Turbulent Combustion: Theory and Modelling
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Lecture - 20
Laminar Non-Premixed Flames (cont…)

Okay welcome back, let us continue the discussion on the laminar non-premixed flame. So, we are looking at the jet diffusion flame and doing the detailed analysis and trying to find out the profiles and all these things.

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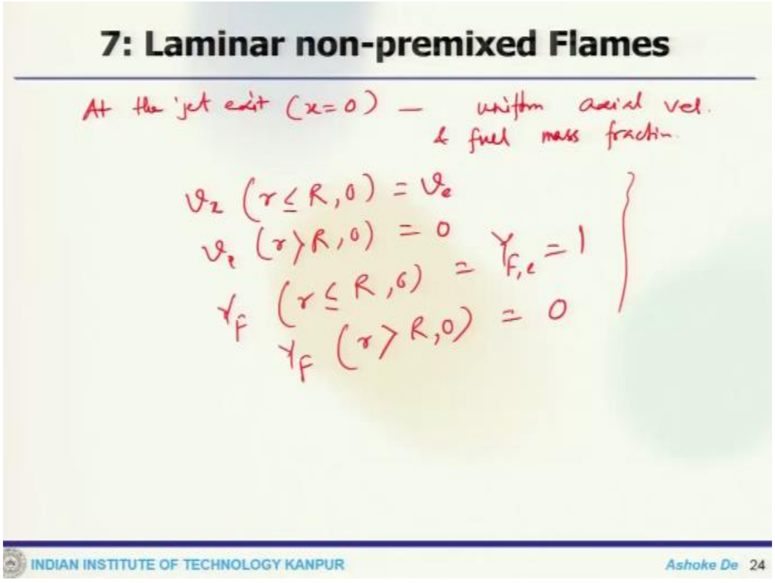

So, this is where we stopped on the boundary conditions and all these things.

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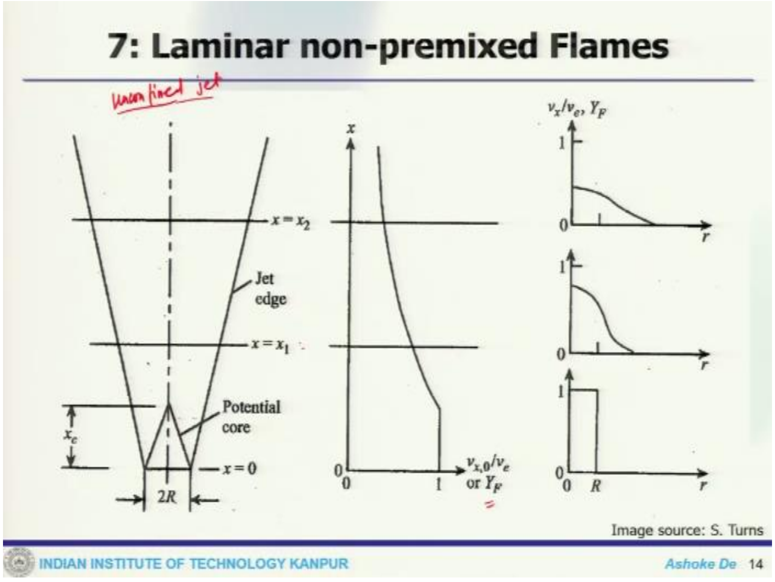

So, this is the jet, unconfined jet. We had fuel coming in and then with some of the assumption.

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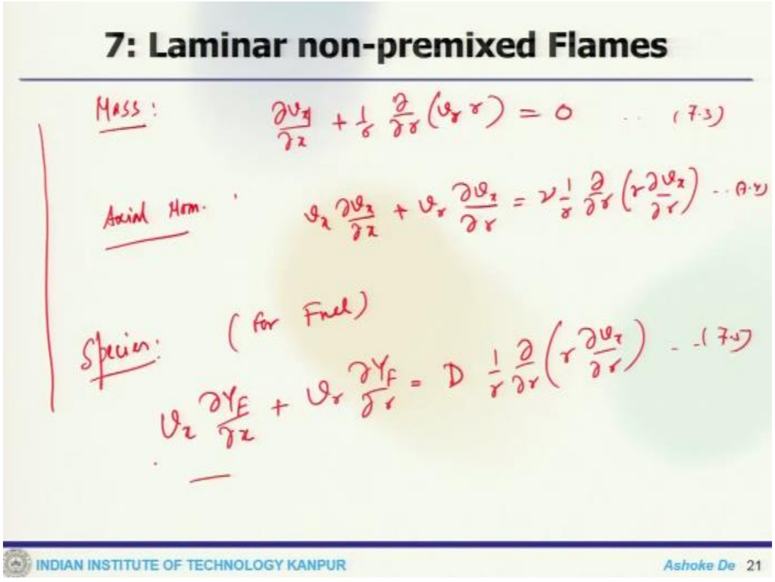

We got to the conservation laws and mass, axial momentum, species.

