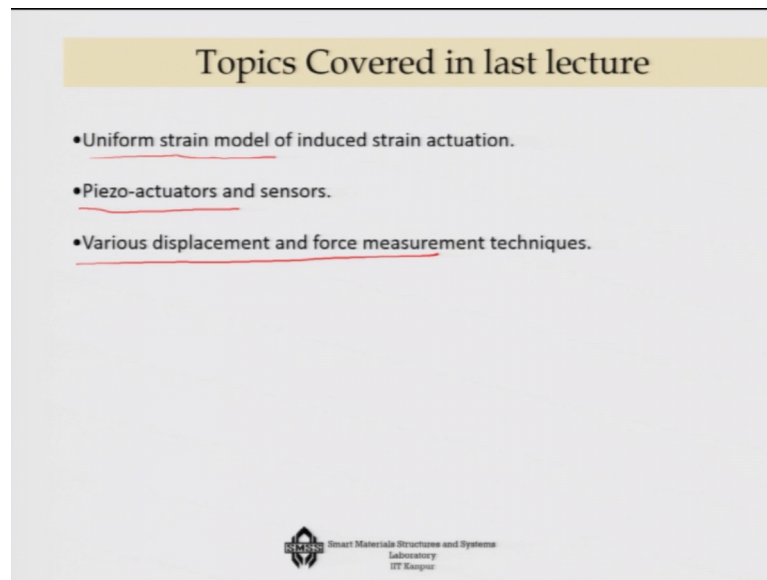


Smart Materials and Intelligent System Design
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Lecture – 13
Modelling of Magnetostrictive material

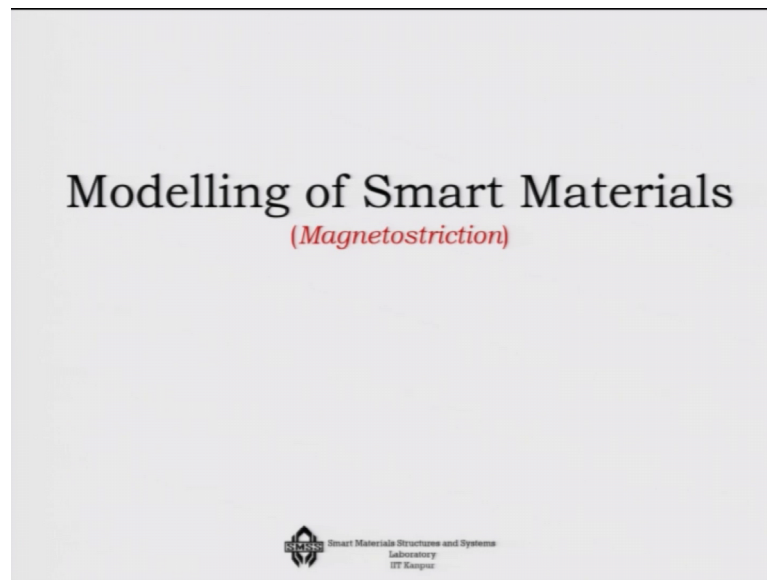
Good morning everybody, welcome to the course of Smart Materials and Intelligent System Design. In this particular module, we are learning how to model the induced strain actuation mechanisms.

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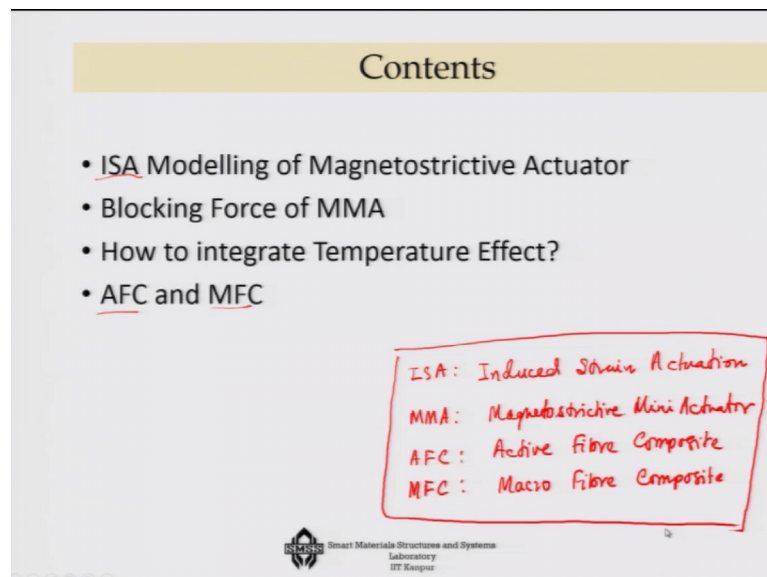
So, towards that direction I have already told you about the uniform strain model of induced strain actuation which I am going to apply today for a different material. And also we have talked about the piezoelectric actuators in that context and various displacement and force measurement techniques in piezoelectric actuators. So, today our focus will be another smart material which is magnetostrictive material.

