Smart Materials and Intelligent System Design
Prof. Bishakh Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 11
Modelling of piezoelectric material 1

Good morning, welcome to the course on Smart Materials and Intelligent System Design now we are coming to the module 3 of this course. So, in module 1 I have given you the overview of the smart materials and in module 2 I have talked about the composite materials also, module 3 I have told you about the mechanics of composite materials. So, this is module 3 in which we are going to now, aided by all the backgrounds that we require, we are having now the how to model we have reached the stage that how to model the piezoelectric materials ok.

So, this is I think that all over background knowledge is particularly the mechanics of composite materials and the overview of smart materials, will help us to rebuild the equations the governing equations now.

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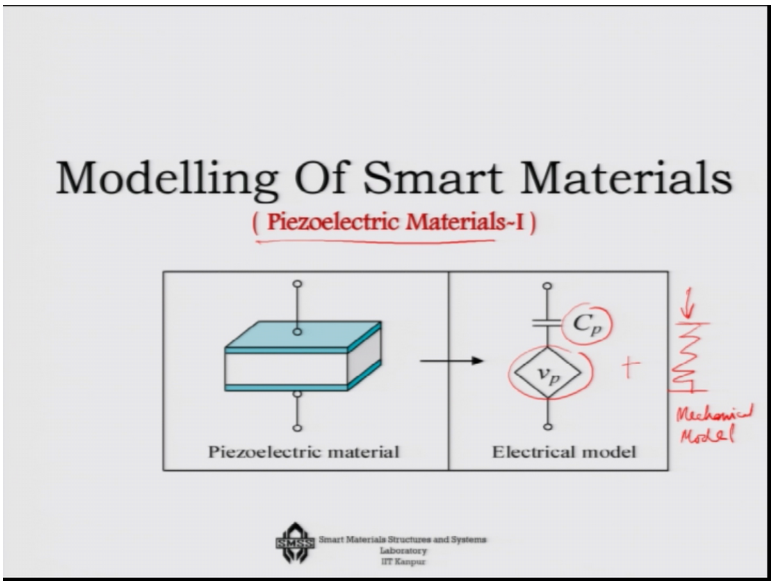

So, when we will talk about modeling of such smart materials like we will start with the Piezoelectric materials, we have to keep in our mind that a Piezoelectric material has both an electrical model in which you can consider it to be a voltage source along with a you know capacitance you know attached in it and also you can also consider the mechanical model of it. So, in the mechanical model you know it is a some kind of a spring system. So, you have a mechanical model and you have a electrical model of the system. The point is that when we will consider the piezoelectric material as an actuator these two has to get integrated with each other.

So, we have to look into it how it gets integrated with each other with the use of constitutive equations and that is what we will be looking at this stage.

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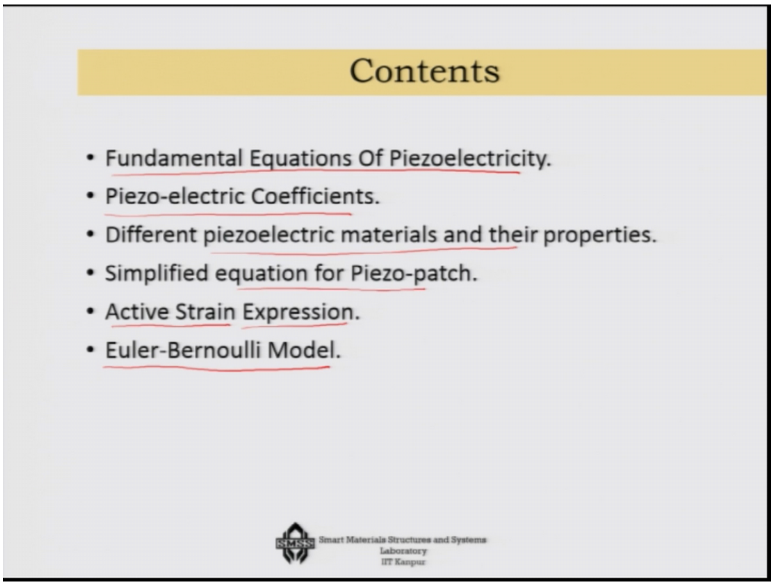

So, I will first talk about remind you about the fundamental equations of Piezoelectricity, that we had initially discussed. We will reemphasize on the Piezo-electric coefficients some of the coefficients are very important for us, then different piezoelectric materials and their properties and we will get simplified equations for a Pezoelectric patch, active strain expressions how to get the Active-Strains and one typical Euler-Bernoulli model of the problem as actuator. So, this is what we intend to do in this short lecture.

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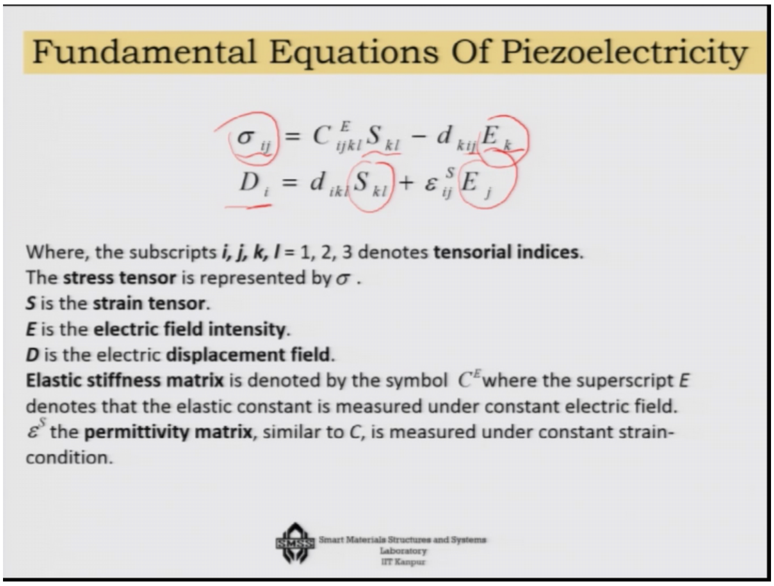

The fundamental equations of Piezoelectricity this time I have represented it in terms of tensorial notations ok. So, accordingly this is the stress tensor which is known to you as sigma i j and S is the strain tensor which is also known to you. The electric field as you can see here that the electric field intensity that is coming up here and d k i j is actually that piezoelectric constants that is there that is relating this electric field intensity with the stress tensor.

