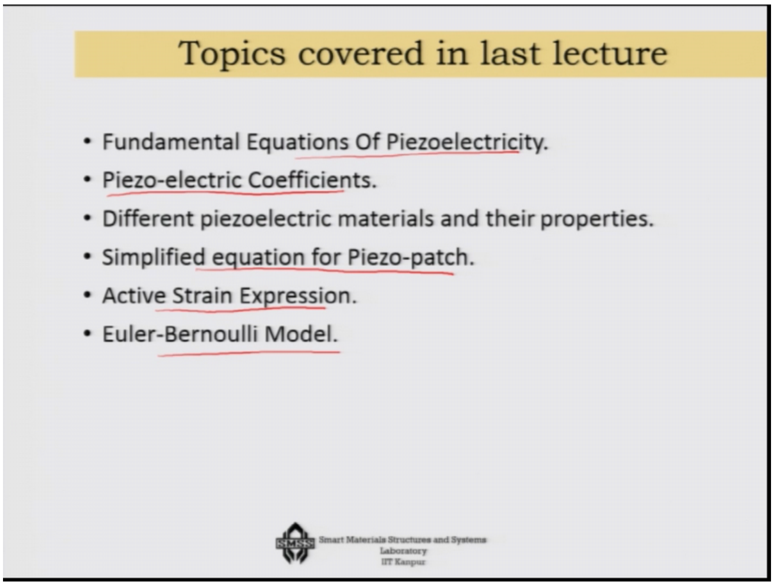
Smart Materials and Intelligent System Design

Prof. Bishakh Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 12
Modelling of piezoelectric material 2

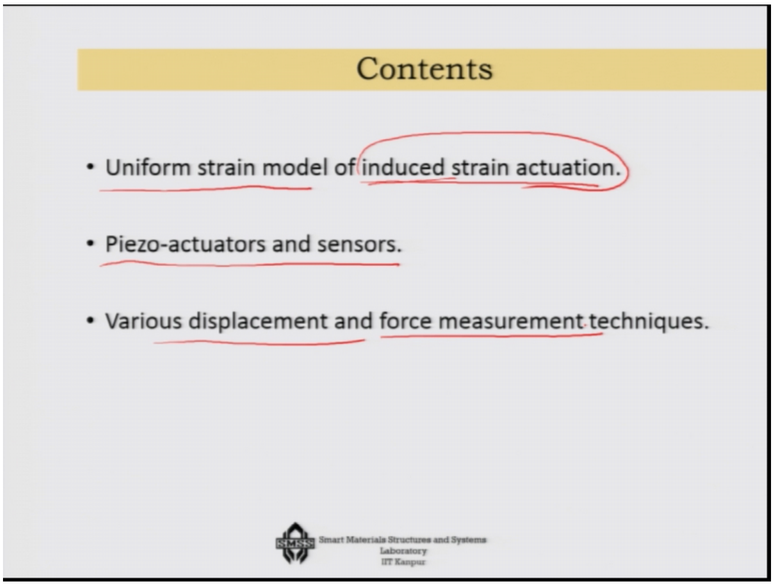
Welcome to the Smart Materials and Intelligent System Design course and we are running now module 3.

(Refer Slide Time: 00:26)



And in this module I have already talked to you about the Fundamental Equations of Piezoelectricity in the last lecture, we have discussed about the Piezo-electric Coefficients and Different piezoelectric materials and their properties, Simplified equation of Piezo-patch. And, also I have introduced you to the concept of Active-Strain Expression and an Euler-Bernoulli model of the same system. Now, today we will go little more in depth in the modeling of piezoelectric actuator.

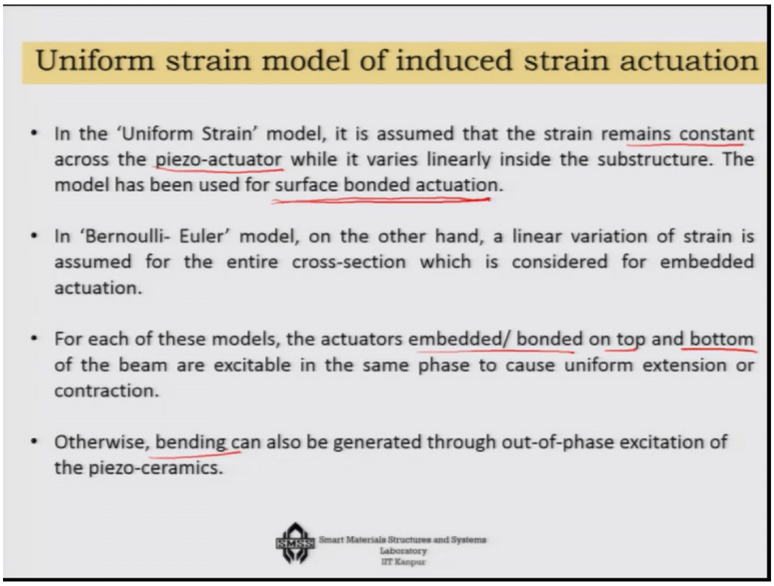
(Refer Slide Time: 01:05)



So, in this while doing it we will be assuming a simpler model to demonstrate the concept which is based on uniform strain model and how this uniform strain model is actually inducing strain in the system. So, the whole essence of the modeling is that the piezoelectric patches when they are embedded or they are surface bonded over a host material, they actually induce strains well you know they are getting actuated. And, that strain is generating further structural strain in the composite body or in whatever the host body with which it is attached or embedded with. And that particular concept that it is not that it is giving a force, but it is inducing a strain that is: what is the essence of this induced strain actuation model.

Because, there are some other models that are available in the open literature where the Piezoelectric material or the or similar type of smart materials their actuation is model in terms of external forces or external movements. But, here we do not do it we are considering these materials to be integrated in a structure, in such a manner that, they are giving internal forces or strain actually not force; they are generating internal strains in the system in the form of induced strain. Also I will talk a little bit about various Piezo-actuators and sensors and some displacement and force measurement techniques.

