Design Practice - 2

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Lecture - 06
Representation of Curves

Hello and welcome to this Design Practice 2 module 6. We discussed in the last module how to make the wireframe representation of different solid objects. Today we will be talking a little bit about what happens when the geometry moves from regular to non-regular that is you know where not all the contours are symmetric in nature and in that even what we normally do is to represent the geometry through functional curves.

So I am going to talk about how such representations can be made using describing curves. I am also going to describe some fundamental equations associated with those curves and the tracing of the curve and then the idea is that once the geometry is very complex not a single curve can actually be able to emulate the whole you know contour of the geometry and so therefore it has to be an array of curves, series of curves which are also known as synthetic curves.

And they should be interconnected to each other in some sense so the way that they are defined makes them aligned, de-aligned with respect to each other so that they can go and map the whole contour of the whole geometry. So let us talk about representation of curves.

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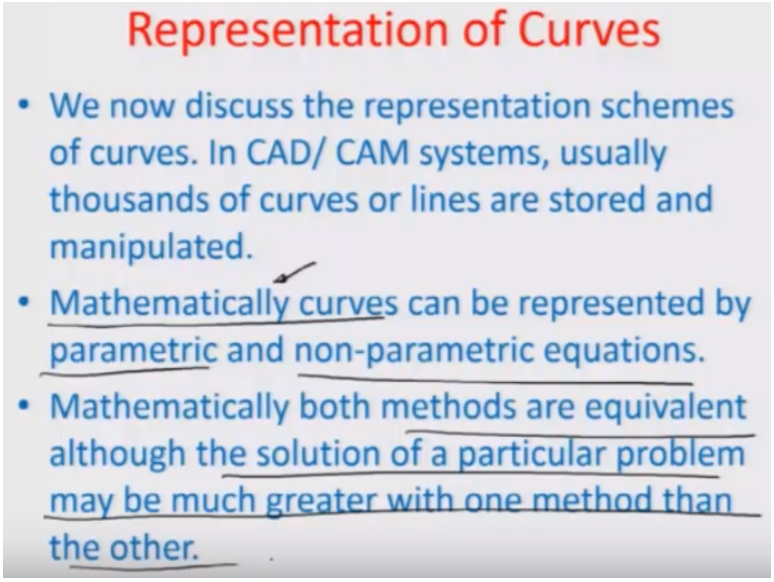

So mathematically if I looked at how curves can be represented, there are two different types, broad categories into which we can split up the governing equations related to these curves. One is called the parametric equations and another is the nonparametric equations. Mathematically both the methods are equivalent. The only difference is that the solution of a particular problem may be much greater with one method than the other.

And we will look into both these methods and try to interpret about what really are the parametric curves and what can be nonparametric representations of different curves.

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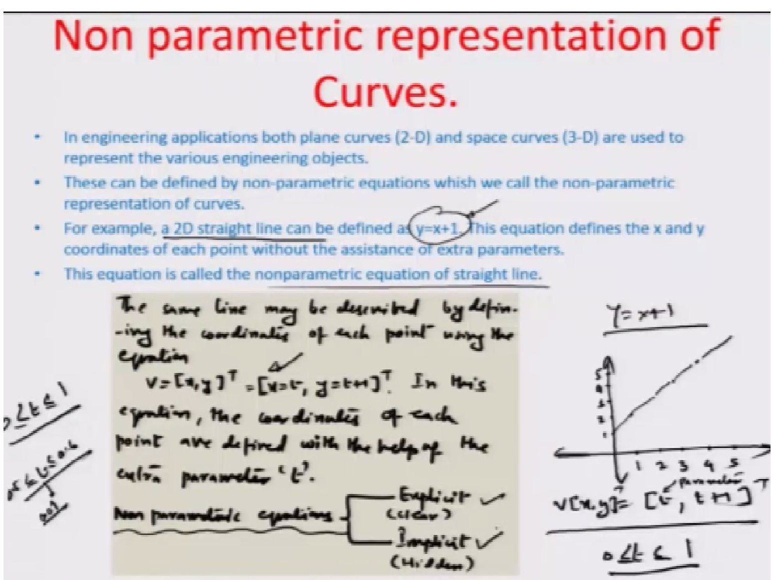

So let us talk about nonparametric representation to begin with. So you know if you for example have a 2D straight line which can be defined by a simple equation here y is equal to x + 1. So obviously if I wanted to plot you know the set of coordinates xy which would follow this line would definitely plot it in the manner by increasing 1 unit along the y direction of whatever be the x coordinate.