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So, we will see that how we can generate induced strain activation from magnetostrictive material and how that affects when I surface bond or embed a magnetostrictive material inside a host structure like a host beam. Now so, I will be talking about induced strain actuation.

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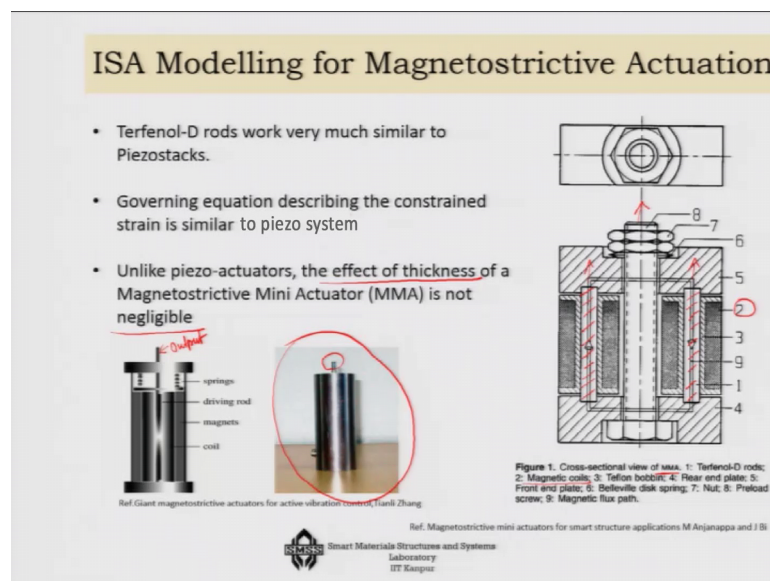


So, that is ISAs, Induced Strain Actuation short form induced strain actuation in short I will call it as ISA and we will also talk about the blocking force that MMA again this is another acronym I have used. So, MMA is magnetostrictive mini actuator,

magnetostrictive mini actuator, so we will see that how the blocking force is generated in MMA. And also we will see that when we will be running this particular system, how you know because the magnetic field is generated by the application of current how the temperature gets generated as a side effect which is not desirable and how that affects the entire constitutive relationship.

So, this is something additional in comparison to the piezoelectric material this is an adverse effect which we have to take into account also we will be talking about two additional things which actually behave in a similar manner. So, I have group them together these are both AFC is our active fibers composite these are piezoelectric both piezoelectric in nature active fibers composite we to keep this acronyms in our mind, because that is the way they are popular in the market and MFC is macro fibers composite ok. So, these are the things that we would like to cover in today's lecture, let us then begin with the ISA of MMA.

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Now, before we start the ISA modelling for magnetostrictive actuation, little bit of generic thing is that the Terfenol D is generally used for this magnetostrictive actuation. I have already talked about Terfenol D to you that what it is it contains of terbium, iron and dysprosium. And the governing equation which we will be described the constrained strain is similar to piezoelectric to piezosystem except that you have to consider the

temperature affect also. So, we will talk about it and unlike the piezo actuators what is dissimilar also is that the effect of thickness of a magnetostrictive mini actuator.

That is not negligible, so in the uniform strain model we cannot neglect that now typically how does a magnetostrictive mini actuator look like as you can see that this is what is the you know this is the way you can purchase it, and essentially each one of them will be having as an output point here. So, this is the point where it is going to apply the force or displacement of the system.