And you have the electric displacement field and once again you have your we are relating it here with the strain tensor and the electric field intensity. So, this is the overall tensorial notations of the system and with this system we will now see that how we can develop the governing equations.

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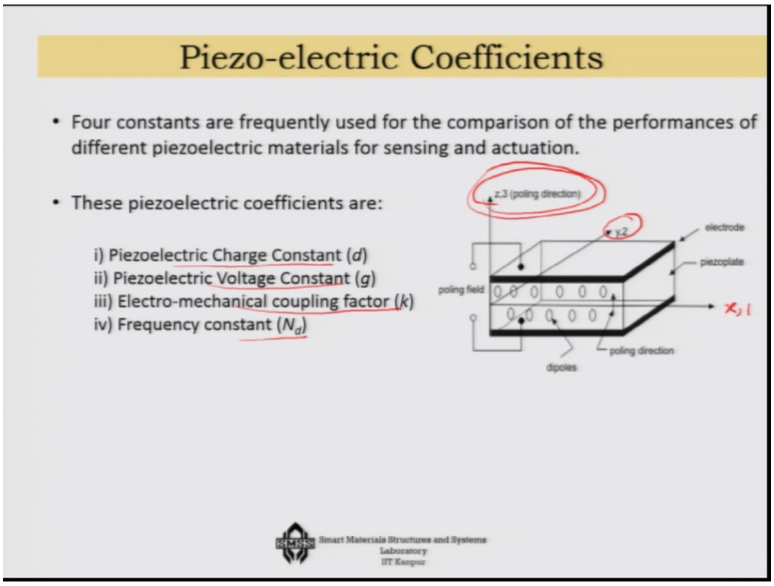

Now, there are four constants piezoelectric coefficients I have to say, which are actually important for us ok; Piezoelectric charge constant, piezoelectric voltage constant, electro mechanical coupling factor and frequency constant. So, when we will be discussing about it keep in mind once again that the planar part is represented by x 1 and y 2 so that is the planar part and the transverse direction, we are representing it by z or 3 that is also the pooling direction.

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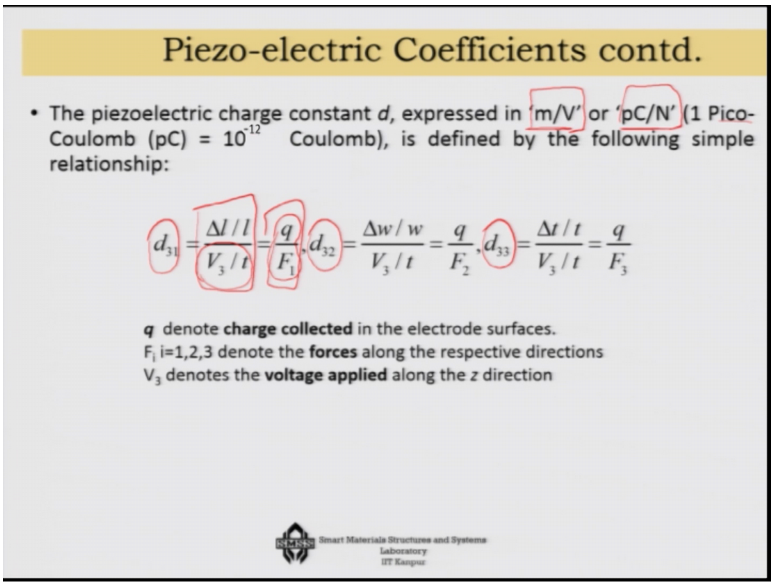

So, with this you know axis system in mind let us look into the piezoelectric coefficients. So, the first important one is the piezoelectric charge constant that is constant d, and in this case is d 3 1. So, what it means is that, I am either applying voltage along the 3 direction, and getting the deformation along one direction or I am applying stress is along one direction and getting the voltage along the perpendicular direction.

D 3 1 and d 3 2 are generally considered to be same, but they may differ because there can be some anisotropic materials where it will be different and d 3 3 of course, in which you are doing everything along the 3 3 direction; that means, along this directions ok. So, you are applying stress along the 3 direction you are getting voltage from the 3 direction or you are applying voltage along 3 direction and you are getting deformation along the 3 direction which is used in the piezoelectric stacks.

So, d 3 1 as you can see will be finally, an unit of charge over force or you can also have the unit in terms of meter per volt. So, basically for all these piezoelectric charge constants the units could be either in terms of meter per volt, which is following this part of the relationship or it can be in terms of the sensor part of the relationship where it is picocoulomb per Newton.

So, the relationship these constant itself is telling us that in one case you know force is generating the charge that is the picocoulomb per Newton part that is where you know we use it as a sensor, and in another case the voltage is developing the deflection in the system. So, the unit itself is amplifying it. The other point is that instead of coulomb per Newton we are using picocoulomb because the capacitance is generally quite low. So, this is a very important constant piezoelectric charge constant that will be using.

