the frozen turbulence and then we go move to the modeling aspect of it.

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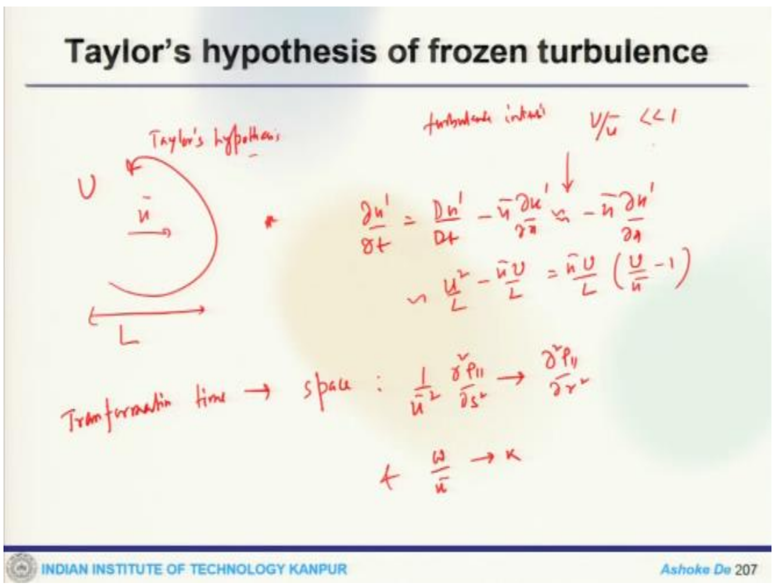

So, this is what we looked at in the frozen turbulence there is a eddy of size this where this is our measurement locations and so, this is what happens which is scales like

$$\frac{u^2}{L} - \frac{\bar{u}U}{L} = \frac{\bar{u}U}{L}\left(\frac{U}{\bar{u}} - 1\right)$$

So, that means eddy quickly addicted by the mean flow says that it does not have time to change. So, the implication is that measure time series is it is in fact a space series. So, the transformation time to space where

$$\frac{1}{\bar{u}^2}\frac{\partial^2 \rho_{11}}{\partial s^2} \to \frac{\partial^2 \rho_{11}}{\partial r^2}$$

and

$$\frac{\omega}{\bar{u}} \rightarrow \propto$$

so, essentially the time transformation into space.

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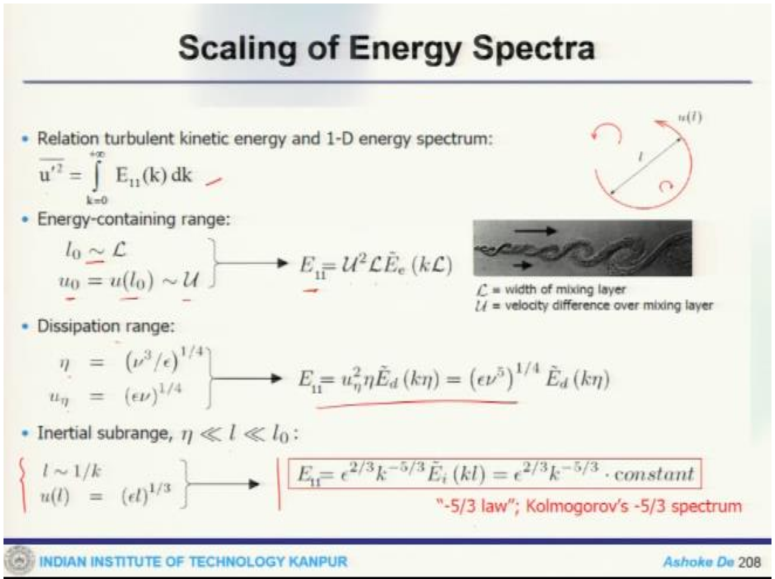

Now, if you look at that energy spectra of that thing. So, this is a relation turbulent kinetic energy and 1-D energy spectra. So, this is our u'^2, then if you look at the energy containing range so, the length scale would be order of L velocity scale is of that so, the energy containing range it will be u'^2 length this. In the dissipation range, length scale and the velocity scale. So, the energy would be like this an inertial range which is in between l and l 0 they will length scale under so, this is what you get in the inertial subject, this is a Kolmogorov's – 5/3 spectrum.

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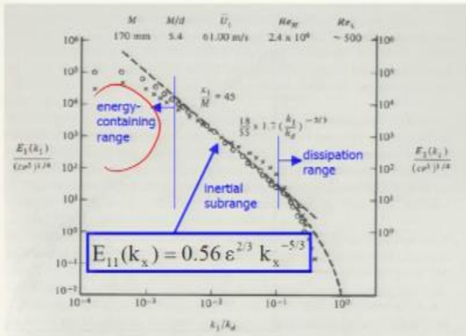
1-D energy spectra with -5/3 behaviour (measured)

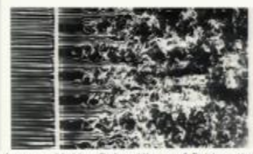


1-dim energy spectra from grid-generated turbulence in a wind-tunnel. Taken from Hinze, Turbulence (p. 255), 1975.

Requirement for -5/3 behavior:

Reynolds number sufficiently large → separation between micro- and macro-structure

$$\frac{\eta}{\mathcal{L}} \sim \left(\frac{\mathcal{U}\mathcal{L}}{\nu}\right)^{-3/4} \ll 1$$



(source: M. Van Dyke, Album of fluid motion)

So, this is very well known that in different and if you put that thing in the spectra, this is your normalized spectra with the wave number space. And that is a different this is our energy containing range, this is the inertial range which follows this and this is dissipation range. So, each range the energy spectra is different. So, the requirement is that Reynolds number is sufficiently large and spectra separation between micro and macro structure.

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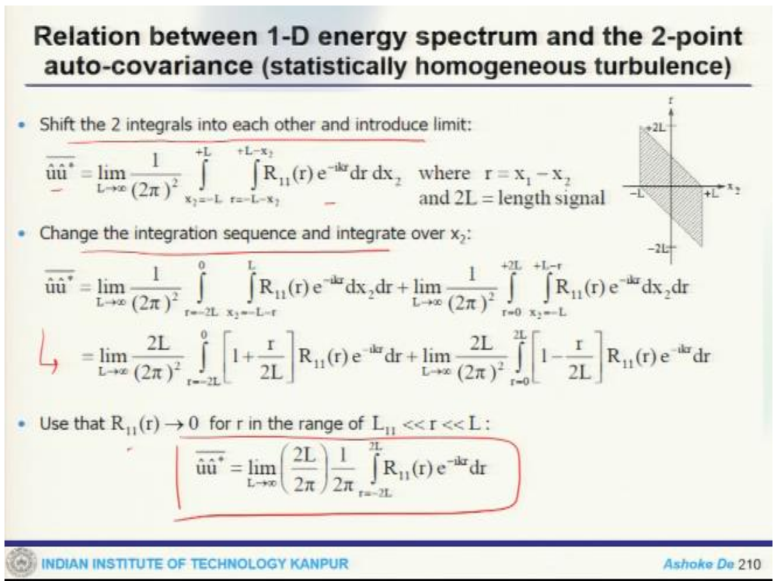

So, now, we can pretty much look at this relation between 1-D energy spectrum and 2 point auto-covariance for a statistically homogeneous turbulence. So, this is shift the 2 integrals into 1 and the limit. So, that $\bar{u}\,\bar{u}'$ will look like this. Now, if you change the integration sequence over this, you do this algebra. Finally, you get back this and if we R_{11} tends to 0 in for r range between these so, this will become a scale like that, okay.