So for example with x equal to 1 the y would be equal to 2; x equal to 2 the y would be equal to 3 so on so forth okay. So this is 1, 2, 3, 4, 5 in the y direction. So I would like to plot this through a straight line which will have again you know a slope of 45 degrees and an intercept of exactly 1 unit to this light. So this is how you represent you know the nonparametric in a nonparametric manner, the equation of a straight line.

Now the same line can be described by a slightly different methodology and the idea here is that if we supposing define the x variable through another variable t which we now try to operate within a certain range okay so obviously the same description of y equal to x + 1 can be arrived at by a set of coordinates V xy where x could be equal to t and the y could be equal to t + 1. And so that is how you can represent.

The only advantage in this case is that you know the t being between certain domains is actually just limited to a certain you know end points or end conditions and so therefore supposing we are able to look at in a very local manner okay by varying the t between some domain to some domain for example within t equal to 0 to 1 t can rhyme between 0.5 to 0.6. It can further have increments of 0.01.

So we can actually do a local plotting using this fundamental global equation between y and x okay. So such kind of representations then which involves the parameter t are known as the parametric representation. So obviously the nonparametric equations can be again represented as explicit and implicit or hidden equations. For example if I were to look at let us say just a certain curve, this was a straight line, so it was very simple to do.

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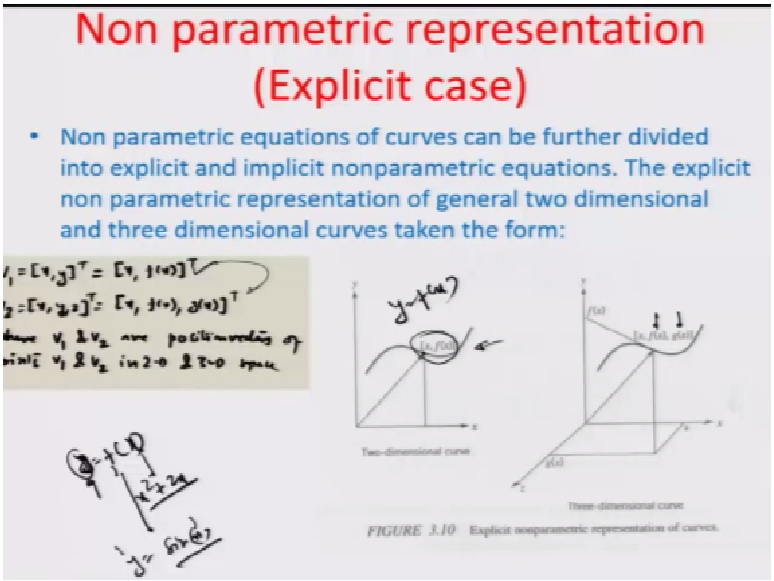

But in a curve for example y equal to fx which describes any general curve as shown in this particular figure. It is a 2-dimensional curve so corresponding to a certain value of x you have a certain value of y okay and if I were to write the coordinates of the vector point v x, y it could be written as x and fx. For example you are writing v1 as x and fx here now if the same curve would proceed to the 3 dimensions.

So you could have actually a single variable and then the fixed effects and gx functions which would be mapping the variable into the various domains along the y and the z direction. So in a way the nonparametric equations can be divided into a very clear representation, the explicit nonparametric representation which talks about okay y equal to f of x. So exactly the y domain is clearly mentioned to be a function. This f can be something like for example x square plus 2x.

Or this f can be something like sin of x okay. So be it whatever the function is clearly expressing a single value of y provided or given value of x okay so periodic probably it is you know along a single x multiple values of y. But as soon as there is a mention of the term x something in the y just props up or just in the y domain props up. So it is dependent in a way to the value of the x. And x is the variable in this particular case but it is very well defined whatever is the dependency. It may not be so fortunate a case where the dependency is exactly established.

