Design Practice - 2
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Lecture - 09
Problem Solving (Based on Bezier Curve)

Hello and welcome to this module 9 of the course Design Practice 2. We were talking and discussing about Bezier curves and the way you know you can fit. The curve typically starts from the first and passes through the last point. But there are certain other control points which fall within the ambit of these two points which define how the curve geometry would be.

And then if supposing those control points are changed and their positions are changed the geometry varies suitably based on that change. So let us actually now try to plot practically from you know a Bezier standpoint.

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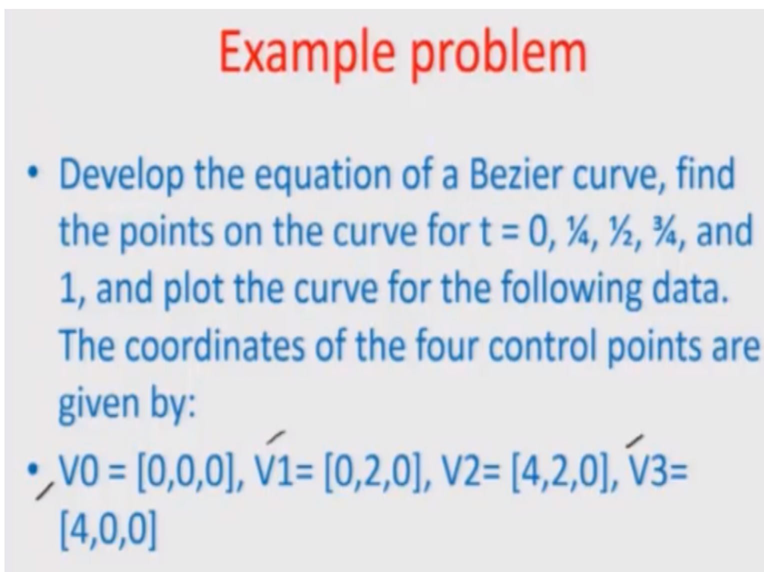

We have these 4 different control points in a particular example V0, V1, V2 and V3 and they are on a 3-dimensional plane. So these are 3-dimensional coordinates with unique value for x, y, and z respectively when you vary between the different points and we want to develop the Bezier curve equation keeping these control points in view and also try to further fit the curve for values of t equal to 0, one-fourth, half, three by four and one and plot the curve okay from the equation on Bezier curve because number of control points is 4.

So it is like a complete treatment as we did in case of Hermitian for the Bezier function. So we want to start this curve plotting exercise.

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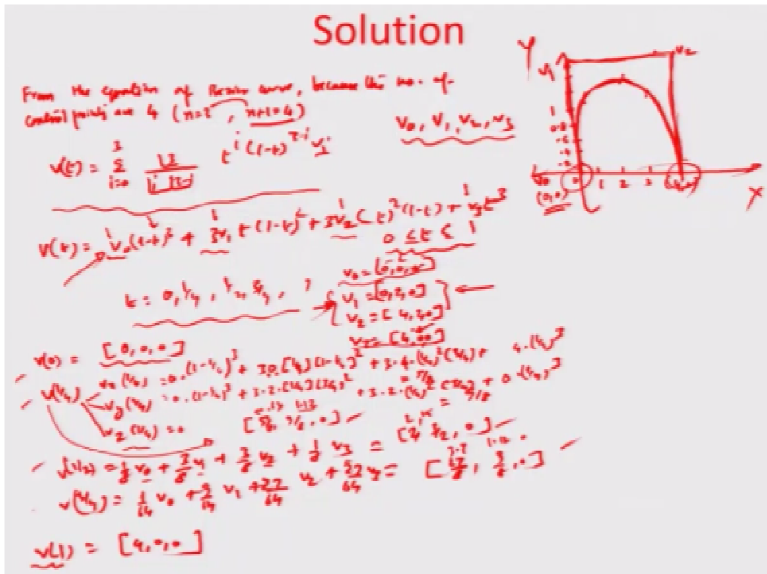

So n becomes equal to 3. You know that n + 1 control points are there, n + 1 = 4 okay. So the curve function that would happen because of that look something like i varying from 0 to 3, a factorial 3 by factorial i, factorial 3 – i, t to the power of i, 1 - t to the power of 3 – i V i where V i is the number of control points. The number of control points you know are V 0, V 1, V 2 and V 3 respectively, 0 to 3. So i varies in that manner.