So, that is what you get. So, that pretty much gives you an idea about scales, spectra how to calculate that and some of the basic example of homogeneous turbulence statistically homogeneous turbulence we have looked at it.

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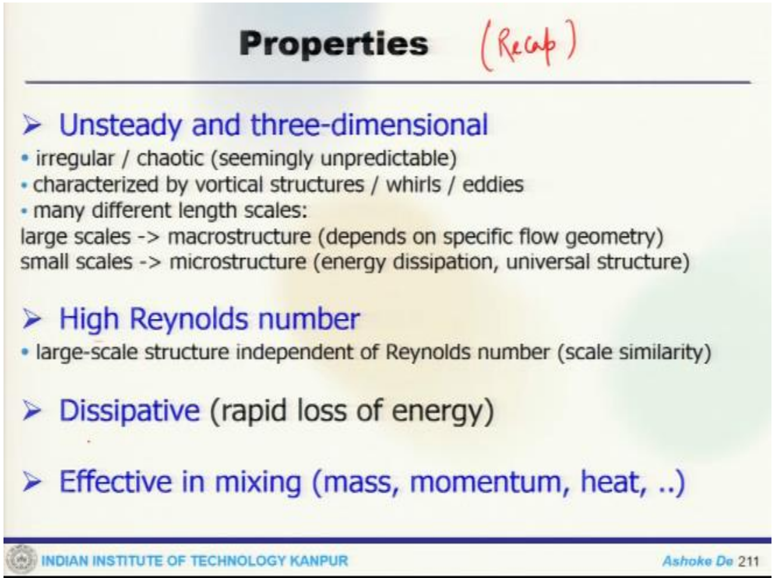

Now, with that information, we can move to the modeling aspect and look at what are the critical features that one has to take care while talking about the modeling part, just to recap about the properties. So, this is again, which we started off, so, the flow feature is truly unsteady and 3 dimensional. So, that means is essentially chaotic. So, there would be different kind of vertical structure, there could be large scale structure, small scale structure, then it is high enough Reynolds number flow, this is dissipated in nature and it allows effective in mixing.

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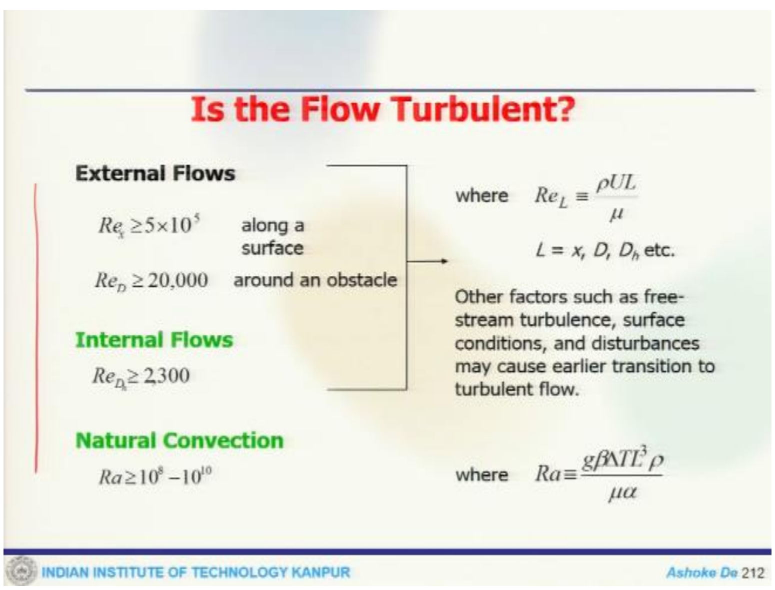

Now, that is how we look at the Reynolds number, which is a non-dimensional number to look at it, whether the flow is really turbulent or not.

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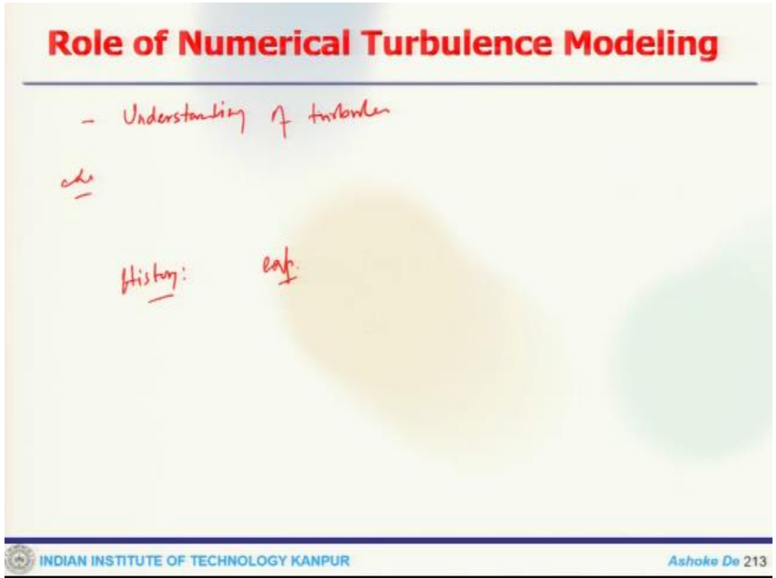

Now, what is the role of this numerical modeling? So, it is essentially require an understanding, understanding of turbulence and the ability to predict the turbulence for any given application. So, for example, one can think about that we can increase the turbulence in chemical mixing or heat transfer, when fluids are dissimilar properties. Also, turbulence increases the drag due to increased external forces.

So, I mean historically only measurements was possible for limited configuration, but now, with the amendment or advancement of the computational fluid dynamics tool or the numerical tool now, the real life complicated problem can be I mean looked at in details.

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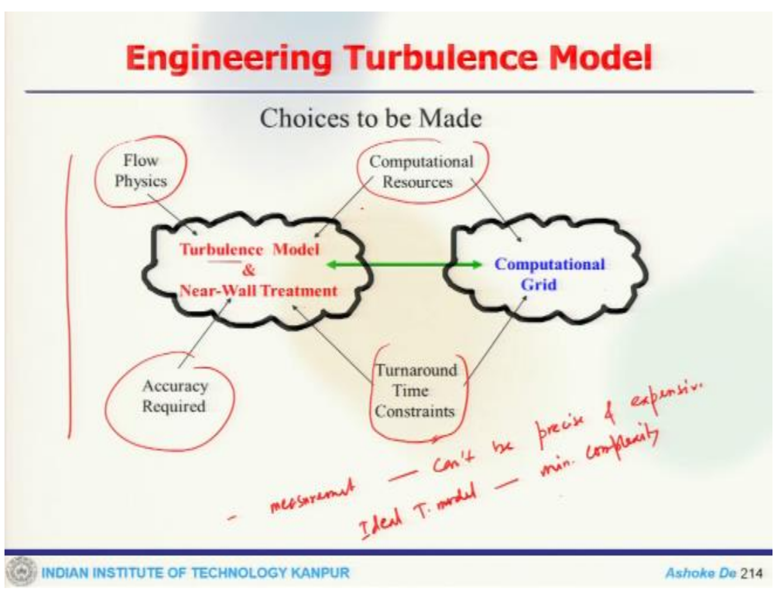

So, what so, one can think about the numerical modeling of turbulence can sort to improve the engineers ability to analyze turbulent flow in design particularly, when the precise measurement cannot be done or rather the measurement cannot be can be precise and also expensive. So, one has to make in choice. So, the ideal turbulence model should introduce minimum complexity while capturing the essential flow physics. So, the ideal model ideal turbulence model that should be with minimum complexity while capturing the relevant physics.