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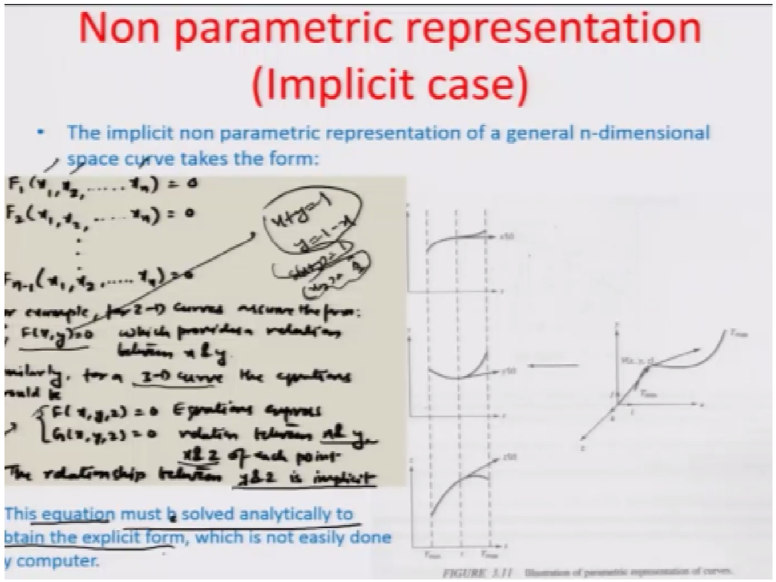

For example let us say when we talk about a family of curves and we are talking about for example you know a 2D curve which is related to a functional representation f, x, and y equal to 0. So let us say there is a representation of the type f x, y as is shown here equal to 0. So this is an equation which relates some kind of a representation or some kind of a form of x with respect to some form of y.

For example this equation can be as simple as x + y = 1 where y becomes equal to 1 - x okay or it could be something like sin x + y equals to probably 0 or let us say 1. So x + y becomes equal to you know multiples of pi by 2. So something like where there is a solution that needs to be developed okay of an equation which comes out because of the way that x and y has been represented. So it is not a direct y equal to f x kind of representation.

But there has to be a solution and this is called the implicit representation of a nonparametric curve. Similarly, in a 3D curve case you might have you know two relations let us say a function of you know capital F x, y, z equal to 0 and capital G x, y, z equal to 0. So you must interpret what is the difference or what is the relationship between y and z given there is some kind of a representation between x, y, z and you know in both the cases.

So the equations express relationship between xy, xz of each point. The relationship between y and z is sort of implicit or hidden in this formulation. Similarly, if there was a family you know of let us say variables x1 to xm we call them dimensions okay and so there are several different functions f1 to f (n – 1) which would be formulated for us to solve a system having variables x1 to xn where xn is the variable and x2 to xn are sort of you know in a way dependent on the basic variable x1.

Although it is in a implicit representation so you will have to solve for all the relationships, interrelationships between x1 to x2, x2 to x3 so on so forth equation like this. So in implicit representation, the equations that are describing curves must be solved analytically to obtain the explicit form which is not easily done by a computer. And therefore it may not be a very good idea to represent you know particularly when we are talking about numerical solutions and kind of plotting, the implicit form may not be a very you know economical way to represent because it will have to then extensively compute.

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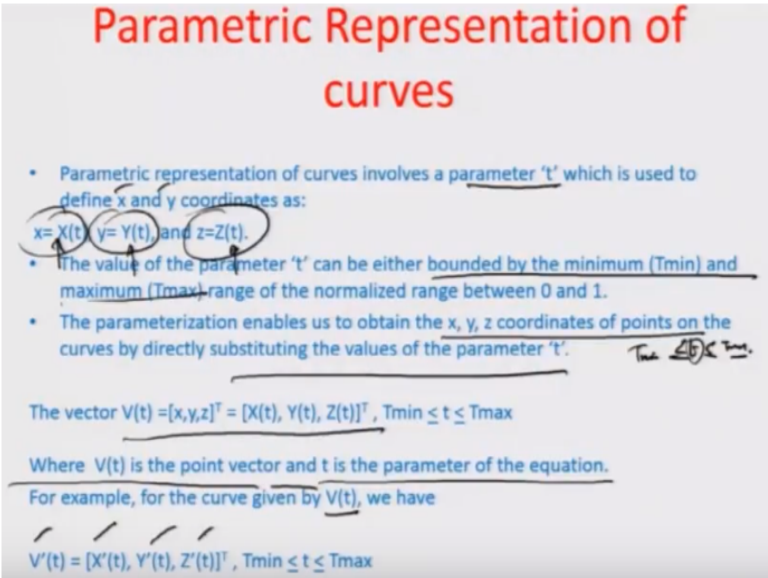

So parametric representation of the curves obviously involves a parameter t which is used to define coordinates x and y. So you can say in a 3-dimensional space both the x or all the 3 x, y and z coordinates are a function of the t; x is for example X t, y is Y t, z is Z t. Further the parameter can be either bounded by a minimum and a maximum value for example t can vary between some t minimum and t maximum range.

