Turbulent Combustion: Theory and Modeling
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Lecture-29
Turbulence (contd...)

Okay. Welcome back and let us continue the discussion on the turbulent flow and we are looking at different situation. So, first we started over with the incompressible Navier Stoke's and where density is constant and the viscosity is constant then we move to a situation where still the dynamic viscosity is constant but there is a variation of the density which essentially gives rise to some effect due to density. And we approximated that one with the Boussinesq approximation.

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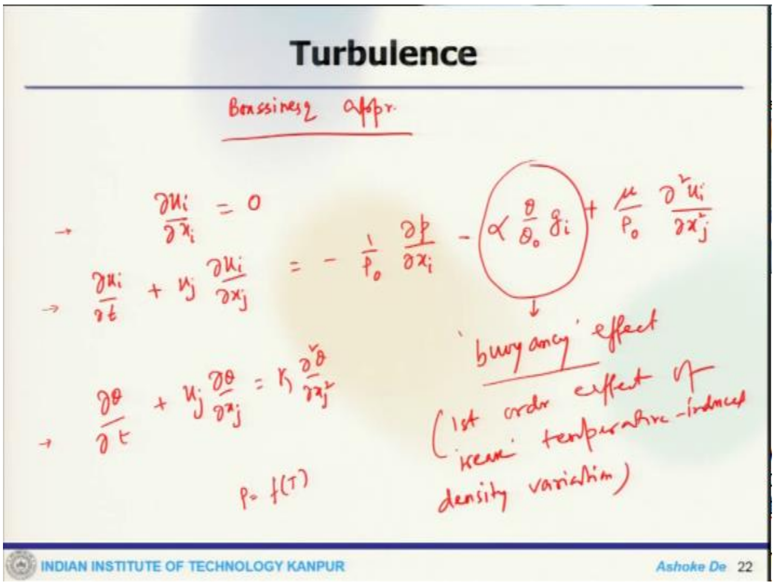

This is what we got it basically when you have the Boussinesq approximation then where you have effect of the density variation and the density variation is only a function of temperature. So, you got your continuity, momentum equation. Now momentum equation will retain a term which is due to the temperature and this is my term energy equation in terms of temperature, so this term is known as buoyancy effect. So, this is nothing but my first order effect of weak temperature induced density variation. So, that is what you get for Boussinesq approximation.

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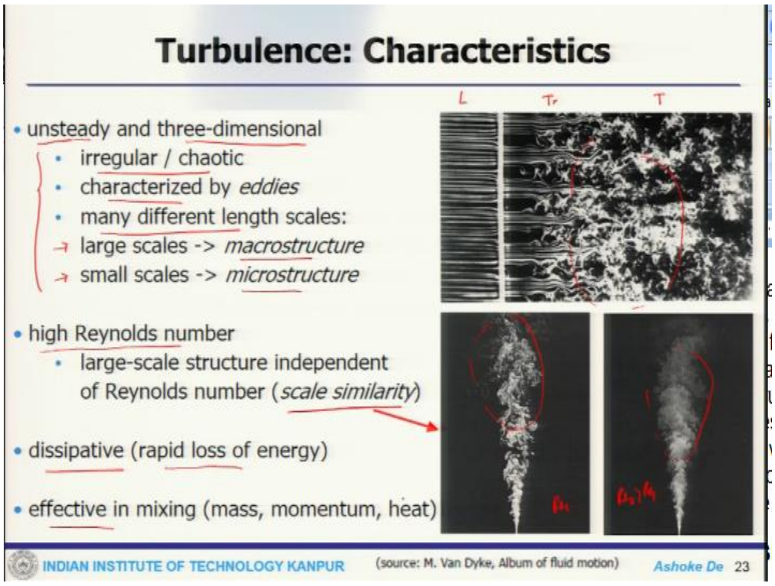