So, you have a turbulence model and near wall treatment if it is a wall bounded flow. So, the and then you have a computational grid. So, these are the turnaround time is in constant computational resources are also an issue for turbulence model and computational grid because, what kind of turbulence model one has one will use. So, what are the resources available then what is the expected time for calculations and all these, but turbulence model or near wall treatment are actually dictated by the flow physics or the accuracy required.

So, this side will actually push you that what model one should use, but then at the same time you have a problem dimension and also the required time estimate and the resources available, which has to be taken into consideration and that is why one has to make a choice.

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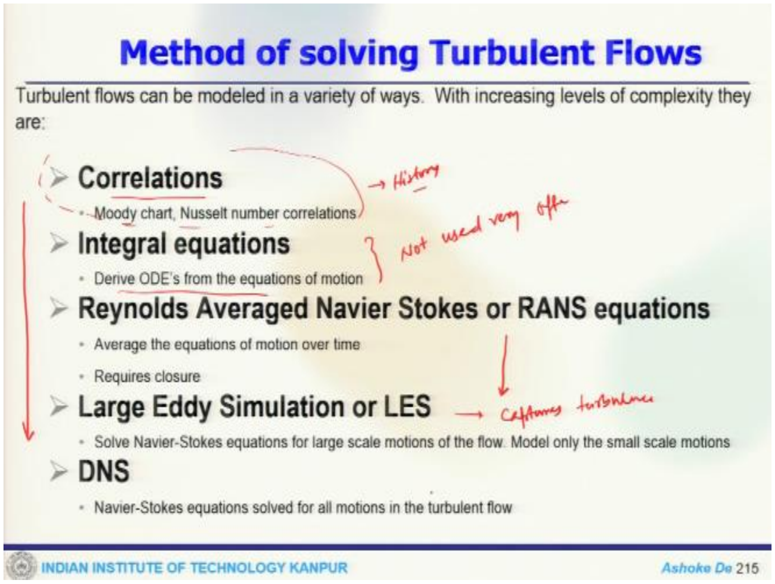

Now, this can be modeled in multiple ways with different degrees of complexity. So, the simple one can use the correlation based calculations which are often I mean used to be used earlier days like Moody's chart, Nusselt number correlations and once you use those correlations and you get the design for data to use that so, that I mean right now, I would say this is a sort of an history, because nobody do that, because the kind of computer or computational facility available these days this one can easily do some simulations.

Now, then next is that, so, this is the increasing order of complexity either integral equations like derive some ordinary differential equations and this is also not used very often. Now, the third option which is RANS equations and this is quite widely used not only in the academic perspective also in the industrial perspective, because the calculation time is quite small and you can get reasonable accuracy accurate results for a large scale applications and that is where people can consider this one or use it for the design calculations.

And then LES is another choice from RANS to LES, which will increase the complexity. But also it captures the turbulence in a better way compared to RANS and finally, if possible like if you have enough resources, I mean in terms of computational power, then you can go for DNS for real life problem, and that is how this is the increasing level of complexity.

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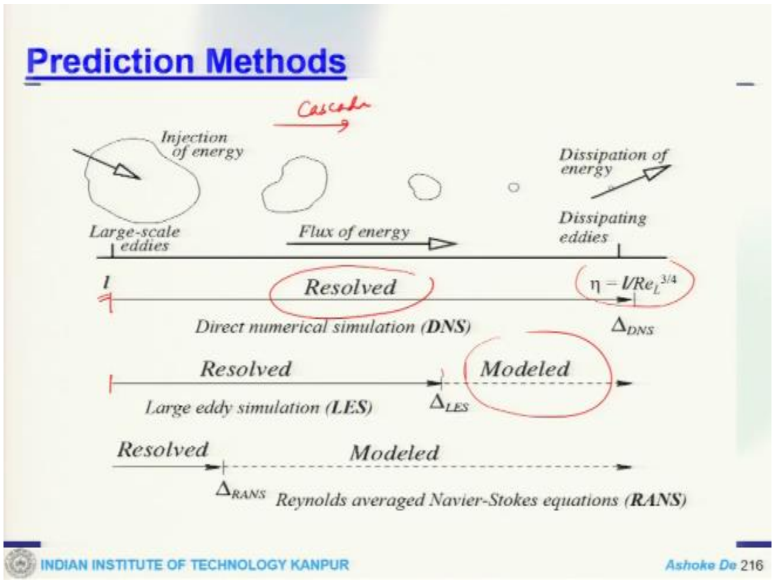

Now, what model does what so, this is mine, if you look at the energy spectra quickly, this is my large scale energy containing structure then energy cascade takes place, this is the energy cascade, and then finally, it comes to small scale where it dissipates. So, this is my length scale at large scale, then this is my smallest scale or the dissipation range and now, when somebody use DNS, it resolves all the scale starting from the large scale to small scale.

So, that is why it is so, expensive computational because one and you can see these scales and already we have seen that they are somehow involved the Reynolds number. And then when you go for the large scale real life problems that you know somebody is quite large. Now, in between LES it resolved most of the energy that in large scale energy and that of intermediary energy only model is small portion of the small scale with the assumption that these scales are universal in nature and RANS pretty much the complete spectrum is modeled.

So, you do not assume or consider the energy cascading effect rather you assume everything is sort of an isotopic equation and mean.

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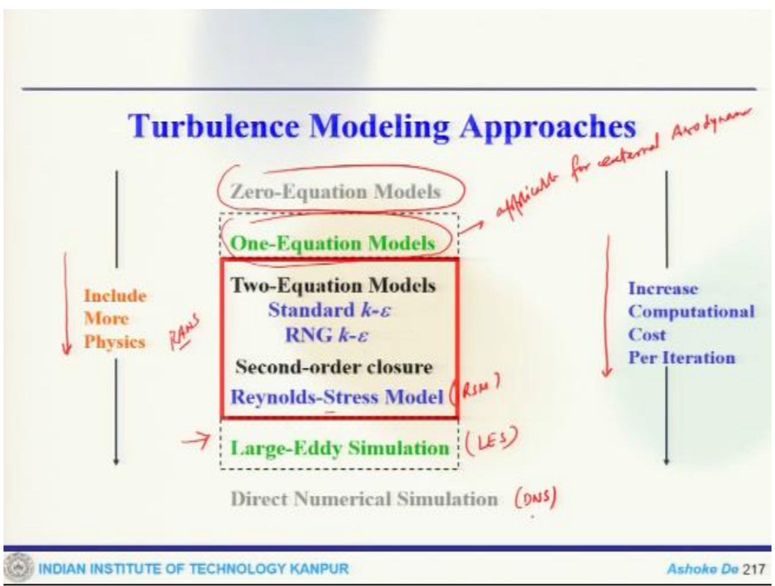

Now, approach one can think about so, this side you think about the increasing the cost in the sense computational cost this at the same times it increases the physics. So, that means, more and more physics you want to resolve or you want to capture, your computational costs will go up. And that is where you one can start of this zero-equation model that is mixing length kind of model, where you have an algebraic equation, but that has a lot of these advantages for a wide range of flows.