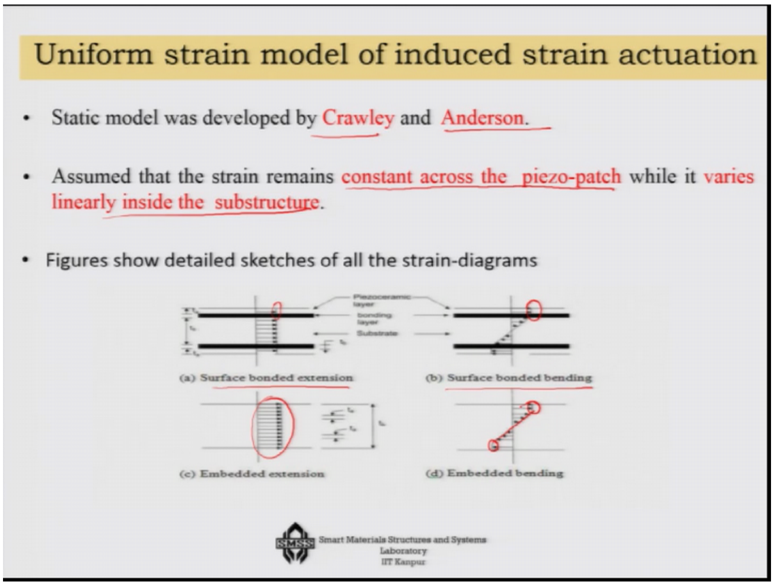
(Refer Slide Time: 02:50)



So, in the uniform strain model that we are going to talk about here, we are assuming that this strain remains constant across the piezo-actuator ok while, it varies linearly inside the substructure. And, that is a valid assumption provided the thickness of this piezoelectric patch is actually very, very thin. And this model is very popular for surface bonded actuation because, they are indeed because this piezoelectric material is not embedded.

So, the variation of strain if you consider as uniforms strain versus that linear variation in the Bernoulli- Euler model, the difference is not very significant. So, this is you know much more useful for surface bonded actuation. Now, for each of these models we have also considered that there is not a single actuator, but a pair of actuators and they are embedded or bonded on top and bottom of the beam. So, that you know they can not only generate in plane strain, but also out of plane bending. So, this bending generation is feasible through this model.

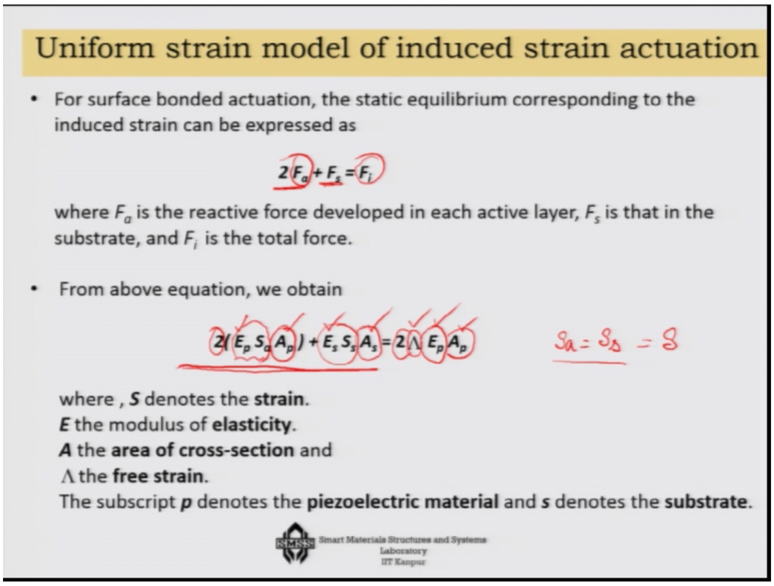
(Refer Slide Time: 04:12)



This model was originally developed by Crawley and Anderson. Also, another thing that you know I have already told you that, the induced strain remains constant across the piezo-patch while it varies linearly, inside the substructure that is the main theme of this particular model. So, you can see that this is for surface bonded extension you can see that, the strain in the piezoelectric layer is constant. This is for surface bonded bending. Here, also the strain in the piezoelectric layer remains constant whereas, in the rest of the part as usual for bending there is this you know linear variation of the strain. Now, it varies from the tensile to the compressive strain.

Now, if you look at the embedded extension there also for extension uniform variation and if it is embedded bending here also for the piezoelectric part it is uniform and then the linear variation starts. So, this are all the relevant sketches that are important for us to get the you know model of this induced strain actuation.

(Refer Slide Time: 05:23)



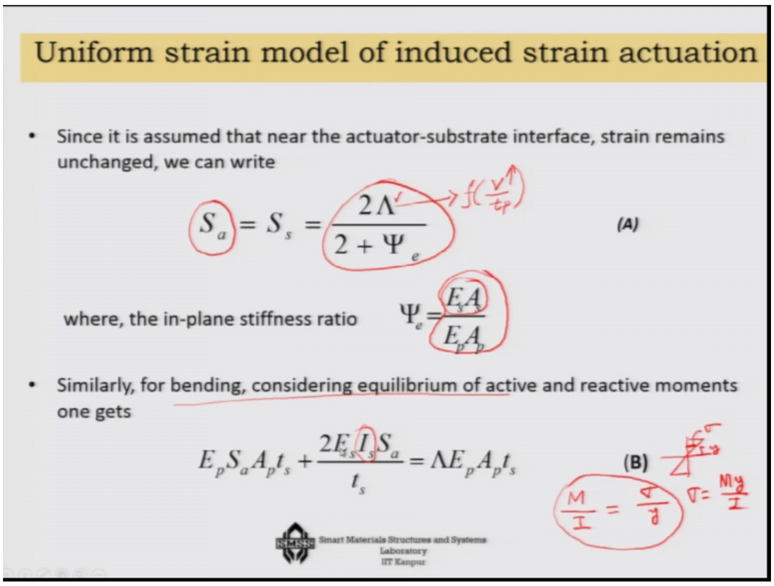
Now, if you consider any one of this model what you are going to see is that there are 2 piezoelectric actuators: one in the top, another in the bottom. So, each one of them let us say is generating a force F a. So, 2 F a they are identical plus F s that is the force in the substrate. And that should be equal to the total force in the system that is simple force balance of strength of materials.

Now, what is the expression of this F a? Well, it must be having a stress times the area. So, this is the stress part of it, if we assume that the strain in the active layer is S a then that stress is E p times S a that times A p gives us the force that is each one of the forces in the piezoelectric material. So, there are 2 of them in case of you know simple extension in this case the forces are adding up so, there are 2 of them. So, that is what is the force from the active part and if you look at the substrate part once again the stress is E s S s times A s so that is the force is a substrate.

Now, that these are the forces that is generated inside the structure as a response to what, as a response to the induced strain. So, the induced strain the free strain is lambda and then you have 2 of this. So, it is 2 lambda 2 of these piezoelectric material generating double the strain then you multiply that with the modulus of elasticity of the piezoelectric material E p; you get the stress multiply that with the area you are going to get the force. So, that is the force which is as a result of the induced strain actuation and that force has to be equal to the total force or the resistance that is coming from the structure.

