Turbulent Combustion: Theory and Modeling

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Lecture-26
Laminar Non - Premixed Flames (contd.) and Turbulence

Welcome back, let us continue the discussion on Rankine Hugoniot relation. So, we will finish this particular one which we are discussing right now on deflagration wave and detonation wave. So, the deflagration wave we have done the analysis for a simple system and now we are in the middle of the detonation, okay? So, this is where we actually stopped.

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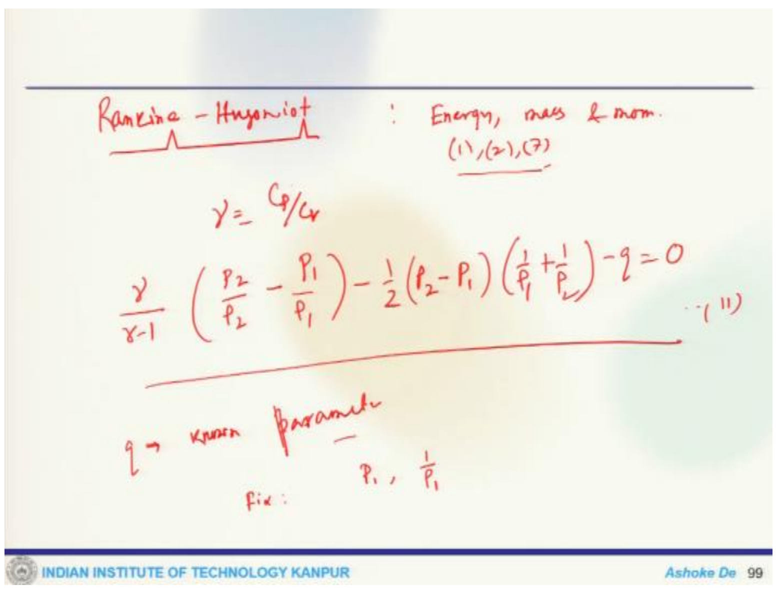

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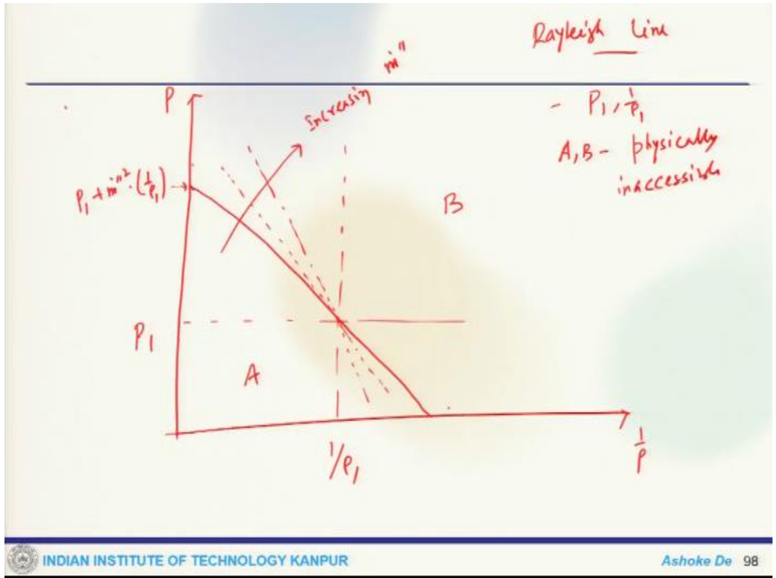

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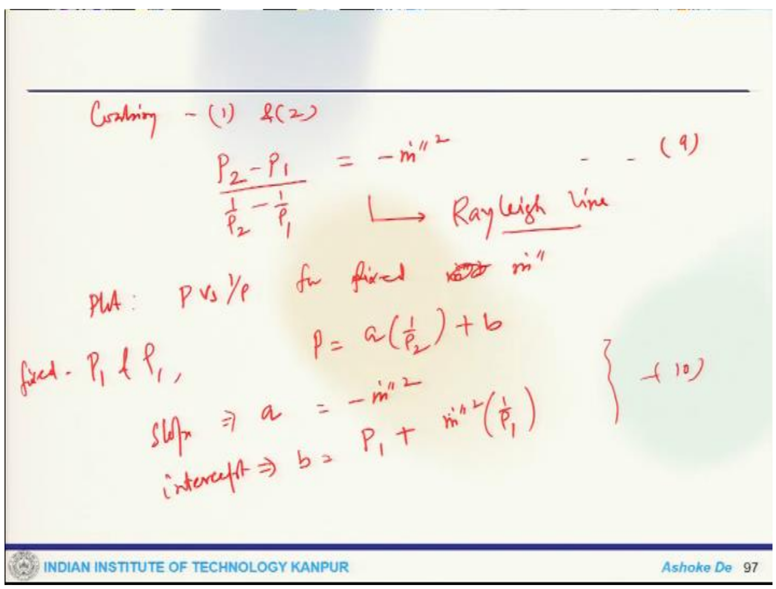