So it kind of gives you, you know calculations related to a local domain provided the whole global representation of certain curve using the nonparametric x, y, z kind of representations. So the parameterization enables us to obtain x, y, z coordinates of points on the curves by directly substituting the value of the parameter t; V t becomes equal to X t, Y t, Z t where t varies between a minimum and a maximum value.

So it is bound okay on both sides and V t is a point vector. So t is the sort of you can say parametric equation. Representation in this t is sort of parametric equation where t is the parameter and the curve V t is defined in a manner so that V dash t becomes equal to exactly X dash t, Y dash t, Z dash t transpose. Again t varying between the same upper and lower bound limits. So let us develop now a real example of how we can represent something through parametric and nonparametric representation. We will use a very simple geometry like a straight line to represent it.

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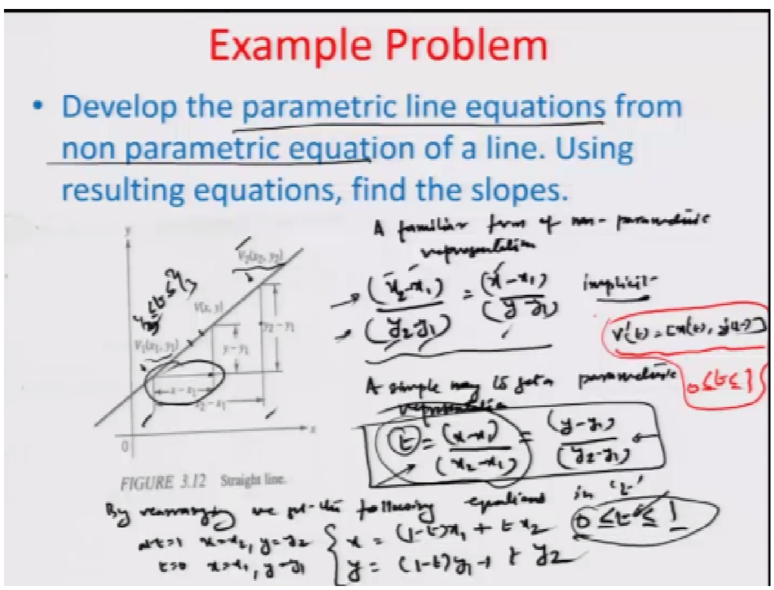

So let us say we want to develop the parametric line equation and we want to develop it again further from the nonparametric representation of a line so that both the forms are clear to you. So we will first define the line through different points, the variable x, y which is varying between lower limit and upper limit. So the vector in the lower limit is V1 having coordinates x1, y1. The vector in the upper limit is V2 having coordinates x2, y2.

And we want to find out how we can relate x, y to x1, y1 and x2, y2 as if these are the two bounds within which the variable x, y is moving. So let us say if we wanted to represent this using a familiar form of nonparametric representation. In case of a line it can be x2 - x1 by y2 - y1 = x - x1 by y - y1 okay. So in other words this equation is a sort of a hidden or a implicit way of representing nonparametrically the relationship between x and y.

We know that the values x1, y1 or x2, y2 on one hand and then there are variables x and y on the other hand. So x and y should have some relationship in terms of the values mentioned here and you know between how algebraically x goes with respect to y. So a simple way to get a parametric representation here maybe something like defining a parameter t which is a fraction of where x and y are placed along the x and y coordinates with respect to the initial point x1 and the final point x2 along the x and y1 and y2 along the y.

So we, say there are fraction representing how much, what is the extent at which x would be positioned with respect to the whole domain x2 - x1 is given by x - x1 by x2 - x1 and similarly this fraction will not be different and so this can always be again represented by y - y1 by y2 - y1. So this fraction would remain same and this is not the slope though. This is a sort of a fraction of the length where the x and y are positioned with respect to x1, y1 and x2, y2.

So given that if I wanted to represent this fraction through the parameter t then I can have 2 set of equations rolling out from this whole equation that has been formulated here for the fraction and by rearranging we get the following equations in t namely x = (1 – t) x1 + t (x2) you just solve this section of the equation and y = t times of or (1 – t) y1 + t (y2).