Now, in this particular case for the uniforms strain model we have assumed S a equals to S s. So, in this particular equation if you look at it E p is already known to us modulus of elasticity, area of the piezoelectric material known to us, modulus of elasticity of the substrate known to us. A s is known to us, lambda is known to us that is, characteristics of piezoelectric material, modulus of elasticity is known to us, area of the piezoelectric material is known to us. So, what is unknown to us? This is one single equation where we had 2 unknowns S a and S s, but since S a equals to S s. So, we should be able to get you know if I just represent it as S we should be able to get S from this one single expression itself.

(Refer Slide Time: 08:25)



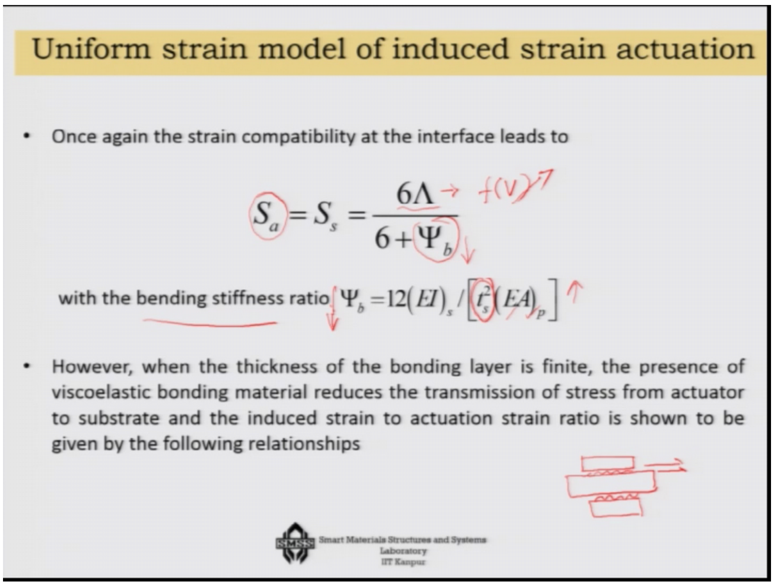
And if you do a little bit of algebra now you we will see that that s will come out in this beautiful form that is 2 lambda over omega 2 plus psi, where psi E can be written as the in plane stiffness ratio that is E s A s over E p A p. So, essentially if I know what is the free strain that my piezoelectric crystal is generating I can actually set this in plants ratio because area of the piezoelectric material is fixed, but area of you know this part E s A s the host part is actually you know according to our necessity we design the system.

But, if we can do that and we can get a stiffness ratio and that stiffness ratio will finally, dictate that what will be the strain that will be getting in the system. Not only that you can also where E lambda if you consider lambda if you look at it that lambda is nothing but it is a function of actually electric field intensity. So, it is a function of voltage over the thickness of the piezoelectric material and this voltage can be actually varied. So, this voltage is variable so I can vary V, I can vary V increase or decrease V and as I change this V I can change this lambda, as I change this lambda I can change, the strain and as a result, I can change the deflection of you know such kind of a system.

Now, similarly you can also balance in these earlier cases, we have balance the force you can also balance the bending moment versus the bending resistance. So, in this case instead of you know you we will be having this area moment of inertia which will come into picture. And this is just that is simple rule of bending that we use if you remember that forms strength of materials that M by I equals to sigma over y. Where, y denotes the distance from the neutral axis in terms of bending ok.

So, if I consider any point which is at a distance y and the stress is there sigma then, the moment to area moment of inertia that is the in this ratio will be equal to sigma over y and that can be used so; that means, sigma is your M y over I. So, that has been used in this case in terms of developing the equilibrium.

(Refer Slide Time: 11:07)

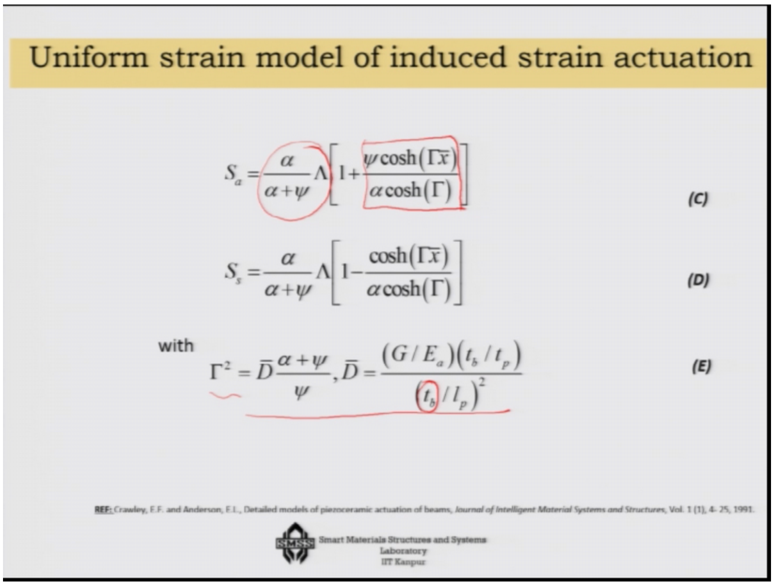


So, once we have this moment equilibrium corresponding to bending we are going to get a very similar expression in the last case, the factor was 2 lambda. Now, it is 6 lambda and 6 plus psi b. So, in earlier case it was stiffness ration now, it is the bending stiffness ratio psi V which equals to 12 EI s over t s square times EA p. So, here also if you look at it carefully that if you know E p and A p these are all fixed ok. So, what is there in our hand one is that t s square we can design in such a manner that, if t s square is more, then psi b will be low if t s square increases very fast psi b will decreases at a very fast way. And if psi b is decreases then, this in this entire summation will be less. So, the denominator will be becoming smaller and as a result you will be getting more and more deformation in the system.

So, thus you can actually control it through the passive part of the design. In the active part once again lambda as a function of the voltage and you vary this voltage as you vary the voltage, you are varying the lambda and you are varying the active strain that you are generating the system. So, in a sense you are varying the nature of extent of bending in the system. So, there are 2 ways in which you can control the bending, either by controlling the voltage in the active manner or the passive part in the basic design itself you control your t s square in such a manner or your I s is such a manner that you get a low psi b and that low psi b will give you I S a; that means, high strain. So, these are the things that you know we are using in terms of the uniform strain model.