So that is we have already looked at this picture at different situation. This is a picture where your laminar transition and this is a turbulent flow this is a flow over a perforated flat plate and these are jet flow this is Ru_1 this is Ru_2 which is greater than Ru_1 is at the jet flows. Now if we combine or accumulate all these characteristics that we have discussed so far then we can see this is unsteady flow and 3 dimensional. So that is quite obvious this flow field has to be completely unsteady and 3 dimensional and when it is unsteady and 3 dimensional, we can say it is quite chaotic and irregular in nature then we can characterize it by lot of eddies these are the eddies in every flow you can see but they have different scales sometimes they are large-scale sometimes they are of small scale and there could be a wide range of scales and that depends on the Reynolds number, what kind of scale we have whether it is a multiple scale or single scale or something like that then as I said there are multiple length scale primarily one could be large scale which we term as macrostructure there could be small scale which we term as microstructure.

So, combined all these features that are exhibited due to unsteady and 3 dimensional flow field then also the flow field is at high Reynolds number so there is a scale similarity that means the large scale structure are independent of Reynolds number. So, if you look at the large-scale structure or the macro structure they are independent of the Reynolds number that gives you scale similarity. This is a very important characteristics then there is dissipative in nature, the flame flow field is dissipative in nature, so that means there is a rapid loss of energy and is effective in mixing and mass momentum heat etc. So, these are the characteristics that we have seen.

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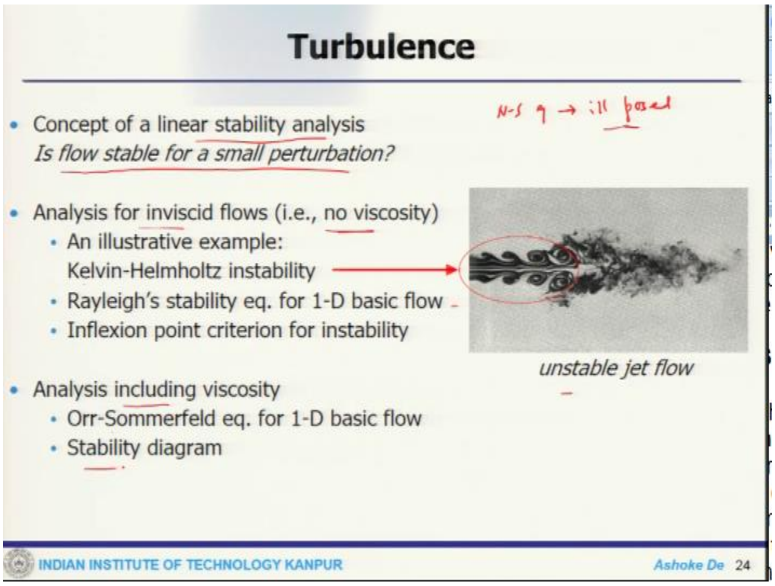

Now we have already said that for a turbulent flow the Navier stokes equations are ill-posed that because if we make some small perturbation to the initial condition or boundary condition the flow does not remain any more stable. So this is the where we can use our concept or linear stability to see whether the flow is stable or not for some small perturbation. Now this is quite important to identify the zone where you can see the stable flow field and the zone where it is unstable.

So you can do some analysis for inviscid flows that means no viscosity one could be type of Kelvin Helmholtz instability this kind of this is an image of unstable jet flow, so this is an image of KH instability or Kelvin Helmholtz instability then we can look at a Rayleigh's stability equation for 1D basic flow and then in second point criteria for instability. Now similar exercise can be extended by including the viscosity in that case we will get Orr-Sommerfeld equation and we can find out our stability diagram.

