**Turbulent Combustion: Theory and Modeling**
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Lecture-19
Laminar Non - Premixed Flames (contd…)

Welcome back, let us continue the discussion on laminar non-premixed flame. So, what we started off this that the looking at different flame structure and we have seen how the flame structure changes when the flow become turbulent starting from laminar. And now we will go into some detail analysis of those kind of structure.

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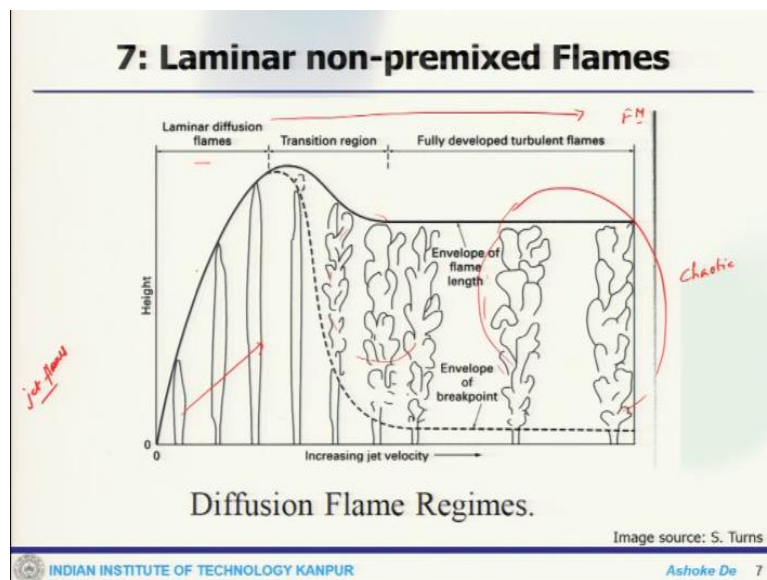
So, if you recall, this is the flame regime that we talked about starting from laminar diffusion system to the completely turbulent flame you can see nice structure here then when it goes to and this is primarily because of your fluid mechanics, how the underlying flow is behaving, if it is laminar you get nice flame structure and also the increasing emphasis and this is obviously for a jet flame, simple jet flame it is increasing, but then once it becomes transition takes place.

So, the structure gets distorted in this zone and it becomes completely curve take when it goes to turbulent zone, so, we can do some analysis of that.

Now the simple approach for a simple laminar diffusion flames on circular nozzle which is similar to a candle kind of flame. So, what we use is the flame height to characterize it. So, this is the important parameter that is used to characterize the flame. Now for simple treatments, reaction zone is defined as the region where the fuel and air mixture is stoichiometric.

So, that will tell you how you define the reaction zone that means, this is your fuel coming in, this is air comes in and when the reaction takes place, that is the zone of stoichiometric obviously, the assumption lies there. When this will be occurring over an extremely wide range of fuel and air ratios. Now diffusion process is also correct determining so that the rate of reaction is directly related to the amount of fuel and oxygen diffusing into the reaction zone that primarily happens because we inject fuel and oxidizer or the air separately into the system.

So, once we do that, fuel and air. So how quickly they diffuse to each other that will actually determine the rest of the reaction characteristics and all these.

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Now we assume a simple conical laminar diffusion flame, here molecular diffusion is considered only in radial direction. So, what is considered is that that molecular diffusion is restricted to radial direction only. So, that is what we start off. Secondly, take a conical flame where diffusion is restricted to radial direction only now, average square displacement which is also one can gain form the Einstein diffusion equation. So, this is the average distance is 2Dt. So, where height of the flame is taken as the point where average depth of penetration. So, average depth of penetration equal to tube radius. So now why one can approximate this y to the radius of the tube, if R is the radius of the tube then one can write this as:

$$t = {R^2}/{2D}$$

So, why, here y is approximated to the radius of the tube.

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Now we have since we have some other guys that time is (L_f/v). Then we can write L_f as:

$$L_f = \frac{vR^2}{2D}$$

So, once you get the flame height, you can find out the volume flow rate. So, volume flow rate estimation can be made which is:

$$Q_F = v\pi R^2$$

Now, if you use these to approximate the flame height is the volume flow rate divided by πD. So, that means if you know the volume flow rate and know the radius of the tube, you can find out the flame height.