So t of course is a normal variable varying between upper limit 0, lower limit 0 and upper limit 1. For example if I would have put the value of t equal to 1, I would get x equal to x2 and y equal to y2 at t equal to 1. And similarly at t equal to 0, I would get x equal to x1 and y equal to y1 which are the upper and the lower limits of the particular line okay.

Which means that in this particular parametric representation you kind of representing the whole domain of the line where if I supposing now wanted to look at a local domain rather than going from x equal to you know t equal to 0 to t equal to 1 I can always go between t one-third and two-third okay or half and you know half or two-third. So we can actually zoom down on various sections of the line with assuming different bindings for the parameter t okay.

The overall way that axis y related to y through t will not vary because of that. So obviously if I wanted to see if the other you know basic premise for parameterization is followed namely V dash t equals x dash t y dash t. So we will try to now prove why this how this V dash t would be again defined by x dash t and y dash t which is a basic premise the t again varies between the same maximum minimum values. In this case it is 0 and 1. So this is a sort of a basic premise needed for the parameterization process to be well defined.

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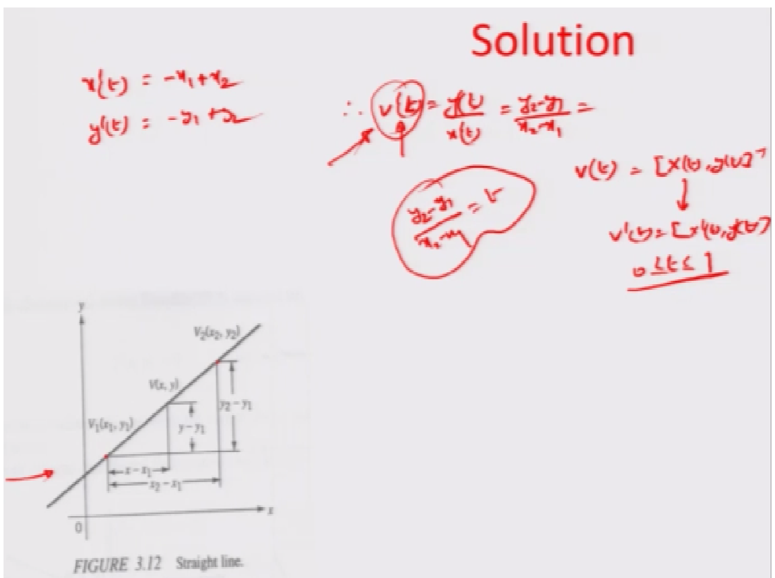

So here the x dash t will be equal to - x1 + x2 and similarly y dash t should be equal to - y1 + y2 and obviously the V dash t becomes equal to y dash t by x dash t you know let say the ratio which is actually equal to y2 - y1 by x2 - x1 which is actually the slope of the curve at certain value of t. So for you know assuming the straight line form of the geometry the slope whether it is at any of the ends, whether it is x1y1 or x2y2 happens to be the slope y2 - y1 by x2 - x1 okay.

And the first derivative of the function describing the point that is x and fx or parameterization like xt and yt is actually nothing but the ratio between the independently x dash t and the y dash t. So the basic idea of Vt being represented by xt and yt automatically implies V dash t equals to x dash t y dash t. This is good enough premise provided t varies between some maximum and minimum value for a proper parametric equation, set of equations to come out. So you understand I guess what we mean by parametric representation and nonparametric representation.

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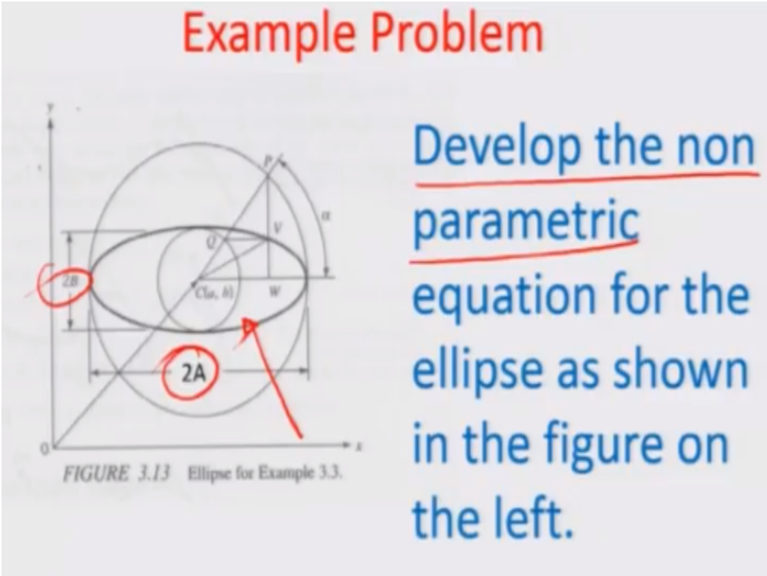