Now, in this model there is one basic assumption that is that interfacial thickness. So, there is a piezoelectric material that we have a piezoelectric-patch and there is this you know host layer ok. And the other side also there piezoelectric material, how do you fix them? By using some kind of a groove we fix. Now, this groove if it is very, very thin then this model that we have discussed is fine, but if this groovy is a finite thickness then, the you know if this part thus, whatever is the strain you know we assume the strain to be same that is not going to happened ok. So, there will be a kind of a gap in terms of the strain transfer and that you know so, that is to be considered in terms of a factor which is called shear lag.

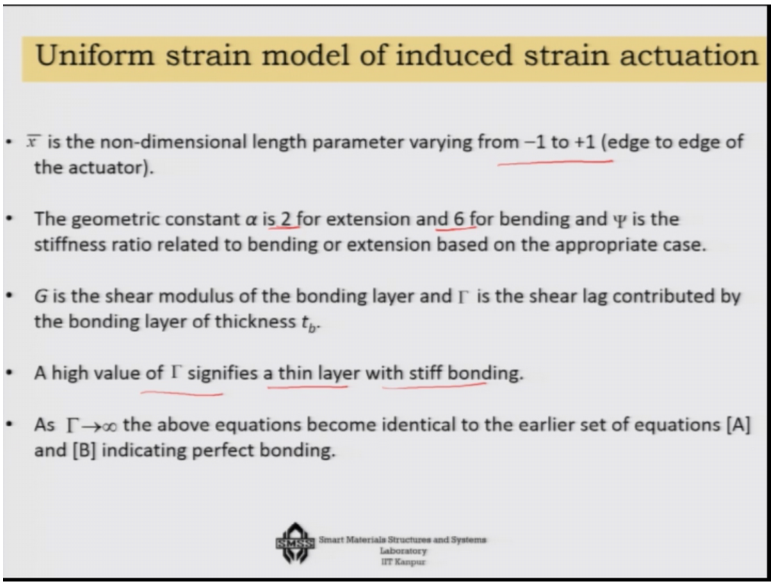
(Refer Slide Time: 14:05)



So, this is the expression you know that has been derived in crawlers paper. So, here we have this part as usual ok, but the additional part that is coming up is considering this shear lag that will come into the system which has a gamma terminate. And this is how the gamma term is defined.

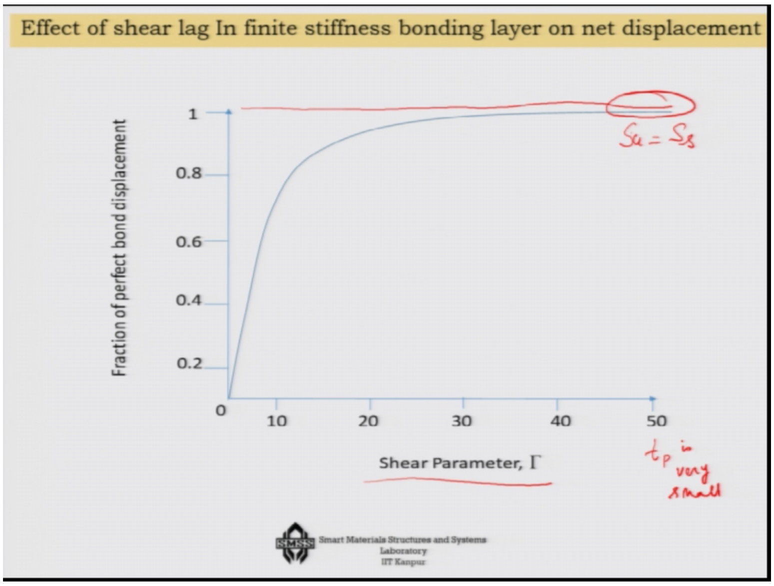
So, if you look at it that if the bonding layer thickness is very, very small then, you know the t b if here denotes the bonding layer thickness. Then gamma will actually enormously and as gamma increases this ratio of cos hyperbolic function, that will actually reduce and then it will become simply unity and as a result, our initial model will be valid in this case.

(Refer Slide Time: 14:58)



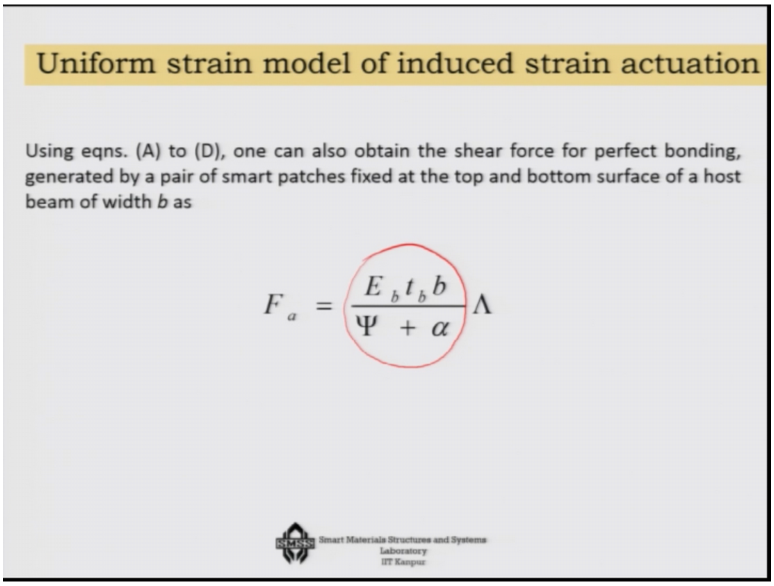
Also, this you know is the point that we have to keep in your mind. That gamma is the non-dimensional length parameter which is varying from minus 1 to plus 1 and alpha is 2 for extension and 6 for bending that I have already told you that, you know you can see it that here alpha is 6 for bending. And, the same model you can use alpha as a parameter where alpha is 2 for the extension and G is the shear modulus of the bonding layer gamma is the shear lag contributed by the bonding layer of the thickness t b. And I already told you that a high value of gamma signifies a thin layer with stiff bonding and as gamma tends to infinity we will get back the perfect bonding equations which we have earlier derived.

