**Turbulent Combustion: Theory and Modeling**
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Lecture-16
Laminar Premixed Flames (Contd...)

Welcome back and let us continue the discussion on this premixed flame analysis where we are doing the analysis of a simplified system and based on the assumption we are taking a 1D premixed flame.

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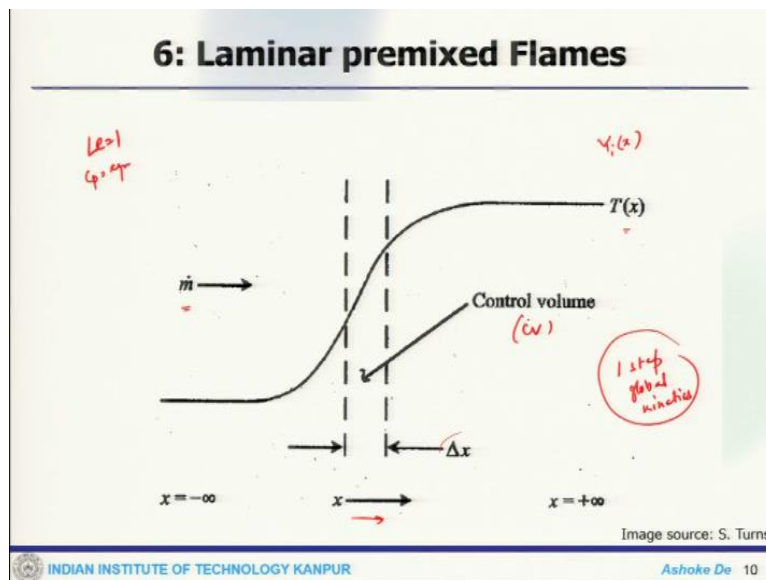
And this is where we stopped and just to quickly bring about there are certain assumptions and which are there one dimensional steady flow there is no pressure change across this flame front. Then Lewis number is 1, C_p all are equal single step chemistry and this is my control volume.

Now so first thing that will start our mass conservation which is an important thing so it will be:

$$\frac{d}{dx}(\rho V_x) = 0$$

Or one can write as to:

$$\rho V_x = constant$$

Now that is my mass conservation. Now we will look at the species conservation, so this is for individual species this is my species conservation because individual species they will react and now when I apply now Fick's law of diffusion.

So if we apply Fick's law of diffusion what we get with the application of Fick's law of diffusion we get:

$$\frac{d\left[\dot{m}''Y_i - \rho D \frac{dY_i}{dx}\right]}{dx} = \dot{m}_i''$$

Now what we have assumed it is a single step reaction, so that is our simplified analysis single step global reaction, so you can think about 1 kg of fuel, ν kg of oxidizer that will give ν + 1 kg of product. So, I can write:

$$\dot{m}_F''' = \frac{1}{\nu}\dot{m}_{ox}''' = -\frac{1}{\nu+1}\dot{m}_{pr}'''$$

Okay? So you get this fuel oxidizer mass flux and the products mass flux equated.

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Now we can write also in terms of individual species because now individual species mass transfer equation if we write, so how it looks like for fuel it would be:

$$\dot{m}''\frac{dY_F}{dx} - \frac{d\left(\rho D \frac{dY_F}{dx}\right)}{dx} = \dot{m}_F'''$$

And for oxidizer if we write:

$$\dot{m}''\frac{dY_{ox}}{dx} - \frac{d\left(\rho D \frac{dY_{ox}}{dx}\right)}{dx} = \nu\dot{m}_F'''$$

So one can notice here in the diffusion law we have D which is constant because that is one thing that we have assumed they are going to be equal it.

Now similarly we can write for the products, so this would be:

$$\dot{m}''\frac{dY_{pr}}{dx} - \frac{d\left(\rho D \frac{dY_{pr}}{dx}\right)}{dx} = -(\nu+1)\dot{m}_F'''$$

So everything is written in terms of $\dot{m}_F'''$. That means if we know one of the mass flow rate whether it is a fuel or oxidizer something we can find out the ratio, so we got species mass conservation equations in terms of individual species like fuel oxidizer and product, now we will write the energy conservation equation which is:

$$\dot{m}''C_p\frac{dT}{dx} - \frac{d\left(\rho D C_p \frac{dT}{dx}\right)}{dx} = -\sum h_{f,i}^0 \dot{m}_i'''$$

So that is the contribution for the species consumption or due to the reaction, so that is mine now this right hand side of this equation one can write in a slightly different format.