Then one can do one equation model for this solve for the eddy viscosity using a transport equation and this is quite applicable or handy I would say applicable for external aerodynamic problem. Now, then the second layer of complications which one can take here is the series of two equations model like standard k-epsilon model, RNG k-epsilon, SSD k-omega there are a series of two equations model which could be used and then we can look at the Reynolds stress model which is in second order closure.

And where the stress individual transport equation for the Reynolds stress in this RSM model, then after RANS one can include more physics resolve more scale so, one can go for LES and then if you want to increase more complexity and more physics into it then one has to go for DNS.

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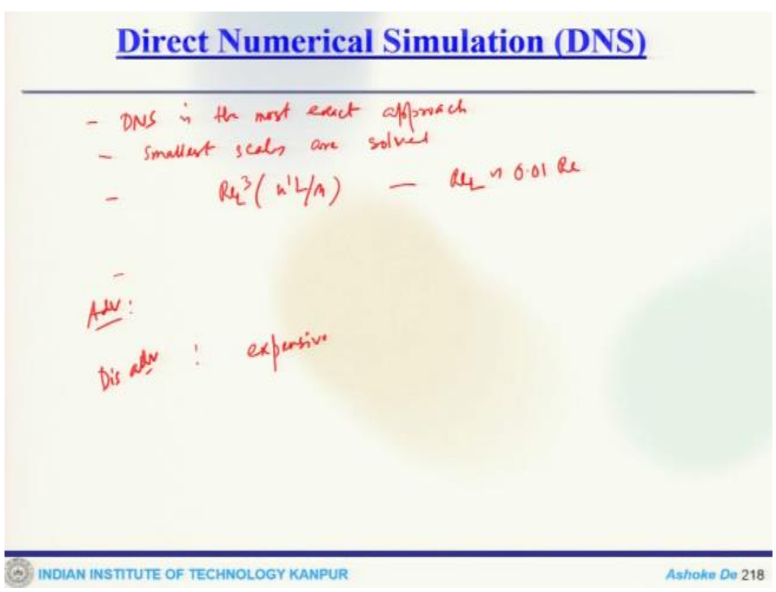

Now, what are the issues with DNS? So, currently DNS is the most exact approach for modeling turbulence because no averaging or no approximation has met. So, the smallest scales are smallest scales are solved due to resolution strategy up to the Kolmogorov's scale it is resolved. So, the DNS simulations scale with $Re_L^{3}\left(\frac{u'L}{\eta}\right)$ where Re_L is order of 1% of Re. So, if you take an example of a simple flow fasten cylinder, it will take some 8 million cells like that, but, as I said given a current processing time and memory one it is still possible.

So, there are some advantages, there are some disadvantages. Advantages it can be used as numerical flow visualization can provide more information compared to experimentation and it can be used to understand that turbulence or mechanism or turbulence production and dissipation the disadvantage primarily computationally expensive and it is still limited to small geometries.

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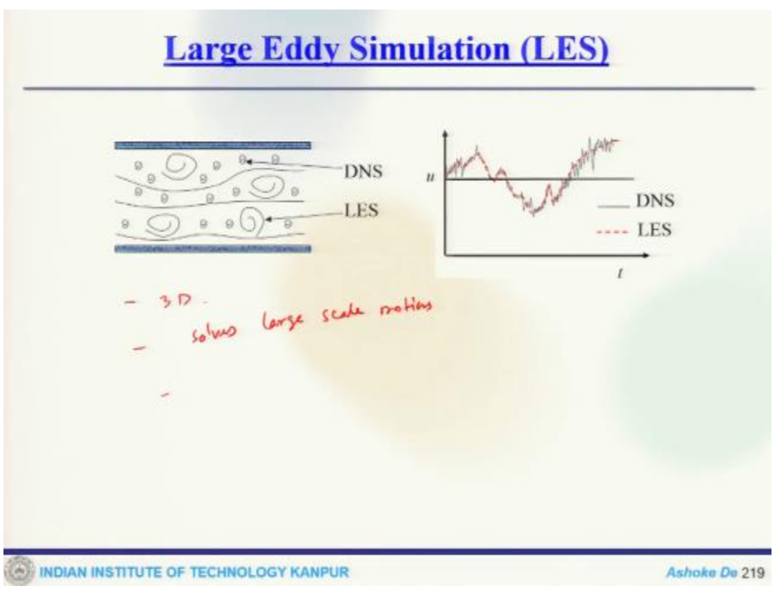

Now, second approach is LES which is in between DNS. So, this is also 3 dimensional equally, I mean it is not like DNS but it is also computational very expensive especially wall bonded close. So, in LES it solves for large scale motions and models the small scale motions and the premise of LES is that the large scale motions content the most of the energy so, and the small scales are more universal in nature. So, that is why?

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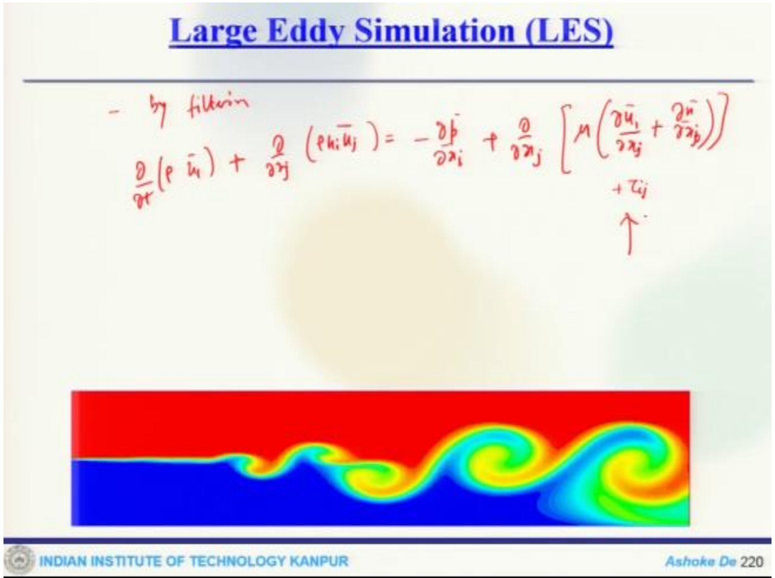

Now, the LES equations are obtained by filtering operation and the filter produces the equations where you get like if you say this is my filtered equation

$$\frac{\partial}{\partial t}(\rho\bar{u}_i)+\frac{\partial}{\partial x_j}(\rho u_i\bar{u}_j)=-\frac{\partial \bar{p}}{\partial x_j}+\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}}{\partial x_j}\right)\right]+\tau_{ij}$$

so the τ_{ij} is your sub grid scale term which is model. So, different sub grids model sub grid scale models are available.