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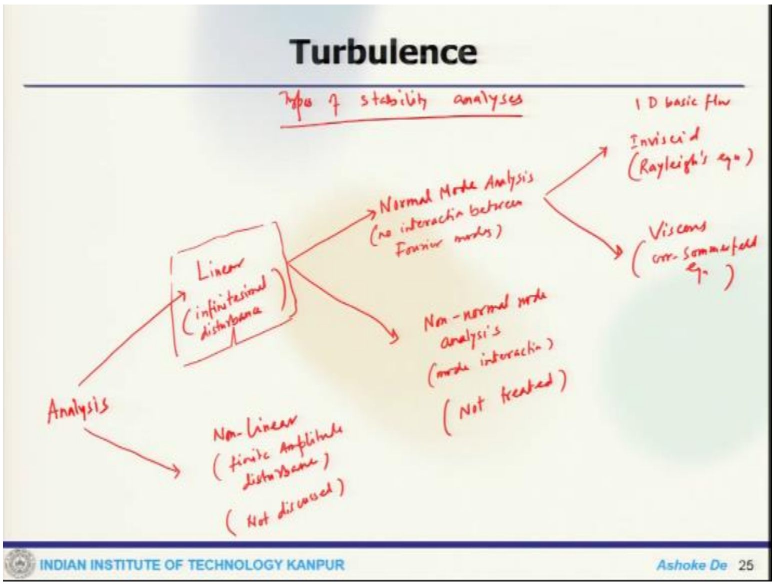

So the types of stability analysis one can do if you start write bottom here the analysis let us say this can go to one direction where it is a nonlinear. So which is something called finite amplitude disturbance or other one is the segment where we can say it is a linear that means infinitesimal disturbance okay. Now we will do or restrict our analysis for the linear system only, so this one will not going to be discussed.

So you look at only the linear stability system now there are also different approach of this one can look at either normal mode analysis which is no interaction between Fourier mode so that is normal mode analysis or it could be non-normal mode analysis that is mode interaction, so even under the basket of linear stability analysis we will only be doing the normal mode analysis. We will not treat this non-normal mode.

So now under the normal mode analysis there could be two categories. So we will say 1D basic flow either it could be inviscid which is essentially analyze equation or we can do viscous which is Orr-Sommerfeld equation. So we will look at these two type of instabilities.

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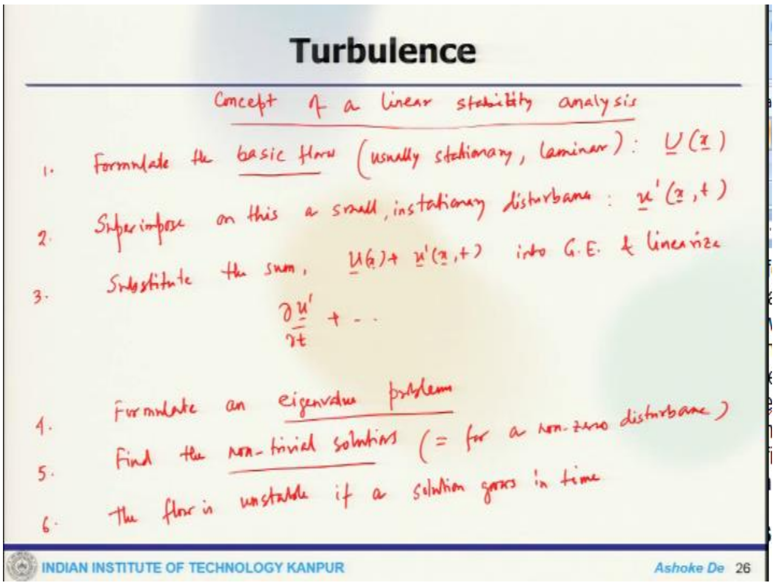

Now before doing that we can look at some of this concept of a linear stability analysis. So, first thing is that one has to do formulate the base flow or basic flow. So, which is usually stationary and laminar. Then you can super impose on this a small instationary disturbance that is u'. Then you can substitute the sum which is plus into governing equation and linearize like $\frac{\partial u'}{\partial t}$ and so on. Finally from that linearized equation, one can formulate an eigenvalue problem.