Like I can simplify that thing like:

$$-\sum h_{f,i}^0 \dot{m}_i''' = -\left[h_{f,F}^0 \dot{m}_F''' + h_{f,ox}^0 \nu \dot{m}_F''' - h_{f,pr}^0 (\nu + 1)\dot{m}_F'''\right]$$

Which in term becomes:

$$-\sum h_{f,i}^0 \dot{m}_i''' = -\dot{m}_F'''\Delta h_c$$

So that is in simplified way one can basically represent that right hand side where Δh_c is nothing but your heat of combustion and one can write it as:

$$\Delta h_c = h_{f,F}^0 + h_{f,ox}^0 \nu - h_{f,pr}^0 (\nu + 1)$$

So this is how one can represent that now we have another assumption which is Lewis number 1, so that tends you κ is $\rho D C_p$. So, using that we can rewrite 6.7 a.

And what one can write now:

$$\dot{m}''\frac{dT}{dx} - \frac{1}{C_p}\frac{d\left(\kappa\frac{dT}{dx}\right)}{dx} = -\dot{m}_F'''\Delta h_c \frac{1}{C_p}$$

So using the other thing now whole idea here is to or the rather the objective is to find an expression for laminar flame speed. Expression for laminar flame speed that is what we want to do now one can correlate that or relate to that the mass flux $\dot{m}''$ like:

$$\dot{m}'' = \rho_u S_L$$

This is a very interesting expression where ρ_u stands for density of unburnt mixture and S_L is your laminar flame speed, so approach is to assume a temperature profile that must satisfy the boundary condition and then we can integrate this equation, so you can assume a temperature profile and which will obviously satisfy the boundary condition then this temperature equation if we integrate then we can get that.

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Now what are the boundary condition if we look at those boundary conditions first at the upstream you have T which x extends to negative infinity is T unburnt and gradient would be when x tends to negative infinity gradient is zero. Now that is enough upstream, now downstream at the product side you have T extends to positive infinity equals to T burnt and gradient while extends to positive infinity is zero.

So 6.9 c to d, so this is the situation this is your upstream and this is your this is for example temperature profile this is downstream. So, you get these 2 condition.
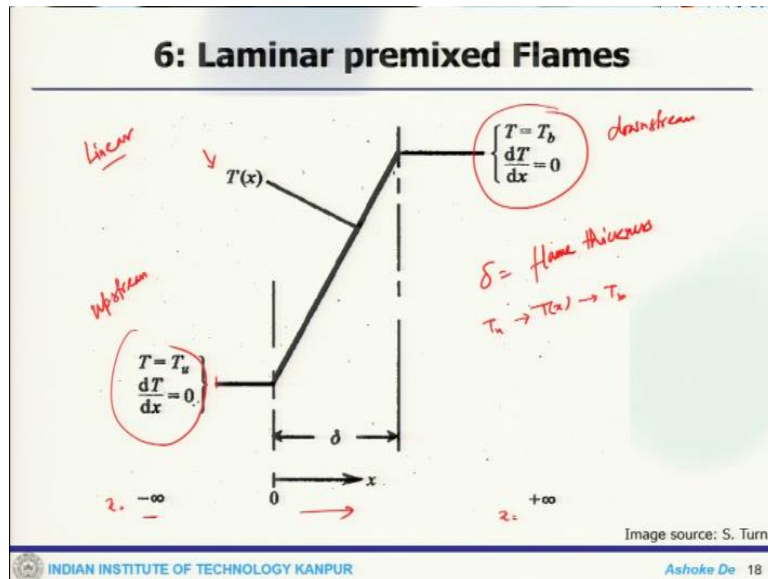
So if you put them this is my x negative infinity this is x positive infinity and this is the direction of the x and we assume in temperature profile, so the one which represented here is a linear profile so this is the upstream boundary condition this is the downstream boundary condition and this is the δ, see this δ is the flame thickness where you have this temperature now the assumption here is that this T(x) is linear.