And so if I wanted to actually try to write this in a manner Vt would look something like V 0 (1 – t) whole cube plus 3V 1 t(1 – t) square plus 3V 2 square of t times of (1 – t) I am sorry plus 3V 3 times of t cube. T of course will vary between 0 and 1. That is how the parametric form of equation is defined and we now start substituting the different values. You know that there are all different x, y, and z coordinates in case of this is I am sorry this is not 3V 3 but just V 3 t cube.

So when we substitute the x values y and z values respectively and for the different values of t that have been given in the question namely 0, one-fourth, half, three-fourth, and one we want to find out the different points of this Bezier curve you know which will give you an idea of how the curves would pass through or what points the curves would pass through in space where the first and the last points are similar to V 0 to and V 3 but the in between points may typically be different than the points which are there actually on the Bezier curve.

So let us algebraically numerically try to solve this example here. Let us say when we do for V 0 you know that V 0 has coordinate value which is given by 0, 0, 0 because this is the first point and typically the Bezier curve's first point also passes through the point which is outlaid you know across both ends of the curve that is V 0 and V 3 in this particular case. So we know V 0. We need to find out what is V one-fourth and for doing that we first put only the x values okay so we now split this up into 3.

So you have a V x of one-fourth. You have V y of one-fourth and you have a V z of one-fourth and you know all of them are amounting to solving the equations with different substitutions for V 0, V 1, V 2 and V 3 and so forth. We know that the values of V 1 are 0 to 0 that of V 2 are 4, 2, 0 and that of V 3 are basically 4, 0, 0 okay. So the V 3 here in this case in any event is going to be 4, 0, 0 which is corresponding to I am sorry this is V 1 okay so that means corresponding to t equal to 1.

This is necessarily where the curve should pass through. And let us look at what are the V x, V y, and Vz. So we take the x value of V 0, V0 being equal to 0, 0, 0 okay so 0 times of 1 – t cube. So when we are talking about one-fourth, this is 1 – 1/4 that is 3/4 cube plus thrice the x coordinate of V 1 which is 0 again times of the value t times of square of 1 - t so you have both these terms are eliminated because of being multiplied with 0.

The third term has a 4 so V 2 x coordinate is 4. So 3 times of 4 times of square of t which is one-fourth square times of 1 – t which is three-fourth plus 3 times of again the value of the x coordinate 4 times of one-fourth cube okay. So that is how the V x one-fourth happens to be and this can be calculated as in this particular case 5/8th okay. So let us if we just calculate V x 1/4 this is 5/8th.

Similarly, if you wanted to put all the y values here so obviously the first terms would or first term would be 0 in that particular case okay times of so V 0 is 0 for all the 3 coordinates so we are now calculating y so we put the y equal to 0 value here times of 1 - 1/4 cube plus again three times of 2 times of one-fourth times of three-fourth square plus three times of in this particular case the value happens to be 2 times of one-fourth square times of three-fourth plus again I am sorry this is not, this happens to be only coefficient 1 not 3 okay.

So times of V 3 which in this case, in this particular case is 0 okay y corresponding to the point V the last point V 3 is actually 0 times of one-fourth cube okay. So this becomes equal to 9/8th and similarly if you wanted to look at 0s we know that all these values here V 0 to V3 are being 0 so it becomes 0. So if I wanted to describe the coordinate of V one-fourth in space this can be written down as 5/8th, 9/8th and 0.