So this is quite important and then we can find the non-trivial solutions which is for a non-zero disturbance and then finally the flow is unstable if a solution grows in time. So, that is how we can do that.

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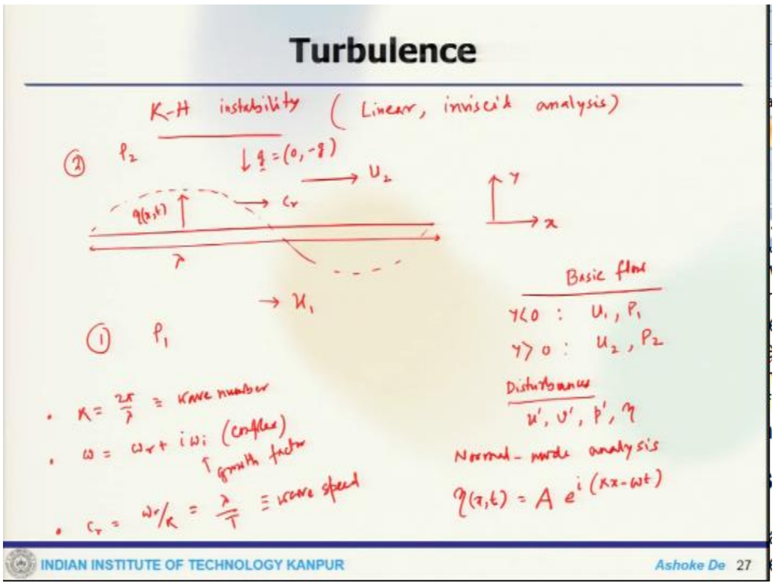

So we start with a K H instability which is linear and inviscid analysis in nature so we can have a fuel like this and this is λ. So this is my state 1: ρ_1, u_1. This is how I defined x and y this is u_2 this is my gravity 0 and state 2: ρ_2. This is η(x, t), this is my C_r. Now here some definitions which are quite important to note one is that:

$$K = \frac{2\pi}{\lambda}$$

Which is wave number then we get this:

$$\omega = \omega_r + i\omega_i$$

Then this is my growth factor and C_r is:

$$C_r = \frac{\omega_r}{K} = \frac{\lambda}{T} = wave\ speed$$

Now if you look at the base flow or the basic flow there y is less than 0 you have u_1 and P_1. Y greater than 0: u_2 and P_2. Now if you look at the disturbances, then they are u', v', p', η. Now for normal mode analysis my η(x, t) is $Ae^{i(Kx-\omega t)}$. So this is the set of variables. Now we can derive the governing equation.

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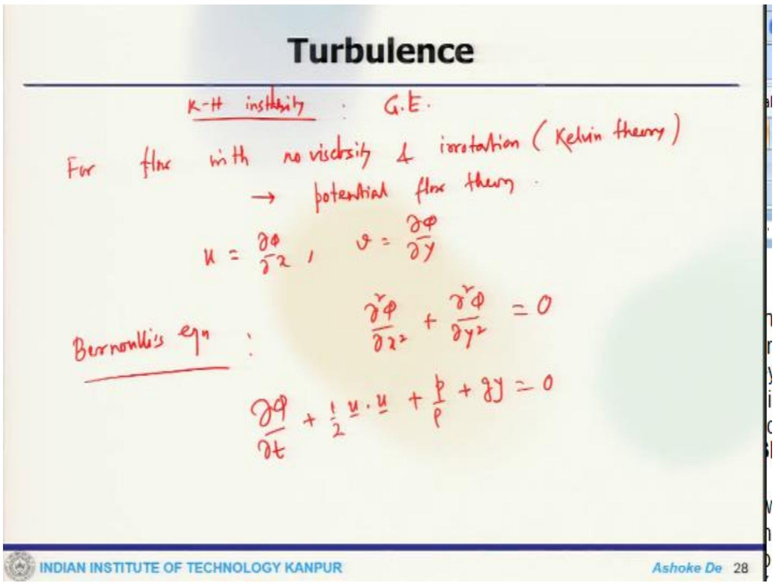