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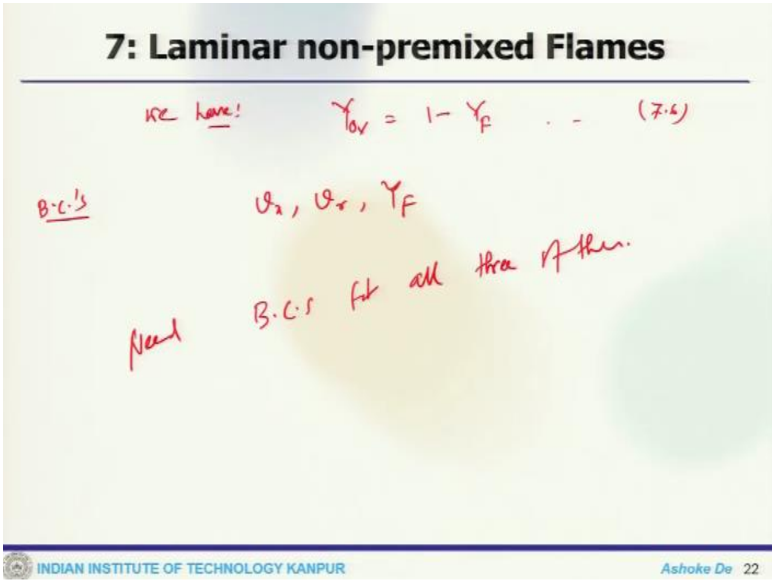

And then we have one from the oxidizer.

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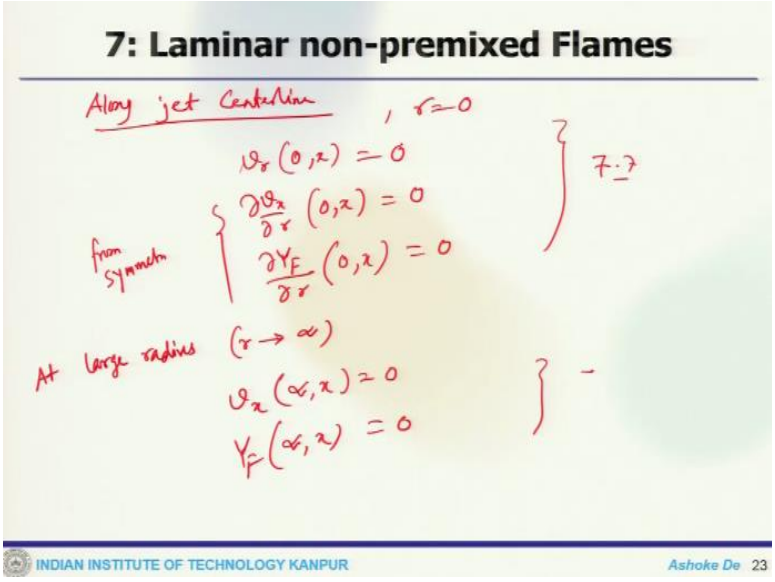

And then we noted down the boundary condition. One of the boundary condition is the along jet center line and then at the large radius, jet center line we have radial velocity is 0 and the derivative said 0 which is from symmetry. At large radius axial velocity is 0, fuel mass fraction is also 0.

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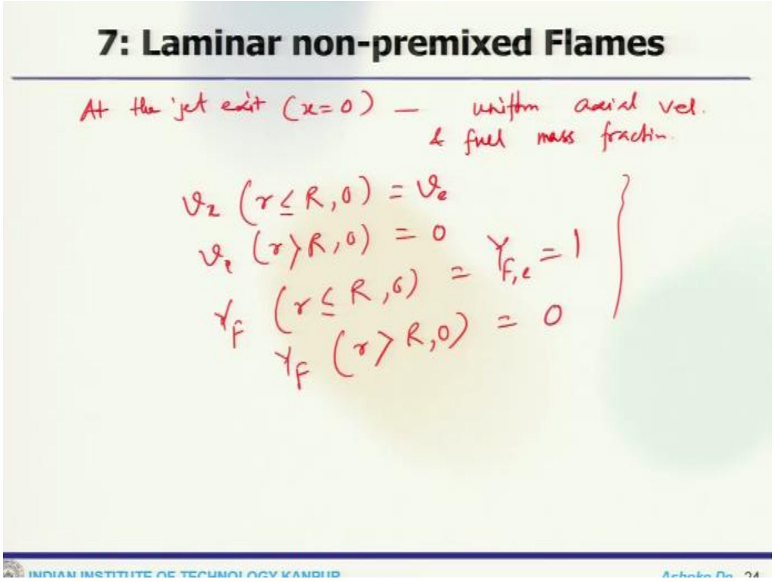

And then at the finally, we have this jet exit where we assume the uniform axial velocity and fuel mass fraction and 0 so that will get you these boundary conditions.

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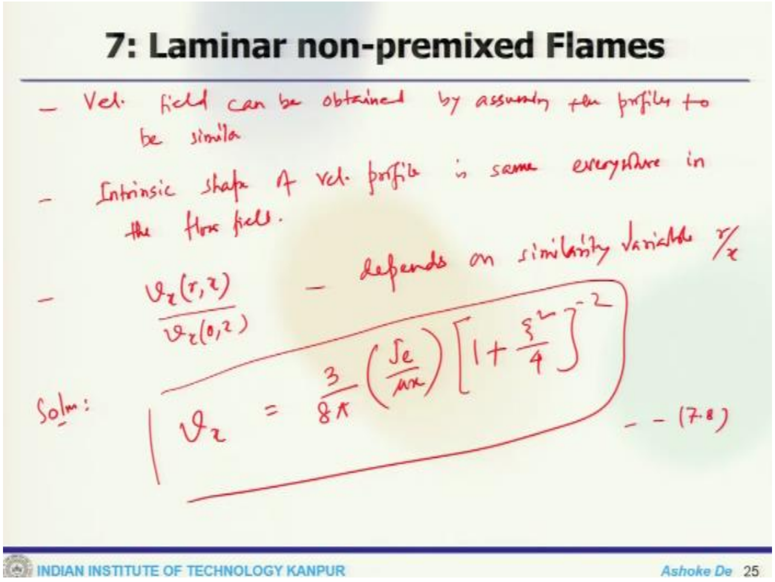

Now for the solution, the velocity field can be obtained by assuming the profiles to be similar. Secondly, the intrinsic shape of velocity profile is same everywhere in the flow field. Thirdly the radial distribution of V_x (r, x) is normalized with the local center line velocity like this is a universal function that only depends on similarity variable that is (r/x).