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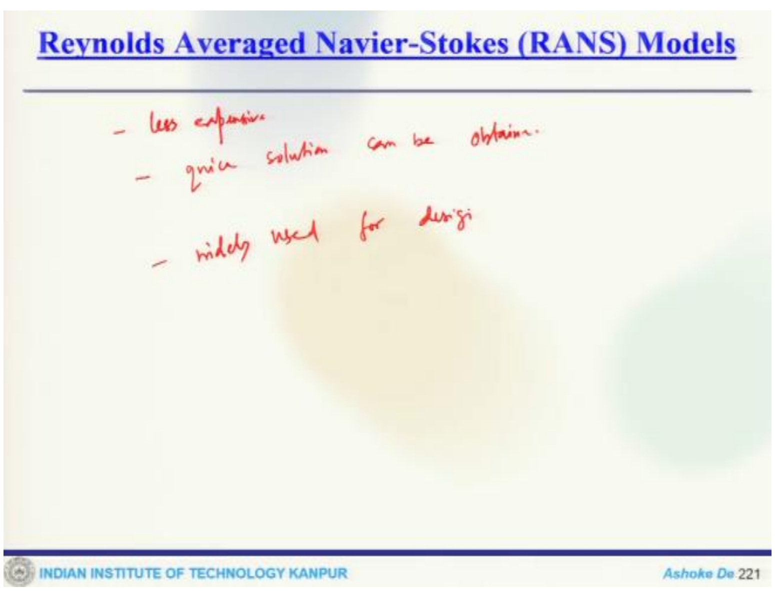

Now, if you go to RANS, RANS has quite a bit of advantage over DNS and LES. So, this is less expensive typical Indian strokes can be quickly solved quick solution can be obtained but this approach would be more turbulence by averaging the understanding of turbulence. So, averaging process has some problem because it creates terms that cannot be solved analytically, but this is quite widely used for design purposes okay.

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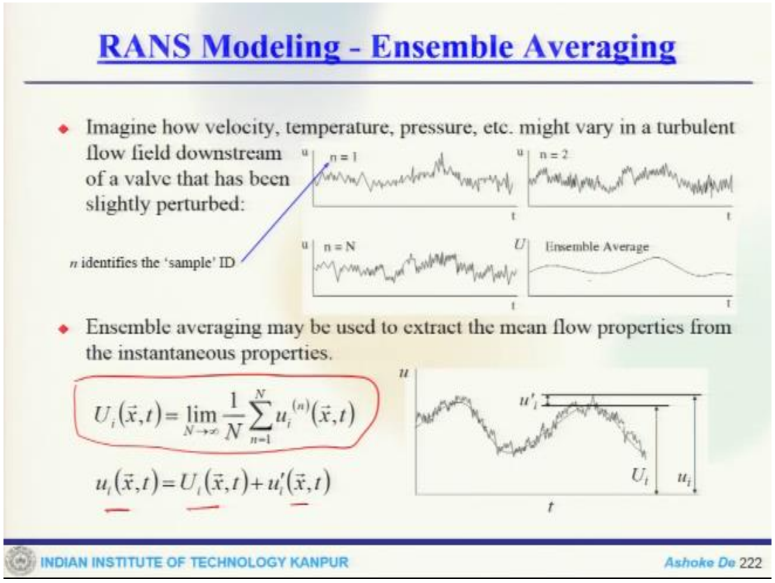

Now, one can think about how we do the and this is already we have discussed in details in RANS you take the ensemble average. This is your velocity signal and then this is an ensemble averaging signal. This is how the ensemble average is so, you take number of samples and ensemble average becomes and the decomposition is the instantaneous flow field is the mean plus the fluctuating component and if any small there could be error which is the statistical error. If N is large, then this can be minimized.

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RANS Equations

- Velocity or a scalar quantity can be represented as the sum of the mean value and the fluctuation about the mean value as:

$$\phi = \bar{\phi} + \phi'(t)$$

- Using the above relationship for velocity(let $\phi = u$) in the Navier-Stokes equations gives (as momentum equation for incompressible flows with body forces).

$$\frac{\partial(\rho \bar{u}_i)}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho \bar{u}_i \bar{u}_j + \rho \overline{u'_i u'_j}\right) = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\right]$$

- The ***Reynolds Stresses*** cannot be represented uniquely in terms of mean quantities and the above equation is not closed. Closure involves modeling the Reynolds Stresses.

So, if you put these things together so, any variable in RANS, there is a 2 component means plus fluctuating component and these phis could be anything and then when you look at the Navier Stoke momentum component. This is what you get back this is your Reynolds term and there are closure which involved in modeling the Reynolds stress term and that gives rise to the different kind of model that we have already discussed like RANS space model.

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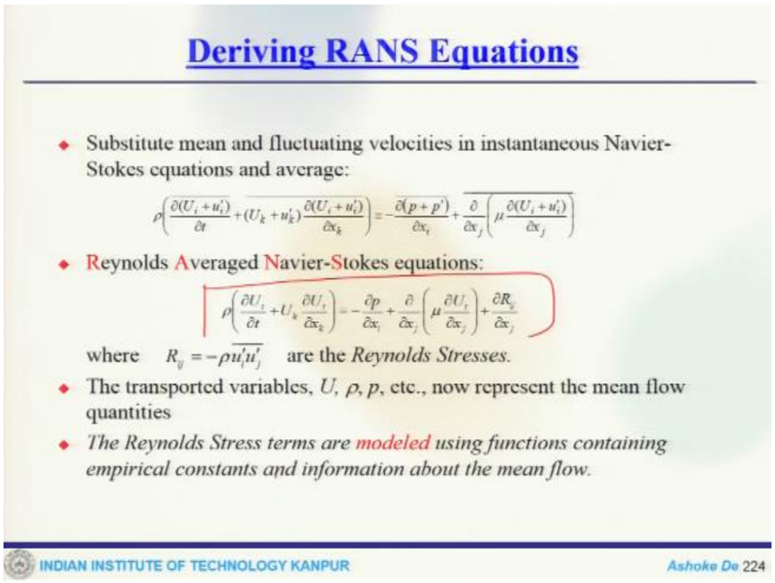

And once you substitute this mean and fluctuating velocity in the instantaneous Navier Stokes equation. So, this is what you get the RANS equation and these Reynolds stress terms are modeled and that will give you different equations.

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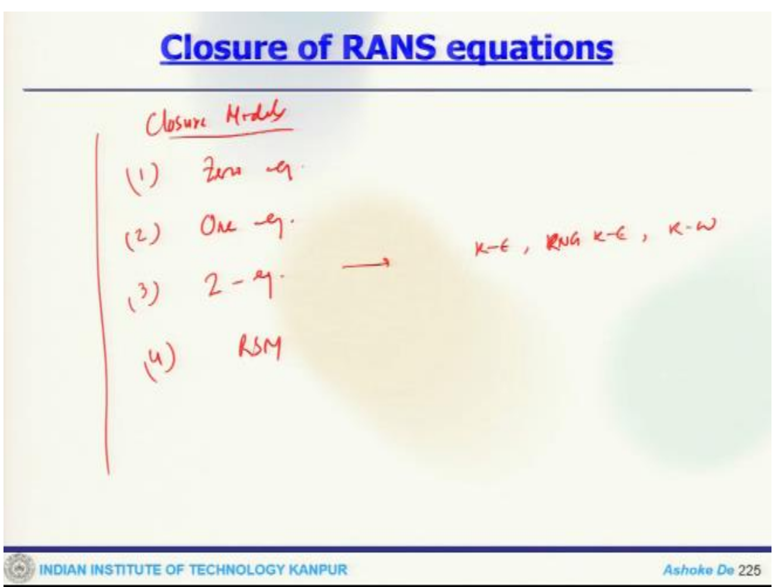

So, the closure models are essentially that is one can have zero equation which is basically mixing length model no transport equation then one equation, which is transport equation for the either kinetic energy or the Eddie viscosity, two equation models which is have all sort of k-epsilon model, RNG k-epsilon, k-omega model so, all two equations based model and the second order closure like RSM based model.

