Turbulent Combustion: Theory and Modelling
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Lecture-32
Turbulence-Chemistry Interaction (Contd...)

Okay, welcome back. And let us continue the discussion on the scaling of the burger equations. So, we were looking at the kinetic energy budget.

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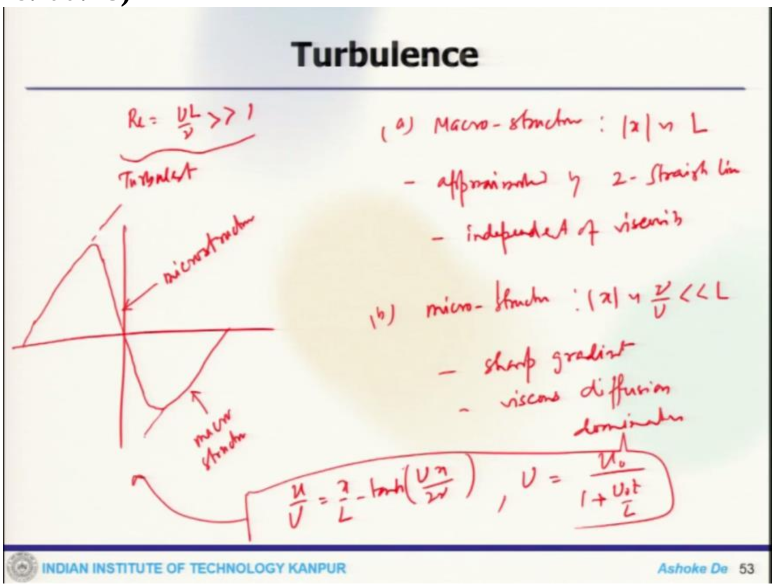

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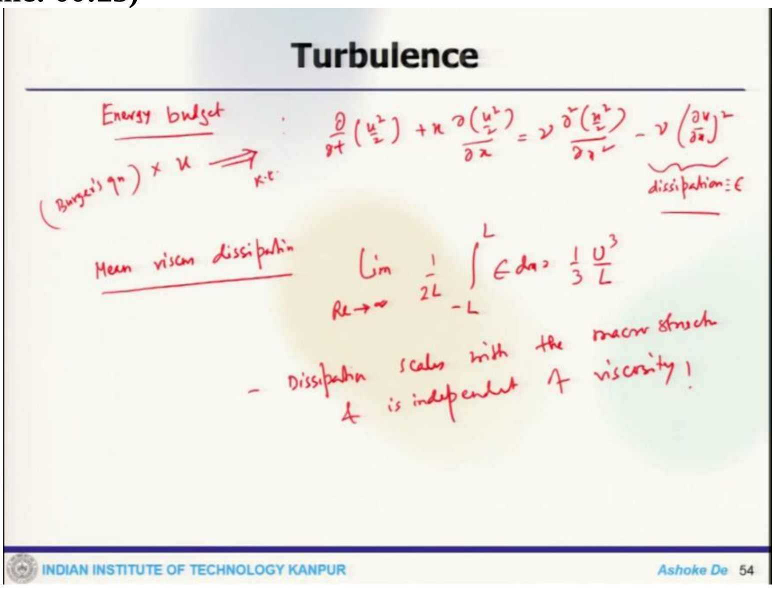

Kinetic Energy budget

So, how we can get the kinetic energy equation. We have the burger equation and we can multiply that with u and we get an equation in this form.

$$\frac{\partial}{\partial t}\left(\frac{u^2}{z}\right)+x\frac{\partial\left(\frac{u^2}{2}\right)}{\partial x}=\nu\frac{\partial^2\left(\frac{x^2}{2}\right)}{\partial x^2}-\nu\left(\frac{\partial u}{\partial x}\right)^2$$

The last term which is an extra term which comes because of the multiplication of that velocity factor U, and this is known as dissipation, defined by ε. When burger's equation is multiplied with u, we get this energy budget equation or rather kinetic energy equation.

Now, mean viscous dissipation, we can also compute. We can compute from the exact solution by

$$\lim_{\mathrm{Re}\to\infty}\frac{1}{2L}\int_{-L}^{L}\varepsilon \mathrm{dx}=\frac{1}{3}\frac{u^3}{L}$$

if you look at it, the dissipation scales with the macro structure and is independent of viscosity. So, this is another term we now come across, but so far, we are looking at the kinetic energy.