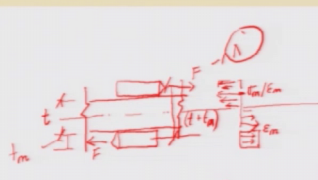
Now there are systems in which instead of a single rod like as we can see in this case it will be something like where you have two Terfenol Ds. So, you can see here that this is a rather more you know involved more indicate piezo of an MMA. So, you can see that there are Terfenol D rods here, so these are I am just you know for your understanding and just hashing it, so there are two Terfenol D rods here as you can see ok.

Now, this is one model some models use one Terfenol D dots some use two naturally if you use to it will be small, but compact and it will give you more force. Now these Terfenol D rods are first biased by these you know field. So, you are going to have both permanent magnet as well as electromagnet in the whole system. So, as you can see here that number two is here the magnetic coils. So, number two this is the magnetic coil that is surrounding this system ok, you what this we expect is that as we apply you know the current here we expect that this Terfenol D dots are going to expand ok.

Now, as they are expanding, so as you can see from this particular construction that they can only you know push the whole system towards this direction. So, this way it can come out, so that is your output direction ok. So, these two forces are basically going to push this system in the upward direction and this whole system is fitted with a Belleville spring in order to give us more or less a linear actuation system. So, that is the point we have to keep in our mind when we will be designing or modeling magnetostrictive actuation.

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Induced Strain For Magnetostrictive Actuation



$$\frac{M}{I} = \frac{\sigma_m}{t/2}$$

$$M = \frac{2\sigma_m I}{t}$$

$$= \frac{2E\epsilon_m I}{t}$$

$$\frac{2E\epsilon_m I}{t} + E_m \epsilon_m A_m (t + t_m) = \Lambda E_m A_m (t + t_m)$$

$$E_m \left[\frac{2Et^3}{12} + E_m A_m (t + t_m) \right] = \Lambda E_m A_m (t + t_m)$$

$$E_m \left[\frac{2Et^3}{12} + 12E_m A_m (t + t_m) \right] = \Lambda 12E_m A_m (t + t_m)$$

$$\frac{\epsilon_m}{\text{induced strain}} = \Lambda \frac{12E_m A_m (t + t_m)}{2Et^3 + 12E_m A_m (t + t_m)}$$

$$= \Lambda \frac{1}{1 + \frac{2Et^3}{12E_m A_m (t + t_m)}}$$

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Now, how do we model the induced strain actuation? let us try to work it out we have done it earlier also, so let us consider that we have a beam system like this. So, we have a beam on that beam I have fixed this magnetostrictive mini actuator for the time being let us also neglect the shear lag effect this you know anyway once you know the basic relationships how to add the shear lag effect.

Now I told you that the magnetostrictive actuators give us output from one direction. Let us say that this is what is the direction and in this case is the reverse way. So, I am going to get force in this way, here and this way here, which means I have basically producing moment external moment in the system. Now, let us say that this particular beam is of thickness t and this magnetostrictive mini actuators each one of them are identical and they are of thickness t_m ok. So, this is what we will need and this is the force that is getting generated by the magnetostrictive mini actuator and that force equal and opposite force is going to create a moment in the system.

So, if I look at the moment diagram then we need to first see that the bending moment will create a bending stress in the system which will be basically opposing these you know direction of the forces. So, essentially we expect that we will be having stresses here like this in the material linearly varying the stresses, and here in this direction we are going to have stresses in the opposite direction in the system right. So, we are going to have stresses which will be like this, so at this point we have the maximum stress right

σ_m and what will be the stresses inside the magnetostrictive material well this will be uniform in nature in here also it will be uniform in nature.

So, these are uniform stresses that is our uniform stress model. So, the strain here basically is the same let us call that as ϵ_m and here also the corresponding strain is ϵ_m . That is the maximum strain that is getting generated in the system, now if I try to do a force balance in the system. So, what is the internal movement resistance that is happening in the system well we can use the bending relationship simple you know bending relationships which tells us that M by I , for the beam equals to m which is the maximum stress divided by t by 2 this is occurring at this particular point, so it is t by 2.

So, in other words our bending moment that will be generated is actually $\sigma_m I$ and 2 times this divided by t . So, this since we know now we can also write this as 2 what is the maximum stress that is getting generated that is E times ϵ_m I over t . So, that is the bending resistance moment, so that if I put as one of the resisting moment I by t plus we add with this the resisting moment that will be there in this magnetostrictive mini actuators itself.