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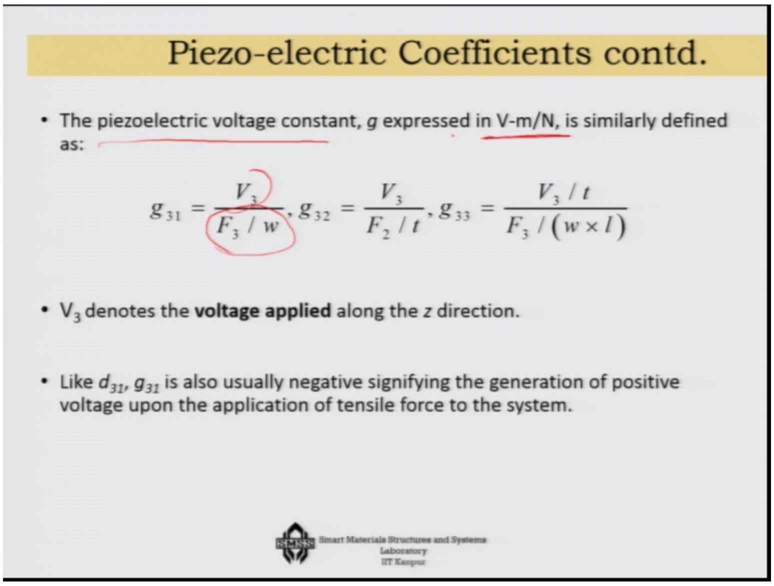

What is the other constant? Well the other one is g which is piezoelectric voltage constant. In fact, this is used more in terms of sensors. So, in this case the ratio that we take is voltage over force per unit width of the system or force per unit thickness of the system for g 32 or g 3 3 is voltage over time and the force per unit area of the system. So, this is used mostly for the sensorial equations and in this case the unit is voltage meter per Newton

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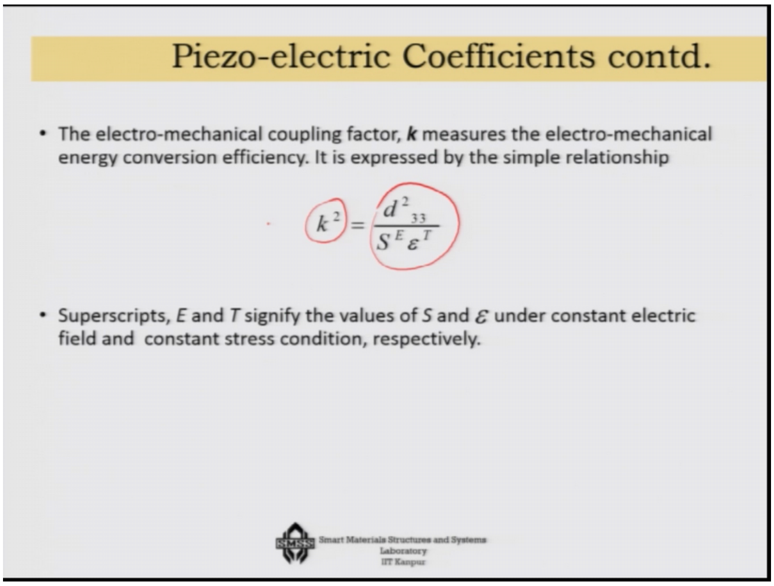

Then another important constant I also told about it to you earlier it actually depicts the energy transfer from electrical to mechanical energy or vice versa. So, in this case it is the ratio of electrical energy to mechanical energy, which is given by k square and k is called the electro-mechanical coupling coefficient. If k is very close to one which means the complete energy is getting transferred if k is less much much less than one it means it is not the material is not very much piezoelectric. So, k actually tells us what is the degree of Piezoelectricity, that is there in the material.

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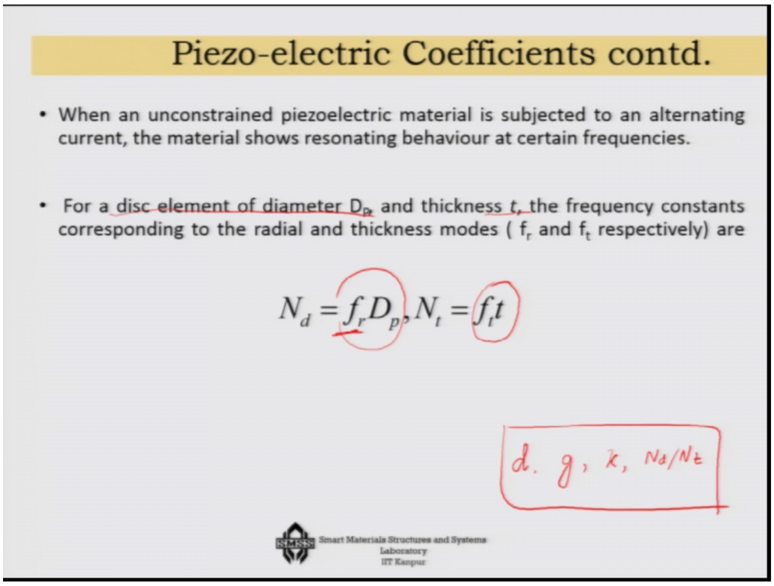

Now one last constant we will talk about it is known as the frequency constant and this is more related to when we actually take a disk of a piezoelectric element let us say of a diameter D p, and thickness t and then we actually apply the voltage and we get it you know vibrated at its resonating frequency.