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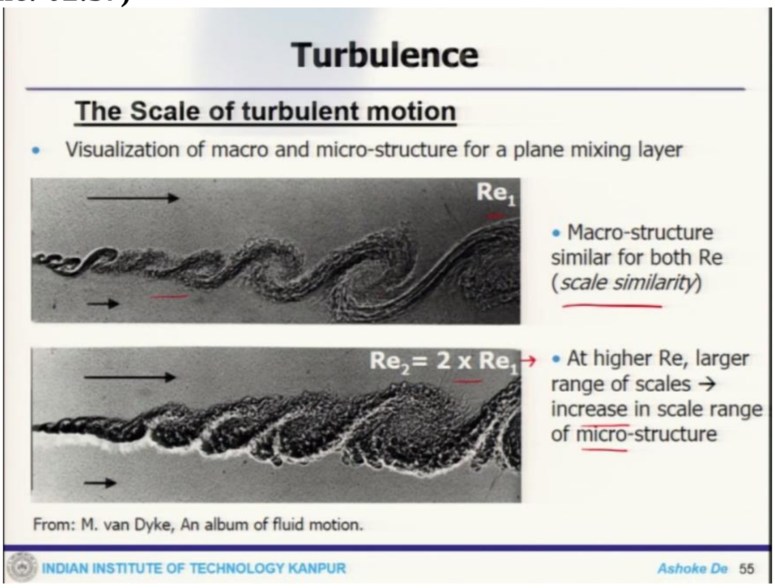

The scale of turbulent motion

So, we can visualize this macro and micro structure and that will give you the different scales. Turbulent motion is shown above for one with Re_1 and the second one is $2\times Re_1$. Scale similarity says that, for both the Reynolds number, the large scale structure of the macro structure are of similar nature, but at high Reynolds number, you can see the small structure that are having a larger range so, an increase in the scale of microstructure.

On one hand you can think about the scale similarity where large scale structure of the similar order that is this what you can see the that is the scale similarity which talks the large scale sector, but when you increase the Reynolds number, then your range of microstructure increases.

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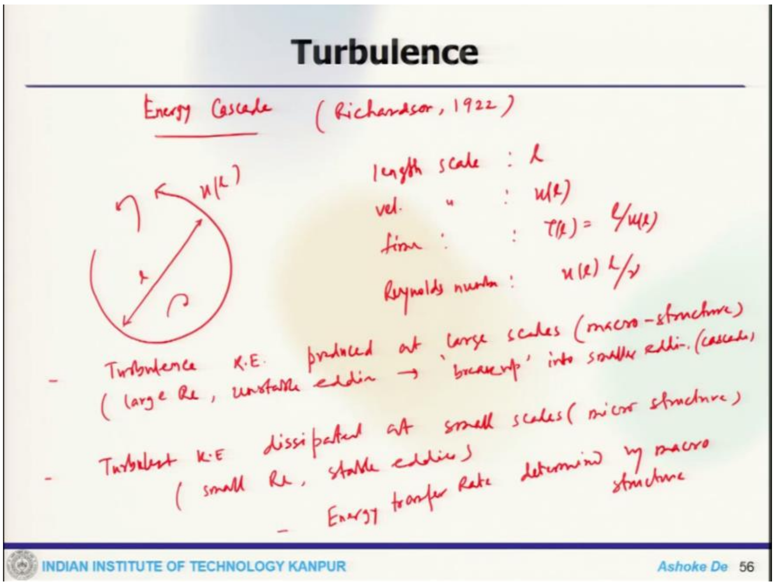

Now, that is where the concept of energy cascade comes into the picture and this is proposed by famous scientist Richardson in 1922. So, what it talks that the turbulent flow is composed of a lot of eddies of different size. So, we can represent an eddy like that shown which is order of l, We can have length scale that is l then velocity scale which is $u(l)$ we can have timescale

$$\tau(l) = \frac{l}{u(l)}$$

and Reynolds number which is

$$\text{Re} = \frac{u(l)\, l}{\nu}$$

It says that, turbulence kinetic energy produced at large scale or macro structural level. so, the energy production is essentially taking place. If large Re, that means unstable eddies which will break up in into smaller eddies that is what is called cascade. Now, turbulent kinetic energy dissipated at small scales at the scales of micro structure level. Macro structure contributes to the production of the kinetic energy and at microstructure level the dissipation takes place.

For small Re, stable eddies. So, essentially, if you look at the burger equation, the energy transfer rate is determined by macro structures. That means, there are 3 different things which happens here. One is the energy production at the large-scale structure then energy dissipation of the small-scale structure and then energy transfer rate by the macro structure.

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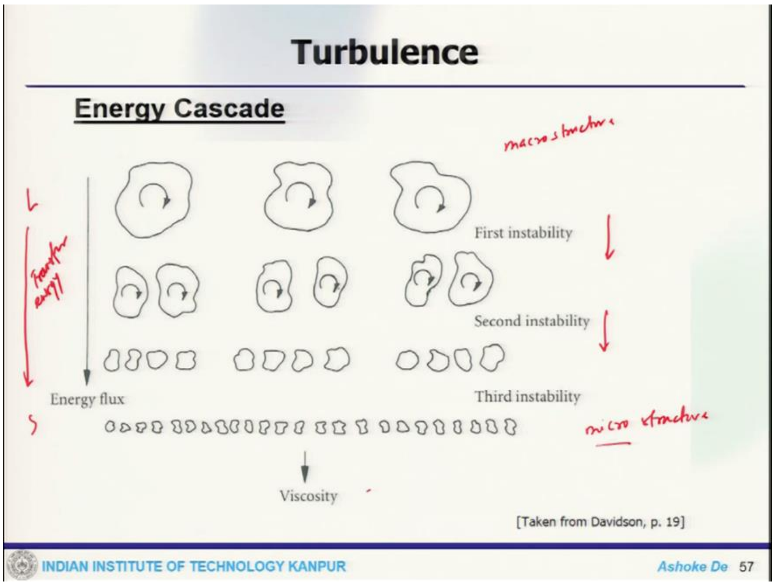