So, this is how you go the increasing complexity where you solve the individual stress component and it does not use any Boussinesq hypothesis. So, many turbulence models are based on this Boussinesq hypothesis and it has observed that if the case analysis there is a shear in isothermal incompressible flow.

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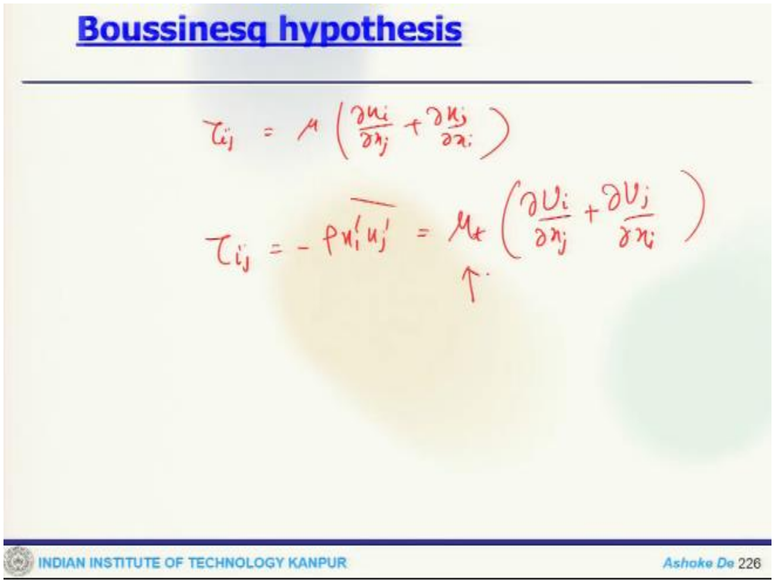

So, it has found that turbulence actually increased the mean date of deformation and the discussed this is were given by

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\tau_{ij} = -\rho\overline{u_i' u_j'} = \mu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

So, this is the mean component here mean standard plan where μ_t is the eddy viscosity.

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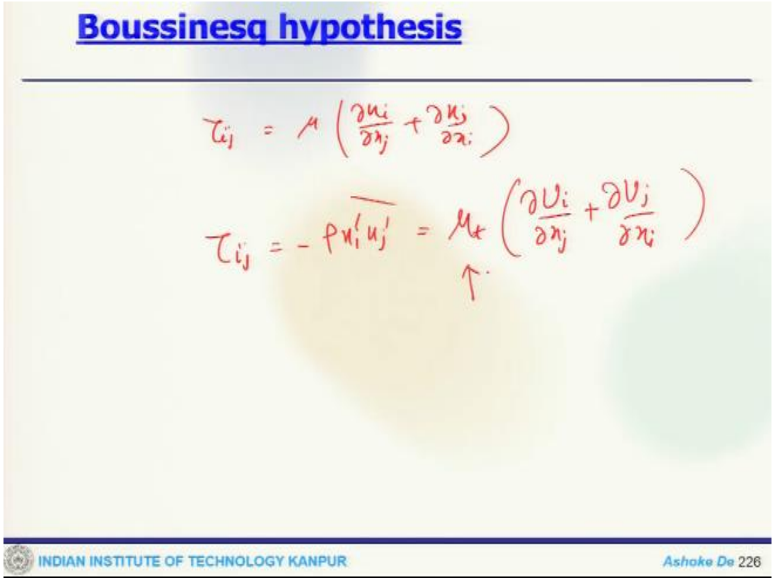

So, this μ_t is the turbulence viscosity or eddy viscosity term okay. So, this can be correlated with your density and the kinematic viscosity. This is essentially not homogeneous in nature it actually varies in space. However, if you assume to be isotropic it is the same in all the direction. So, that is quite I mean often used for some of the application.

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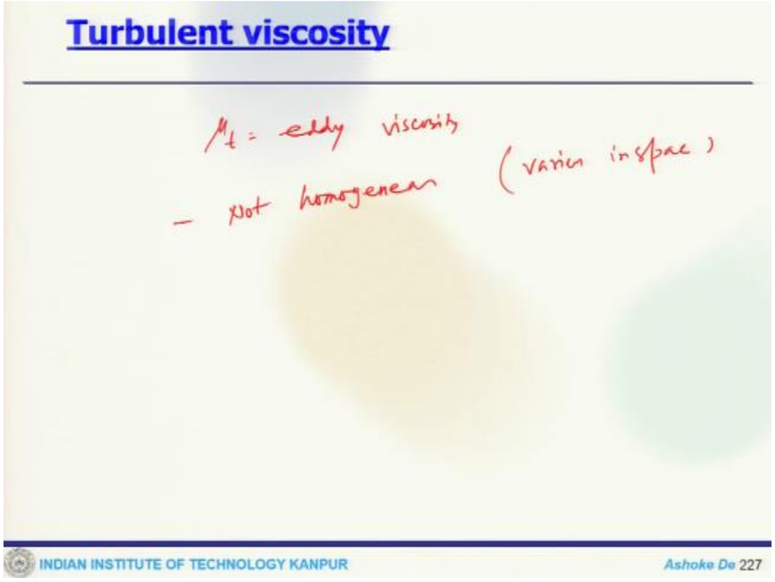
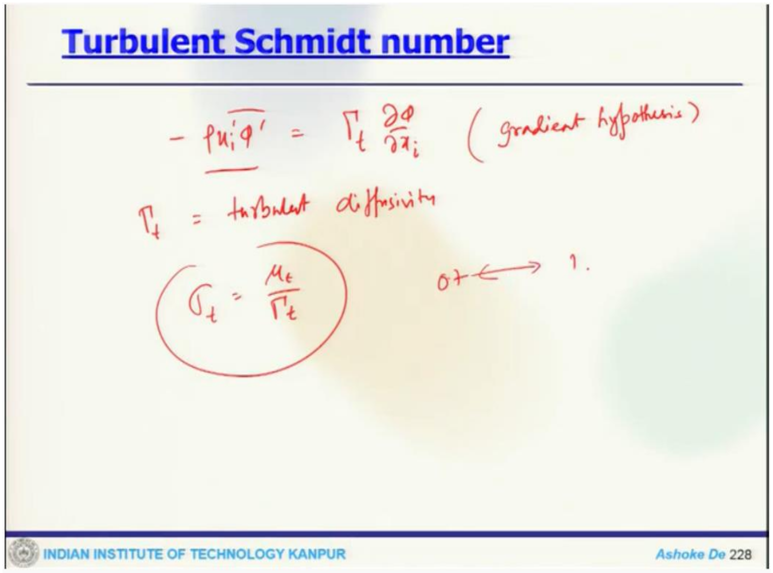


Then you have Turbulent Schmidt number.



For scalar property or any scalar field so, we have a unclosed term for the scalar flux. So, this is what we close the scalar term stress term through the gradient hypothesis .This is called gradient hypothesis and this Γ_t is your turbulent diffusivity. Okay. Now, the turbulence diffusivity this is calculated from the turbulence viscosity using a model constant which is known as turbulent Schmidt number like

$$\sigma_t = \frac{\mu_t}{\Gamma_t}$$

So, this Schmidt number there is for 0.721 so, that remains pretty constant for large scale problem.

