Design Practice - 2
Prof. Shantanu Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology-Kanpur

Lecture - 07
Parametric Representation of Synthetic Curves

Hello and welcome to this Design Practice 2 module 7. Here we will talk about synthetic curves.

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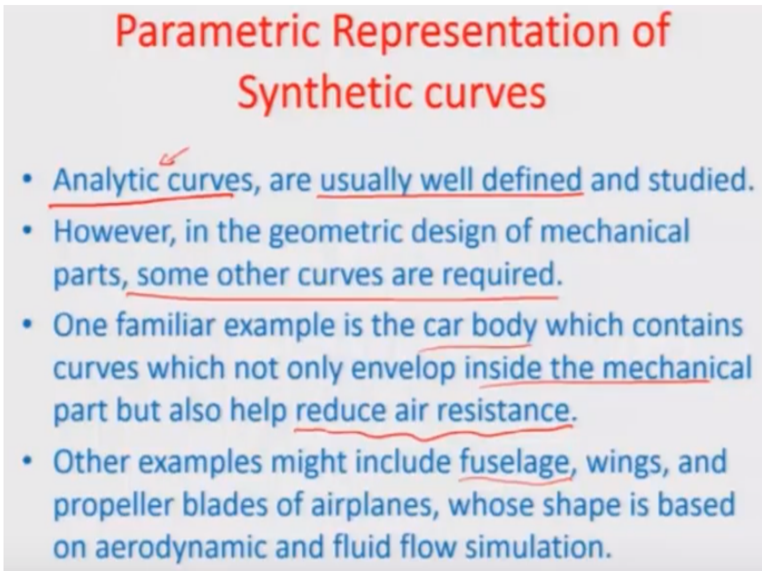

And while coming into the topic, so analytic curves as you have seen earlier are usually very well defined in terms of the functional relationship as well as you know the symmetricity which exist in terms of usually the parameter which is taken and so the definition whether it is parametric representation or nonparametric representation is quite simple. However, in the geometric design of mechanical parts there can be an extremely complex contour which maybe plotted by a bunch of different curves rather than these sort of standard analytic curves.

And one fine familiar example for this can be something like the aerodynamic design of a car body or that of an airplane which would contain curves which not only envelop inside the mechanical part but also has a functional aspect for example in car or whether it is you know the fuselage or wings or even the propeller blades in an aircraft. Each of it has a different functional aspect by virtue of the contouring that it defines.

So essentially it guides the fluid basket in a certain manner which will result in either reduced air resistance in some cases or you know getting the corresponding lift in some other cases so on so forth. So in order to represent such curves we use not a single curve but a bunch of different curves okay.

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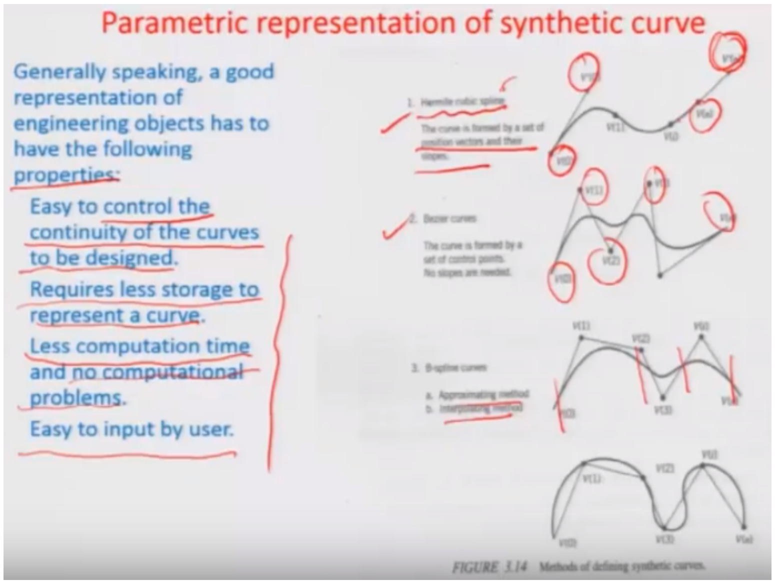

And generally speaking a good representation would be something which will have properties related to controllability okay as regards the continuity of these curves. So whatever system we want to design should have this aspect. We should typically require less computational effort, less computational space to plot a curve, whether you know it is about storage to save data so on so forth, less computational time, maybe very simple computations.