So, for that it is simple that the stress here is uniform, so life is simpler for us. So that means, that force that it will be generating will be something like σ_m which we can write it as magnetostrictive actuators say modulus of elasticity equivalent modulus of elasticity, $E_m \epsilon_m$ that times let us consider that A_m is the equivalent area considering all the rods that we will be having.

So, this is the force and then what is the momentum for this here it is not negligible this distance which we have neglected for the other cases this momentum will be t plus t_m . So, it will be t plus t_m and that equals to the other side also if we consider that the force is generated by the free strain λ then we can also write this as $\lambda E_m A_m t$ plus t_m . So, that is what is the external moment, so I have balance the internal and the external moment.

Now, so a little bit of algebraic 5 2 so it will be 2 let us take the ϵ_m the maximum strain you know that we are trying to find out let us take it in the outside. So, it will be 2 E by 12 d t cube times t plus $E_m A_m t$ plus t_m which equals to $\lambda E_m A_m t$ plus t_m right. So, I can actually right this as ϵ_m and this t is cancelling t square. So, I

can write this as $\epsilon_m = \frac{2 E b t^2}{12 E_m A_m t + t_m}$ plus Λ that must be equal to $\Lambda = \frac{12 E_m A_m t + t_m}{2 E b t^2}$ on the other side right.

So, essentially what is the constrained strain or the induced strain this is induced strain right this is the final induced strain. Even though you are applying Λ as a free strain because you are constraining it the actual induced strain at that point of transfer that equals to what Λ times a factor, and that factor is $\frac{12 E_m A_m t + t_m}{2 E b t^2}$ plus Λ .

This can be you know we can just simply organize it in a manner that this becomes $\frac{1}{1 + \frac{2 E b t^2}{12 E_m A_m t + t_m}}$ over ok, $\frac{1}{1 + \frac{2 E b t^2}{12 E_m A_m t + t_m}}$ ok. So, this is how we can derive try that what will be the actual induced strain in the system for magnetostrictive actuation, and you see the difference is that we have to consider this Λ plus Λ in this particular case.

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ISA Modelling for Magnetostrictive Actuation contd.

- Induced strain related to the free strain by the following relationship.

$$\epsilon_c = \left(1 + \frac{2 E b t^2}{12 A_m E_m (t + t_m)} \right)^{-1} \Lambda$$


$\epsilon = \sigma + d_m H$
 $\Lambda = d_m H$
 $= d_m \left(\frac{N}{L} \right) i(t)$
 $= d_m q i(t)$

b and *t* are the width and thickness of the host beam
A_m and *t_m* are the cross-sectional area and thickness of the MMA
E denotes the modulus of elasticity of the host-beam
E_m denotes the modulus of elasticity of the magnetostrictive material.
- Expression for the free-strain Λ , contains an additional term - the thermal effect due to current passing through the solenoid and hence Λ is expressed as

$$\Lambda = d G i(t) + \alpha' K \int_0^t e^{-\frac{t}{C_2}} i^2(t) dt$$

\uparrow
 Thermal effect

G - parametric Constant
 α' - Effective coefficient of thermal expansion
K, C₂ - thermal constants
i - current passed



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So, that is what is the expression here for the constraining strain or the induced strain that is happening in the system. Now, what is the Λ ? Next question will be where am I going to use the constitutive relationship in the system? Now Λ , as you know that it is the free strain in the system.

So, what was the strain expression? If you just try to recall it was σ plus d of the magnetostrictive material times H . Now, Λ is the active free strain which comes

from this point ok. So, this is lambda, so lambda is actually supposed to be d m H now because we are working with magnetostrictive material this m I did not write many a times. So, this is basically d and H we will write it and what is H if there are number of terms N and if there is L as the length of the coil.

So, that will give us that you know that times I that will give us the what you call free straining the system. And this whole parameters you can write it as d G I in that is the parametric constant G, now in addition to that you see I have added some more parameter here ok.

Now, what is this is because of the thermal this is because of the thermal effect ok. So, how does these thermal effect comes into picture? Let us just spend 1 or 2 more minutes on that ok.

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Blocking Force of MMA

- Blocking force attainable is 2 times the blocking force required for each individual Terfenol-D rod to block.

$$F = 2A_m \sigma = -\frac{2d}{S} \underbrace{GA_m i(t)}_{\text{magnetostriction}} - \frac{2\alpha' KA_m}{S} \int_0^t e^{\frac{-t}{\tau}} i^2(t) dt \quad \leftarrow \text{Thermal effect}$$

- Force of the Rod is directly proportional to current at the initial phase.
- The temperature effect is non-negligible later.