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Now, although it is quite good approximation, it has some predictions. Number 1: at a given flow rate, flame height is independent of the burner diameter. Now, secondly, since the diffusion coefficient D is inversely proportional to pressure. So, the height of the flame that is a L_f is also independent of pressure at given mass flow rate whereas, this guy is directly proportional to the volume flow rate of the flame. So, Q_F is the volume flow rate of fuel. So, that is what this F stands for.
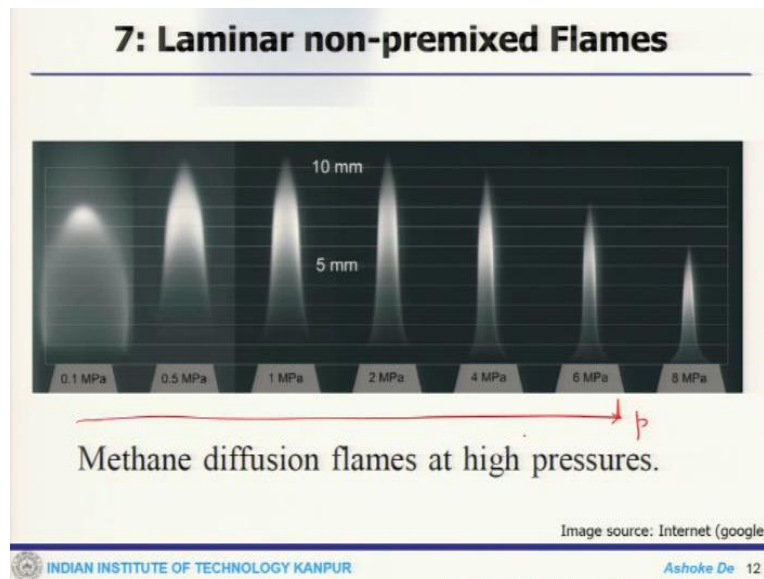
Now, this is an example of Methane diffusion flame at high pressure. Now, you can see how this side, the pressure is increasing and then you can see the different flame structure. So, this clearly shows you the effect of the pressure on the flame structure and also on image.

Now, we can consider the non-reacting constant density laminar jet. So, this is again a jet flame that will consider and try to do some analysis. Now, one of the thing is that the analysis that we looked at just previously that provides only some qualitative feature of laminar diffusion flame. So, it is not that very quantitative, but one would like to always have some quantitative description of the flame.

Now, to have a detailed understanding of this reacting jet. So, we will consider this constant density situation and then do the analysis. One important point here to be noted the basic flow and the diffusional process.