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$$C_pT_1 + \frac{v_1^2}{2} + \underbrace{\underbrace{\sum Y_i h_{f,i}^\circ}_{\text{state 1}} - \underbrace{\sum Y_i h_{f,i}^\circ}_{\text{state 2}}}_{\text{Heat addition, } q} = C_pT_2 + \frac{v_2^2}{2} \quad \cdots (6)$$

After simplification,

$$C_pT_1 + \frac{v_1^2}{2} + q = C_pT_2 + \frac{v_2^2}{2} \quad \cdots (7)$$

Using ideal-gas assump.,

$$\left.\begin{aligned} P_1 &= \rho_1 R_1 T_1 \\ P_2 &= \rho_2 R T_2 \end{aligned}\right\} \quad \cdots (8)$$

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Mass

$$\dot{m}'' = \rho_1 v_1 = \rho_2 v_2 \quad \cdots (1)$$

Mom.

$$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2 \quad \cdots (2)$$

Energy:

$$h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2} \quad \cdots (3)$$

$$h(T) = \sum Y_i h_{f,i}^\circ + \sum Y_i \int_{T_0}^{T} C_{p,i}\, dT \quad \cdots (4)$$

$$h(T) = \sum Y_i h_{f,i}^\circ + C_p(T - T_0) \quad \cdots (5)$$

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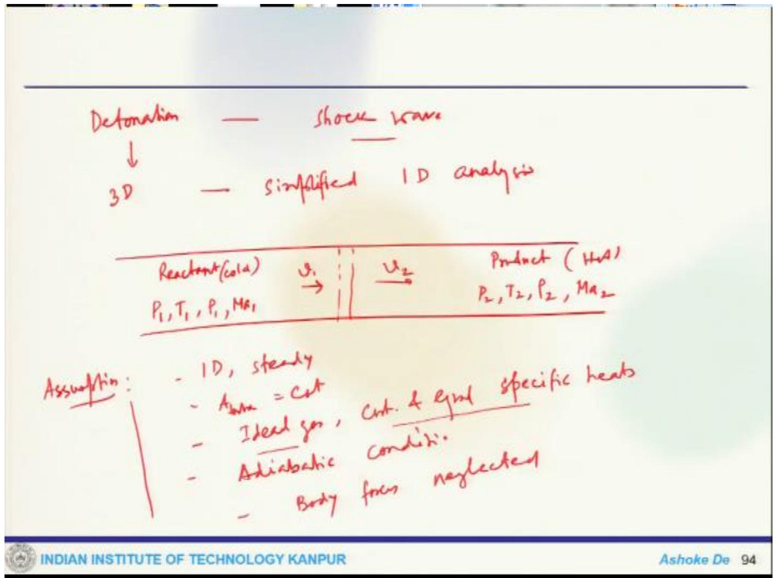

This started with a particular system like this. And we had an upstream condition, we have some downstream condition, and these are the set of assumptions that we had something like 1D, steady, tube area is constant, ideal gas assumption and constant and equal specific heat and adiabatic conditions. So, all these are taking into and obviously no body forces which are considered. So, considering these assumptions we have looked at our mass conservation, momentum conservation, energy than using the expression for enthalpy we finally got this for the Rayleigh line where with the different mass flow rate we get this.

