Engineering Metrology

Prof. J. Ramkumar
Dr. Amandeep Singh Oberoi
Department of Mechanical Engineering & Design Programme
Department of Industrial & Production Engineering
Indian Institute of Technology, Kanpur
National Institute of Technology, Jalandhar

Lecture - 48
Quality Control, control charts for attributes

So, Good Morning. Welcome back to the second part of lecture on the Quality Control. So, we were discussing the Quality Control charts. So, I will discuss the attribute charts in this lecture.

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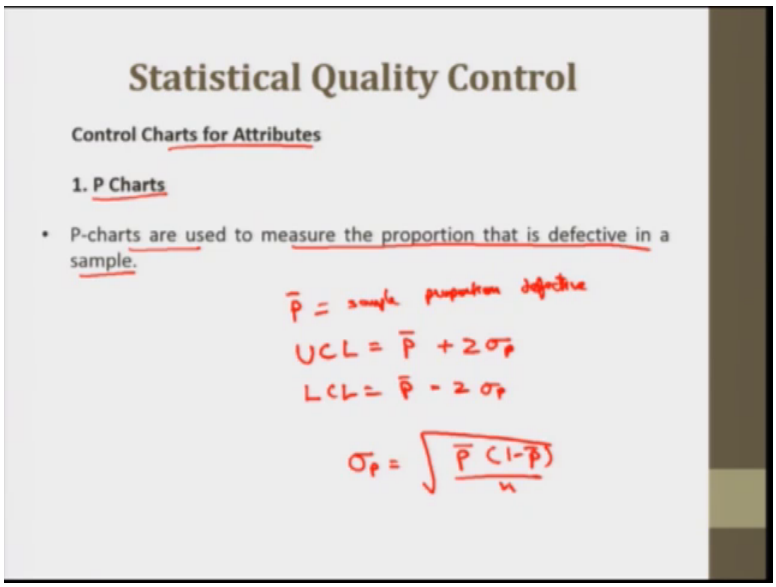

So, Control Charts for Attributes, first chart I will pick is the P charts. P-charts are used to measure the proportion that is defective in a sample. Proportion defective means, the computation of central line as well as the upper and lower control limit is similar to the computation of the other control chart, but the centerline is computed as the average proportion defective in the population that is denoted by P bar. So, it is obtained by taking the number of samples of observations at a random and computing the average P across the values. Upper control limit is equal to the central proportion plus z into sigma P ok.

Similarly, lower control limit is equal to P bar minus z into sigma p. What is sigma, but what is standard deviation here? So, P bar is here my sample proportion defective. So, that is a standard normal deviate and sigma P is a standard deviation of the average proportion defective where the value of sigma P can be calculated as the square root of P bar into 1 minus P bar by n. where n is the sample size.

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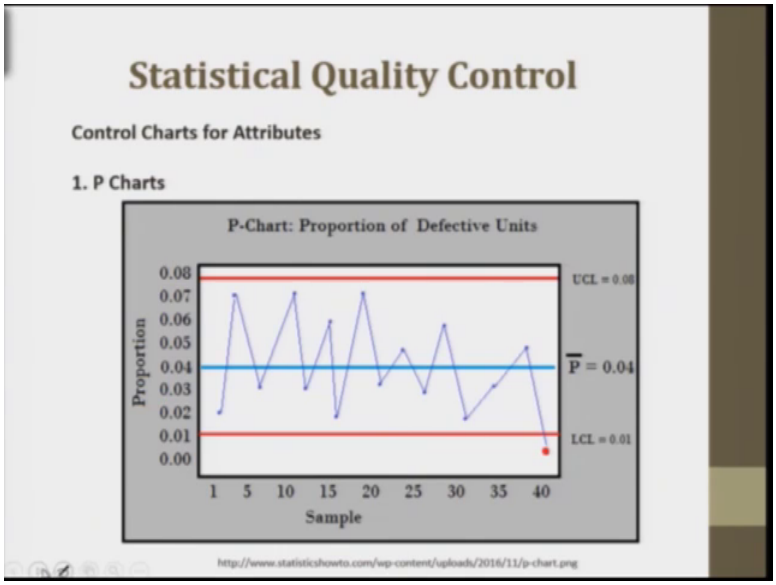

So, only this is a plotted P chart. That is, pick from one of the references here. So, it is showing the proportion of defective units. Upper control limit and lower control limit is there.