Now, if I know this radial resonating frequency ok. So, the product of that with the you know diameter that gives us one constant and the thickness wise frequency product of that with thickness gives me another frequency constant. So, essentially these two frequency constant tells us that for a particular diameter or for a particular the thickness the higher the N d or the higher the N t the higher will be the resonating frequency of the system. So, thus we are having four constants with us please keep in mind that d piezoelectric charge constant, and g that voltage constant and then you have piezoelectric you know electro mechanical coupling coefficient k and finally, this N d N t that is the frequency constant of system. With these four different types of constants you know we should be able to define the behavior of a piezoelectric material.

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Different piezoelectric materials and their properties

Property	PZT (Hard)	PZT (soft)	PZT-PVDF	PMN-PT	$LiNbO_3$	PVDF
d_{33} (pC/N)	190	425	120	1240	6	30
d_{31} (pC/N)	-55	-170	-	-	-0.85	-16
g_{33} (mV-m/N)	54	27	300	43		150
g_{31} (mV-m/N)	-16	-11	-	-		-150
k_{33}	0.67	0.70	0.80	0.92	0.17	0.11
E_p (GPa)	63	45	~30	100	20	2.7
Density (ρ) (Kg/m^3)	7500	7500	3300	8120	4600	1760
Λ	1500	1980	400	3100	1210	700

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Say for example I have a PZT hard or PZT soft PZT PVDF you can see that how these d 3 3 is changing. So, you can see that PMN pt shows us the largest value of this ok. You can also see that d 3 ones are actually smaller than d 3 threes and you can also see that g 3 from the sensing point of view PVDFs are actually much better in comparison to both g 33 and g 31 in comparison to the piezoelectric material PVDFs are much better. And you can also see that from the you know coupling constants point of view PMN and PT which shows the largest piezoelectric effect actually has the largest coupling coefficient. From the modulus of elasticity point of view also it is true, but however, this is more dense. So, denser and also it is something you know which is very brittle it is not very easy to use ok.

Free strain is once again very similar to the larger value of d 3 3 or d 3 1. So, you can see that free strain is maximum here because it has the largest coupling coefficient it has largest d 3 3 is etcetera. So, it has the largest free strain and the smallest free strain is if you make a composite of PZT PVDF or otherwise PVDF is also having low free strain, but it is good in terms of voltage generation.

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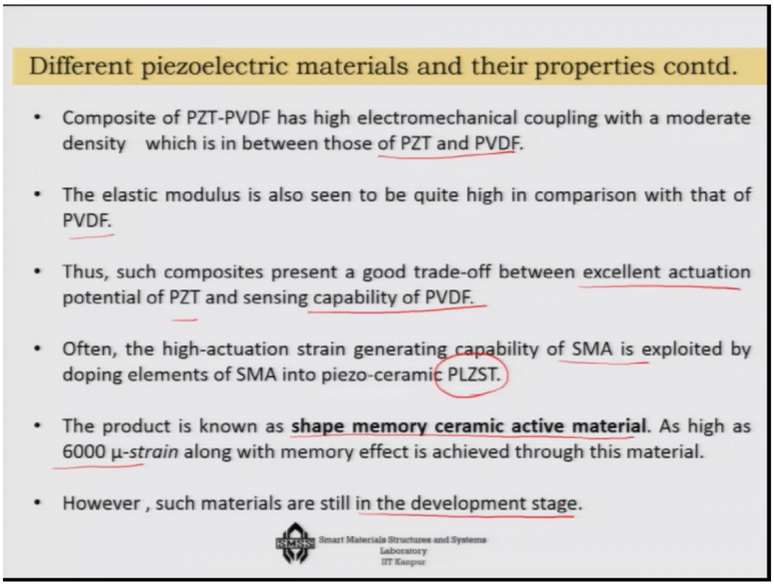

What are the peak points here? Composite of PZT PVDF it has high electro mechanical coupling with a moderate density, which is in between PZT and PVDF elastic modulus is quite high in comparison to PVDF. So, you again if you make a composite and if you know make a composite it will be a tradeoff between excellent actuation of PZT and excellent sensing of PVDF. So, it is a composite that is a desirable composite you are taking the good parts of these two systems.

often we also put SMA in such systems and make PLZST which is a SMA based piezo ceramic composite, but that part we have not touched here. And this which is known as shape memory ceramic active material, it actually shows large strain and however, such materials are still in the developmental state.

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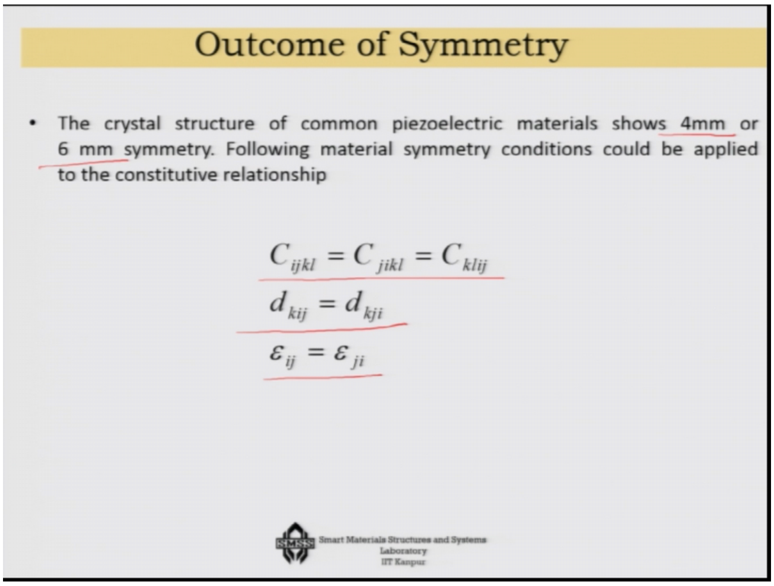

Now we know the constitutive relationship we can we have more symmetry based on the materials that we observed. In fact, we see that the common piezoelectric materials shows 4 mm or 6 mm symmetry. So, the outcome of these symmetries that our elastic constants have this symmetry and also piezoelectric charge constant and also our you know strain tensor.