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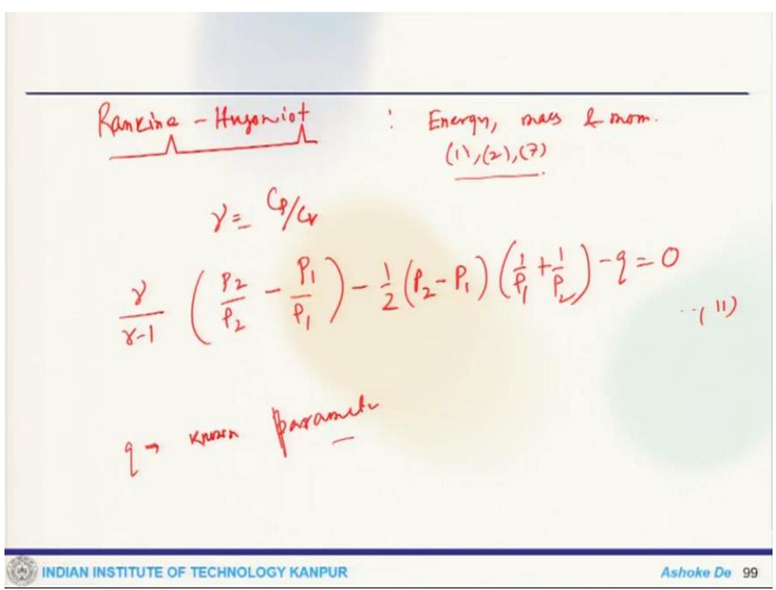

And then finally we got from the energy, mass and momentum when you combine and also when you write:

$$\gamma = {}^{C_P}/_{C_V}$$

γ is the ratio of the constant pressure specific heat to constant volume specific heat. This is what we get now, where q is the known parameter. Now we will further fix now ${}^{P_1}/_{\rho_1}$.

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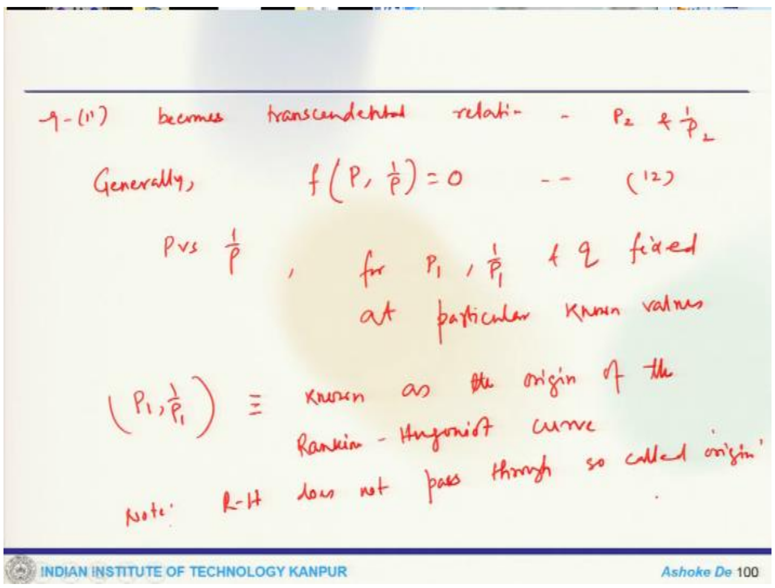

So once we do that, now this equation 11 becomes transcendental relation between P_2 and ${}^{1}/_{\rho_2}$ or rather in generically one can write this to be zero. Now one can plot this pressure versus 1 by ρ for let us say P_1, ${}^{1}/_{\rho_1}$ and q to be fixed. At particular known values and the point or the coordinate of let us say P_1, ${}^{1}/_{\rho_1}$. This is a sincerely known as the origin of the Ranking Hugoniot curve.

So what one can see that or rather a note that the RH curve does not pass through the so called origin. So, RH curve does not pass through, so called origin, which is essentially the intersection of x and y.

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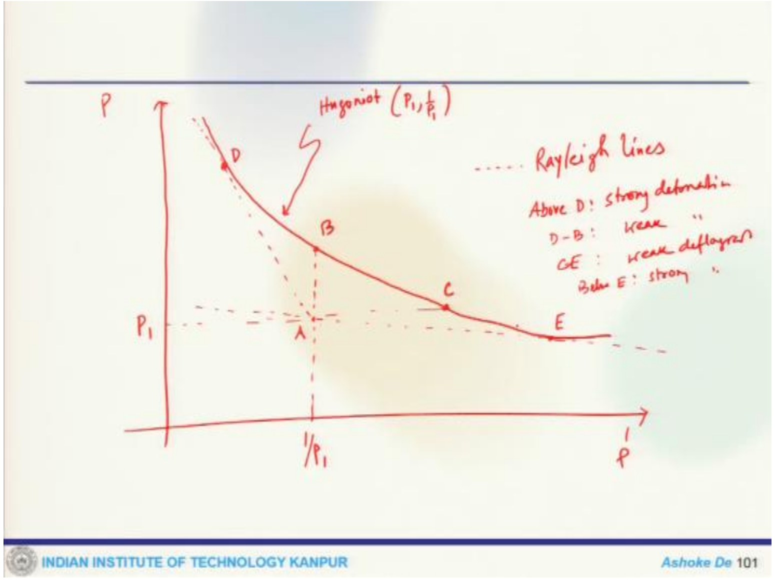