So, this is a standard jet flow. So, the analytical solution exists for this solution for the axial velocity is:

$$v_x = \frac{3}{8\pi}\left(\frac{\sqrt{e}}{\mu x}\right)\left[1+\frac{\xi^2}{4}\right]^{-2}$$

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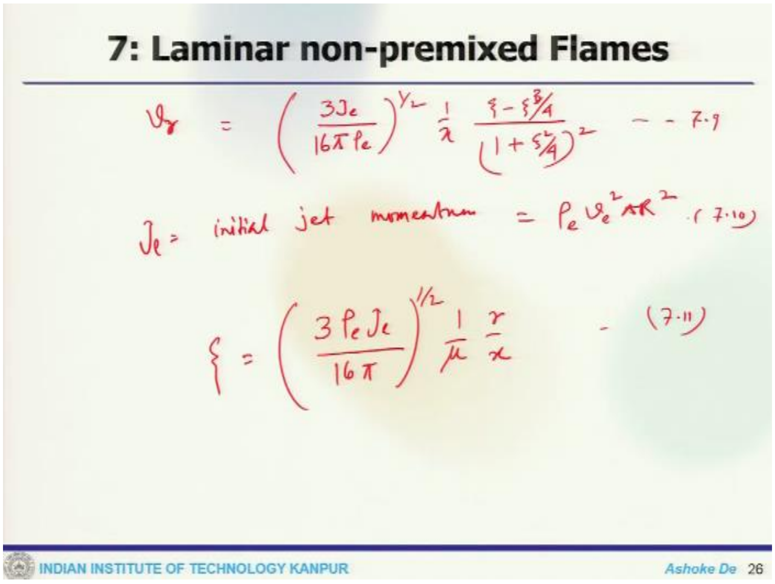

Now, similarly, for radial velocity solution is:

$$v_r = \left(\frac{3Je}{16\pi\rho_e}\right)^{1/2} \frac{1}{x} \frac{\xi - {\xi^3}/{4}}{\left(1 + {\xi^2}/{4}\right)^2}$$

So, Je we have already defined it is the initial jet momentum which is $\rho_e v_e^2 \pi R^2$ and ξ is defined as:

$$\xi = \left(\frac{3\rho_e Je}{16\pi}\right)^{1/2} \frac{1}{\mu} \frac{r}{x}$$

So, these are the definition for initial momentum and the ξ.

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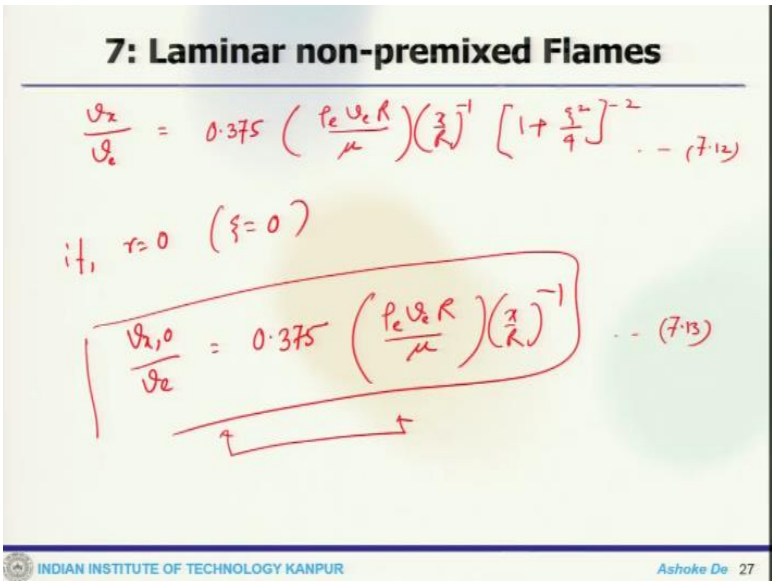

Now, one can demand the axial velocity that means in terms of (v_x/v_e) which is an exit velocity of the jet. So, if you do that this dimensions form will look like:

$$\frac{v_x}{v_e} = 0.375 \left(\frac{\rho_e v_e R}{\mu}\right)\left(\frac{x}{R}\right)^{-1} \left[1 + \frac{\xi^2}{4}\right]^{-2}$$

Now, the dimensionless center line velocity indicates okay, now this can be obtained by setting r = 0 if we put these were ξ = 0:

$$\frac{v_x}{v_e} = 0.375 \left(\frac{\rho_e v_e R}{\mu}\right)\left(\frac{x}{R}\right)^{-1}$$

So, this is what we get now, what it clearly says or shows that this relationship how the center line or velocity decays. The velocity decay actually inversely proportional to the axial distance and jet Reynolds number.