$\frac{1}{S} = E_m$



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So, before we do that another thing is that what is the blocking force in the system what is the blocking force in the system is 2 times m times sigma ok. If suppose both of them you know work in the same direction and it if it opposes then you know the negative signs will come up. And that blocking force you can see will be having two part in it one is because of this active part ok. And this should be clear to you that in the last expression if you remember that we have you know denoted this part as the lambda part ok.

So, that is what is coming up here the only thing is that $1/S$ is actually nothing, but E m ok. So, that if you keep in mind you will get this expression which is simply that the blocking force is estimated by putting both the you know magnetostrictive effect as well as the thermal effect. So, this is due to magnetostriction and this is also as s adverse by product of the whole thing which is due to the thermal effect, as I will be applying current in the coil to magnetize this current will have a heat which will be generated and that will create this part in it.

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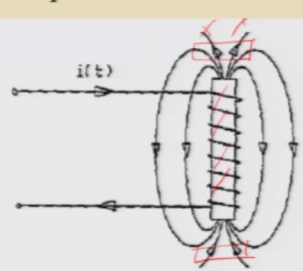
How to Integrate the Temperature effect?

Constitutive Equation

$$\varepsilon = S\sigma + dH \quad (1)$$

$$B = d\sigma + \mu H \quad (2)$$

d Matrix of Magnetostrictive constants
 μ Permeability at constant stress
 S Compliance at constant magnetic field
 H Magnetic Field



Single Magnetostrictive Actuator

Ref. A theoretical and experimental study of magnetostrictive mini-actuators M Anjanappa and J B

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How to integrate this temperature effect? We have come across this, let us think of it. First of all you have to keep thing in mind that you have these two basic relationships with you as I have told you earlier. Epsilon is s sigma plus dH and B is the you know magnetic flux intensity that is d sigma plus μH . Now as I am applying current in the coil what is happening? It is generating the magnetic field all around. And that magnetic field is part of it is coming back to the Terfenol D rod, suppose it is a Terfenol D rod and what we try to do is that we try to stop this entries by giving a return path here. So, if we keep you know 2 iron or ferromagnetic blocks there you can give a return path, so that most of the magnetic field remains content in the system.

(Refer Slide Time: 18:45)

How to Integrate the Temperature effect contd.

$$q_s(t) = C_1 R i^2(t) \quad (3)$$

$$q_M(t \rightarrow 0) = \frac{I}{R_1} q_s \quad (4)$$

Where, R_1 is effective thermal resistance considering conduction and convection

$$R_1 = \frac{\frac{1}{k_c} \ln\left[\frac{(r_1 + r_2)}{2r_1}\right] + \frac{1}{k_c} \ln\left[\frac{2r_2}{(r_1 + r_2)}\right] + \frac{1}{h_c r_2}}{\frac{1}{k_c} \ln\left[\frac{2r_2}{(r_1 + r_2)}\right] + \frac{1}{h_c r_2}} \quad (5)$$

Ref. A theoretical and experimental study of magnetostrictive mini-actuators M Anjanappa and J B

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Now, with this model if you look at it that the first point is that how the heat is getting generated in the system ok, that is happening because I have a coil if you imagine that this is the coil part of it and the coil has is passing a current I. So, because of this there is a $i^2 R$ effect that is coming into the picture multiplied by a suitable material constant. What is the heat flow that is in the magnetostrictive material, because this is our magnetostrictive material right this is our Terfenol D rod let us say.

So, that q_M in the you know in t equals to 0 at the very beginning we are use an a you know ambience is same as the ambient temperature. So, at that point of time if the effective thermal resistance, you know consider both conduction and convection, because as you can see that these are the terms related to you know conduction and this is relate to convection. So, you can get the expression of q_M by simple heat equation that is what is the heat flow rate at the you know when t is 0 and now this is the initial condition and this is what happens at the steady state.

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How to Integrate the Temperature effect contd.