Now, we can plot this pressure versus. So, this x is it would be pressure, this is $^1/_\rho$, this is pressure. Now, we can have a curve like this, where this is 1 point and we can draw a tangent to that point then this will cut through then this will also cut through a point like this and this will also pass through the tangent of this point. So we can have a curve which is going through like these. So this point you say let us say you start some point, this is D, and this point is B. This is a point which you call C then there is this is the tangent.

Let us say E and this is a point which passed through called A and this is your P_1 and this is $^1/_{P_1}$ and this particular curve viewer Hugoniot for P_1, $^1/_{P_1}$ and this dotted line are all Rayleigh lines. So, this is what you get. Now, the point is that question which may arise what point this Hugoniot correspond to realizable physical state. Now, one can see any real process which pass going through the point 1 and 2 must satisfy this Rayleigh and Hugoniot relation. So, there are a few limiting situations.

This is above D this is the situation where you have strong detonation, D to B this is weak detonation, C to E weak deflagration and below E this is strong deflagration. So these are the sort of different one is above this point, then one segment is D to B, another segment B to C or C to D then below E.

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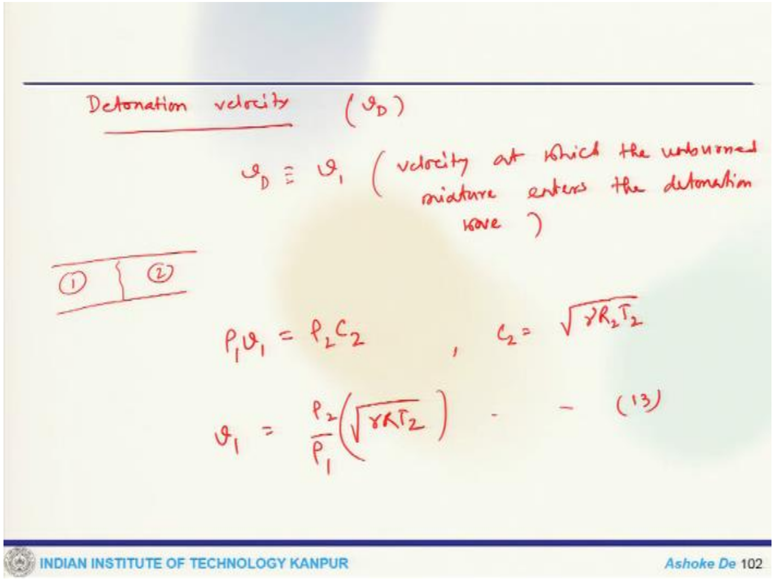

Now, we can define our detonation velocity as v_D Okay? So, this should be equal to v_1 which is essentially the velocity at which the unburned mixture enters the detonation wave. Now, if you recall from that particular figure you have these then all these state 1 and this is state 2, now detonation for state 2 will be the upper CJ point at which is velocity sonic so, then we can write:

$$\rho_1 v_1 = \rho_2 c_2$$

Where your c_2 is $\sqrt{\gamma R T_2}$ is the sonic velocity.

So, then one can write:

$$v_1 = \frac{\rho_2}{\rho_1}\left(\sqrt{\gamma R T_2}\right)$$

This is our equation 13. Now, we try to relate this with the density ratio $\left(\frac{\rho_2}{\rho_1}\right)$ and T_2 for state 1 or any other available known quantities, so then what we can write that we can write from our momentum equation, this equation number 2.