No computational complexities or problems and generally an easy to input functionality by the user and so in line with it there are certain very nicely defined curves which have emerged to define the basic element of such synthetic curves, a repetition of which in 3D space would lead to the whole contour of the whole profile sometimes. So the first representation is known as the Hermite cubic spline representation. It is formed by a set of position vectors and slopes.

So when we talk about such a you know representation of a fit there is let us say two end points below I mean between which you are trying to fix this particular family of curves and what you do is that with these end points and end slopes into consideration you can vary either one of them or multiple so that it generates a bunch of different contours one of them can suit eventually the part of the profile that you are wanting to fit.

So it is a, it is sort of a representation in a synthetic manner so that you are force fitting a section of the curve on the topology that you want to map so that that representation of the topology can be accurately done in a functional manner so you can induce parameterization and you could actually have you know good effects in terms of easy computability or a comfortable you know input defined by the user and some of these properties would control for example less storage of data so on so forth.

So the second representation is that of a Bezier curve which happens because of a variety of different control points. There are no needs for slopes which were earlier there in the Hermite cubic spline fit and I will talk about these in details when we actually evaluate how with a certain equation to govern the trajectory we without having a slope or with having a slope will be able to plot a bunch of different you know curves.

So in the Bezier curve the curve is formed without the slopes but on the using the control points and then there is of course the B spline curve which is an approximatic method it is an interpolating method where you could have sort of more flexibility than the Bezier curve in terms of the various sections of convexity and concavity okay associated with a single curve.

In fact there is a form of B spline curve better known as rational B spline where you can in fact vary the different sections and the way they vary by putting some weightages across those sections. So these are some of the basic units of the synthetic curves that I am talking about.

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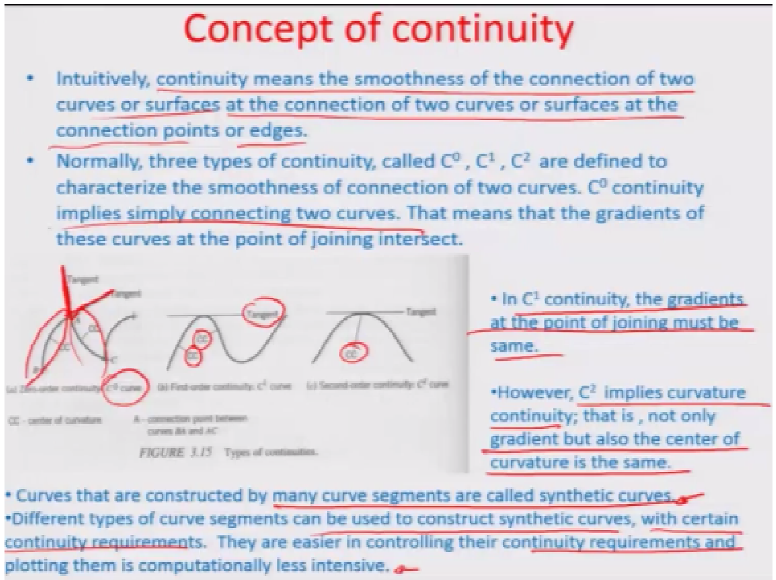

The other issue is how to make them continuous, how to connect them together so that they can have a geometrical definition of the surface, connecting curves is a very important aspect of defining geometry. So intuitively continuity actually means the smoothness of the connections of two curves or surfaces, add the connections of two curves or surfaces particularly at the connection points or the edges.

Normally there are three different kinds of continuity which we consider. One is the zero order continuity C0 where the implication is just simply connecting the two curves without caring about how smooth they are getting connected. So it is about an intersection between the two end tangents. In one of the cases, the section AB for example is an end tangent defined by this line right here and the section A that is the next start in the section AC the next starting section has a start tangent defined by this particular line in the other direction.

And typically if these two tangents intersect each other you define a continuity among the two segments which we regards as C0 continuity of zero order continuity. Similarly, if we have a case where maybe two different segments of a you know curve of a synthetic set of curves have different center of curvatures but a line along the same tangent as they match each other at a certain point they fall in the domain of C1 continuity.