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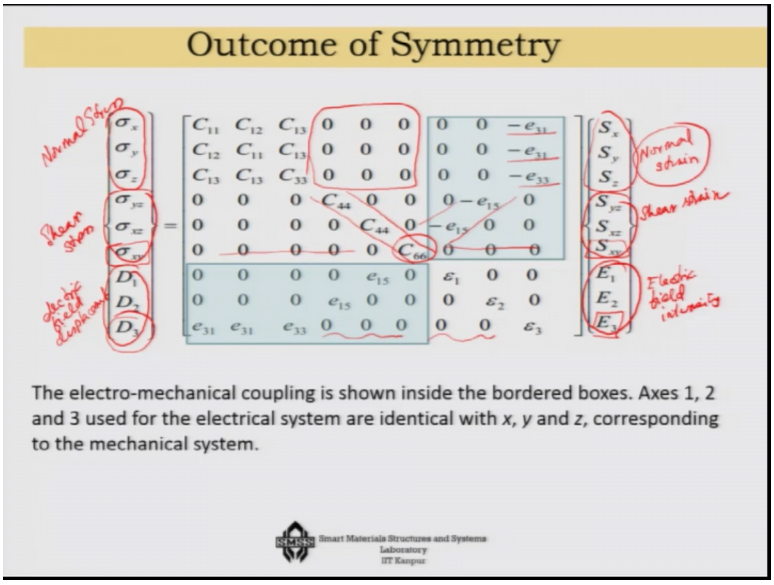

So, if I use this symmetry condition then actually our relationship becomes much simpler this is the generic relationship. So, if you look at this relationship very carefully, that you have the first 3 terms sigma x sigma y sigma z which are normal stress right. So, these are your normal stress and this is: what is your shear stress. So, this is your shear stress. And D 1 D 2 D 3 they are electric field displacement. In fact, if you electric field is if you integrate each one of them you will be getting charge that will be you know stored in that area.

Now so, you have how many you have total nine you know six stress components and 3 electric field displacement components and here you have this as the naturally the normal strain. So, these are the normal strain and then you have the shear strain this is the shear strain and then you have this part, which is electric field intensity electric field intensity. So, you have basically supposed to have a 9 by 9 matrix, but in that matrix because of the symmetry conditions and other you know material conditions that you observed, a good part of it is actually you know 0’s full of 0’s. So, that will not give us coupling. In fact, if you look at it that all normal stresses are coupled with the piezoelectric effect ok.

Only through E 3 1 E 3 2 and E 3 2 is E 3 1 itself. So, only through that constants. So, that is how electric field intensity are coupled with the normal stress ok. In fact, that the normal stresses are not coupled, they will not generate in this particular piezoelectric material you know the shear strains. So, shear stains are generated by the shear stresses right that is diagonal part, and also some part of the electric field takes part in terms of generating the shear strain, but this shear strains are mostly sigma y z and sigma x z. You can see that sigma x y shear strain in this kind of materials cannot be generated by the electric field, because there is all the other terms are 0 here all terms are 0 except C 66.

So that means, this part is non smart in this group of materials then from the electric field displacement also we see a similar situation that the electric field displacements can be generated from say for example, if we are mostly concerned with D 3 in the perpendicular direction electric field generation. So, that can be done from the 3 normal strain components if I deform, but not from the shear strains not from the electric fields at the other two directions except from the electric field at the 3 direction can generate D 3. So, these are the things that we have to keep in mind when actually using them as actuator or as sensor.

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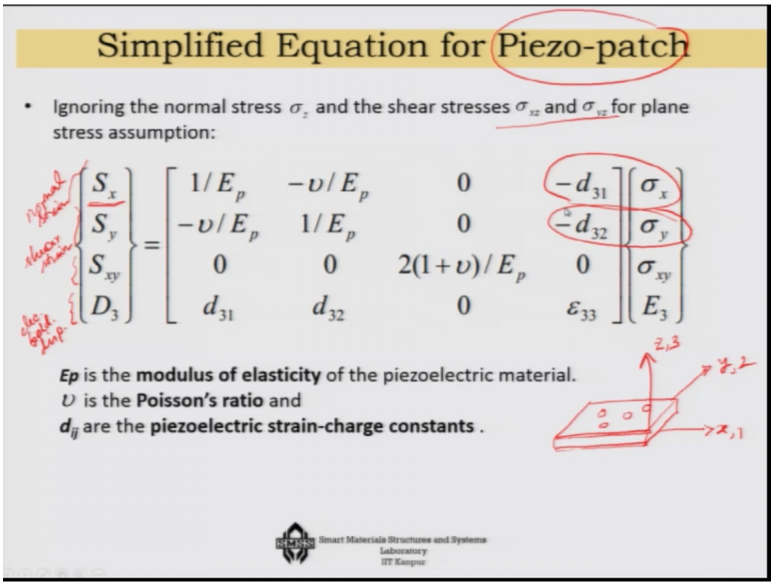

If I ignore all other stresses that is the normal stressing was z and the shear stresses sigma x z and sigma y z for a plane stress situation, then we have only two you know of the normal strain and one shear strain. So, this S x S y as the two normal strain and S x y as the shear strain and D 3 we get which is electric field intensity electric field displacement. So, this electric field displacement is actually in the perpendicular z direction.