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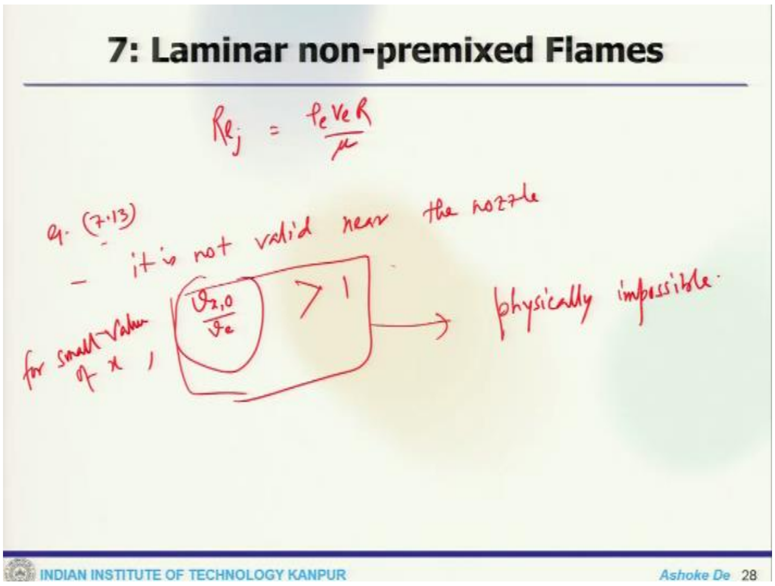

So, the jet Reynolds number is defined as $\left(\frac{\rho_e v_e R}{\mu}\right)$. So, if you put it so, this is proportional to the jet Reynolds number and inversely with the axial distance. Now, from this expression, one can also see that this is the 7.13, it is not valid near the nozzle. Why? Because at a small value of x, the dimension less centerline velocity that is this, this is for small values of x.

This quantity becomes greater than 1 and this is a condition which is actually not possible or rather one can say it is a physically impossible scenario. So, this cannot happen.

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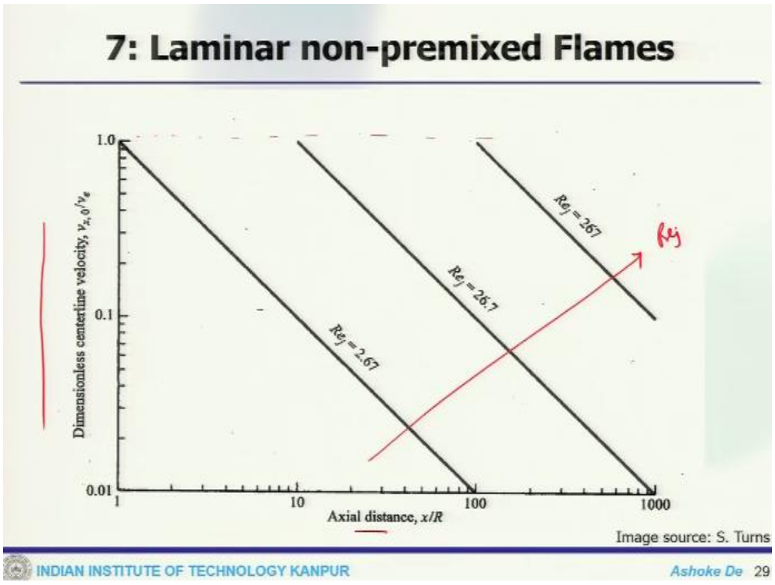

So, that one can see these dimensionless centre line velocity along this axis which will plot dimensionless centre line velocity and the axial distance. So, you can see for this size is the increasing jet Reynolds number and with the increasing jet Reynolds number, you can see the pattern because it should not increase or goes beyond one which is not physical and also feasible too.

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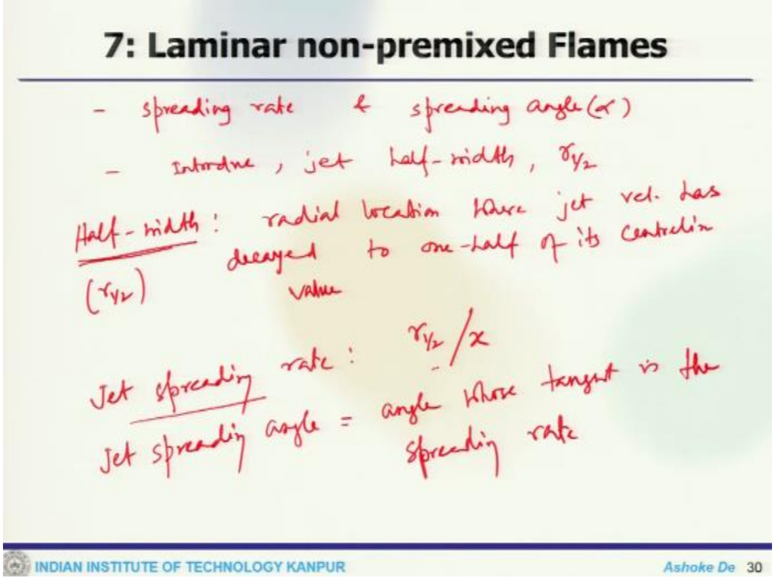

Now, so, the other parameters which are used to characterize the jets are the one is spreading rate and spreading angle which is α. So, these are 2 more important parameters which are often used to define or characterize the jet spreading jet. Also we can introduce jet half-width which is $r_{1/2}$. Now what is half-width? Half-width is the radial location where jet velocity has decayed to one half of its center line value.