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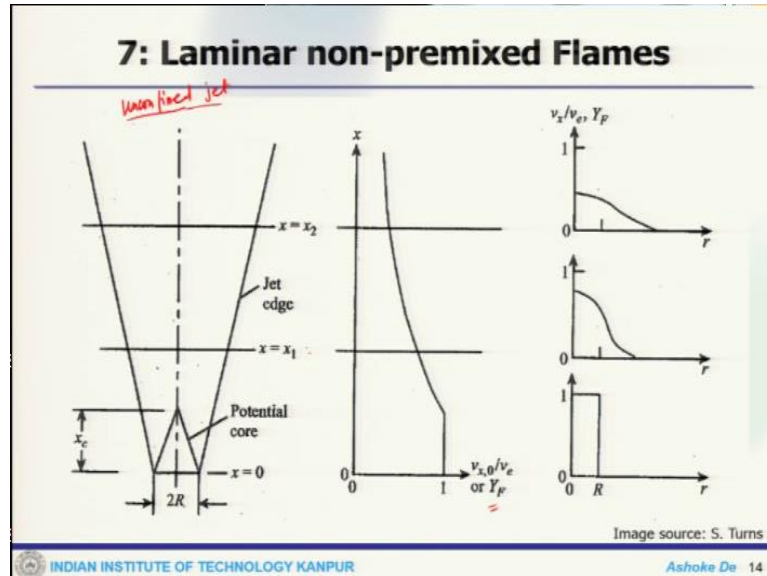
So, if you try to define the problem schematically this is how one can do. This is the diameter of the jet, this is how the jet core evolve and this is again the edge of the jet because there would be some shear layer because fuel free jet or unconfined jet. So, if you have unconfined jet it actually fuel this free shear layer and this is the height of the potential core and this is the potential core and this is the burner to get exit where the location starts. Now, we can look at this fuel mass fraction how velocity varies at the different downstream location for varying x.
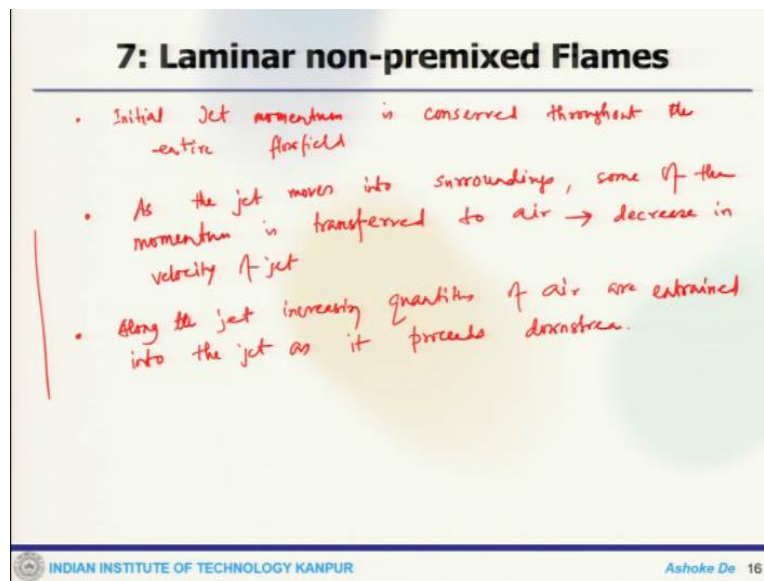
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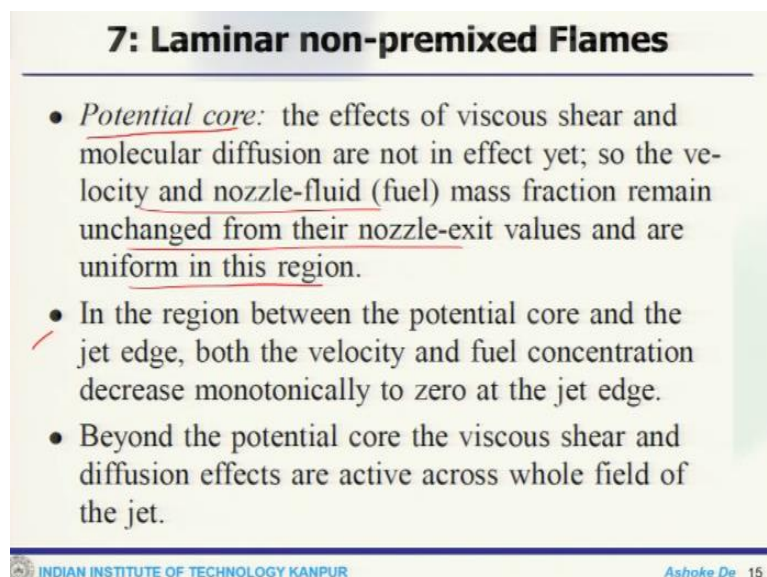
Now, potential core, this is the height of the potential core. What does that mean? Physically, potential core means that effects of the molecular diffusion are not in effect yet. So, the velocity and nozzle fluid mass fraction remain unchanged from their nozzle exit values and are uniform in this region. So, this is how one can define the potential core.

Now, the region between the potential core and the jet edge means, this region between the potential core and the jet edge where the velocity and fuel concentration decreases monotonically is zero at the jet edge. So, this is how one can look at it the velocity and the fuel must record when it comes to the jet edge the monotonically decreases now, beyond the potential core. So, this is what it is beyond the potential core that we discuss here and the diffusion effects are active across whole field of the jet that means, beyond this, this will be quite dominated by both the viscous effect and the diffusion effect.

(Refer Slide Time: 13:21)

Now, what happens that your initial jet momentum is conserved throughout the entire flow field now. Secondly as the jet moves into surroundings, some of the momentum is transferred to air. So, which will lead to the decrease in velocity of jet. Third along the jet increasing quantities of air are entrained into the jet as it proceeds downstream. Now once you have this, you can mathematically express all these things using the simply mathematical expression from the momentum equation.