Steady State Heat Flow

$$q_M(t \rightarrow \infty) = 0$$

$$q_M(t) = \frac{e^{-\frac{t}{C_2}}}{R} q_s(t) \quad (6)$$

Energy Balance

$$E_{st} = q_M \quad (7)$$


$$E_{st} = \rho V C_p \frac{dT}{dt} \quad (8)$$

Simplifying the equations 3,6,7,8
Change in Temperature of rod

$$\Delta T = K \int_0^t e^{-\frac{t}{C_2}} i^2(t) dt \quad (9)$$

where

$$K = \frac{C_1 R}{\rho V C_p R_l} \quad (10)$$



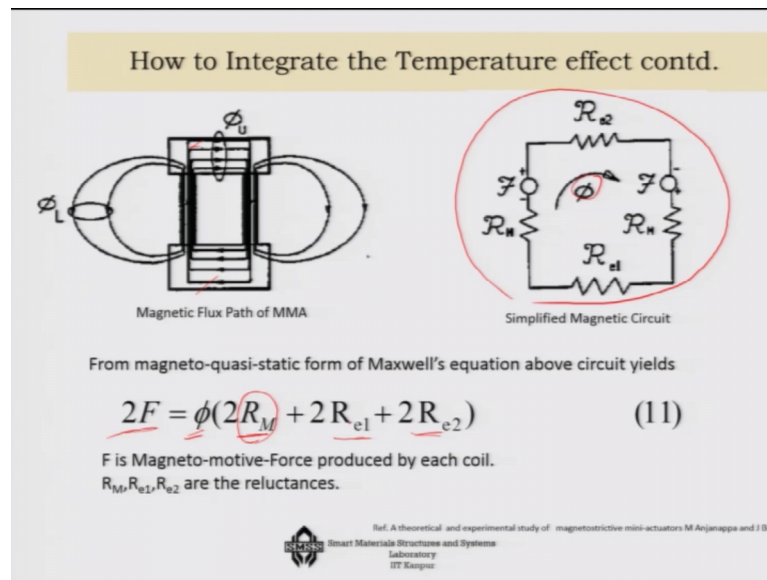
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So, if I you know solve this I am going to get q_M at any point of time as an exponential function which is again you know governed by a parameter C_2 ok. So, I am going to get this $q_M t$ as a function of you know $q_s t$ multiplied by a exponential envelope.

Now, if you also look at the energy balance, so the energy balance of the steady state which is q_M and that equals to you can actually get it from the change of temperature in the system. So, by applying this equations you should be able to find out that what is the change of temperature that is happening in the system. Because now you know this dT at E between say t equals to 0 to t equals to some finite time you can actually integrate and you will get this ΔT and then will give you this you know integration effect that is coming into the picture.

So, essentially what is happening as we are passing the current initially the ambient temperature and the temperature the Terfenol D rod are same. The moment I start to pass the current the coil is getting heated up and a part of it based on the effective resistance thermal resistance a part of it is going inside the Terfenol D rod. And that you know is actually heating up until and unless you reach a stage when it reaches steady state with the ambient system. And that is the part that we are deriving through this particular expression, and so this we can now integrate in our effect.

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the only now is that you earlier remember the expression of blocking force now the magnetic point of view the from the Maxwell's equation the magnetomotive force can be written in terms of magnetic flux and in terms of the reluctances of the system ok.

So, what are the sources of reluctances for us we have this magnetostrictive materials if there are two of them? So, it is going to give 2 times R_M and then we have this you know the end plates there in both the sides. So, I am getting the effect of each one of them, so thus we are getting all these and the magnetic flux ϕ is passing through the whole system. So, through a simplified magnetic circuit model which is very much similar like our electrical resistance model the electromotive force EMF there that is replaced by the magnetomotive force. So, you should be able to calculate the two you know the in this case what is the force that is happening in the system.

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How to Integrate the Temperature effect contd.


Equation 11 can be written in terms of magnetic field intensity as

$$2Ni(t) = 2H(t)l_M + H_{e1}(t)l_{e1} + H_{e2}(t)l_{e2} \quad (12)$$

Where N is number of turns in each coil
i (t) current in each coil
H(t) Mean magnetic Field intensity
 l_M is the length of each rod
 H_{e1} and H_{e2} are the average magnetic field intensity in rear and front plates
 l_{e1} and l_{e2} are effective lengths of rear and front plate respectively

Magnetic flux is continuous, so

$$\mu H(t)A_M = \mu_{e1}H_{e1}(t)A_{e1} = \mu_{e2}H_{e2}(t)A_{e2} \quad (13)$$

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Now if you consider this equation itself then the f is can be replaced by these n which is the number of turns in each coil times i t. And this actually is divided by the length of the coil and that you can take into account here. So, that is for the magnetostrictive material and these two are for the end plates that you have rear and the front plate in our system.