So, if you look at the energy cascade you can see in above slide., where first row is the large-scale structure, these are the macro structure and last row are the micro structure. So, when you increase the Reynolds number, that range of scales that the micro structure level that increases. So, this is how energy transfer actually takes place. You have large structure, which produce the energy and then from there it actually goes to the second level. It breaks down into the smaller eddies and finally through the dissipation, viscous dissipation the energy transfer takes place. So, this is what that very common and well know concept of energy cascade that means large scale structure to small scale structure it transfers the energy and at the smallest level through viscous dissipation or heat dissipation, energy dissipates.

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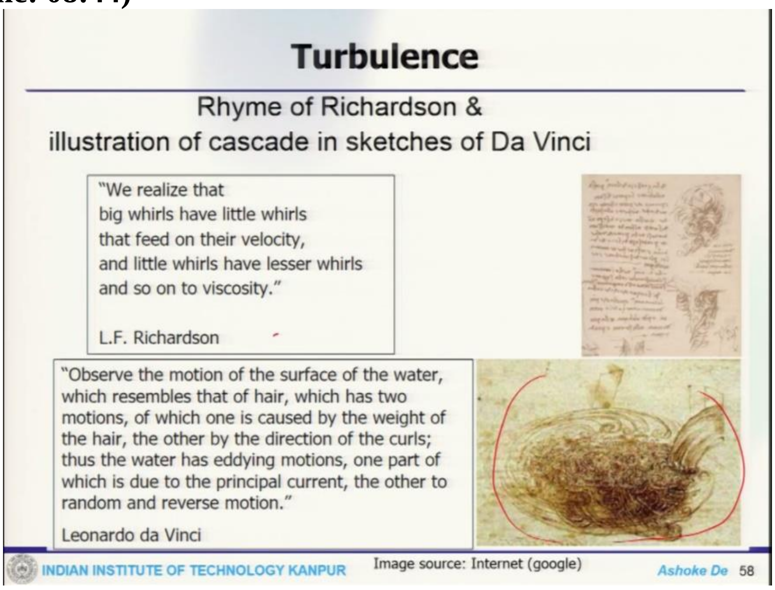

This is a very well-known quote by Richardson, it says that, we realized that big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity. And this is a famous picture by Leonardo da Vinci who says the “Observe the motion of the surface of the water, which resembles that of hair, which has two motions of which one is caused by the weight to the hair, the other by the direction of the curls, does the water has eddying motion, one part of which is due to principal current, the other to random and reverse motion.”

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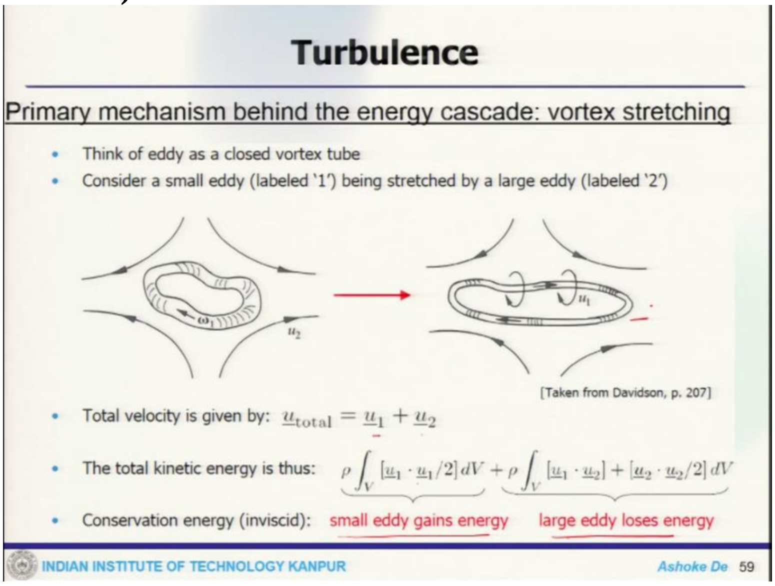

Energy cascade mechanism

Now, we can come to the primary mechanism of energy cascade, which is essentially due to vortex stretching. We can say that in the first picture, it is a closed vortex tube you think about this as an eddy and there is this small eddy levelled 1. So, it is stretched by the large eddy and then it becomes like shown in second picture. So, total velocity is given

$$u_{total} = u_1 + u_2$$

and the total kinetic energy is given by

$$\underbrace{\rho \int_V [u_1.u_1/2]dV}_{\text{small eddy gains the energy}} + \underbrace{\rho \int_V [u_1.u_2] + [u_2.u_2/2]dV}_{\text{large eddy loses the energy}}$$

So, that means, whatever energy is lost by the large eddies it is gained by the smaller eddy. So, primary mechanism that works behind the energy cascade is the vortex stretching mechanism. So, you have the eddies due to the small level of eddies. due to the large level eddy it gets stage like this and then the energy keeps on transferring from one to another.