And then if the center of curvatures are also being similar it induces a little more smoothness than the first order and so this is called C2 you know continuity case. So we would like to whatever equations either the Hermitian equation or the Bezier equation, try to plot you know in a manner so that we obey one or more of these connection methodologies to ensure that the curves, the synthetic curves are actually continually able to define the surface that is in question.

So C1 continuity, the gradients at the point of joining must be same, C2 implies curvature continuity that is not only gradient but also the center of curvature which is the same and curves that are constructed by many curve segments. They are actually called synthetic curves and the different types of curve segments can be used to construct synthetic curves with certain continuity requirements. They are easier in controlling their continuity requirements and plotting them is computationally less intensive.

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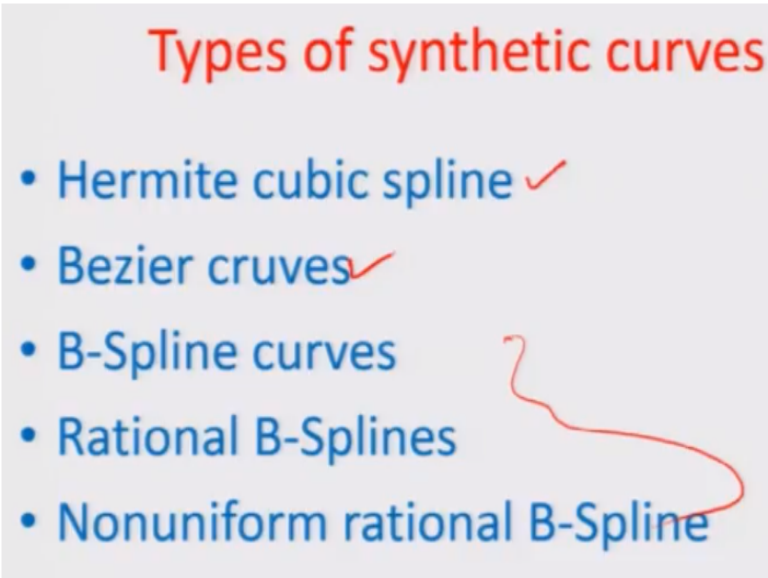

So once we define the continuity between the curves let us actually look at some individual forms of representations like the Hermite cubic spline and the Bezier curves. I will leave these other three food for thought you know for people who are interested to go a little bit higher in this domain. My idea here is to just give you an example of how computationally we can handle complex contours, what we call in the forms or organic shapes.

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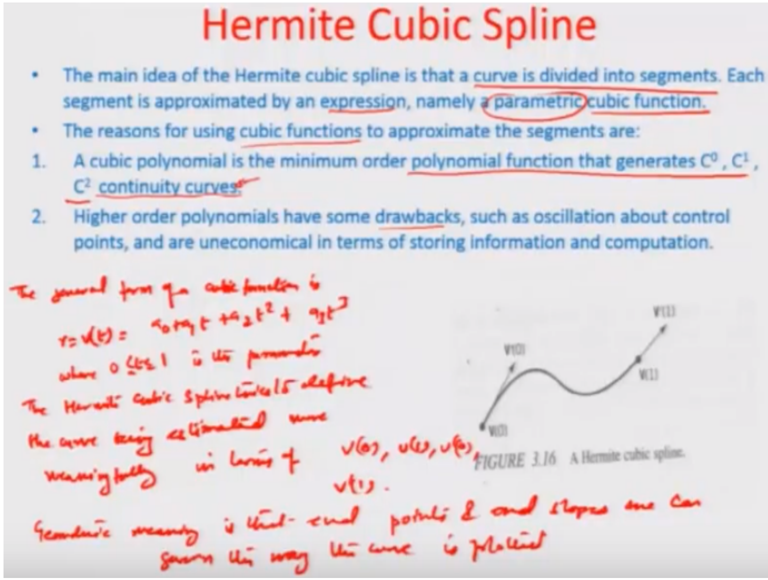

So what is a Hermite cubic spline curve. So the main idea behind this particular geometry, it is the curve that is divided into segments again. Each segment is approximated by an expression, namely a cubic function. In this case it is a parametric cubic function and there are variety of reasons for cubic functions to approximate the segments. One of them is that cubic polynomial is the minimal order polynomial function that generates all the three continuity curves.