(Refer Slide Time: 15:53)



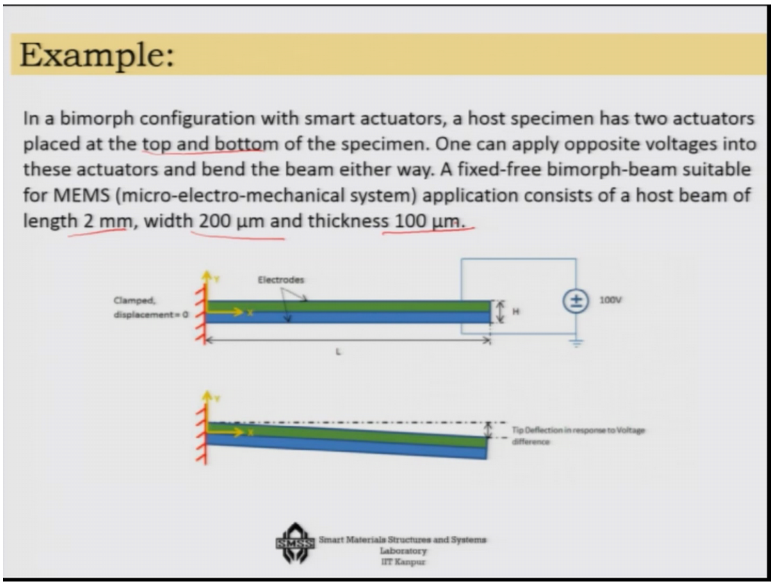
Now, if you actually try to visualize that, that how the shear parameter gamma increases you would see that the fraction of the perfect bond displacement that gradually approaches unity. Meaning thereby that our assumption of S a equals to S s is varied in this range where gamma is very, very high and that actually indicates that the thickness of the bonding layer t p is very, very small; t p is very small then only this particular thing will approach unity and as a result the effect of shear lag can be neglected.

(Refer Slide Time: 16:34)



Now, this uniform strain model can also be used in terms of getting the force ok. So, we can you know just extending the same thing we can find out the shear force that, is there in the system and you know, you can get it just simple extension by using equations that I have already discussed.

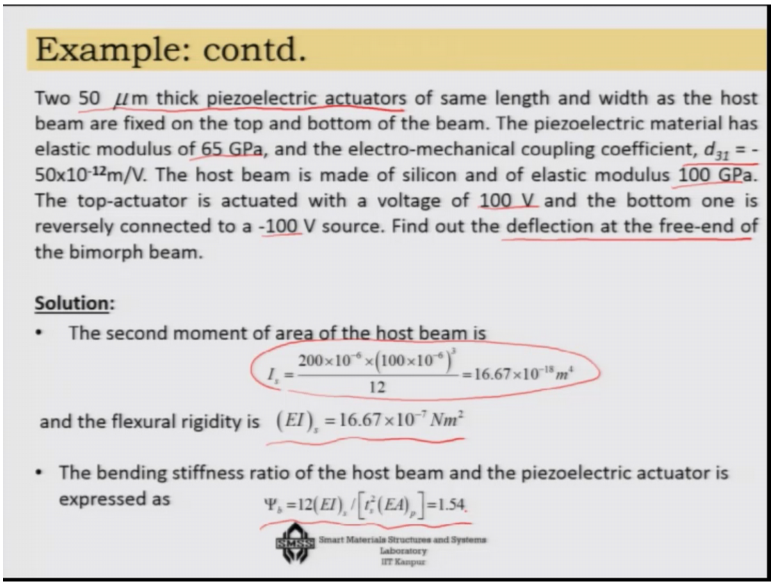
(Refer Slide Time: 16:58)



So, let us try to discuss this whole concept with the help of an example. The example that we have chosen is that of a bimorph ok. So, you can see a bimorph here and in this by bimorph, what you have is a you know piezoelectric material 2 piezoelectric layers which are joined together. 2 actuators placed at the top and bottom of the specimen and we can apply opposite voltage to each one of them.

So, that we can actually generate a bending out of the situation and this is a typical MEMS system we have discussed about that kind of flow control. Where, this kind of MEM system can be used and the length of the beam is 2 millimeter width is 200 micro n meter thickness in about 100 micron. So, it is a very, very small MEMS type of applications.

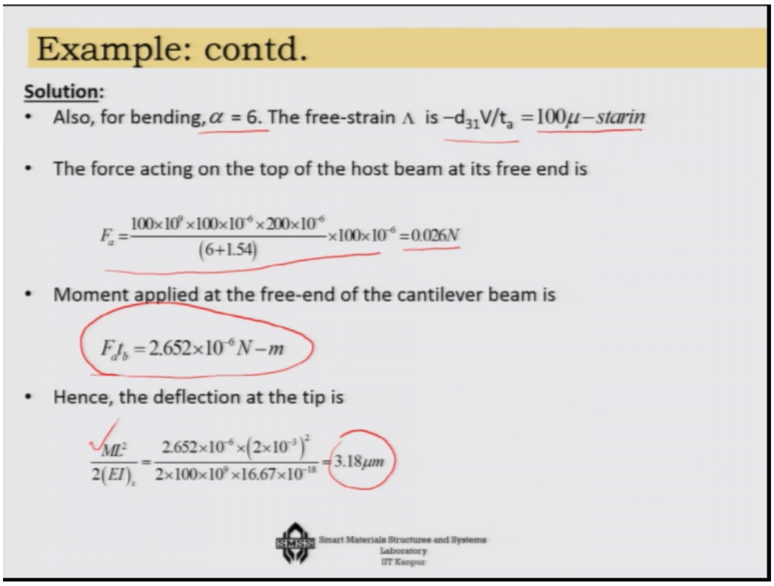
(Refer Slide Time: 17:53)



Now, we are using 250-micron meter thick piezoelectric actuators in this system ok so, they are grove together. So, total you are getting 100-micron meter let us say some modulus of elasticity is known to me 65 GPa coupling coefficient d 31 which is important in this case because we are applying voltage from the 3 direction and we are getting you know in plane actuation. So, that is known to be that is relevant here and also the elastic modulus you know host beam is made of silicon. So, that is elastic modulus is known to me top actuator has about 100 volts and the bottom is minus 100 volts.

So, the first thing so what we have to find out is, the deflection at the free end of bimorph beam. So, in order to do that the first thing that I have to know is that: what is the area moment of inertia of the host beam because, I know the whole beam size. I can very easily find it out as one twelfth bhq and then, I can find out what is the flexural rigidity EI s the moment I will find it out I can actually find out, what is psi b that is this ratio I can find it out which will come out to be about 1.54.