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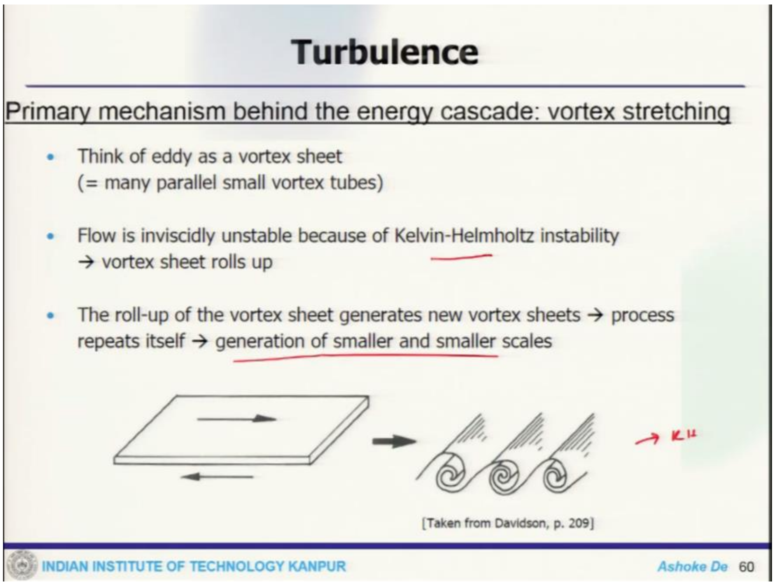

Vortex stretching

Now, you can think about eddies like a vortex sheet. which are many parallel vortex tubes. So, flow is in viscidly unstable because of the Kelvin-Helmholtz instability. These vortex sheets actually have a gradient between the upper and lower surface so they roll up. This is your K-H kind of instability. Now, these rolls of the vortex sheet generate new vortex sheets and process repeats itself. Which will lead to generation of smaller and smaller scale that means, through the stretching mechanism or vortex sheet mechanism that smaller and smaller scale structures are always formed. We can scale these macro structure and also find out the energy transfer rate.

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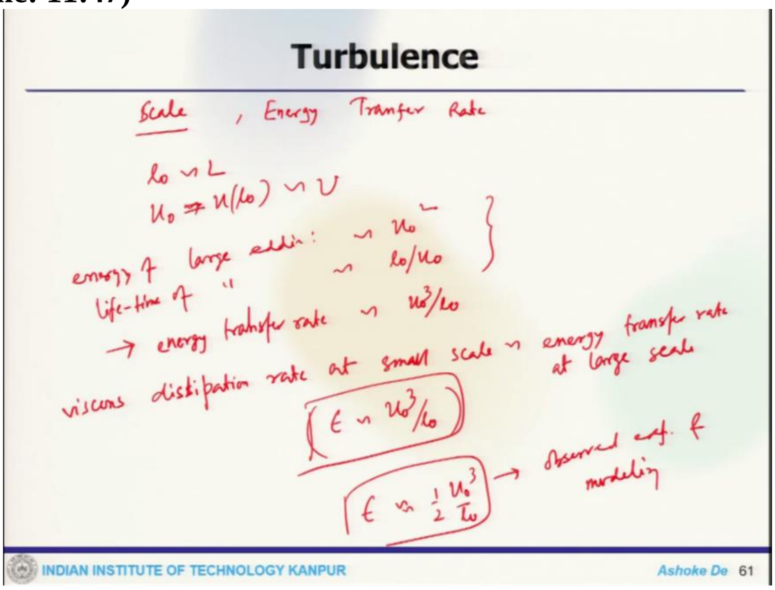

So, typical scales of macro structure are order of $l_0 \sim L$

the velocity is $u_0 = u(l_0) \sim U$

energy of large eddies is $\sim u_0^2$

Life time of large eddies is l_0/u_0

energy transfer rate is u_0^3/l_0

So, essentially that is my viscous dissipation rate at small scale is energy transfer rate at large scale.

$$\varepsilon \sim u_0^3/l_0$$

So, it has been shown both experimentally and through modelling also that epsilon typically found to be order of $\varepsilon \approx \frac{1}{2}u_0^3/l_0$

This is observed experimentally and modelling, that this is the order of dissipation. Important information here is that the viscous dissipation rate at the small scale is of the order of the energy transfer rate at the large scale.