So you can connect them for the C1 continuity or the C0 continuity and even the curvature continuity so there is enough flexibility of a cubic polynomial to append to these all different connects. There can be still higher order functions more than three which can serve the purpose but you know they have some drawbacks particularly they have oscillations about control points and sometimes the computations maybe uneconomical in terms of storing of information and computation.

And so therefore it is preferred to operate at a cubic function level. So let us look into exactly what function we are talking about so the general form of a cubic function is a 0 + a 1 t + a 2 t square + a 3 t cube where t varies between 0 and 1 is the parameter. The Hermite cubic spline tries to define the curve being estimated more meaningfully in terms of V0, V1, V dash 0, V dash 1.

So the geometric interpretation or the geometric meaning is that by end points and end slopes one can govern the way the curve is plotted. In fact by changing either the end slopes or the magnitude of the end slopes the family of curves can be generated one of which maybe able to suit the topology which is being fitted through this particular method. So as I told you this gives the user a way to connect two curves and assures a certain degree of continuity between the connects.

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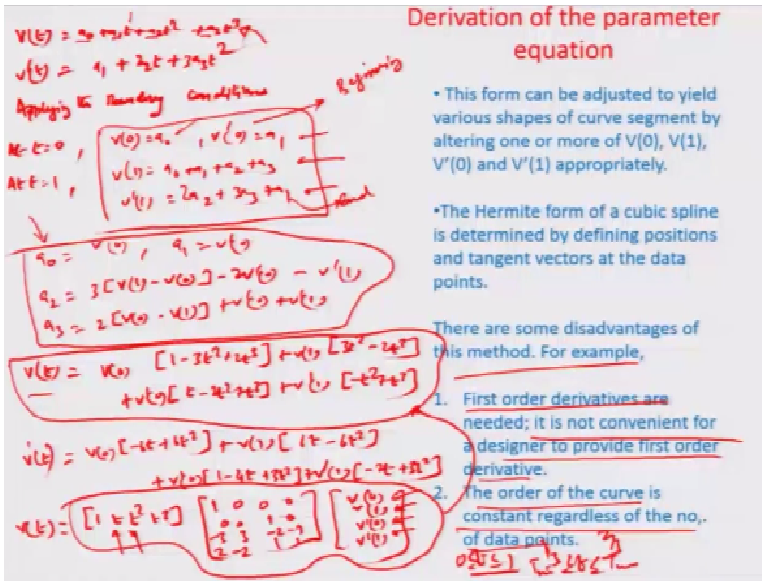

Let us look at physically what is going on here upon differentiation. Let us say this is the parametric representation a 0 + a 1 t + a 2 t square + a 3 t cube. So physically if I wanted to differentiate once with respect to t, the form that the equation will take is a 1 + 2a 2 t + 3 a 3 t square. So when we apply the boundary conditions here we know that at let us say t equal to 0 suppose, V t is equal to a, V 0 is a. So let me call this V of 0. So it is a 0.

Just substitute the value of t into this particular equation and similarly V dash 0 becomes equal to a 1 and at t equals 1 V1 would become equal to a 0 + a 1 + a 2 + a 3 and V dash 1 simultaneously would become twice a 2 plus thrice a 3 + a 1. So that is how you will define the boundaries and if we solve all these four set of equations the four variables and four equation so you should have unique solution; a 0 can be represented as V 0 already, a 1 as we can see is represented by V dash

0 and a 2 is represented by thrice V 1 - V 0 - twice V dash 0 - V dash 1 and a 3 is represented by twice V 0 - V 1 + V dash 0 + V dash 1.

So this just comes through the way that you know linear systems are solved. The only aspect here which was unknown in the equation that we first assumed from a 0 to a 3 is being you know tried to we tried to solve for all these unknowns through the boundaries which are the points at t equal to 0 corresponding to one end of the curve, the beginning of the curve and t equal to 1 which is the end of the curve.

And all we have is the beginning and the end point and the slopes V dash at the beginning and the end points respectively. So we can actually represent now the function V t with respect to all this different solutions which have been obtained a 0 to a 3. This can be represented as V 0 times of 1 minus 3 t square plus twice t cube plus V 1 times of 3 t square minus twice t cube plus V dash 0 times of t minus twice t square plus t cube plus V dash 1 times of minus t square plus t cube which is actually how the equation which was assumed earlier will be represented.