So that means, if I consider a plate like system like this and if I denote this as x axis as y axis x or one y or two as I told you and you know this axis z as 3, then in this case you can see that we are only considering the charge accumulations at the top or the bottom. Remember I am always assuming a very thin layer of electrodes a very thin layer of electrodes as the top and bottom, where this charges are actually getting accumulated ok. So, this is the simpler relationship and in the plane stress conditions which is what we normally use for a piezoelectric patch kind of a situation.

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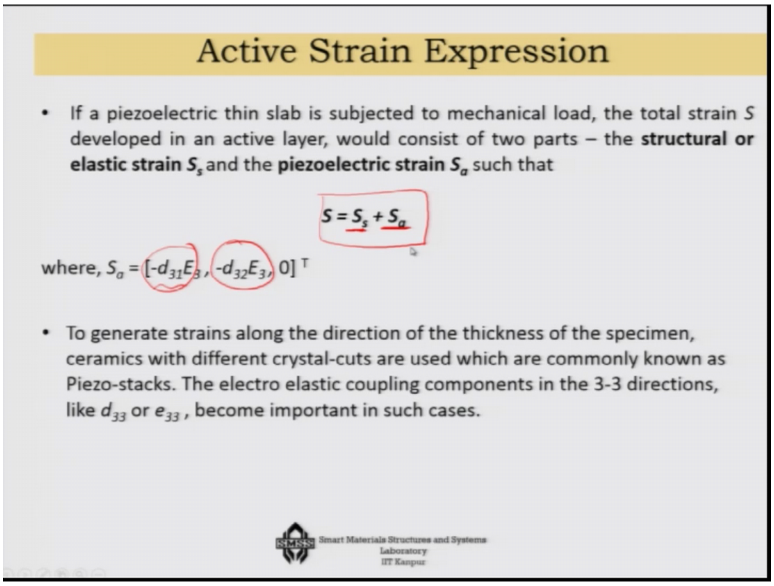

Now, if we consider the strain that is generated in the system, then the strain will be having two parts in it one is the structural strain which is happening because of the force that you are applying, and the active strain. And the active strain component if you look at it from the last equation, what are the active strains in the case of S x the active strains is minus d 3 1 sigma x, in the case S y it is minus d 3 2 sigma y; S x y there is no active strain. So, that is what is minus d 3 1 E 3 minus d 3 2 E 3 and there is no active direction in the shear direction. So, that is you know this is the total strain that actually comes into the system.

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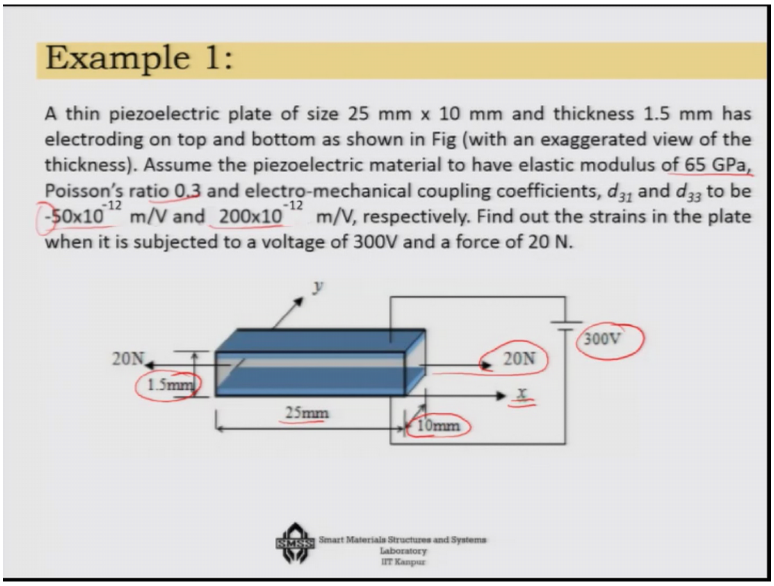

Now let us do one small example let us consider a thin piezoelectric plate, which is of size 25 mm by 10 mm and thickness is 1.5 mm has been shown here, and this is 25 mm length and 10 mm is the width of it ok. Now let us say that in this one and the piezoelectric material has an elastic modulus of about 65 gigapascal; Poisson's ratio of about 0.3 d 31 d 33 are known to us ok. Remember d 31 we give it is with a negative sign meaning thereby that positive voltage we will be generating compressive stress in the system.

Now, find out the strains if the plate is subjected to a voltage of 300 volt across and a force of 20 Newton along the x axis ok. So, I am applying a voltage and of 300 volt, I expect that it will expand and I am applying a force of 20 Newton which will also expand and let us try to see that you know: what are the strains that will happen if both of them are working on the system.