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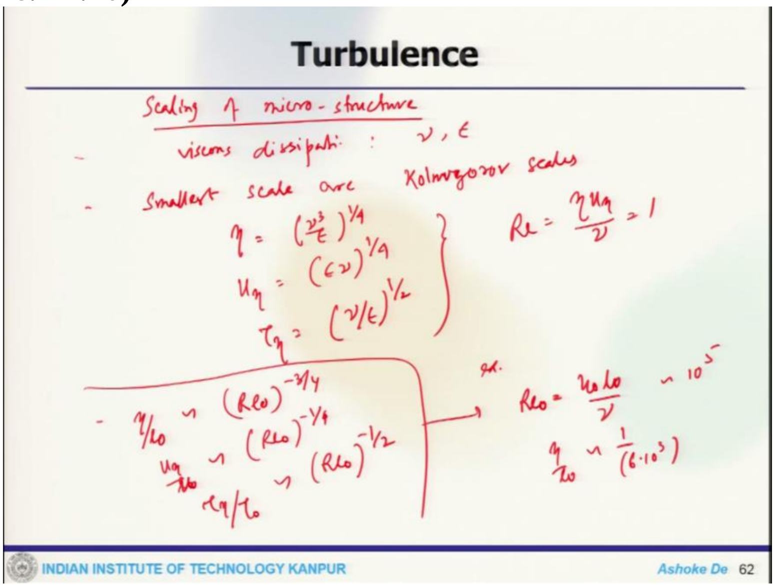

Kolmogorov scales

Now, we can scale the or we can find out the scaling of micro structures or small eddies. Now, essentially the dynamics of the small scale these are dominated by viscous dissipation. So, once it is dominated by viscous dissipation that means, the parameter like v, ε, these are going to a play a critical role. The smallest scales are the Kolmogorov scales and that is given by

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$$

$$u_\eta = (\varepsilon\nu)^{1/4}$$

$$\tau_\eta = \left(\frac{\nu}{\varepsilon}\right)^{1/2}$$

where Re here is defined

$$Re = \frac{\eta u_\eta}{\nu} = 1$$

A low Re consistent with dominance of viscous dissipation. Now, we can find out the ratio of the Kolmogorov scale to the macro structure and that ratio one can find out

$$\frac{\eta}{l_o} \sim (\mathrm{Re}_0)^{-3/4}$$

$$\frac{u_\eta}{u_o} \sim (\mathrm{Re}_0)^{-1/4}$$

$$\frac{\tau_\eta}{\tau_o} \sim (\mathrm{Re}_0)^{-1/2}$$

So that is where

$$Re_0 = \frac{u_o l_o}{\nu}$$

If $Re_0 \sim 10^5$ then $\frac{\eta}{l_o} \sim \frac{1}{(6\times10^3)}$

So that is the scaling. So, this is a connection between my macro structure with the Kolmogorov length scale.

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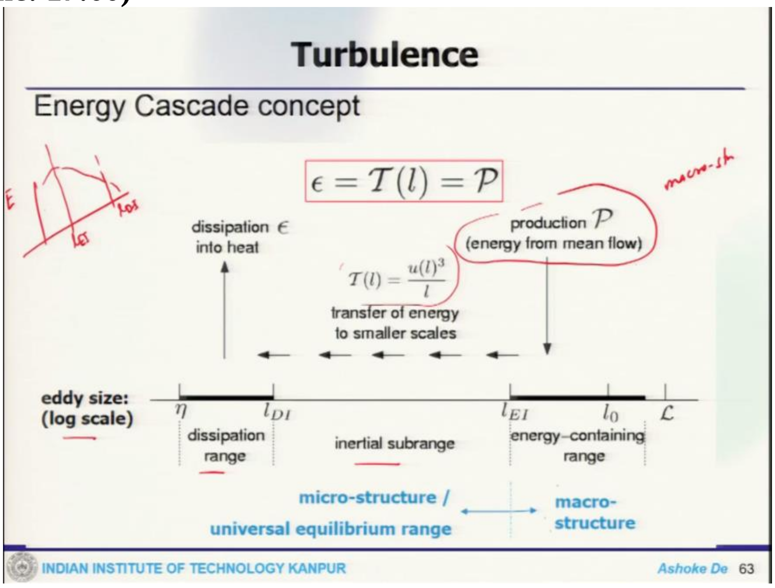

So, at the large-scale structure and the small-scale structure, the scaling parameters are different. Now, if we put that energy cascading concept in a platform.