So this is what we assumed that temperature profile and the temperature goes from T unburnt to T burnt and the distance between which this temperature is, so that is the δ or the flame thickness. So, within this distance temperature goes T unburnt to T burnt.
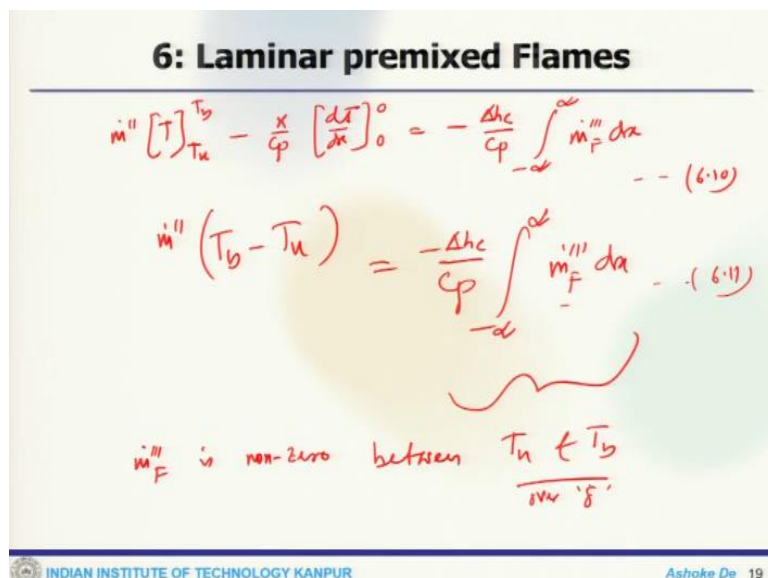
Now we integrate 6.7 b and get:

$$\dot{m}''[T]_{T_x}^{T_b} - \frac{\kappa}{C_p}\left[\frac{dT}{dx}\right]_0^0 = -\frac{\Delta h_c}{C_p}\int_{-\infty}^{\infty}\dot{m}_F''' \, dx$$

So if we write:

$$\dot{m}''(T_b - T_u) = -\frac{\Delta h_c}{C_p}\int_{-\infty}^{\infty}\dot{m}_F''' \, dx$$

So now the limits of the reaction rate integral this is the one can be switched to the temperature from space coordinate the reason is that m triple dot this $\dot{m}_F'''$ is this guy is non-zero between T_u and T_b so over δ.

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So:

$$\frac{dT}{dx} = \frac{T_b - T_u}{\delta}$$

Or

$$dx = \frac{\delta}{T_b - T_u}dT$$

Now we switch these things, now this one if we used here in 6.11 then what we get now using these in 6.11 we get:

$$\dot{m}''(T_b - T_u) = -\frac{\Delta h_c}{C_p}\frac{\delta}{T_b - T_u}\int_{T_u}^{T_b}\dot{m}_F''' \, dT$$

So we switch the limit of the integration from the spatial coordinate to the temperature coordinate.

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Now the average reaction rate which one can define this is the average reaction rate which is now:

$$\overline{\dot{m}_F'''} \equiv \frac{\delta}{T_b - T_u} \int_{T_u}^{T_b} \dot{m}_F''' \, dT$$

So we turn that thing is a average reaction rate, so if you use this information of the average reaction rate we can finally simplify that term to be:

$$\dot{m}''(T_b - T_u) = -\frac{\Delta h_c}{C_p} \delta \overline{\dot{m}_F'''}$$

So that is the expression one gets, now here in this particular equation if you look at it there are 2 unknowns number one is $\dot{m}''$ and second is δ. So these are the 2 unknowns, so we have right now one equation and 2 unknowns so that means to get the complete solution we need one more equation.