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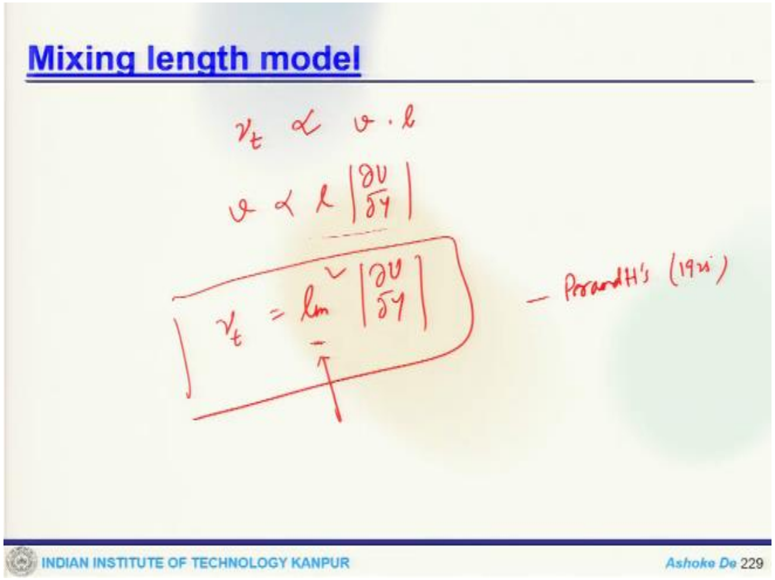

Then once you go to mixing length model where mixing length model this already we have seen ν_t is essentially velocity scale into length scale ,where velocity scale varies that $\frac{\partial U}{\partial y}$. So, this is proportional to the gradient of the velocity. So, finally, the

$$\nu_t = l_m^2 \left|\frac{\partial U}{\partial y}\right|$$

this is what Prandtl proposed in 1925. Prandtl's mixing length model and this mixing lengths constant which one has to provide in his calculation and that is why the biggest disadvantage of this particular model though it is simple that this mixing length definition is not known a priory.

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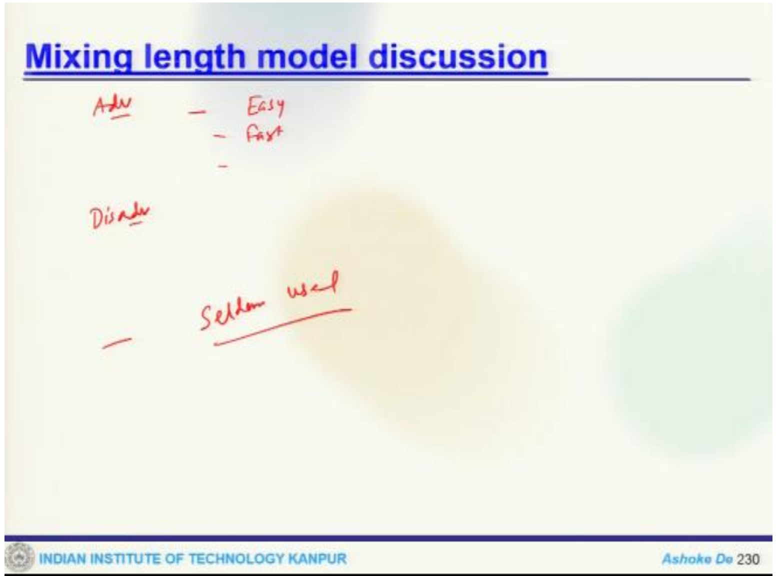

So, the advantage there are certain advantage like easy to implement fast calculation time. So, it can provide you reasonably good prediction for simple flow disadvantage as I said that it is not capable of solving the largest problem for a wide range of RANS number. And that mixing length has to be provided a priory which is a problem. But it is used sometimes for simple external flows also. But, these days, it is seldom used, because there are advanced models which are available. So that is why?

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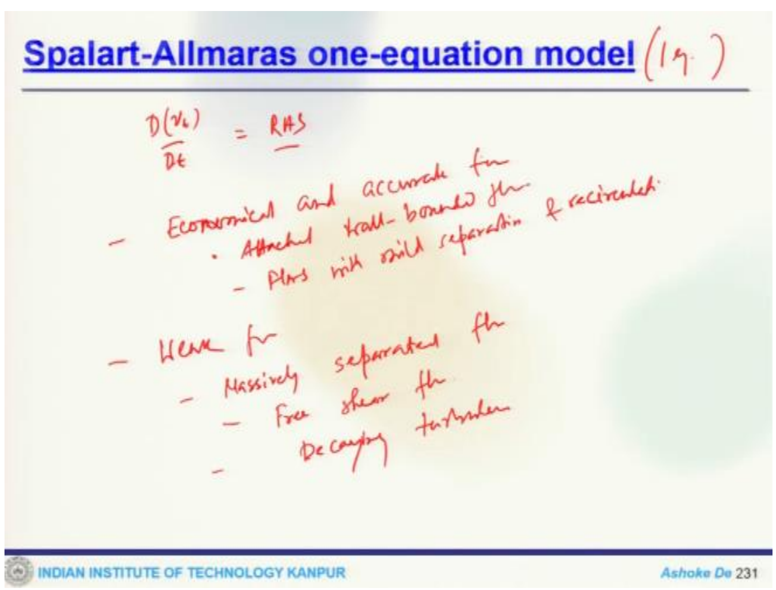

Now, there is this is one equation model. One equation model, where it is a single conservation equation for turbulent viscosity. So, that contents there would be an equation which have a right hand side term. So this is quite often used in external almaras calculation .This is economical and quite accurate for attached wall bounded flows, flows with mild separation and recirculation but this is quite weak for massively separated flow free shear flows and decaying turbulence. So, this has quite a bit of limitations and that is why it is not it is not often used extensively.

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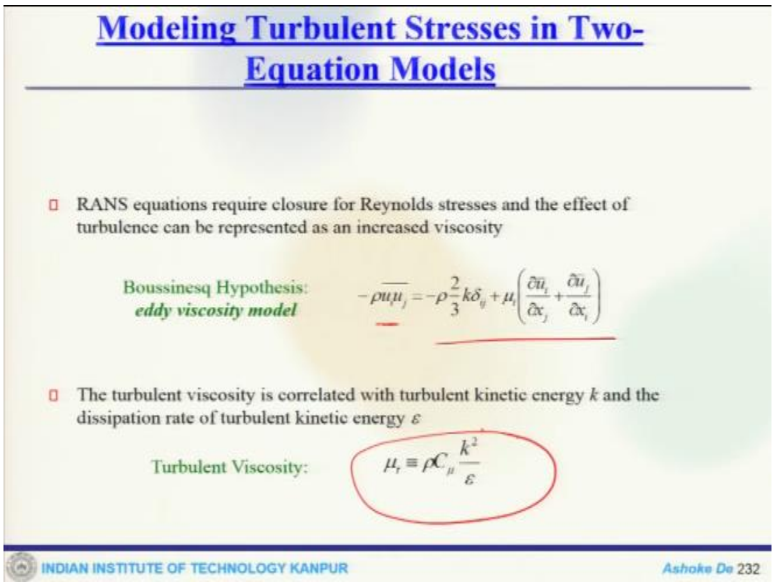

Now, then one can move to the two equation model where again using the Boussinesq hypothesis. The Reynolds stress term can be symmetric like these are the turbulence viscosity is approximated using some model constant and in the presence of kinetic energy and the dissipation.