In a similar way we can do this for V half and for the V half the equation happens to be 1/8 V 0 + 3/8 V 1 + 3/8 V 2 + 1/8 V 3. I am calculating pretty much in a similar way by substituting all the values of t corresponding to half in this particular expression, earlier it was one-fourth. And so in that event if I put the different values of V 0 to V 3 in the x, y, and z directions we get again another 3 set of coordinates 2, 3/2, 0.

And similarly we could do the same for V 3/4 where the final formulation would be in the manner 1/64 V 0 + 9/64 V 1 + 27/64 V 2 + 27/64 V 3 and then if you substitute again x, y, and z values of all these 4 different points we are left with coordinates 27/8, 9/8, and 0. So we have now plotted whatever falls on the curve these again for the illustration of all readers are basically the points which are control points. The curves would not pass through them necessarily.

They will only pass through the end points but in order to define the Bezier function etc. they are absolutely necessary but when the Bezier values actually prop up they correspond to the values taken by different values of t that is one-fourth, half, and three-fourth and when we solve these problems we get a set of coordinates here all in three you know x, y, and z directions corresponding to the different incremental t okay which is there in the parametric form of the equation.

So if i plotted all this together on xy plane, xyz plane obviously one thing that is being seen here is that whether it is V 0 or V one-fourth or V half or V three-fourth or V 1 all these z values are 0. So therefore it is essentially an xy plot okay. There is no plot along the z direction. That is how the problem has been defined in this particular case. But if i wanted to plot these values together 5/8th happens to be about 0.63 and 9/8th happens to be 1.125 maybe.

And similarly this is 21.5 this is again 3.7, 1.12, 0 and with all these plots here we would be able to get you know sort of a circle okay in this manner where we are talking about the first point which is 0, 0 of course. The last point which is 4, 0 and in between there are set of points here one of them is being described by 0.63 along the x direction, 0 along the y direction and 1.12 so somewhere here maybe.

Let us say we have a scale that we want to plot in a manner so that you have 1, 2, 3, and 4 okay. So these are the different values starting from 0 and similarly we have another scale here so you have 0.2, 0.4, 0.6. Let us actually plot this figure with respect to the scale that I have drawn 0.8 so on so forth. So here we are talking about a y increment of in this particular case 1.12 and then x increment of 0.64.

So corresponding to somewhere here the curve somewhere here so we are talking about this kind of a curve okay starting from here then the next coordinate corresponding to V half is 2, 3/2. So 2 and then 1.5 so somewhere here okay. So passing through this and then again 3.7, 1.12 so similar level but again 3.7 is somewhere here so again somewhere here okay and then finally 4. So that is how the curve is going to be placed.

So it is like a if i looked at the geometry it is going to be like an arc okay in this manner and that is how the basic curve would like typically in this particular case. So the Bezier curve is defined by now a polynomial or polygon described through connecting all the different coordinates and if I looked at whether the condition of the Bezier is being set here namely the first point would align with the tangent.

So the first line segment of the polygon of a curve would align with the tangent to the first point and the last line segment would align with the tangent to the last point. So it is actually true in this particular case because when we start with the different points, control points you have 0, 0 here then 4, 2. So 4, 2 is something like I am sorry 0, 2; 0, 2 is something like here. So we are talking about a point somewhere here as V 1. This is V 0. Then similarly we have V 2.

V 2 is corresponding to 4, 2. So somewhere here again 4, 2. So this is V 2 okay. So we join these two points and then finally V 3 to V 4 okay. So this is the polygon which we are creating with respect to this Bezier curve and as you see here the tangents of the first and last points are parallel to the line segments which define the polygon which is going to define the curve. So in a way that is how Bezier curve is defined.