So, if someone write that:

$$2\pi \int_0^\infty \rho(r,x)v_x^2(r,x)dr.r = \rho_e V_e^2 \pi R^2$$

Here, this is the momentum flow of the jet at in x-direction. So, one can define this one as this is the momentum flow as J and this is momentum flow is in it from the nozzle. So, J_e here e actually stands for the jet exit or the nozzle exit condition. Now, these processes that control the diffusion and convection a momentum are similar to that process which controls the fuel concentration field. So, essentially that is your convection and diffusion of fuel mass.

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Now, one can look at the distribution of fuel mass traction, which will be a profile of Y_F function of r and x. So, this should be similar to the velocity distribution (v_x/v_e), so this is a normalized velocity profile. Now fuel molecules actually diffuse rapidly outwards as per Fick's law of diffusion. Now the effect of moving downstream is to increase time value for diffusion.

So, this will happen. So, that effect for moving downstream is to increase. Now the width of the region containing fuel grows essentially with x and centerline fuel concentration decay also the mass of the fuel issue from the nozzle is obviously constant because fuel mass that is also conserved otherwise that will violate the mass conservation principle.

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So, if we derive that it should be:

$$2\pi \int_0^\infty \rho(r,x)v_x(r,x)Y_F(r,x)r\,dr = \rho_e V_e \pi R^2 Y_{F,e}$$

Here $Y_{F,e}$ is 1 because the fuel mass fraction is 1. So, now to get the velocity and fuel mass fraction to get the analysis term we need some assumption or we simplify this. Number one: molecular weight at the exit will be equal to the molecular weight infinity pressure is constant, temperature is also constant and uniform density flow. Secondly, species transport is by Fick's diffusion only.

Third: momentum and species diffusivities are not only constant, they are equal. So, which means there is a non-dimensional number called Schmidt number which is $\left(\vartheta/D\right)$ is 1. So, that means the mass diffusion and the momentum diffusion are of the same order or they are equal. Number 4: diffusion is only considered in radial direction not in the actual direction. So, which means the axial diffusion is neglected.

Now, when there is a jet flow this particular assumption is not in very good assumption spatially this will incur some error near jet exit. Since, near the jet exit it is expected that the axial diffusion will be significant, this assumption will not be good there, but otherwise downstream of the flow field, this seems to be a reasonable assumption. But one has to be careful where it actually plays so close to jet exit. It is primarily dominated by axial diffusion. So there this will not one good or rather this is not a good assumption.

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Now you can write our laws. First thing is the mass conservation, so we write:

$$\frac{\partial v_x}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(v_r r) = 0$$

Now, axial momentum conservation equation which will be:

$$v_x\frac{\partial v_x}{\partial x} + v_r\frac{\partial v_r}{\partial r} = \vartheta\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_x}{\partial r}\right)$$

And species conservation equation that is only for fuel which will be:

$$v_x\frac{\partial Y_F}{\partial x} + v_r\frac{\partial Y_F}{\partial r} = D\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)$$

So, these are the conservation equation we get and only thing we write here the species conservation equation for fuel only.

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So, we will get in addition to that we have one more thing is the oxidizer which will be (1 – fuel). Now to solve this setup equation one need boundary condition. So, the boundary conditions are there are unknown in these equations v_x, v_r and Y_F. So, we require boundary conditions each for v_x, v_r and Y_F. Otherwise we cannot solve it. So, we need boundary conditions for all 3 of them.

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Now we go at a time fasting along jet center line which is r=0, this is v_r is 0, $\frac{\partial v_x}{\partial r}$ is also 0. $\frac{\partial Y_F}{\partial r}$ is also 0. Last these 2 things you can write from symmetry. Now, at large radius that means r tends to infinity, v_x will be 0, Y_F will also be 0.

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Now, at the jet exit we get x equals to 0. So, here what we assume is that uniform axial velocity and fuel mass fraction and 0 values or else. So, what do you get:

$$v_x(r \leq R, 0) = v_e$$
$$v_r(r > R, 0) = 0$$

Similarly, for fuel mass fraction:

$$Y_F(r \leq R, 0) = Y_{F,e} = 1$$

And

$$Y_F(r > R, 0) = 0$$

So these are same. Now these are the set of boundary conditions that you need. And now we can look at the solution procedures. So, we will stop here today and take it on the next lecture.