That means along the radial directions it is basically width, so you find the radial width where the jet velocity decays to one half of the center line value, this is what one can form as a half-width. Now, this $r_{1/2}$ or half width can be derived by setting this ($v_x/v_{x,0}$) So, then one can say jet spreading rate is nothing but $\left(r_{1/2}/x\right)$. So, one needs to find out the half-width and jet spreading angle is the angle whose tangent is the spreading rate. So, Jet spreading rate would be $r_{1/2}/x$ and get spreading angle would be angle whose standing the jet spreading rate.

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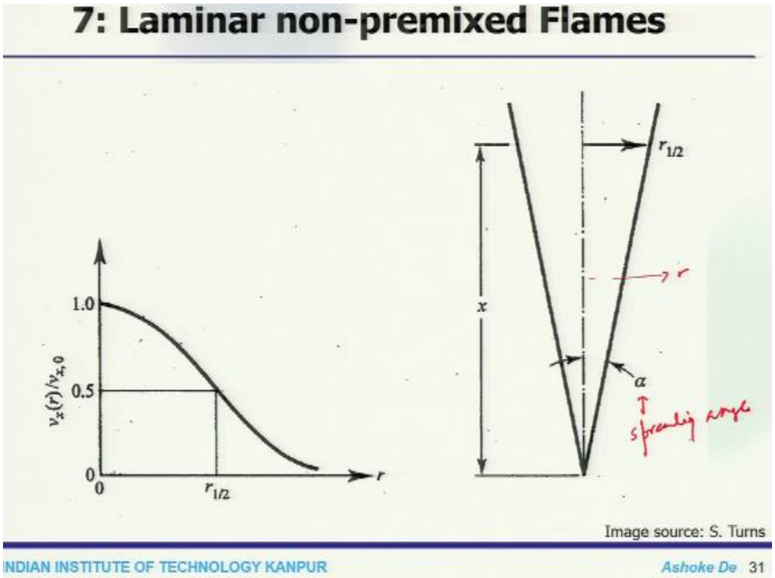

So, you can see this is our spreading angle and this is the half with and then along these are $r_{1/2}/x$ will give you that jet spreading rate. So, one can look at that thing and this is my $v_x/v_{x,0}$, how it varies with width r that means along these directions, radial direction. So, these 2 quantities, I mean rather I would say 3 quantities, one is the spreading rate, spreading angle and half width they are very important when you characterize the jet or other jet close.

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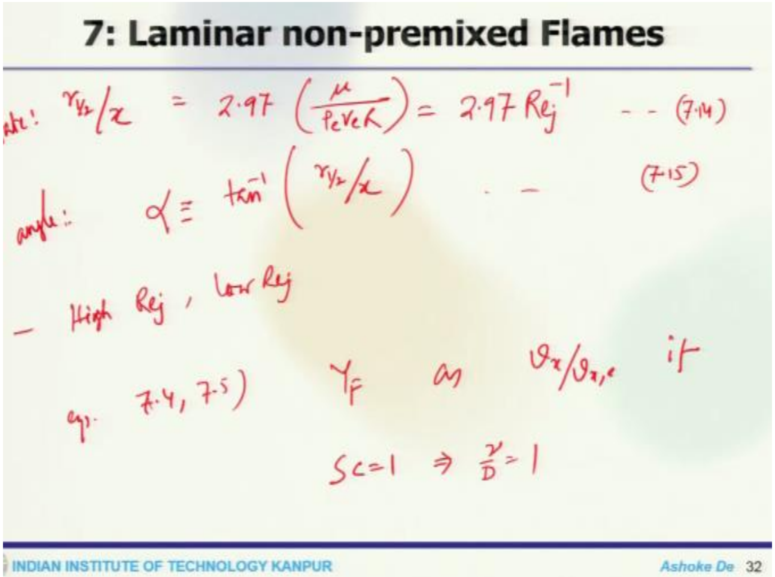

So, the spreading rate one can estimate it would be:

$$\frac{r_{1/2}}{x} = 2.97\left(\frac{\mu}{\rho_e v_e R}\right) = 2.97 Re_j^{-1}$$

And the spreading angle will be:

$$\alpha = tan^{-1}\left(\frac{r_{1/2}}{x}\right)$$

So that is how you get one is rate spreading rate, one is spreading angle. So, at high Re_j that means jet Reynolds number on narrow while low Re_j are white which one can see because this is inversely proportional.

So, if you have high Re_j and low Re_j you can see the difference between the spreading rates for a low Reynolds number case. The spreading would be more for high Reynolds number case, the spreading would be low that primarily because it is inversely proportional. Now, we also know that when you look at the 2 equations 7.4 and 7.5, Y_F plays a same mathematical role as $v_x/v_{x,e}$ if SC is 1 that is the Schmidt number which means ${}^{\mu}/_{D}$ is 1.

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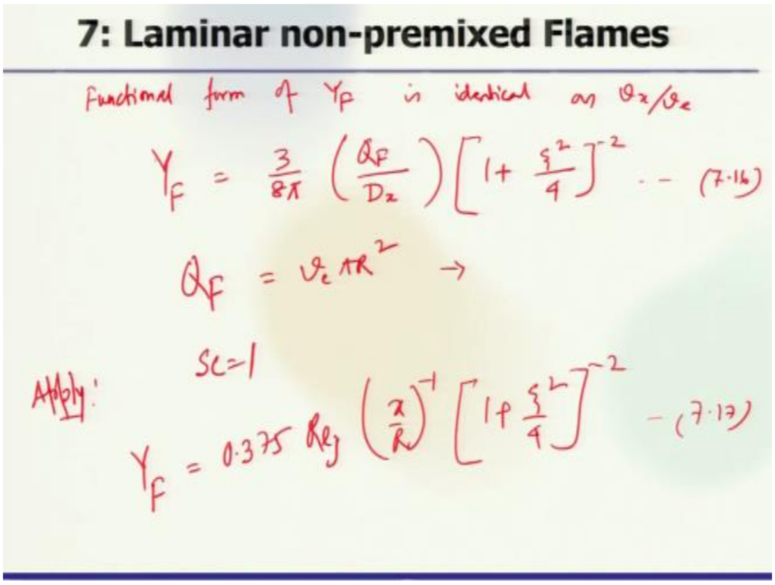