I will have you solve another problem example where we are wanting to develop the nonparametric equation in this particular case of this ellipse right here which is having a major axis of 2A and a minor axis of 2B and compare that with the parametric form. So in doing it we all know that the major and minor radii of the ellipse being A and B respectively.

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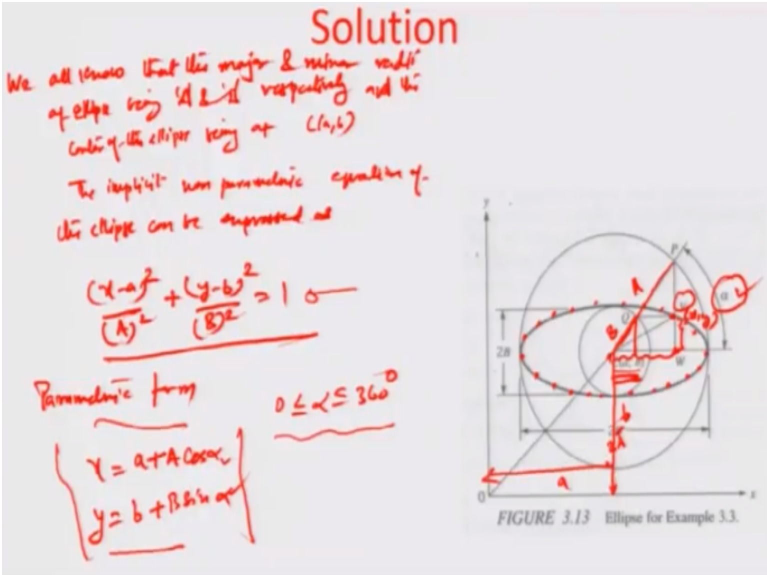

And the center of the ellipse being at some point let us say in this particular case C a, b okay. Center of the ellipse being at C a, b. The implicit nonparametric equation of the ellipse can be expressed as x - A square by capital A square plus y - b square by capital B square equals to 1, right? So if you wanted to parameterize this and represent the corresponding parametric form we will use a variable angle alpha here given here which corresponds to defining this point V in a manner so that there is a component of the angle which is being used alpha.

And then there is a component of the basis coordinates A and B which are being used from where you are trying to plot the V x, y. The V x, y in this case for example this particular point can be represented as you know the amount of translation which happened in the x coordinate because of the point C or being at the point C which is A plus the x coordinate or let us say the x intercept of the radius here of the circumscribing circle which is actually the coordinate A.

In this case it has just been wrongly printed here, it is 2A which is the major axis of this particular ellipse. So when we are talking about the point x which is pretty much the point here in space, x can be represented in terms of the parameter now alpha where alpha varies between 0 and 360 degrees okay? As the coordinate A plus the component of the angle alpha that you know it describes A cos of alpha I am sorry, cos of alpha.

So that is how you represent the x coordinate. In a similar manner the y coordinate could be represented through how much translation has happened while going to the point C from the origin in the y direction that translation being B and this component right here which is actually again a component of the inscribed circle which has the radius equal to the minor radius of the ellipse okay B. So you can write y in this case geometrically as small b plus capital B sin of alpha.

So these are the parametric representations of the same equation nonparametrically represented above here through an angle alpha which can be varied between 0 and 360 degrees for full rotation which would correspond to an array of points V x, y on this ellipse which follow these two parametric representations of small a small b capital A capital B and an alpha. So this is how all the geometries are represented into parametric and nonparametric form.

So I am going to close this particular module here but in the next module we will look at a slightly more complex problem where we talk about a contour which cannot be defined any longer by simple shapes like circle, ellipse or maybe just straight lines and so on so forth and it needs basically an integration of multiple curves and we will start our section on synthetic curves in the next module. Thank you very much.