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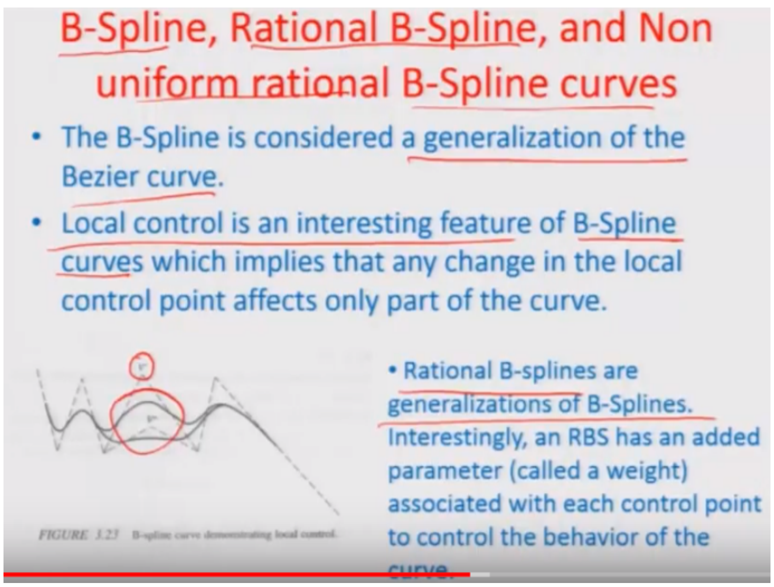

So practically you have now seen how different fits like Hermitian as well as Bezier curves are aligned or we can actually try and look at some other varieties of what are the essential differences so we have different you know curve fitting geometries like the B-spline fit, the rational B-spline, the nonuniform rational B-spline curves. The B-spline is considered a generalization of the of the Bezier curve.

Local control is an interesting feature in such a B-spline plot or B-spline curve. It implies that any change in the local control point affects only that part of the curve. So therefore just as in the

Bezier you saw that if even one of control points vary the overall shape and feature of the size of the curve changes and in this particular case if a single point varies it only concerns the overall curve variation to be in that particular segment of the polynomial which has that particular point okay.

So you can locally vary rather than globally varying the whole curve as was the case for the for the Bezier. So there is a further degree of control that can be given through rational B-splines which are a sort of a generalization of B-splines with an added weight. So now each section would have a different weight. So if I wanted to vary one section and you know typically I can put a higher weight to that so a little bit variation in the control point would give a lot of variation in that particular section and vice versa.

So that is how you have the different features of a curve fitting in 2D plane. We will go to a little bit interesting you know a different dimension altogether or the problem which is about surface modeling. So far you have been looking into how 2-dimensional curves we should get into organic 2-dimensional forms let us say would be able to get created in terms of data structure within a computer.

Now looking ahead I am going to talk extensively about how you know given a particular curve or given a set of different curves how could we align them together or integrate them together to form a surface because so far even if with line geometry or even with let us say the wireframe model we are able to define the overall shape and feature. It is very difficult to see how the surface behavior would happen particularly in 3D solids.

And when we are talking about organic solids, organic shapes even the complexity is much much higher. So we need to one option is of course we need to represent. But another option is how we actually do a data structure so that it can be used for representation. So we are talking about slightly higher order you know analysis in comparison to just merely representation of shapes or you know 3-dimensional objects through textures and lines. So let us go to surface model.