Now, if that is the situation, we can find out the functional form of the solution for Y_F. So, the functional form of Y_F is identical as $v_x/v_{x,e}$. So you can write:

$$Y_F = \frac{3}{8\pi}\left(\frac{Q_F}{D_x}\right)\left[1 + \frac{\xi^2}{4}\right]^{-2}$$

Where fuel flow rate is $v_e \pi R^2$. So this is your volumetric fuel flow rate. Now we apply the condition Schmidt number = 1 to this equation. So, that gives back:

$$Y_F = 0.375 Re_j \left(\frac{x}{R}\right)^{-1}\left[1 + \frac{\xi^2}{4}\right]^{-2}$$

So, that's the functional form.

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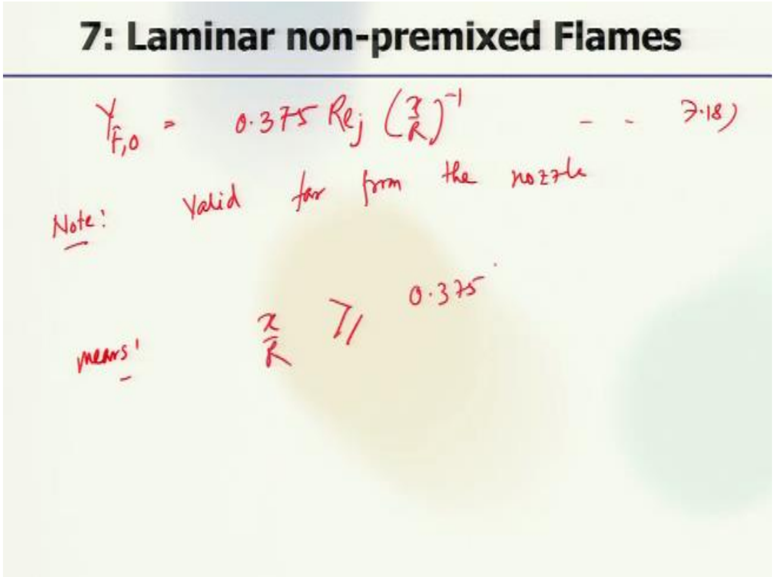

Similarly, one can find out the center line values of the mass fraction. So, the center line values of mass fraction would be:

$$Y_{F,0} = 0.375 Re_j \left(\frac{x}{R}\right)^{-1}$$

It is similar to the velocity program. Now again, one has to note all these solutions are valid far from the nozzle. The dimensionless distance downstream or the solution is valid must exceed the jet Reynolds number, so which means x/R should be greater than this.

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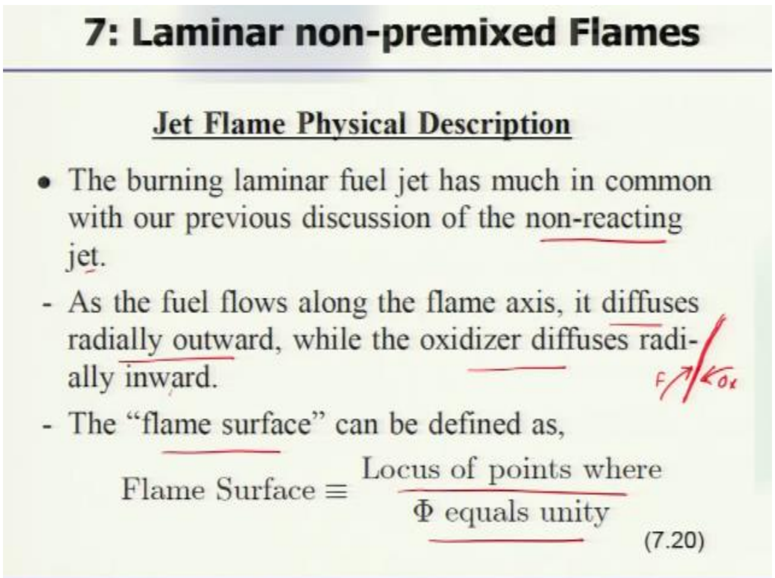

Now, once we do this, we can look at the jet flame behavior. So, now the burning laminar fuel jet is quite similar to the discussion that we have done for the non-reacting rate and to get into this burning jet that we have first looked at how the jet flames and the flows looks like, how central line velocity profile changes and all these characteristics. Now, since the fuel is flowing along the axis, it obviously diffuses radially outward.

And the other hand oxidizer diffuses inward. So, that means, if fuel goes like this oxidizer come like this. So, in that contact point you get the flame. So, this is exactly what the flames surface is the locus of points were φ equals to unity that means along the stoichiometric condition so when one goes towards outside one come towards inside.