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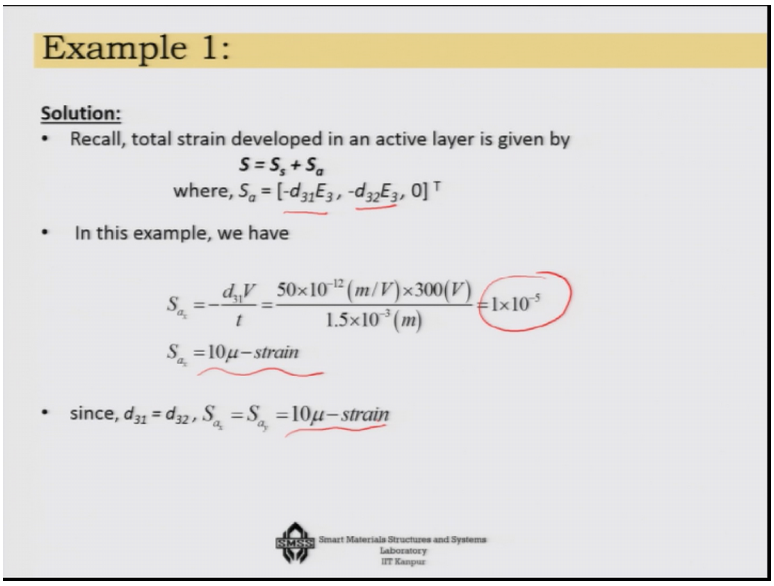

So, we have a structural strain to calculate we have an active strain to calculate. So, this is the active strain expressions that I already told you. So, by applying the voltage constant etcetera we can get the active strain, which will come out to be about 10 micron strain. And the other direction also it is same so, it will be about ten micron strain.

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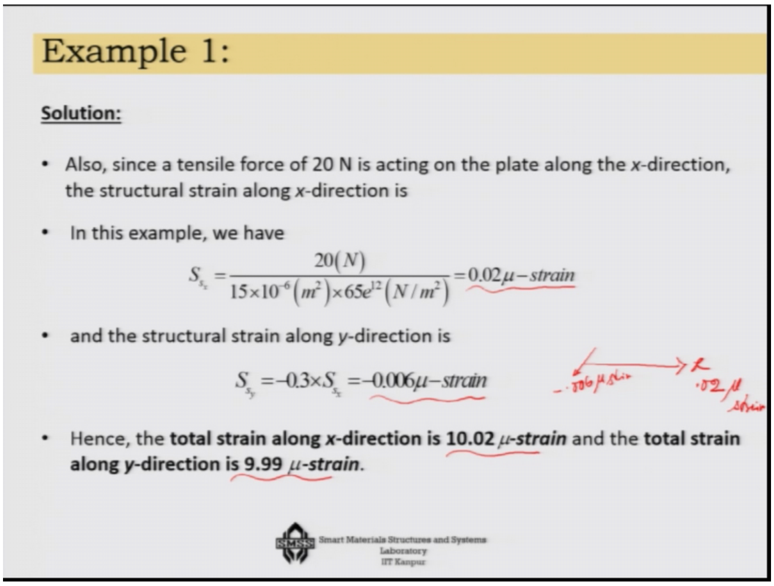

Now, we are applying the force of 20 Newton. So, that will give us in the longitudinal direction about 0.02 micron strain and because of the Poisson's effect in the also in the x direction it is expanding this is x direction 0.02 by 0.02 structural strain the other direction it is actually contracting ok. So, it is that contraction. So, is happening in this direction. So, the here it is minus 0.006 micron strain ok.

So, this is because of the Poisson's effect. So, in one direction it is expanding and in other direction is actually contracting. So, now, but in both the directions we have the active strain which was about 10 microns. So, in one direction it will be added up it will be 10.02 and in the other direction it will get subtracted it will become 9.99 micron strain. So, this example thus tells us that how to calculate the strain in the system.

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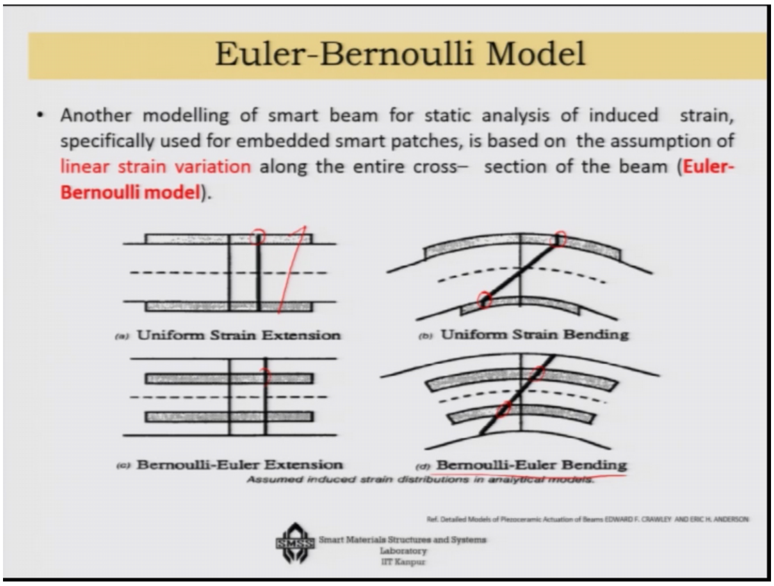

Now, we come to model of the system. Let us say we are talking about two different ways in which piezoelectric patch can be model one is called uniform strain model note here that you have two piezoelectric patches one at top one at the bottom of a particular beam type of a structure, but during deformation, this strain is not changing in top and bottom. So, this is also known as uniform strain a extension this is extension time and this is the bending time ok.

Even though the stress this strain reversal is taking place, in the rest of the beam, but here the patches it is uniform. The other model is that particularly when it is not surface, boundary it is embedded in side during extension this strain is same, but during bending there is a variation inside the piezoelectric material also. So, that is the Bernoulli Euler bending. So, we have a uniform strain bending and a Bernoulli Euler bending these are the two models that we can actually think of.