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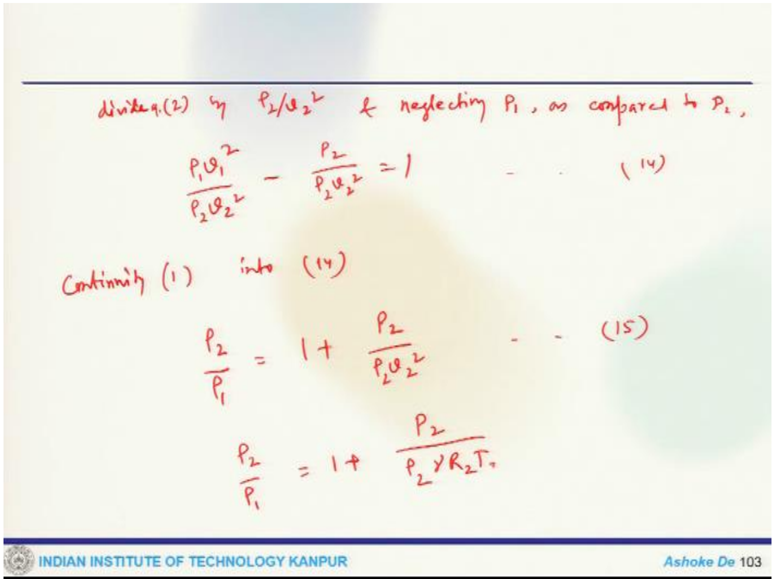

So, that equation we divide that momentum conservation by ρ_2/v_2^2 and neglecting P_1 as compared to P_2 we get:

$$\frac{\rho_1 v_1^2}{\rho_2 v_2^2} - \frac{P_2}{\rho_2 v_2^2} = 1$$

This is equation 14. Now we apply the continuity equation continuity that equation number 1 into 14 what we get?

$$\frac{\rho_2}{\rho_1} = 1 + \frac{P_2}{\rho_2 v_2^2}$$

Now we can replace v_2 to c_2. So, this will become:

$$\frac{\rho_2}{\rho_1} = 1 + \frac{P_2}{\rho_2 \gamma R T_2}$$

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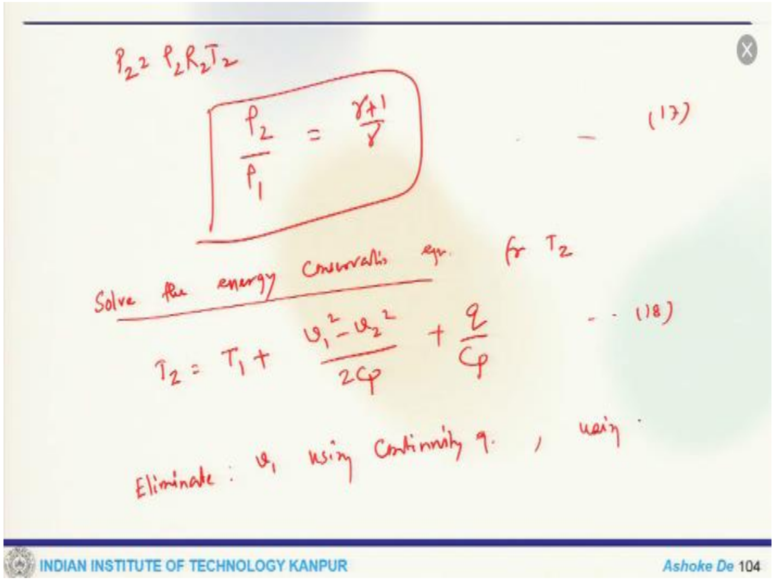

Now use some ideal gas law that is:

$$P_2 = \rho_2 R T_2$$

Which will get us:

$$\frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma}$$

So, that is our equation number 17. Now, once we solve the energy conservation equation, if we solve the energy conservation equation for T_2 what we get:

$$T_2 = T_1 + \frac{v_1^2 - v_2^2}{2C_p} + \frac{q}{C_p}$$

Again here, if we eliminate now we can eliminate, view on using continuity equation. And also using c_2 equals to v_2.

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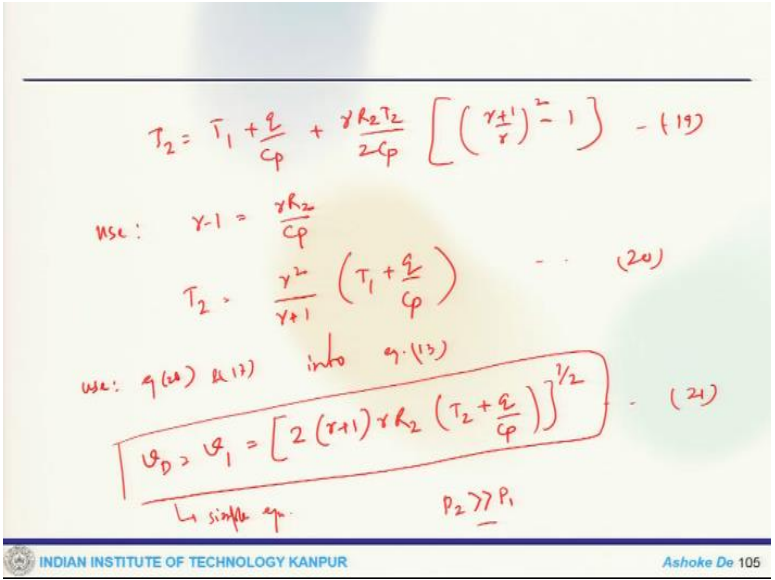