So, also we have to keep in mind that the total magnetic flux is continuous right is the same 5 we have assumed is going to the system there is no leakage in this system. So, you can get this relationship because the total magnetic flux is the same, so that will essentially give us a relationship between what is H e 1 in terms of H t and what is H e 2 in terms of H t, provided you know the magnetic field permeability's etcetera, and the area of cross section of both the systems.

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How to Integrate the Temperature effect contd.

$$\underline{x(t)} = \left(\underbrace{S\sigma}_{\text{Mech. stress}} + \underbrace{dGi(t)}_{\text{Active stress}} + \underbrace{\alpha' K \int_0^t e^{-\frac{t}{\tau_c}} i^2(t) dt}_{\text{Thermal stress}} \right) l_M \quad (14)$$

$$G = \frac{N}{l_M \left(1 + \frac{\mu A_M l_{e1}}{2\mu_{e1} A_{e1} l_M} + \frac{\mu A_M l_{e2}}{2\mu_{e2} A_{e2} l_M} \right)} \quad (15)$$

$$\alpha' = \frac{\alpha l_M + \alpha_e l_{e2}}{l_M} \quad (16)$$

$x(t)$ is total change in actuator length.
 G is parametric constant which characterizes actuator's geometry and material properties.
 α_e Coefficient of thermal expansion of the front end plate.
 α' effective thermal coefficient of thermal expansion.

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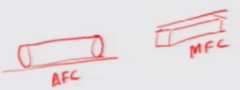
So, then I can use all this equations now, together and I can get the relationship idea I got a relationship in terms of strain. Now I can get a relationship in terms of the actual displacement that is going to happen in a magnetostrictive material ok, which is first of all a function of a stress that it is subjected to and then this is the free strain part. So, this is the mechanical part due to mechanical displacement mechanical stress and then this is due to active stress and this is due to thermal strain that it is going to affect the system.

So, what is the G here that is a little bit complicated as you can say that she is actually in over this particular thing here that is a little bit complicated as you can see here? That G is actually N over this particular thing which takes into account all the important factors like the coefficient of thermal expansion and the length of the magnetostrictive materials the length of the end effector all this things together we get this particular expression in the system.

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AFC and MFC

- Two types of smart composites have been developed using piezo-ceramic fibres; these are **Active Fibre Composite (AFC)** and **Macro Fibre Composite (MFC)**.
- AFC fibres are developed using standard sol-gel technique, the MFC fibres are essentially chopped from PZT blocks.
- MFC fibres are rectangular in cross-section and hence it offers better electrical contact between the fibres.

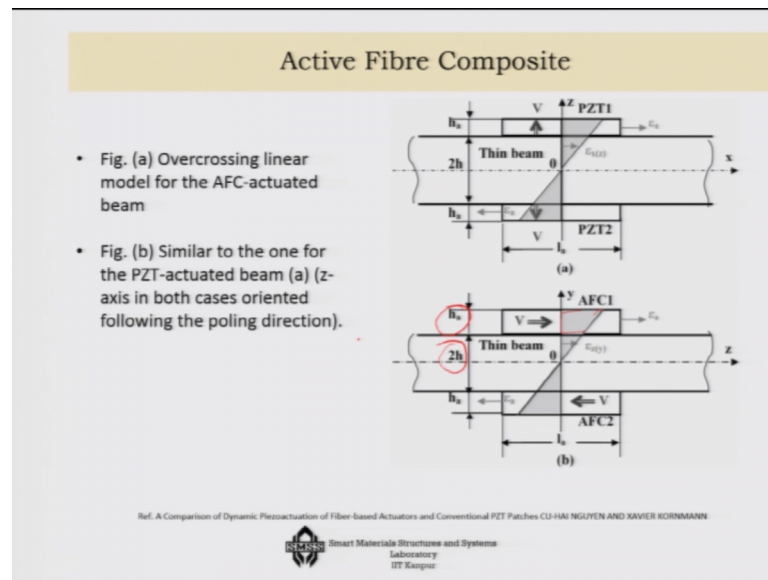


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So, we have seen that how we can actually model our magnetostrictive mini actuator. Similar things can be done on active fibers composite and MFCs, but these are consisting of piezoceramic fibers. Now, this AFC fibers are developed using standard sol gel technique whereas, MFC fibers are essentially chopped form of the PZT blocks. So, MFC fibers are rectangular in cross section and that offers you know better contact between the fibers ok.