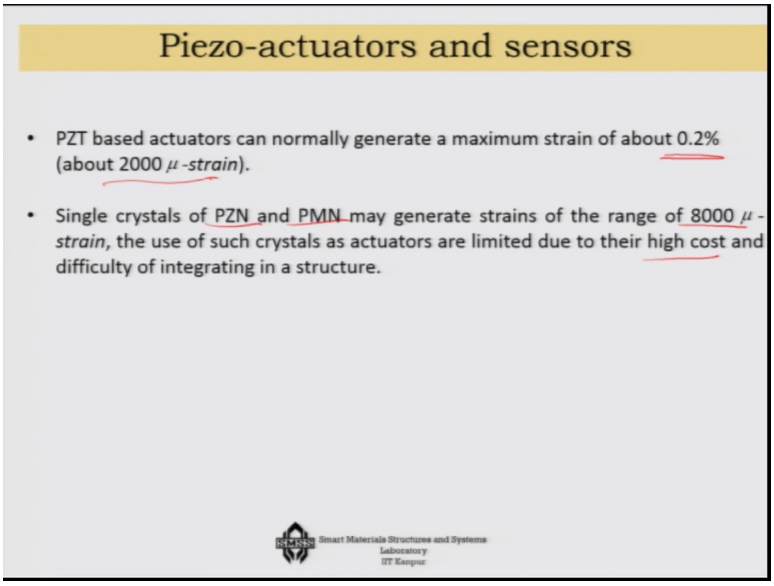
(Refer Slide Time: 19:14)



Now, for bending we know that alpha is equals to 6 and free strain lambda is also known to us. We can find it out because the voltage is known to us which will give us lambda to be 100-micron strength. So, you can use this data and we can find out what is the force that is acting on the top of the host beam which will come out to be 0.026 Newton exactly an opposite force is there in the bottom. So, this actually comes up in the form of a couple.

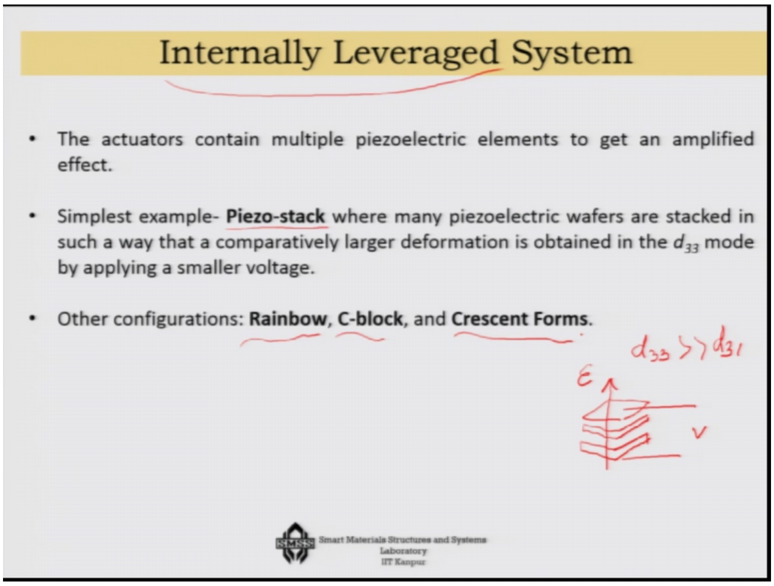
So, if we know the thickness of the beam we can find out that what is the moment, that is working on the system. Now, that the moment is known to me a simple and the length of the beam is known to me ML square by 2 EI is going to give us. What is the you know deflection at the tip of the beam. So, that you know that is what then we can find out that is this is going to the tip deflection then we can design the system accordingly in terms of some control.

(Refer Slide Time: 20:22)



This is you know so can be used for various Piezoelectric systems and a maximum strain lambda that, you will be you can use is for 2 is about 2000micron strength. In some cases, in some cases one can go for PZN and PMN crystals where, you can get up to 8000 microns strength, but that is not very you know kind of commercially available and even if it will be a very high cost. So, generally the lambda max is about 2000 microns strain and based on that we have to make the design of such MEM systems.

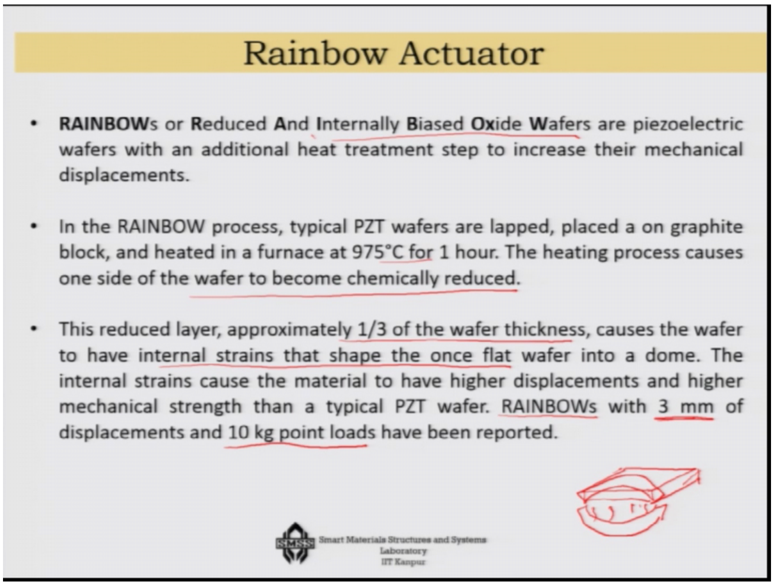
(Refer Slide Time: 22:01)



Well can we not because 2000-micron strain is very low I told you last time you know I have shown you through the graphs in magnetostrictive materials. How low is this kind of micros this 2000 into 10 to the power minus 6. So, you can imagine how small it is can we not increase this well there are 2 ways of increasing it one is called internally leverage system; that means, you do something on this system you know while making. This piezoelectric actuators itself simplest example is that, used stack lot of this piezoelectric you know wafers very, very thin wafers you stack this wafers ok.

So, if you have many of these things and if you are using d33 parameter; that means, you are applying voltage across and you are measuring the deflection along across the you know resistance transfer direction because d33 is much much higher than d31. So, we intend to get a large you know strain along this direction. So, this is one you know internally leverage system.