So what do we get:

$$T_2 = T_1 + \frac{q}{C_p} + \frac{\gamma R T_2}{2C_p}\left[\left(\frac{\gamma+1}{\gamma}\right)^2 - 1\right]$$

So this is what we get, now we can use another relationship that is:

$$\gamma - 1 = \frac{\gamma R}{C_p}$$

If you use this, the above solution turns out to b:

$$T_2 = \frac{\gamma^2}{\gamma+1}\left[T_1 + \frac{q}{C_p}\right]$$

Nnow you will use this one and the previously obtained equation. So, if we use equation 20 and this one 17 and 17 into relationship of v_1 that is in 13 into equation 13, so that's get me:

$$v_D = v_1 = \left[2(\gamma+1)\gamma R\left(T_2 + \frac{q}{C_p}\right)\right]^{1/2}$$

So, if you look at this particular equation is an approximation or approximate equation, because of all simplified assumption employed and also one of the critical assumptions that we have used these P_2 is much higher than P_1. So, this particular one is a simple equation or other approximate equation. Now, if we little bit relax the assumption of let us say constant and equal specification we get more accurate, but still very approximate equation.

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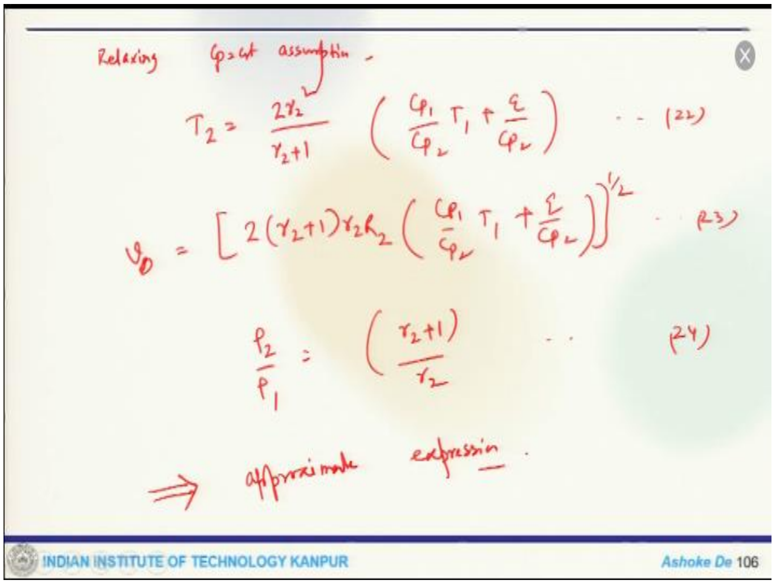

So, if we relaxing C_p equals to constant assumption. So you get slightly more involved or complicated system. So which will get us:

$$T_2 = \frac{\gamma_2^2}{\gamma_2 + 1}\left[\frac{C_{p1}}{C_{p2}}T_1 + \frac{q}{C_{p2}}\right]$$

Which will involve different C_{p1} and C_{p2} which will get us:

$$v_D = \left[2(\gamma_2 + 1)\gamma_2 R_2\left(\frac{C_{p1}}{C_{p2}}T_1 + \frac{q}{C_{p2}}\right)\right]^{1/2}$$

So, that will have a different these things, but still these are all approximate expression. Now, if we go back to this particular curve and look at this, we can comment on these things like.