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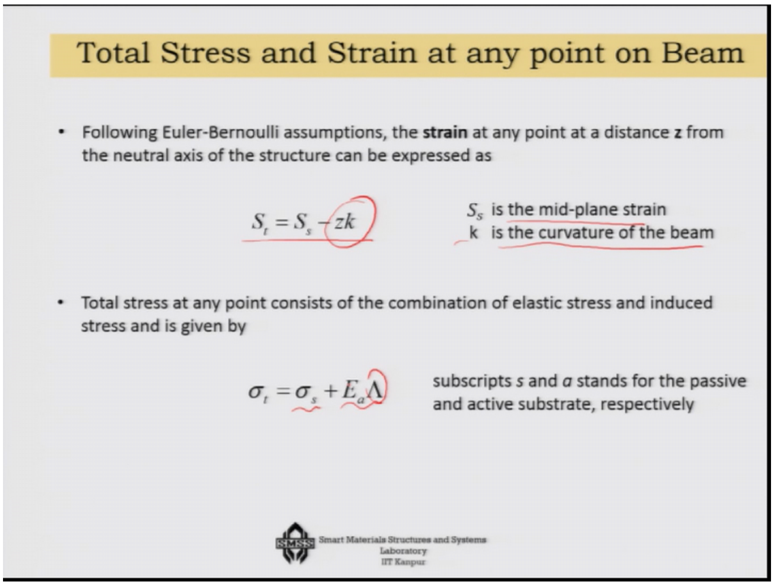

Now if you consider the Bernoulli Euler one, then the total strain as you already know from your ideas of composites has two parts: one is the mid plane strain and another is related to the curvature z kappa where k or kappa is the curvature of the beam. And the total stress at any point will be having the stress that is coming from the structural part and also the stress that is getting generated from the actuator that these you know the free strain blocking A lambda. So, that is the total stress.

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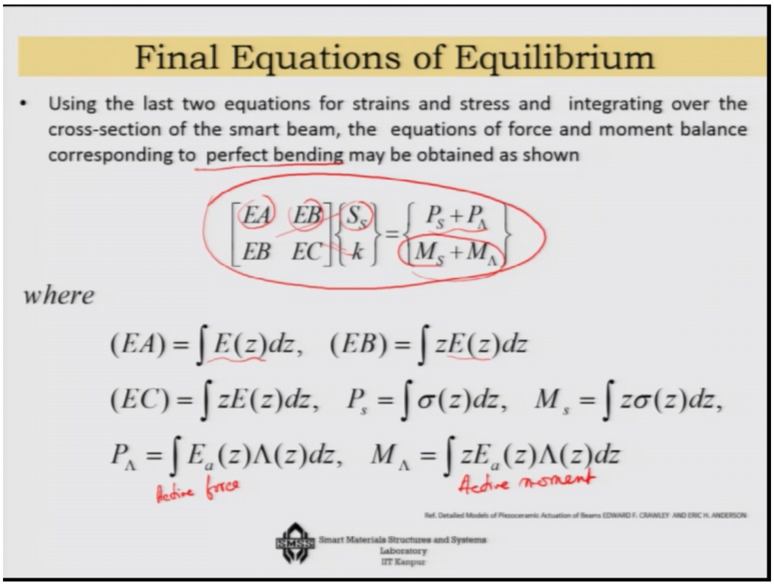

So, we can actually get from this, the final equations of equilibrium by using all these conditions we can get it as the EA a times S s and EB times kappa. So, that will give us the P s plus P lambda. So, that is total you know from the force balance point of view. Similarly, from the moment balance point of view, EB times S s EC times kappa will give us M s plus M lambda. So, keep in mind that P lambda and M lambda this are the active force and the active moment. Whereas P s and M s are the mechanical load and the you know moment the you know also to say you can also talk about in terms of stress resultant due to mechanical deformation and also the moment due to the you know bending of the system.

So, that is your P s and M s. EA EB EC as you can see you can easily find it out from if you know what is the modulus of elasticity and if you know that how this deformation is happening into the system. So, thus you can get an equation of equilibrium in the system one is by the from the force balance and another is from the moment balance assuming of course, perfect bending in the system.

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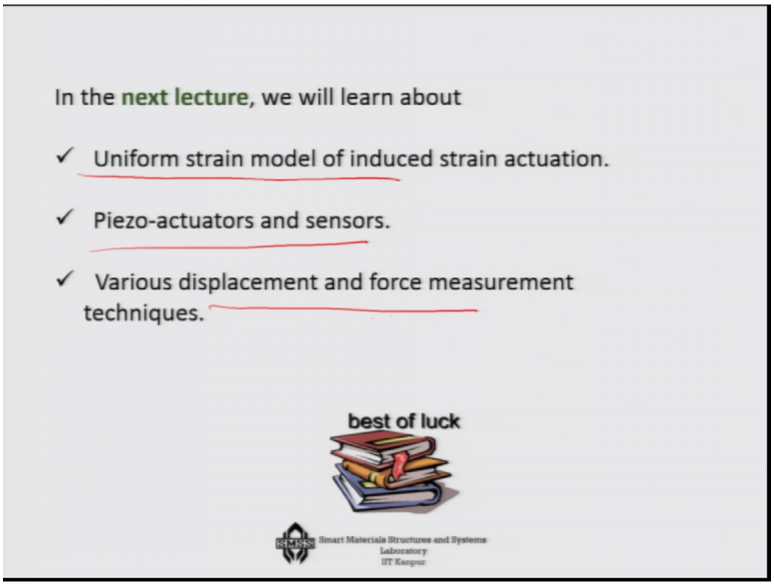

So, this is: what is the first state of equations using Euler Bernoulli model. In the next lecture we will look into much more details of the uniforms strain model, we will also talk about piezo actuators and sensors and various displacement and force measurement techniques.

Thank you.