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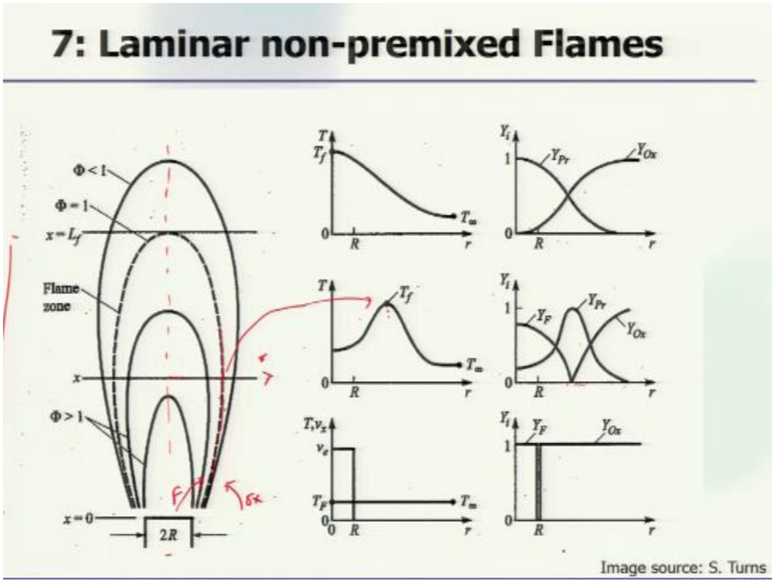

Now, this is the burning jet, this is the jet diameter and jet exit or the nozzle exit and this dotted line. So, fuel is coming from the side. So, fuel goes like this oxidizer come from that side when they meet at a point that is the locus of the flame surface and this is the flame surface where it is stoichiometric. Outside of that, this is lean condition because fuel is going from that side so, it is lean φ less than 1.

These two lines corresponding φ greater than 1 and this height is called a flame height, so this is the flame height. Now, if we look at the temperature profiles at this particular location and this is at the jet exit location. So, jet exit location you have a velocity which is uniform, temperature is uniform, fuel mass fraction is also 1 and this is outside the jet it is all oxidizer so, that is 1 and this position there is nothing.

At a sudden downstream location, this is temperature is varying. So, this position corresponding to the location of the radial location. If we look at here, this is r direction and this is the location which corresponds to T_f flame surface. So, this corresponds to that and this is how my Fuel comes down oxidizer comes down and product forms go further downstream, this is my temperature will vary because now this is exactly at the tip of the flame height.

So, this will be the variation of the product and this is the variation of the oxidizer. So, once you have an idea about the evolution of this jet, you can also identify these characteristics and the profiles how that looked like and this should also help you to understand a lot about laboratory scale burner or the unconfined burners which you use later on for the analysis and see how that evolves and all these things.

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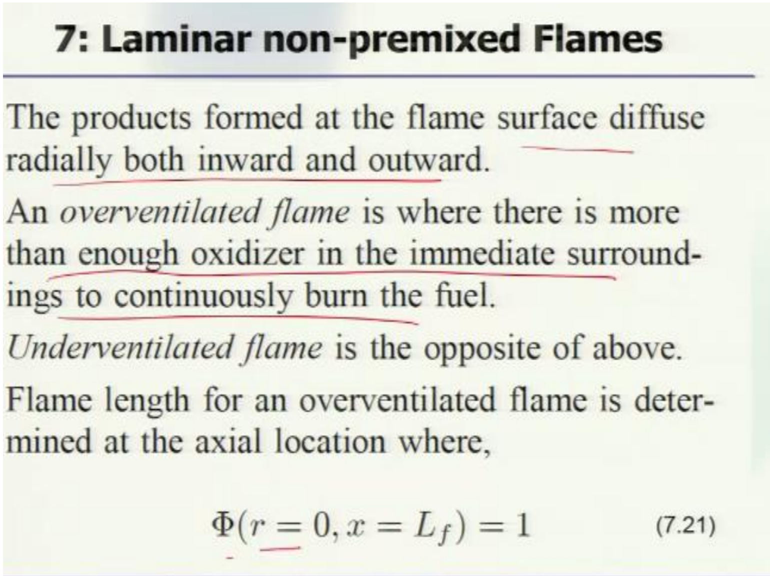

Now, some points which can be noted here. One is that the products which are formed at the flame surface that diffuse radially both inward and outward. Obviously that would happen because products can go either side from this flame surface what is produced. This can go this side this can go that side. Now, over ventilated claim is where there is more than enough oxidizer in the immediate surroundings to continuously bond the fuel and obviously the opposite to that would be the under ventilated flame.

Now, the flame length for an over ventilated flame can be determined at a location where $\varphi(r = 0, x = L_f)$ and the pi is isometric.

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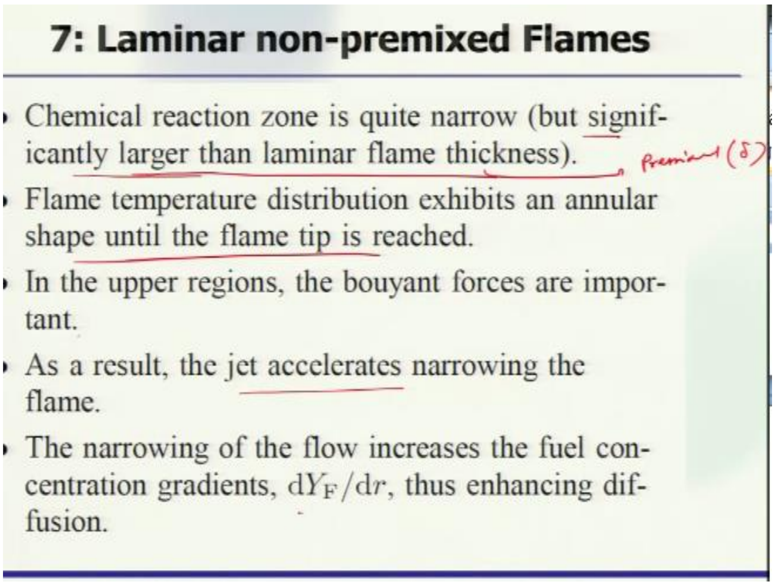