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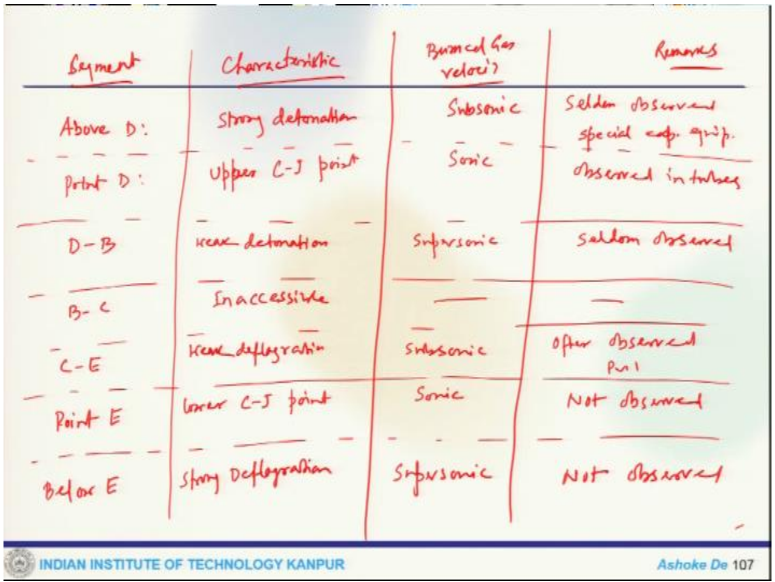

Let us say we take a segment which is above D. So, characteristics is that strong detonation, so you can say segment characteristics burned gas velocity and some remarks it is subsonic. So, you can put some category like this this is seldom observed special experimental equipment you need special experimental equipment to get that particular state. Now, second is we can have at point D which is upper C-J point which is sonic. So, this is observed in tubes. Now the segment D-B, which is weak detonation here speed is supersonic this is also seldom observed.

B-C this is sort of inaccessible location. C-E we get weak deflagration speed is subsonic. This is often observed pressure is order of 1. Point E, this is lower C-J point sonic this is not observed and finally below E strong deflagration. So, this is supersonic, this is also not observed. So, essentially, if you categorize that whole curve that Hugoniot curve, there are different zone and if we just go back to that curve to conclude what we said some point, you will get strong detonation way up and the burned gas velocity would be substantial.

But this is not a situation which is often observed this particular point, this is upon C-J point this is your sonic speed will be the burned gas velocity would be sonic and this is observed, this is your weak detonation D-B. Here, the gas velocity unburned gas will burned gas velocity would be supersonic. This is also not often observed B-C these positions are not accessible at all C-E which is again deflagration wave which is often observed in the system from the burned gas velocity sub sonic and point E this is a lower C-J point so, not observed sonic velocity and below this point rest and strong deflagration of which is supersonic and not observed. So, this is what we have sum ducted in this table. So, that is pretty much what we wanted to talk about this Rankine Hugonoit relation to other that covers all your essential basics for what you think about basic combustion characteristics, thermodynamics and all this?

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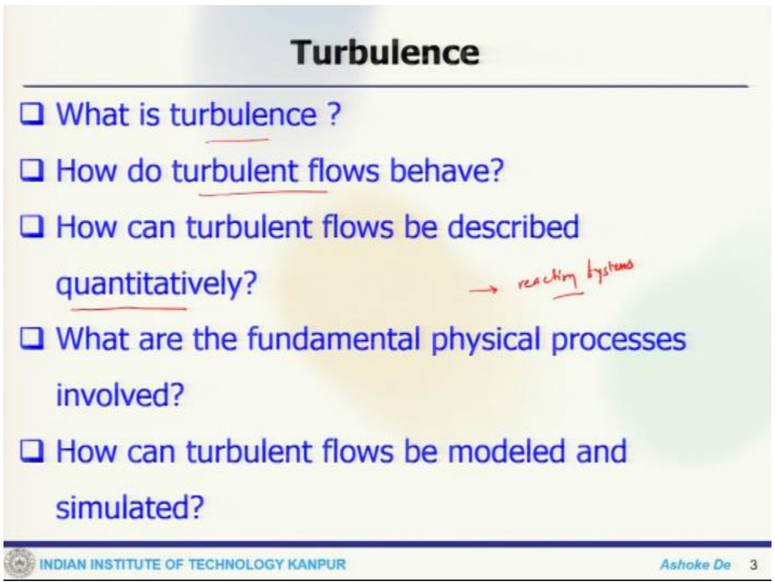

Now we will move to turbulence and there onwards or actually the all discussion would be talking about the turbulent and turbulent reacting system. And to continue our discussion on this, we need to have those basic especially when you talk about how turbulence interact with the reacting flame front. We will see the information of our basic understanding of the combustion and others will come. So, first we will touch upon the turbulence.