$\varepsilon = \tau(1) = P$ this is dissipation. P is the production of the energy at the macro structure level of the large-scale structure. On the length scale and we have an energy-containing range in the macro structure level, from here the energy gets transferred to the small scale and is given as

$$\tau(l) = \frac{u(l)^3}{l}$$

and this is going from the macro structure to the micro structure level or universal structures smaller scale level and these ranges in inertial range and when it goes to the smallest scale, it actually dissipates through the viscous dissipation or heat. So, this range is called the dissipation range. So, there are three range one can think about in the spectrum, there is an energy containing range, so if I put that in a qualitatively, this is energy, there is an energy containing range, there is an inertial range and there is a dissipation range. So, this is how the demarcation would be, L_{EI} and L_{DI}.

So, there is a production, there is an energy transfer to the smallest scales and the amount of energy through viscous dissipation. The total energy balancing is also there. Now, at the different range we can look at the scaling of the eddies.

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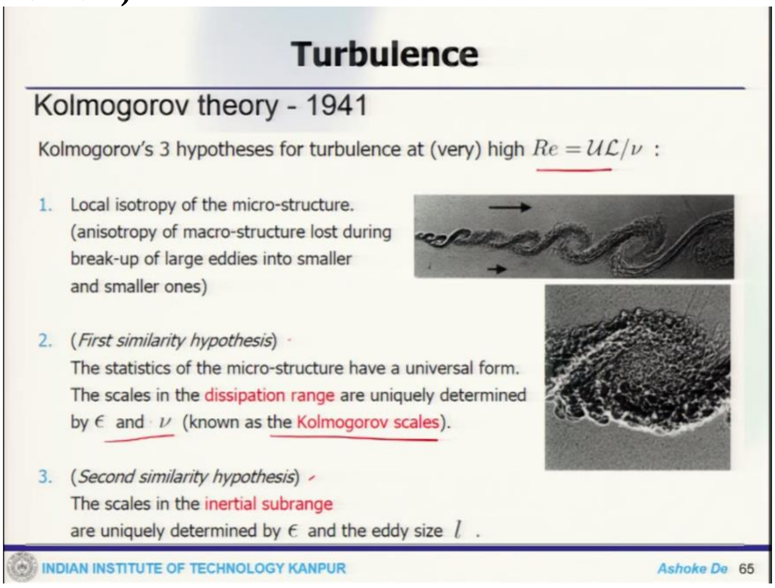

So, first we look at the inertial sub-range. The size of eddies in the inertial sub-range is $\eta \ll l \ll l_0$. The characteristics parameter would be $\varepsilon$and eddies size.

So, the scales are of $\sim l$

the velocity is $u(l) = (\varepsilon l)^{1/3}$

Time scale is of $\tau(l) = \left(\frac{l^2}{\varepsilon}\right)^{1/3}$

Now, if I relate with η and macro scales, what we get is

$$u_\eta \left({}^{l}/_{\eta}\right)^{1/3} = u(l) \sim u_0 \left({}^{l}/_{l_0}\right)^{1/3}$$

$$\tau_\eta \left({}^{l}/_{\eta}\right)^{2/3} = \tau(l) \sim \tau_0 \left({}^{l}/_{l_0}\right)^{2/3}$$

So, this is for

$$u_\eta \ll u(l) \ll u_0$$

$$\tau_\eta \ll \tau(l) \ll \tau_0$$

Kolmogorov hypothesis

So that is how you get the different sub-range scales in the inertial sub-range. So, that brings to an important Kolmogorov hypothesis these are very important three hypotheses which proposed.

This is for high Reynolds number that means that things are in turbulent in nature.

1. The local isotropy of the micro structure that means, an isotropy of the macro-structure is lost during break-up of larger eddies into smaller eddies. So, these are the larger eddies with breaks into the smaller eddies so there is anisotropy of the micro structure.
2. the First similarity hypothesis says that the statistics of the microstructure have a universal form and the scales in the dissipation range are uniquely determined by ε and ν which are known as Kolmogorov scales. We have seen how ε and ν are connected with the scaling parameters.
3. Second similarity hypothesis, which says that the skills in the inertial sub-range are uniquely determined by epsilon and then large eddy size l. So, that is what the Kolmogorov's hypotheses are, and it talks about local isotropy of the micro structure, first similarity hypothesis and the second similarity hypothesis.

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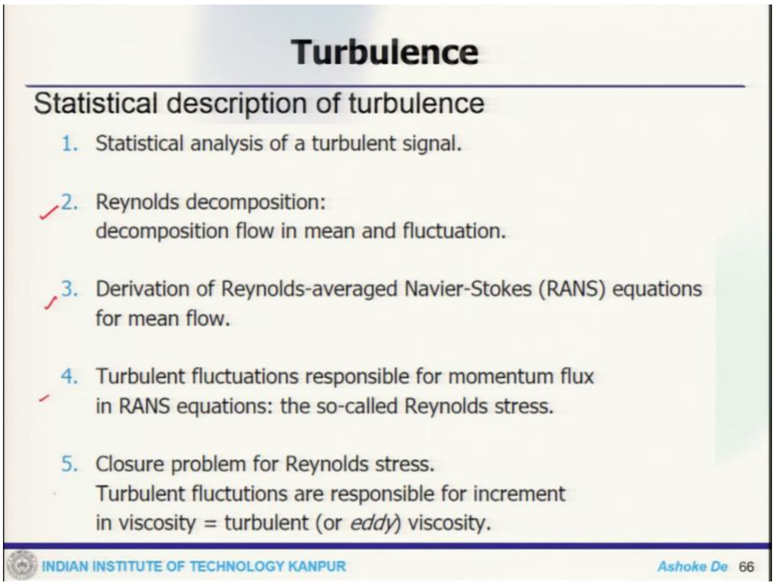