So, that way MFCs are little better in comparison to the AFCs because AFCs are developed using standard sol gel techniques. So, they are basically round in terms of their shape. So, a round shape you know would not give you a good contact area on the other hand if you get a rectangular block then you are going to get a good contact are in the system. So, that is the so this is what is our MFC and this is what is our AFC that difference in terms of the geometry in the system.

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Now however, from the principal in which it works it is somewhat similar. So, here what we try to show you is that in the figure a, you know you have a linear model for the AFC actuated beam ok, and we have just you know just compared similar to the one or the PZT actuated beam ok. So, you can see that you know you can actually consider the variation in the system. So, you can of course, apply the Euler Bernoulli model in this particular case and the beam is of thickness $2H$ and this parts are of thickness H a.

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Active Fibre Composite contd.

- Following is the balance of active and reactive moments, the active strain at a distance z from the neutral axis can be expressed as

$$K_f = \frac{3E_a[(h + h_a)^2 - h^2]}{2\{E_a[(h + h_a)^3 - h^3] + Eh^3\}}$$

$$\epsilon_x = K_f \Lambda z$$

$\Lambda = d_{33} \frac{V}{h_a}$
 $d_{33} \gg d_{31}$
 $h_a \ll h_p$

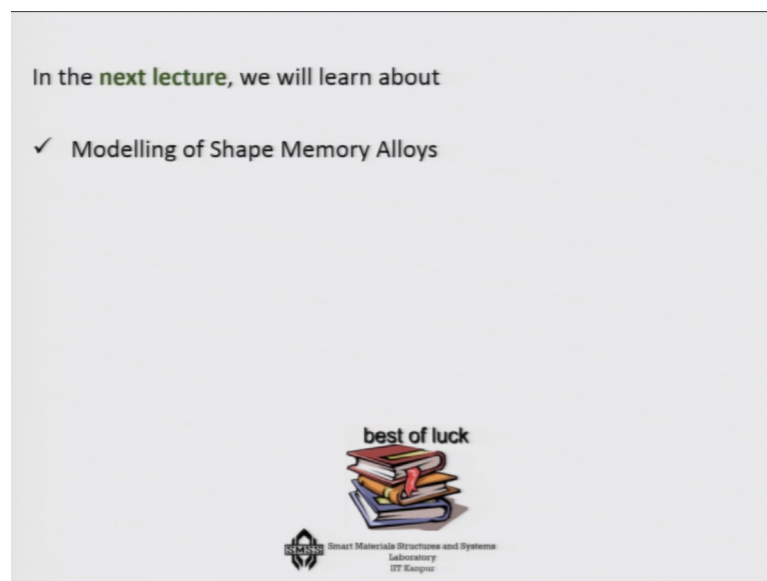
- Voltage is applied along the length of the fibre and hence the free strain Λ is obtained using the piezoelectric constant d_{33} .

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So, with that once again if you carry out a very similar moment balance you will see that the actual strain that it will be giving is $K_f \lambda Z$ and K_f is given by this particular expression and λ that is the most important thing of free strain is actually $\frac{d_{33} V}{h a}$.

Now, you consider all the advantages are actually here because of this λ . Why? First of all d_{33} is much, much greater than d_{31} I told you; so you are going to get strain. Secondly, if you think of the intensity that is important then by using the same voltage because $h a$ is much, much smaller than the you know traditional piezoceramic plates etcetera. So, h piezo, so you are going to get a much higher you know electric field intensity. So, this will be increasing and this will be increasing as a result you will get a very high λ and as you are getting a very high λ you will be able to generate more strain through this MFCs and AFCs. I will show you in the laboratory you know how you can use them for vibration control.

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So, this is where we will come to an end. In the next lecture, we will learn about modeling of shape memory alloys.

Thank you.