(Refer Slide Time: 22:24)

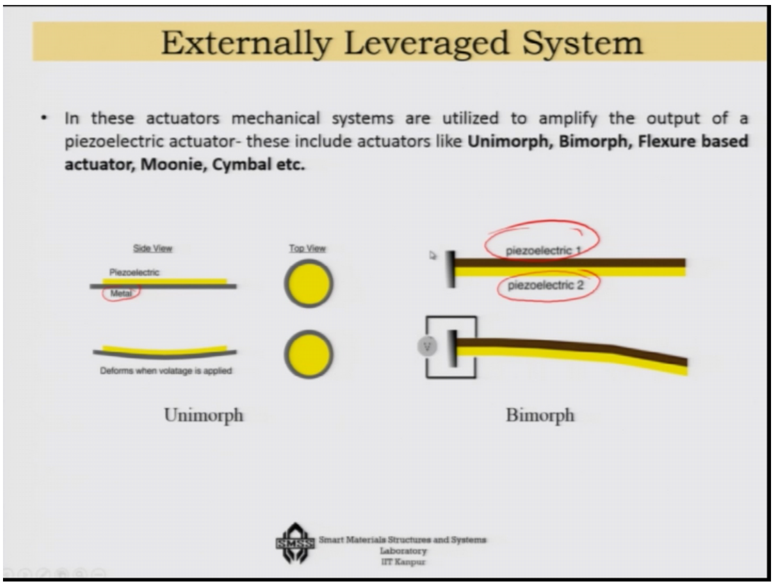


There are other internal leverage configurations like Rainbow, C-block or Crescent forms configurations, all in this kind of configurations like RAINBOW you have Reduced and Internally Biased Oxide Wafer. So, in this case what you do is that you have PZT wafers you just left the PZT wafers and hit them at a high temperature. So, one side of the wafer will get chemically reduced in this heating process. So, this reduce their which will be one third of the original wafers thickness, that will cause the wafers to have internal strains that will shape the once flat wafer (Refer Time: 23:00) into a dome.

So, if suppose you know this is what is my Piezoelectric patch and I am applying you know thermal load to it. So, I am applying temperature and then what will happen is that it will get reduced and it will get a bend shape ok. So, if you have this kind of arch bend shapes and if you have 2 of these then they actually can be very good in terms of you know, whatever is the you know this internal strains will help in terms of amplification of the strain output in the system.

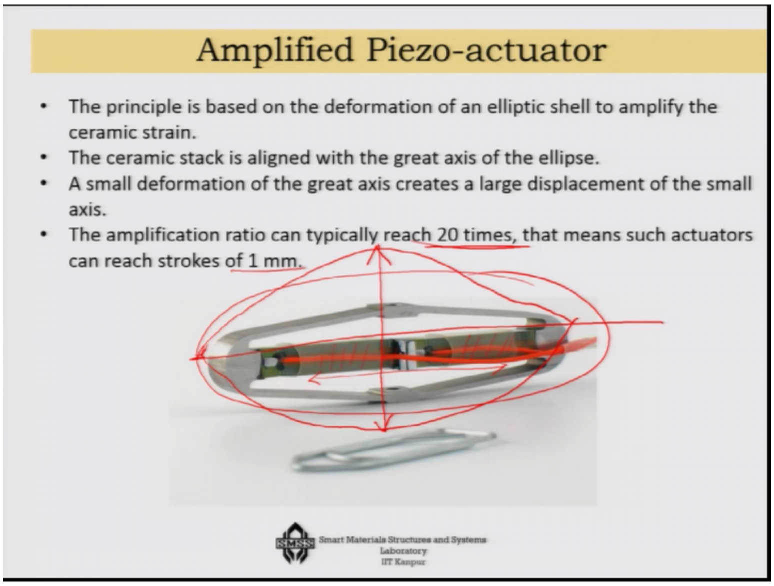
In fact, is standard rainbows that are available in the market they can show deflection of the range of 3 millimeter. Compare that with your you know 2000 parts per million and the amount of deflection. It can generate that is very less, but in the rainbows configuration that same thing can we actually amplified and the load also is about 10 kg point loads. So, this is one way of increasing the deflection from the system.

(Refer Slide Time: 24:16)



The other way is to go for externally leverage system ok. So, say for example, the simple you know Unimorph systems or in which one part is piezoelectric another part is metal that is uniform or both are Piezoelectric. And you apply opposite voltage that is the Bimorph system or you can have this kind of system which is flexure based ok.

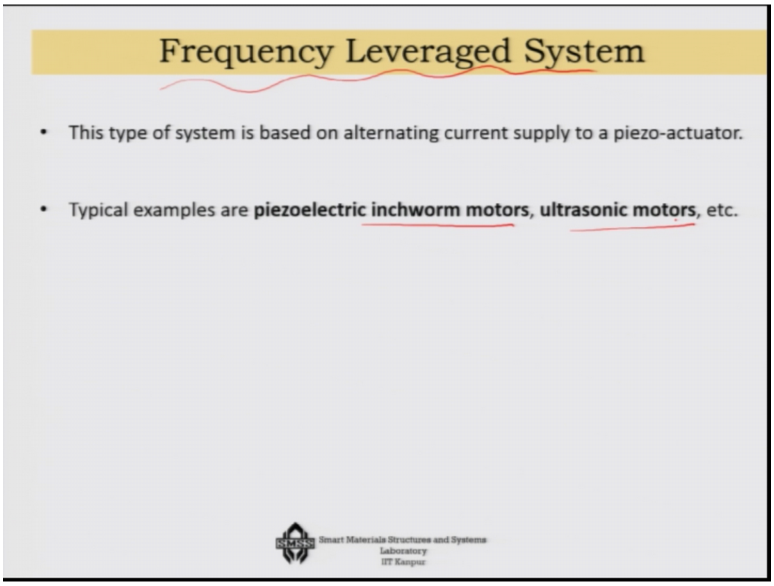
(Refer Slide Time: 24:40)



These type of systems are externally you know developed mechanical arrangement. Through which you can. Actually, you know magnify the very small you know strain or deflection that will be there in the system. So, the this case for example, if you take this one into consideration, you can have many you know stacks layers here ok. So, that you can generate whatever is a maximum strain possible in this system along the axial direction. Now, in the because of this particular configuration the flexure you can easily imagine that it is always stiffer along this (Refer Time: 25:35) imager ok.

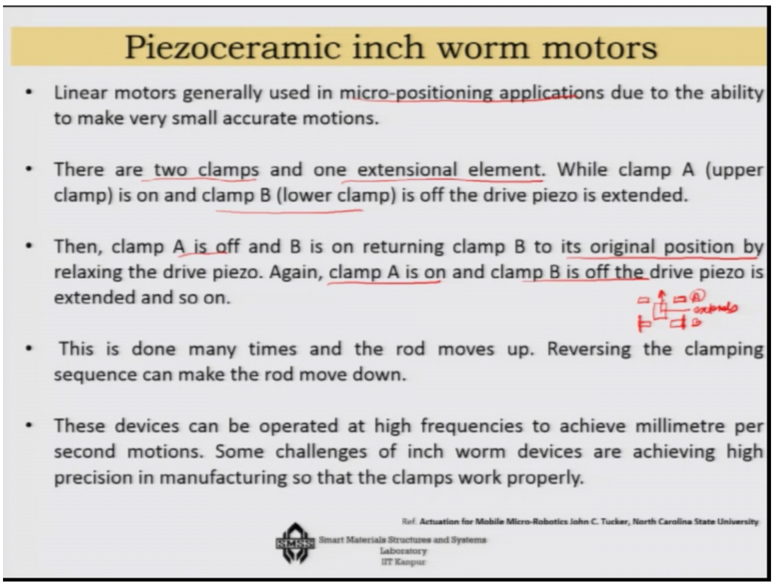
So, whatever is the little deflection here suppose, that deforms system if you look it will be much more amplified. I am just roughly telling you that, it will be much more amplified along these semi mino direction ok. So, that is what is the advantage you get by having a amplified piezo actuator the amplification can be as high as 20 times. So, which means the strokes can reach something like a millimeter or. So, so that is the good part of amplified piezo actuators.