Since this flow is quite random and chaotic in nature will now look at the statistical description of this turbulence. So, we look at some turbulent signal to be analysed, then we look at the Reynolds number decomposition, then from there, we will derive the Reynolds-averaged equation and then the Reynolds stress, and then finally, the closure problem and the issues associated with.

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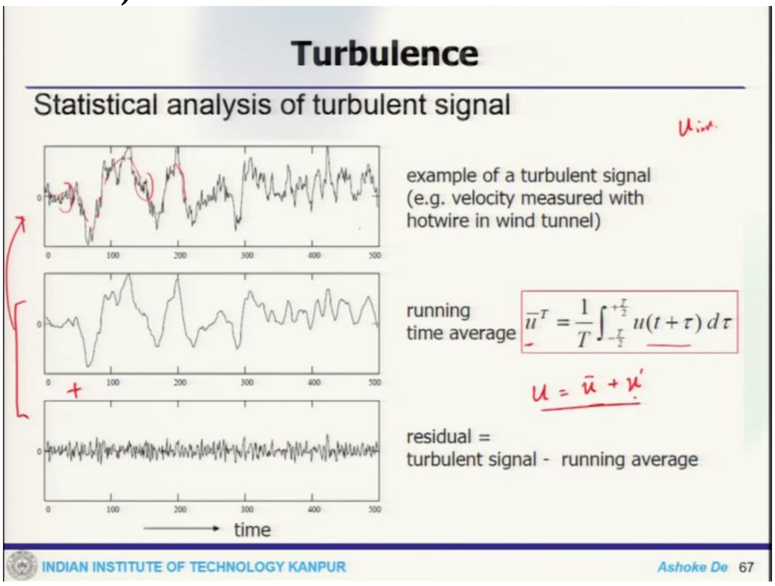

Statistical analysis of turbulent signal

So, when you look at the statistical part of it, that is a typical example of a turbulent signal. So, it is a velocity signal, which is obviously, instantaneous velocity. This is real time acquisition. Now when we do some averaging of the signal, these noises which are shown here, they get smoothened out and get a nice mean signal and from instantaneous signal if this is subtracted, we get this noise back.

So, essentially the mean and the residual shown will combinedly give back the original signal, Original signal has some mean of the signal and then top of that there is where the noise would be sitting there. Now, one can find out this running average over a time

$$\bar{u}^T = \frac{1}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} u(t+\tau)d\tau$$

So that means my instantaneous signal is of some $\bar{u}$ a mean signal and plus some fluctuating components u'.

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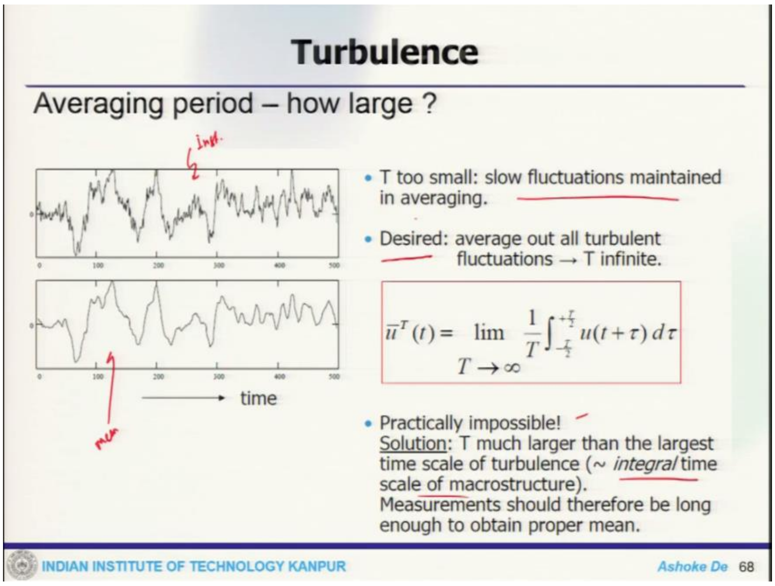

Now, when you do this averaging, the immediate questions which may arise that how long one has to do this energy or how large it could be. Now, depending on the T this mean could be different, for T too small some fluctuations maintained in the averaging and if T is too large and this could be the noise could be smoothened out.