Now what we can assume we can assume the reaction rate is much smaller within the first half of δ that means which essentially says between x equals to - infinity and x equals to $\delta/2$, so we can re evaluate equation 6.10 and from x equals to - infinity to x equals to $\delta/2$ if we do that we get:

$$T = \frac{T_b + T_x}{2}$$

And

$$\frac{dT}{dx} = \frac{T_b - T_x}{\delta}$$

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So now one can write 6.10 we can rewrite that or that becomes:

$$\dot{m}'' \frac{\delta}{2} - \frac{\kappa}{C_p} = 0$$

So here is my second equation. So, we have 2 equations now equations are 6.15 and 6.17 and we have to solve for δ and $\dot{m}''$, so once you solve this 2 equation the solution gives you:

$$\dot{m}'' = \left[2 \frac{\kappa}{C_p{}^2} \frac{(-\Delta h_c)}{(T_b - T_u)} \overline{\dot{m}'''_F} \right]^{1/2}$$

So that is one solution that you get for this and the δ becomes:

$$\delta = \frac{2\kappa}{C_p \dot{m}''}$$

So, you get the solution for these things.

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Now we apply the definition of laminar flame speed, so we applying the definition of a S_L which is:

$$S_L = \frac{\dot{m}''}{\rho_u}$$

where α is:

$$\alpha = \frac{\kappa}{\rho_u C_p}$$

And heat of combustion is:

$$\Delta h_c = (v + 1) C_p (T_b - T_u)$$

So if you use all this expression rather combine them together and you use the solution of δ and $\dot{m}''$ what we get:

$$S_L = \left[-2\alpha(v+1)\frac{\overline{\dot{m}_F'''}}{\rho_u} \right]^{1/2}$$

And δ is:

$$\delta = \left[\frac{-2\rho_u\alpha}{(v+1)\overline{\dot{m}_F'''}} \right]^{1/2}$$

Tthis is my 20 and one can see if you use the information of S_L here in the δ, one can write this δ as $\left(2 \alpha/S_L \right)$. So that is what one can write, it is a nice correlation you will get between S_L that is the laminar flame speed in terms of thermal conductivity and the flame thickness.

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Now we can look at so that one thing there is and this thing is that there will be some factors which are influencing this so we look at base factors that influence S_L and δ. Now one thing immediately you can notice that from this 2 equations 6.20 and 6.21 you can immediately notice that S_L has a dependency on temperature or δ is a function of temperature.

Now we first consider some temperature approximate temperature scaling some approximate temperature scaling which will get you that:

$$\alpha \propto T_u\overline{T}^{0.75}P^{-1}$$

And reaction rate would be proportional to: $T_u\overline{T}_b^n P^{n-1}exp\left[-\frac{E_A}{R_uT_b} \right]$, so here n is the overall reaction order and $\overline{T}$ is the $[0.5(T_b + T_u)]$. So, that is what one can get.

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Now if you combine this scaling which will allow you to get an expression for S_L and that will look liked now it will be:

$$S_L \propto \bar{T}^{0.375} T_u T_b{}^{-n/2} exp\left[-\frac{E_A}{2R_u T_b}\right] P^{\frac{n-2}{2}}$$

There is δ:

$$\delta \propto \bar{T}^{0.375} T_b{}^{n/2} exp\left[\frac{E_A}{2R_u T_b}\right] P^{\frac{-n}{2}}$$

So what one can notice here there is a strong dependence on T, on both T_u and T_b and flame thickness that δ is inversely proportional to S_L if you take some number. For example, let us say n is 2 then S_L becomes independent of P that is a specific case one can look at it that the dependency of S_L on P and all this.