So here we are talking about K H instability. So we derived the governing equation now for flow with no viscosity and irrotational which is a Kelvin theory this is a flow it is insulation potential flow theory which gives you:

$$u = \frac{\partial\varphi}{\partial x}, v = \frac{\partial\varphi}{\partial y}$$

And Bernoulli's equation will give you:

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0$$

And the other term:

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2} u.u + \frac{p}{\rho} + gy = 0$$

So that’s what we get.

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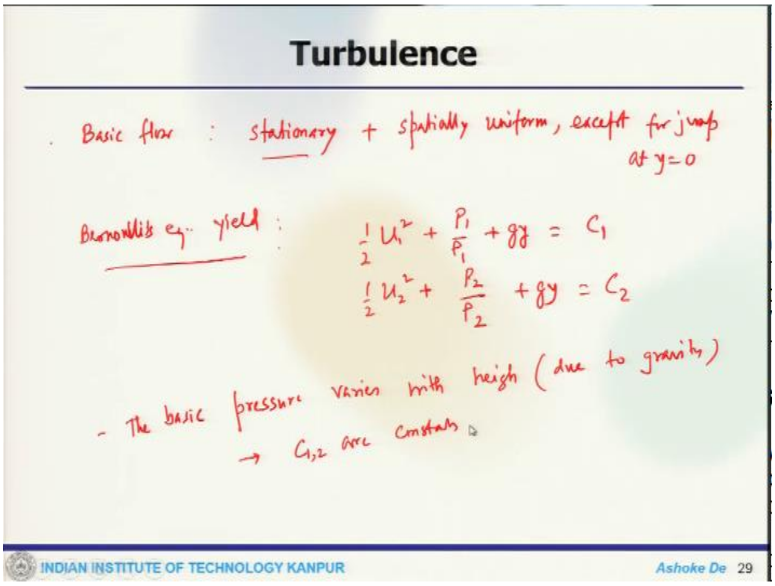

So what is our basic flow is stationary plus spatially uniform except for jump at y equals to 0. So the Bernoulli’s equation yield for this:

$$\frac{1}{2} u_1^2 + \frac{p_1}{\rho_1} + gy = C_1$$

And:

$$\frac{1}{2} u_2^2 + \frac{p_2}{\rho_2} + gy = C_2$$

So this is what we wrote. So this is constant. So then for stationary the unsteady term goes off and from the rest of the term if it is spatially uniform except the point y equals to 0 is it and one can also note that the basic pressure varies with height that is due to gravity which means C_1 and C_2 are constants.

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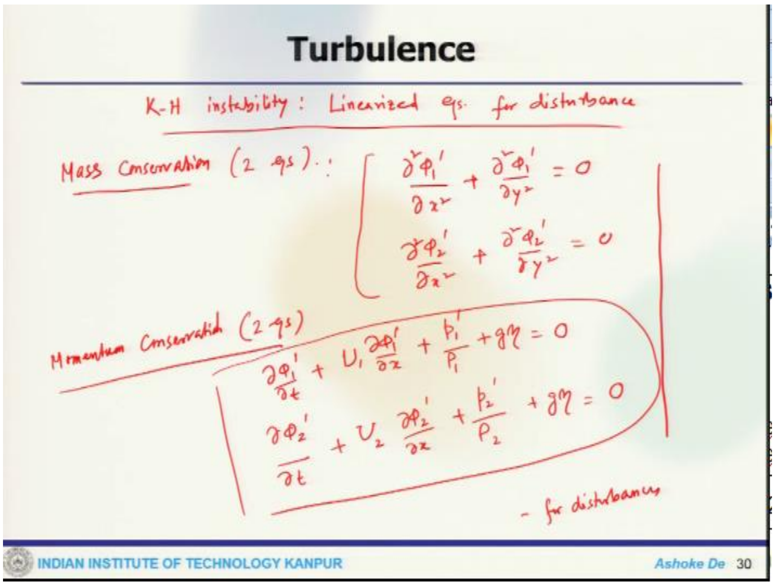