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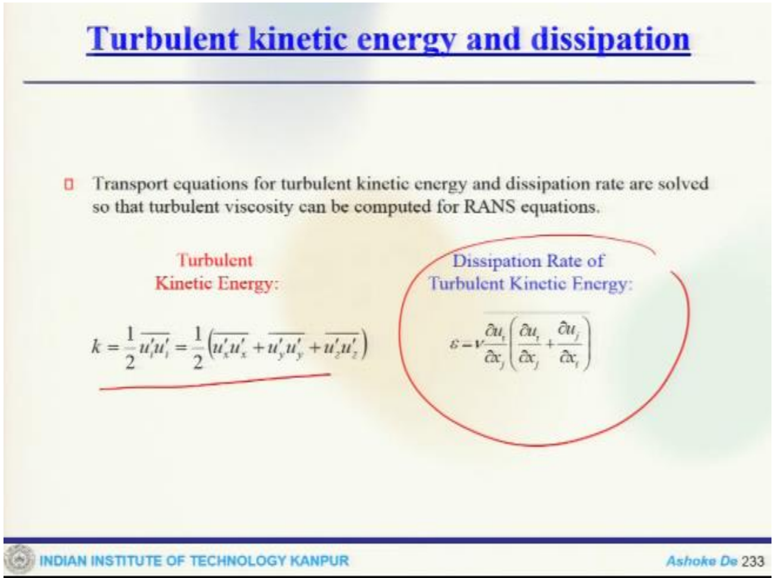

So, this is what we have already looked at in and the turbulent kinetic energy is estimated like that and that dissipation rate of turbulent. So, this is used in two equation turbulence model.

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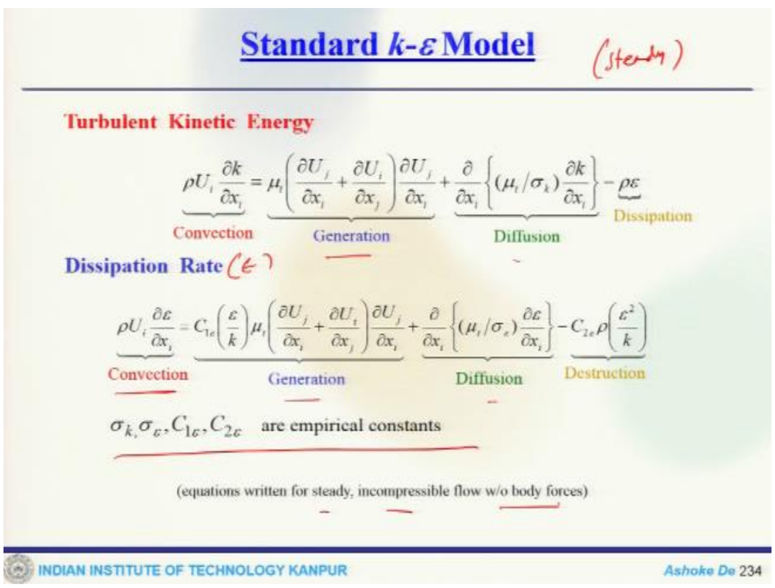

Now, if you look at the standard k-epsilon model, which we have already look, this is the convection term, this is production term, and diffusion term and the dissipation term this is in a steady situation and the dissipation rate which is the solution for epsilon. This is the convection, generation, diffusion and these are the modern constant. So, as I said this is written for steady incompressible with nobody forces. So, that is your two equation model.

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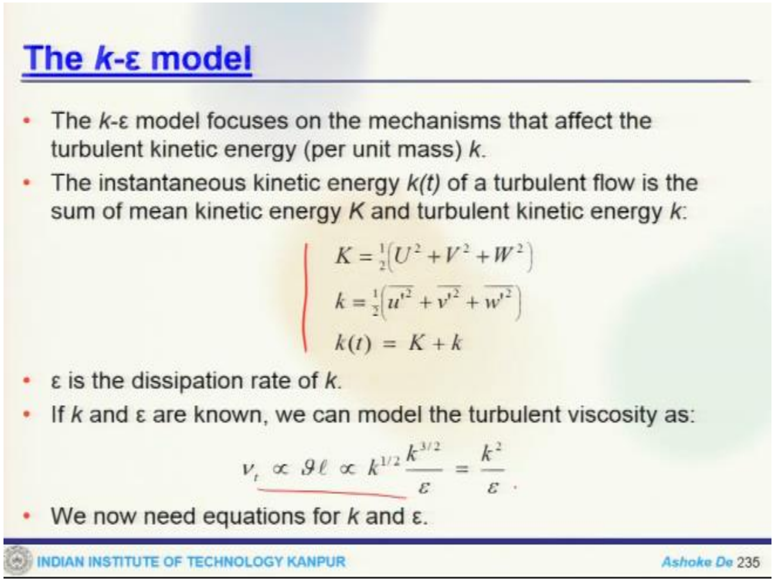

Now, the k-epsilon model it focuses on the mechanism that effect the per unit mass. So, this is how you get the kinetic energy and finally, ν_t is estimated like $\frac{k^2}{\varepsilon}$.

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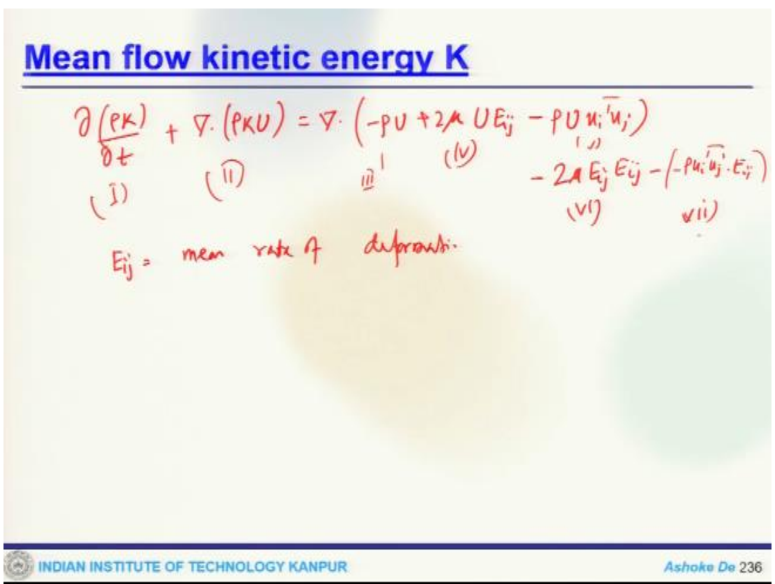

Now, the mean flow kinetic energy equation which is

$$\frac{\partial(\rho k)}{\partial t} + \nabla(\rho KU) = \nabla\left(-\rho U + 2\mu U E_{ij} - \rho U_i u_i' \overline{u}_j\right) - 2\mu E_{ij} E_{ij} - (-\rho U_i u_i' \overline{u}_j E_{ij})$$

So, here this is the first term, second term, this is third, fourth, fifth, sixth, seventh, here E_{ij} is mean rate of deformation. Now, the first term is the rate of change of k, second term is the transport of k by convection term and third term is the transport of k by pressure.

Fourth term is the transport of k by viscous stresses, fifth term is term is the transport of k by Reynolds stresses, sixth term is the is the rate of dissipation of k and seven term is the turbulence production term. So, that is what you look at in the mean flow kinetic energy equation. We will stop here and continue the discussion in the next lecture.