So we will like to also represent the V dash t here in terms of all the you know knows which are the V 0 and the V 1 and similarly the V dash 0 and the V dash 1 and we can represent this in terms of V 0 times of minus 6 t plus 6 t square plus V 1 times of 6 t minus 6 t square plus V dash 0 times of 1 minus 4 t plus 3 t square plus V dash 1 times of minus 2 t plus 3 t square. So in other words if I wanted to represent the point vector V t in terms of a matrix representation I should be able to have a multiplication of three different matrices.

One is the parameter matric 1 t, t square, t cube plus a connecting basis matrix which in this case would be a 4 by 4, 1 0 0 0; 0, 0, 1, 0; -3, 3, -2, -1; 2, -2, 1, 1 and the matrix which is corresponding to the boundary conditions, in this case it is V0, V1, V dash 0, V dash 1 and you can say that you know this multiplication would typically result in this particular equation between which is the relationship between Vt, V0, V1, V dash 0 and V dash 1.

So here computationally what we can do is to actually let off all these different end conditions in terms of the coordinate values in the slopes and vary the t between limits 0 and 1 or maybe a local t which varies between some value to some value a t min and a t maximum, maybe it can be one-third and two-third, or it can be half and 0.67 two-third something like that. And be able to plot on a local basis the section of the curve that we would like to have in this particular case.

So some of the disadvantages of this method are for example the major need of this method is the first order derivative term which has to be there at least till the ends to define the whole geometry and it may not be very convenient for a designer to provide the first order derivative because the slopes are very difficult to be measured at a certain point. Also the order of a curve is constant regardless of the number of data points so that is another issue while obtaining the curves through the cubic fit method. Let us actually physically try to solve one problem.

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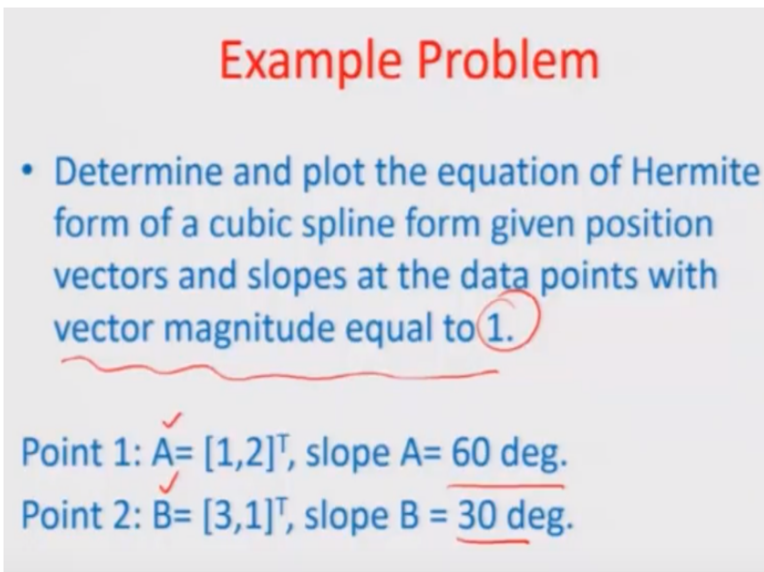

A simple example of this could be let us say we want to plot a Hermitian equation of the cubic spline form between two different points A and B where the coordinates are provided as 1, 2 and 3, 1 and the slopes at the point A and B are provided as 60 degrees and 30 degrees. Further it given that the vector magnitude is equal to 1 for defining the slopes meaning thereby the x component of the slope would be the magnitude times cos of the particular angle and y component would be again the magnitude times sin of the particular angle.

So we have to find out what is the Hermitian fit and generate maybe some tweaking of the end condition so that we can have not one curve but a family of curves one of which can fit to the profile and that is what we mean by when we talk about curve fitting okay. So in this example, we will have to use the cubic polynomial and try to tweak in a manner so that we find all the knowns which is 0 to a 4 as I think I had suggested in the last step.

And then based on that putting that value back and putting a equation in place we should be able to with the parametric form of the equation plot for different values of t varying between 0 and 1 the whole curve itself okay and then vary something so that the curve varies. So I think we will do the solution for this problem in the next module and in the interest of time this module will be closed now. So thank you very much and in the next module we will solve this problem. Thank you. Bye.