So, average out all the turbulent fluctuations when T is actual theoretically goes to infinity, which is essentially a decide phenomenon that means, if I want to get that mean data then theoretically should have these averaging for a sufficiently large amount of time, which is practically again impossible. So, if the T is much larger than the timescale of the largest timescale of the turbulence, which is order of integral timescale of macro structure. So, whether it is a major mean or simulation time these becomes sufficiently large.

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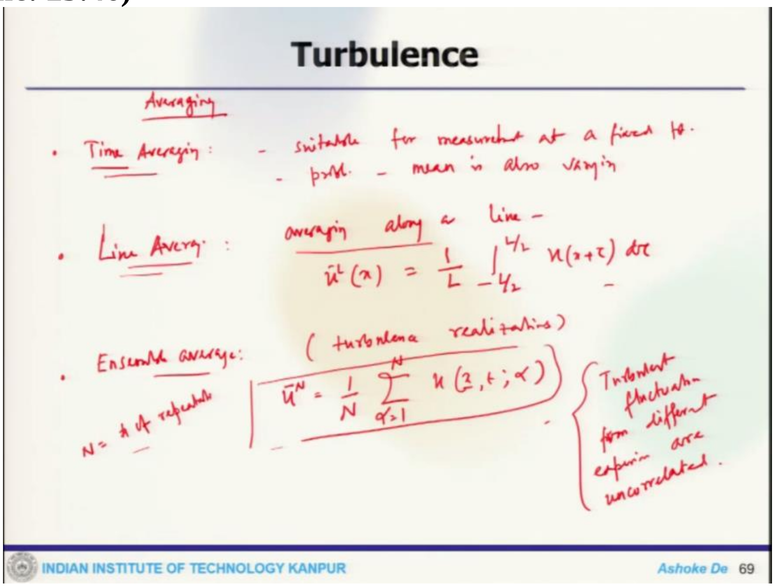

So, now, we have different kind of averaging technique. One is the time averaging. This time averaging this is quite suitable for measurements at a fixed point and this has a problem when the mean is also varying. That means, if you have a time varying mean then there is a problem. So, which clearly an issue if you have a statistically unsteady turbulence that means atmospheric boundary layer day and night cycle seasonal cycle. That kind of situation this time averaging would not work.

Now, one can do line averaging so, this is an averaging along a line and I can get or express this, like

$$\bar{u}^{L}(x) = \frac{1}{L}\int_{-\frac{L}{2}}^{+\frac{L}{2}} u(x+\tau)d\tau$$

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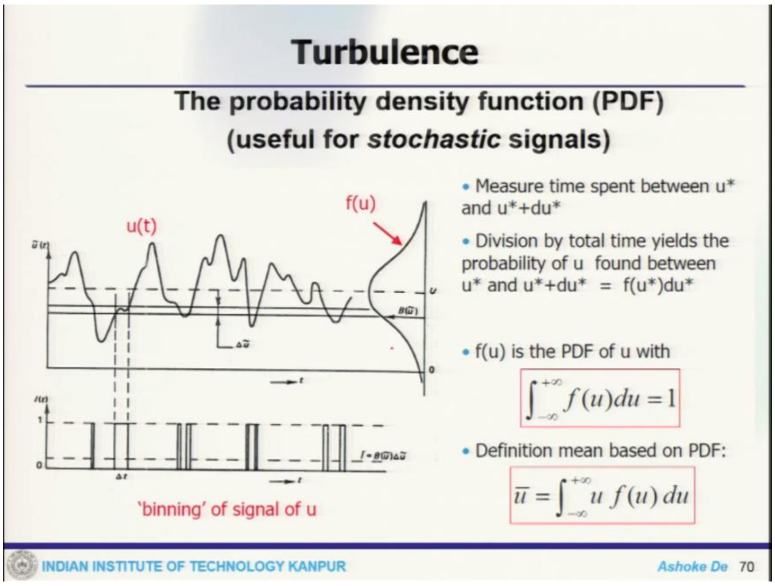

Then finally, we got ensemble average. So, this is something like you can repeat the experiment. If you are doing experiment repeat the experiment multiple time and average out all over the experiments or in simulations also, you can take the averaging for multiple times, this allows you to do turbulence realizations.

So, ensemble averaging, you can represent like

$$\bar{u}^{N} = \frac{1}{N}\sum_{\alpha=1}^{N} u(x,t,\alpha)$$

So, that what you get in in ensemble averaging and N is the number of number of repetitions. Now here important point to be noted here these that turbulent fluctuation from different experiments are uncorrelated.

That means one can do averaging in three different ways, one is time averaging, which is quite suitable for measurement at a fixed point, but this is not suitable if you have statistically unsteady turbulence like where the mean flow itself is having, there it does not like atmospheric boundary layer when it goes day or night cycle then one can do line averaging which is averaging is done along a line or one can do ensemble averaging where actually you can repeat the experiment multiple times and get the, but this is where the turbulent fluctuation from different experiments are uncorrelated.

So, this is how you get the averaging and we stop here today and continue the discussion in the next lecture