Now, the zone of chemical reaction is quite narrow, but one can see that the laminar flame thickness but significantly larger than that. So, this is corresponding to your premixed flame thickness which is δ. Typically, the premixed flame thickness is quite small and that creates some challenges for the modeler. But in the diffusion case you get in bigger reaction flame.

Now, flame temperature distribution also exhibits an annular shape until the tip is reached and the upper sides of the buoyant forces are quite important. So, what it does is jet accelerates narrowing the flame and narrowing the flow increases the fuel concentration gradient. So, that in turn enhances the radial diffuser.

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So, these are some images of flame. This is a nice laminar flame obviously, this is not laminar this is turbulent, but this is laminar and one can see clearly the difference between these 2 flame and what is quite chaotic in nature here. So, you get different reaction zone and in these case you get a nice shape.

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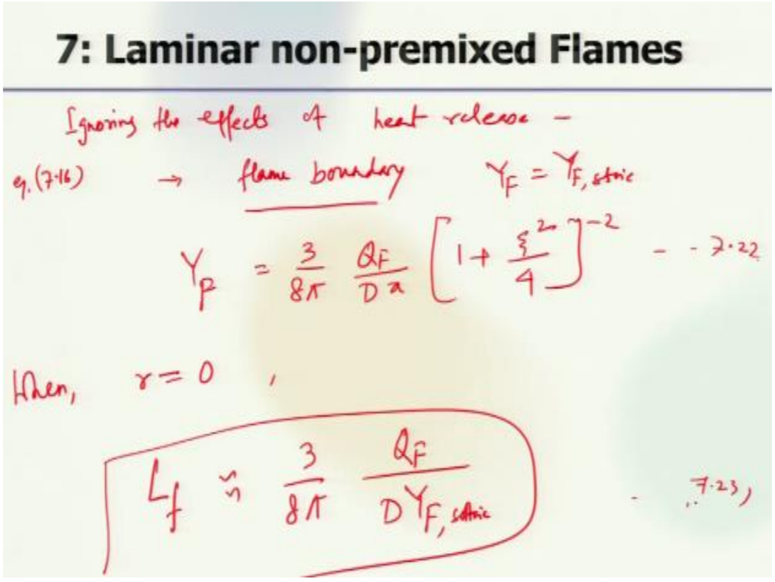

Now we can ignoring the effects of heat released due to reaction. Then from 7.16, one can get in very crude description of the flame boundary when:

$$Y_F = Y_{F,stoi}$$

So, this is a very crude description because what we are ignoring is the effect of the heat release which is not possible in a realistic situation. In that case, this would boils down to:

$$Y_F = \frac{3}{8\pi}\frac{Q_F}{Dx}\left[1+\frac{\xi^2}{4}\right]^{-2}$$

Now, when r equals to 0, we get the flame length and that is:

$$L_F = \frac{3}{8\pi}\frac{Q_F}{DY_{F,stoi}}$$

So that is what you get by assuming that you are ignoring the effect of the flame.

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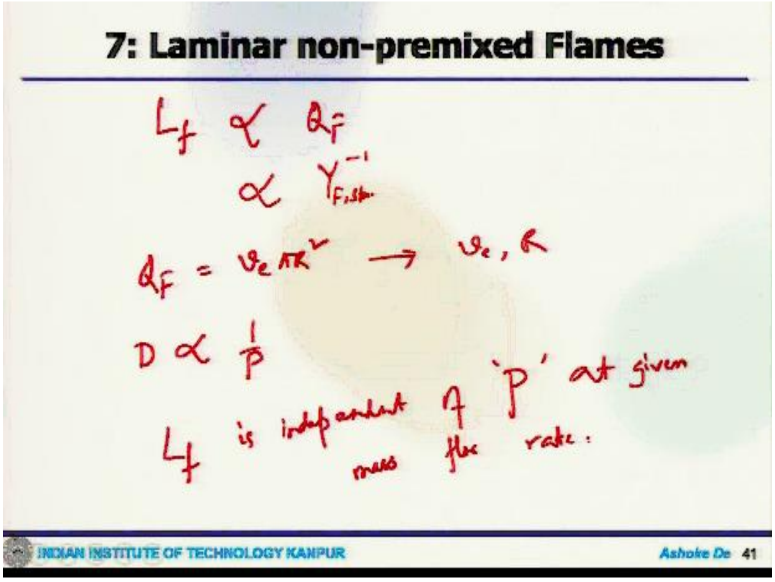

Now the flame length what you see it is proportional to the volumetric flow rate of the fuel. At the same time, it is inversely proportional to the stoichiometric fuel mass fraction. Now, since we have volumetric flow rate of the fuel is $v_e \pi R^2$. So various combination of v_e and R will be the same flame length. Since the diffusion coefficient D is inversely proportional to the pressure, the height of the flame is is independent of pressure at given mass flow rate. So, that is how you get the analysis of the flame height in a jet diffusion flame. We will stop here and continue the discussion in the next lecture.