Now there are different correlations which actually exist some for let us say for stoichiometric methane air mixture the correlation is that:
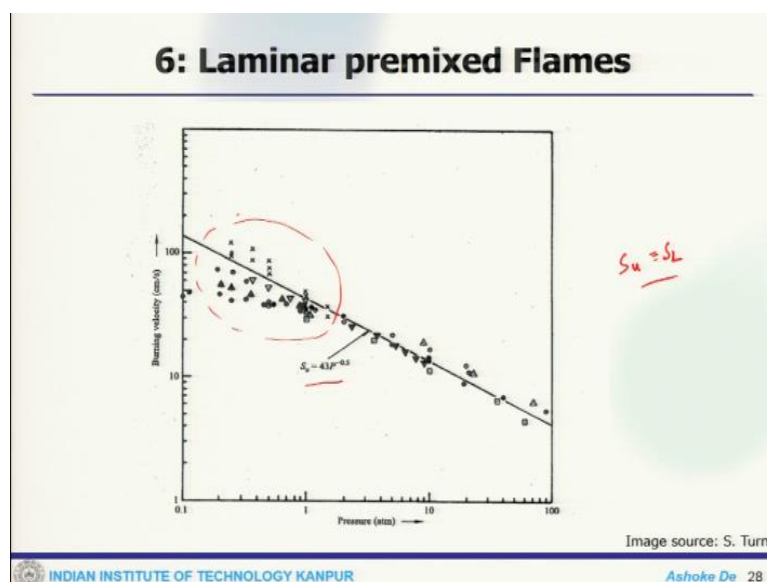
$$S_L = 10 + 3.21 \times 10^4 [T_n(x)]^2$$

And this is based on temperature based on pressure same S_L is:

$$S_L = 43 \times [P(atm)]^{-0.5}$$

So simplified analysis can capture the effect of temperature but not the effect of pressure.

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So you can see these are again based on some simplified analysis there is a correlation S_u is nothing but your S_L that we are discussing so laminar burning velocity the pressure in the atmosphere when it is changes the data is in the lower atmospheric condition data is quite

scattered and then the burning velocity at the higher atmospheric condition follow some correlation, but at the lower pressure there is a quite a bit of scattered mists which is observed.

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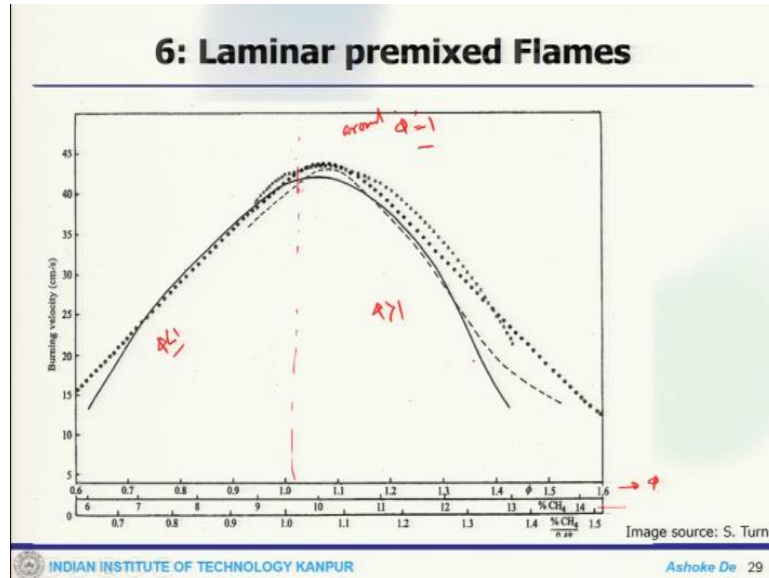
At the same time if you see that this is on the equivalence ratio this is φ curve axis this is percentage of methane and this is normalized and this is again burning velocity what you can see clearly and this is a pretty much and very unique characteristics of hydrocarbon fuel that you can see if you plot the laminar flame speed corresponding to φ, you get some peak around this point 1, that means around φ equals to 1 that is a stoichiometric so there will be a peak before that this φ less than 1 that means the lean condition.

So the lean condition temperature dependency meaning temperature would be less and it is completely burnt. So, it is increasing because as the φ is increasing, fuel is increasing, temperature due to reaction is increasing, S_L is increasing it reaches a maximum limit around φ equals to 1 which may not be exactly 1 then after that this side is the φ greater than 1 where again it starts decreasing this is one of the unique characteristics that you find in the hydrocarbon fuel.

So but other fuel also you can estimate this and again there are analytical expression which are available. So, you can actually take it up as in smaller work assignment and calculate this. So, now stop here and continue the discussion in the next lecture.