And this is what we would go about it. We will first see what turbulence is because this is a very important topic of interest and this is one of the classical still not well understood problems in physics or nature. So we will see how we try to characterize it how do the flows behave? Well, these are important, because once we understand the characteristics of the turbulence flows, then only we can understand how turbulent reacting flows would behave.

So, right now, you may assume what is what we are heading up to basically, we are going to discuss on turbulent reacting system where the important aspect is the turbulence and turbulent flows. Because now, once you have what we have done so far is that all these reacting system under laminar condition. So, the underlying flow physics or the fluid mechanics is quite nicely behaved. But once you come to turbulence, we can see how the behavior is and depending on that behavior, we need to look at how that flow behaves.

Now, then, we can we have to once we understand how the behavior of the flow field, we need to characterize it. So, essentially we have to describe it not only qualitatively also quantitatively, so, that means essentially we need to characterize the flow field quantitatively, which is very, very important because these quantification has a direct impact on the reacting system. So, these quantifications are quite important not only qualitatively it is quantitatively then we will look at what are the fundamental physical processes which are involved.

And then finally, once we are done with this theoretical part of it, we will see the modeling part of it and how things are simulated in the modeling perspective, because this is one of the important things.

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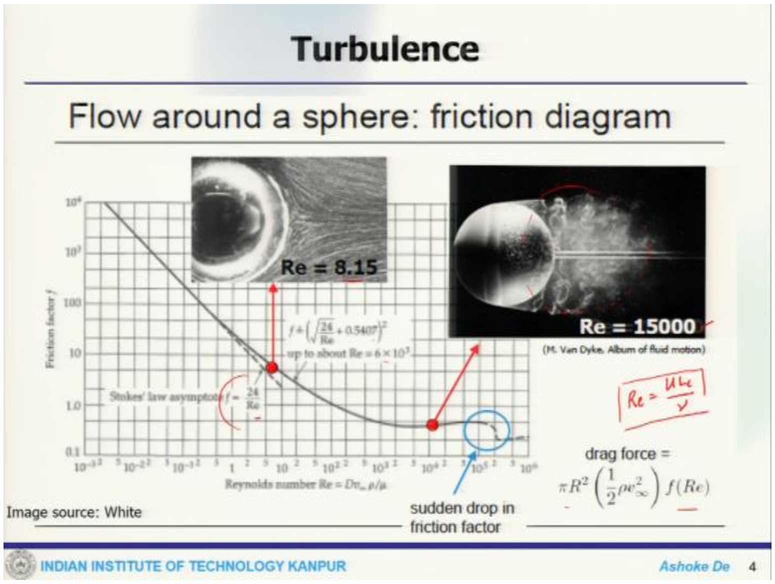

So, this is how one can look at it, it is a simple flow around the sphere and this diagram is well known diagram, if you have done advanced fluid mechanics or something. So, when the flow fast sphere at a high Reynolds number, so, the Reynolds number one of the parameters that we will use upstream velocity characteristics length divided by kinematic viscosity of the flow. So, this Reynolds number is quite important to see the characteristics of the flow whether that is laminar turbulent Re 15000 is absolutely turbulent flow field and if you look at the images.

This is how it looks like there is a lot of this portion if you see it specifically, you can see the randomness in the fluid motion these are smoke visualization. So, which clearly shows the randomness in the motion of the flow field. Now, compared to that if you come to Re is equal to 8.15 if you see this, the flow looks nicely behaved that minutes comes along the sphere go along with the sphere along with a stagnation point and there is a well-known friction curve versus the Reynolds number for this sphere which goes like these are scales like these, then it comes down and then there is a drop.

So, this portion which follows the Stokes law asymptote that is friction factor is 24 by so, these forces which is a linear curve which follow 24/Re. Now, at this point the Reynolds Number laminar and there is a from here to here, this follows in curve, which is:

$$f = \left(\sqrt{\frac{24}{Re}} + constant\right)^2$$

So it is a parabolic curve which follows here for 2 Reynolds Number 6×10^3 then at this position, this becomes completely turbulent. And you can see the friction factor that remains quite flat for some time and this is a sudden drop in the friction factor and the drag force which is experienced by the sphere it would be $\pi r^2 V_\infty$, V_∞ is the upstream velocity and the function of the Re. So, this friction dropping the friction factor is quite important here you start feeling some drag crisis which is often known as a drag crisis.

So, there is a similarity of this particular sphere problem with if you are a cricket lover, there is a nice similarity between the cricket ball and we will discuss as we go along with the discussion of this. We will stop here and continue the discussion in the next lecture.