Now we can formulate the linearized equation for disturbance so still we are doing KH instability this is linearized equation for disturbance. So from mass conservation equation we get 2 equations one is:

$$\frac{\partial^2 \varphi_1'}{\partial x^2} + \frac{\partial^2 \varphi_1'}{\partial y^2} = 0$$

Second is:

$$\frac{\partial^2 \varphi_2'}{\partial x^2} + \frac{\partial^2 \varphi_2'}{\partial y^2} = 0$$

So these are the 2 linearized equation that we get for the mass conservation. Now similarly momentum conservation momentum conservation will get 2 equation. So one is:

$$\frac{\partial \varphi_1'}{\partial t} + U_1 \frac{\partial \varphi_1'}{\partial x} + \frac{p_1'}{\rho_1} + g\eta = 0$$

And:

$$\frac{\partial \varphi_2'}{\partial t} + U_2 \frac{\partial \varphi_2'}{\partial x} + \frac{p_2'}{\rho_2} + g\eta = 0$$

So these are the 2 equation these are from mass conservation there are 2 equation from momentum conservation there are 2 equations and they are linearized equations and these equations are for disturbances.

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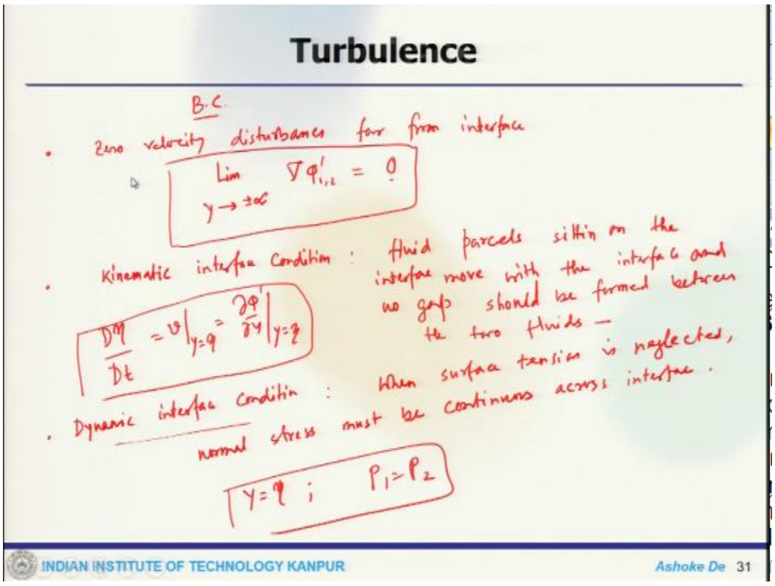

Now we need boundary conditions which are quite important. Number one we can have 0 velocity disturbances far from interface that means one can write:

$$\lim_{y\to\pm\infty} \nabla\varphi'_{1,2} = 0$$

So that is our delay velocity disturbance far from the interface, second we can have kinematic interface condition so kinetic interface condition talked about with fluid parcels sitting on the interface move with the interface and no gap should be formed between the 2 fluids which means:

$$\frac{D\eta}{Dt} = |v|_{y=\eta} = \left|\frac{\partial\varphi'}{\partial y}\right|_{y=\eta}$$

So that is my kinetic conditions and last is dynamic interface condition when surface tension is neglected, normal stress must be continuous across interface. Now for inviscid flows a continuous normal stress reduces to a continuous pressure across the interface which will get at $y = \eta$ and $P_1 = P_2$. So this is what we get. So these are 3 conditions that one have as the boundary conditions. So we stop here today and continue it in the next lecture.