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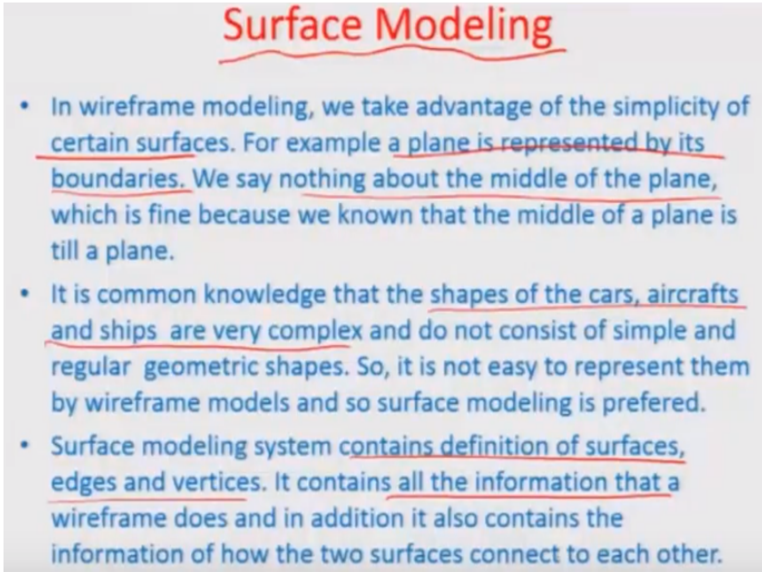

So in a wireframe modeling we take advantage of the simplicity of certain surfaces for example a plane is represented by its boundaries. We say nothing about the middle of the plane, neither we have a data structure for that but now we would like to do that through this approach of surface modeling. So further you know to add complexity we know that shapes of cars and aircrafts are not that simple and particularly ships also are very very complex.

And so they do not consist of simple geometries anymore. And so representing only the whole feature with boundaries is not really a very good idea particularly when we talk about creating a data structure so that a 3-dimensional plane can be seen. See visualization generally has been enhanced because of the CAD technology. So therefore why not utilize it fully is the next question and so complex data structure needs to be developed so that you can plot that in terms of a surface with control of each and every point on the surface that we are constructing.

So surface modeling systems contain definitions of surfaces, edges, vertices. It contains all the information that a wireframe will not provide and in addition also contains you know information of how two surfaces would connect to each other. There are certain entities which can be defined.

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For example you know a plane surface could be defined with 3 lines more simplistically or maybe multiple lines as you are seeing in this particular figure. So you know if 3 points which are non-coincident are in space in near proximity to each other it automatically defines a plane which passes through all the 3 points. There can be surfaces descriptions you know in regular geometries through ruled surfaces.

So typically ruled surface can be defined as a linear interpolation. So there is curve 1 and curve 2 in this particular case and you are having some rulings or some lines in between to connect the curves or multiple points on these curves so that you can have you know a set of plane surface as we should be able to approximate the whole surface which is being formulated by the curve. So informally the straight lines connecting the two rails would form the surface.

There can be surface of revolution created. For example there is a line which sweeps in plane across another line at a certain angle. You know if it sweeps on a full rotation or a half rotation it also creates a surface generated by rotating a planar curve in space and that can be a straight line or any other curve okay at a certain angle. You also have tabulated cylinders. So in this particular geometry we are considering the surface to be generated by sweeping a planar curve in space in a certain direction at a certain distance.

For example above a straight line sweeps for example in this sweeps in a rotational manner and it sweeps about a path you know which is circular in nature with the center. So typically the straight line would then be called the generatrix and the circle would be called the directrix which generates the straight line to move along the circle thus formulating a surface. We also talk about Bezier surfaces or B-spline surfaces which are slightly complex.

They are not that easy as you know the regular surfaces as being shown in the last three examples here, 4 examples here. So they are actually irregular surfaces okay. And so when we come to irregularity and we come to you know patches which are formulated through multiple data structures probably with the Hermitian fit in mind or with let us say the Bezier fit in mind it is very obvious this whole patch is created with the goal of trying to fit on a complex profile.

So that we can get the extract of the data okay of that surface through predictive modes once this fits very well with the profile itself. And instead of going for a sort of a 2-dimensional space curve we are talking about a surface so therefore obviously the degrees of freedom would increase in that case and because of the degrees of freedom increasing by one order there would be a variety of complexity created which should be needed for maintaining and creating such a data structure.