(Refer Slide Time: 26:04)



The other type of this you know, increasing this is actually frequency leverage systems. So, in this case what we do is that we developed a system which works like a interest on you know ac supply, but it works like a oscillatory system. The series of operations it actually develops the or grows the deflection. So, instead of deflection in one shot your deflection you are achieving deflections in many many cycles, but because it is very fast you can operate it very fast. So, this is the concept of inchworm motors or ultrasonic motors etcetera.

(Refer Slide Time: 26:46)

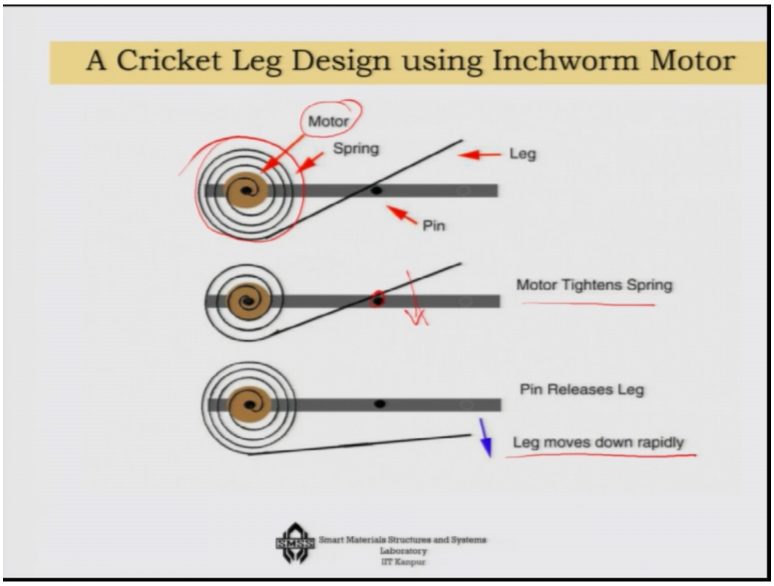


And if you look at the inchworm motors concept, would see that this is mostly used in micro positioning applications. So, the essential concept I have also earlier ones explained you that there are 2 clamps and one extensional element in between. So, the upper clamp n and the lower clamp ok. So, we you know when you are keeping the upper clamp, on the lower clamp is off and then you are you know extending it and then the upper clamp is off; that means, that is loosened and then you know you are actually relaxing the position of B and again the upper clamp is on and clamp B is off. So, that the drive piezo is extended and so on and this is done many many times so that the rod moves up.

So, and if you reverse the clamping sequence you can actually make the rod moving down ok. So, this particular type of a system that clamp A clamp B an extension. So, you can think of it I have shown it to you earlier in terms of pipe crawling that suppose, you have this type of a system which has 2 pair of clamp. So, this is your upper clamp this is your lower clamp and this is the system which actually extends.

So, the trick is that you know initially you put this upper clamp off and while you keep these lower clamping on. So, you are fixing this part and you are releasing it and you are extending it. So, the whole thing moves forward and then you are going to you know clamp this part the A part and then you are taking the B up like that sequential you do it.

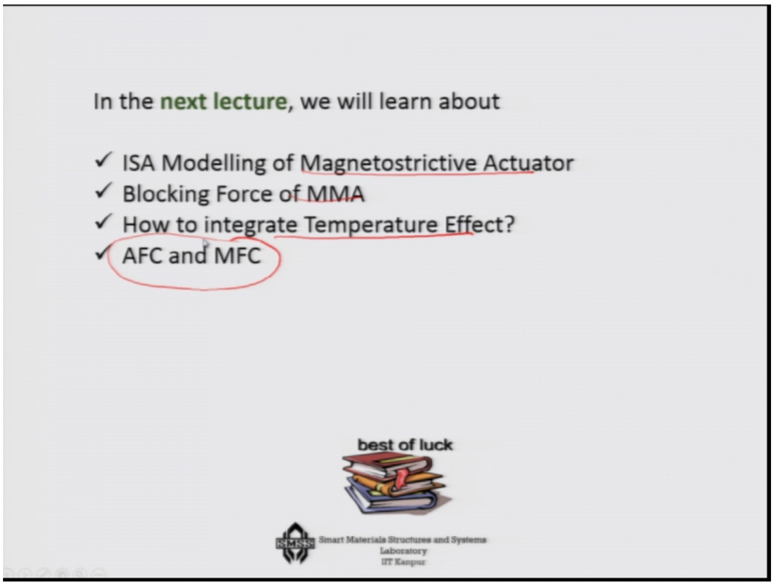
(Refer Slide Time: 28:44)



So, this is the inchworm concept by which you can do it. In fact, a very interesting expansion of the inchworm concept is in terms of a cricket leg design. So, you know in this design, what you can do is that you can use this kind of a motor to actually twist you know generate the tensional torsions in the springs ok. So, you will twist it and the motor is tightening this spring for example, and you keep a pin ok. So, that it cannot retract now once the twisting reaches a particular level then suddenly you take that you know pin you drop that pin. So, that this whole thing can snap back so that is how the leg moves down rapidly and then you can get a high. You know trust all of a sudden and you can use it in terms of generating motions in the system.

So, these are some of the things that a tricks you know that you can use. So, basically once again if you remember that, once you develop piezoelectric activation through the model that we have discussed that, you can amplify using 3 ways internally. Leverage systems like I told you rainbow systems etcetera or external leverage systems like I told you Unimorph, Bimorphs and amplified Piezo actuator etcetera or by a frequency leverage system. And a combination of frequency leverage system and mechanisms. So, like this cricket leg designs etcetera. So, these are the techniques that we use in terms of generating motion in the system this is where we will stop today.

(Refer Slide Time: 30:24)



In the next lecture, we will talk about the magnetostrictive actuator, the blocking force. How to integrate the temperature effect? Also we will talk about two interesting applications of such systems: one is Active Fiber Composite and Macro Fiber Composite.

Thank you.