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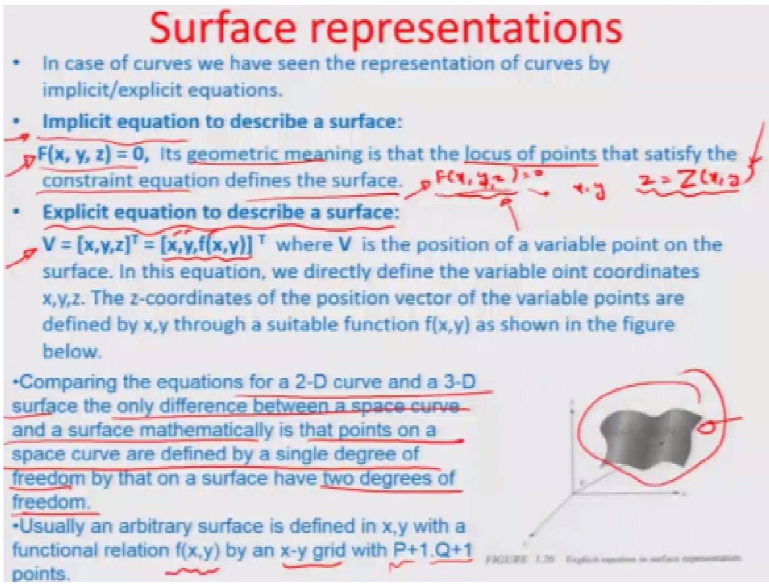

Let us look at one problem example here when we talk about again how to represent surfaces in terms of implicit and explicit equations okay. So because there is a third dimension involved and degree of freedom is increased by 1 we would like to represent in a implicit manner a particular surface through an equation F(x, y, z) = 0 a function of x, y, and z altogether equal to 0 it will define you know geometrically a locus of points that satisfy the constraint equation okay.

And those locus of points if plotted in a 3-dimensional space would result in what we know as a surface. So what it basically means is that you have you know 2 variables in this particular way x and y and we are representing through a function you know F(x, y, z) = 0 a sort of a relationship which is inbuilt you know and it has to be found out so we call this let us say capital Z x and y. So the whole third dimensional z is defined by different you know values of the variables x and y. So the same thing can be written in explicit form.

Remember in the curve case we had talked about two different equations for example you know how x would be independently related to y and z because in this case in that particular case both y and z were dependent variables where x was the independent variable. In this particular case it is that both x and y are the independent variables. Degree of freedom is increased by 1 and z is a dependent variable.

So we can describe the same equation in a very explicit manner, in a very you know direct manner by saying that V the locus of all points V in space would be represented as x, y and function of x, y which is actually the z value. So here the function is very clear. You do not need to calculate as you need to do from this extract here okay and so that is why it is called explicit.

So when you compare the equations for a 2D curve and a 3D surface the only difference between a space curve and a surface mathematically is that points on a space curve are defined by a single degree of freedom and that on a surface by two degrees of freedom remember. New variable y has been added into the x variable which was otherwise used for describing the space curves. So that is the whole difference which happens in case of a surface.

This for example is a representation of a surface. So usually an arbitrary surface in x and y could be defined with a functional relationship F(x, y) equals a third dimension z. And it can be again mapped with an x, y grade where we have independent values of x and y and corresponding to each value or each combination of x and y we plot a z value okay. So there are exactly P + 1 and Q + 1 points in the x and y grade one in the x direction, one in the y direction to define a set of points which would be able to finally formulate the surface.

So we would like to now probably at the next module consider how to represent such a surface mathematically. I would like to just do one case of a Hermitian surface or a Hermitian patch and the idea is that you know a Bezier patch can be an extension of what I do for the Hermetian. The computations will become extensive because obviously the data structure now has one completely new degree of freedom which adds up.

So there are different combinations and combinatorial which will be formulated between the x and y so that there can be a z value which maps the surface. So we will do this in the next module. I am going to close this module in the interest of time. Thank you very much.