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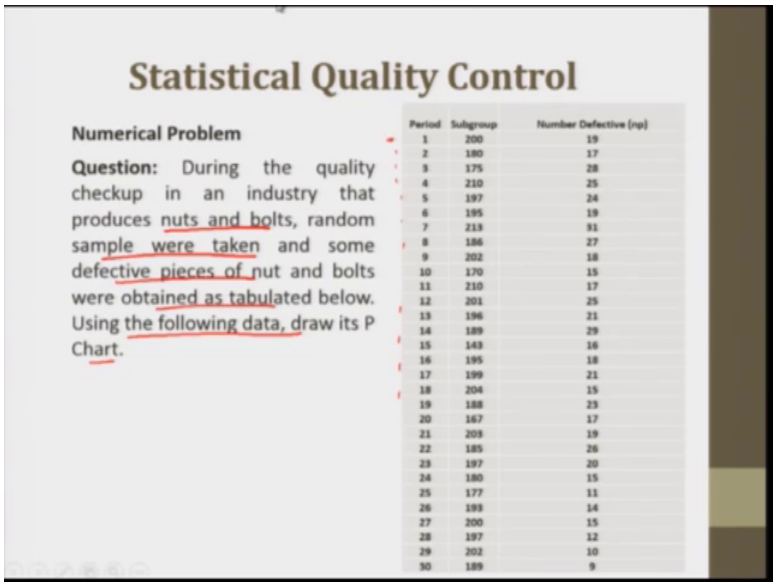

So, I will try to just see this numerical during the quality check up in an industry that produces nuts and bolts. Random sample were taken and some defective pieces of nuts and bolts were obtained and as tabulated below using the following data, draw it is P Chart. So, we have the period from 1 to 30. So, and this subgroup value is there and the number of defective is there. Now, I will just export this data to my excel sheet.

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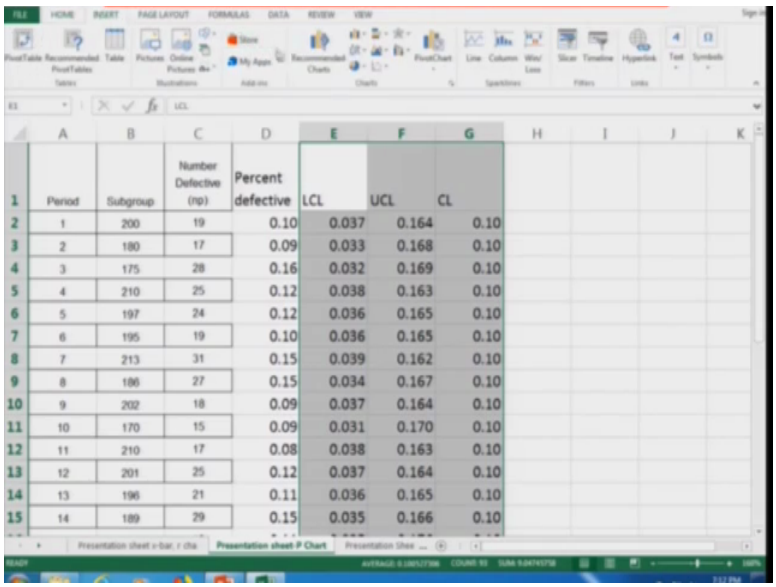

I have already exported that and like to show you that how do we construct the P Chart. First of all, we have number of defective pieces and the period. This is actual period, this

is the when is number selected. So, this is Number Defective np number defective I need to see the proportional. And I also come to the np chart and np chart is will when will plot this directly.

So, number of proportion you have directly would be this is equal to the number of defectives divided by the total number of pieces those are inspected this is 0.95 ok. If it I will repeat number of defectives this is equal to number of defectives is in 19 divided by the subgroup size that is 200 enter.

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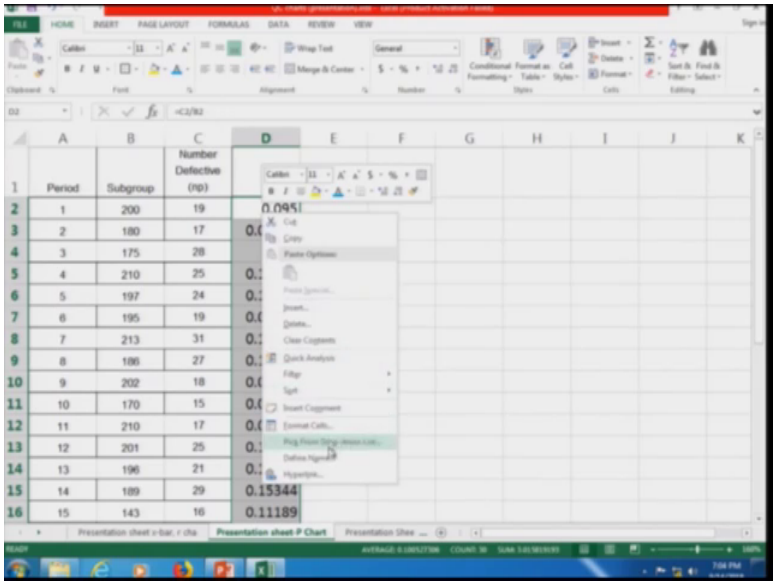

So, I will just drag this to get my proportion defective and format cells, I will bring it to two places of decimal ok.

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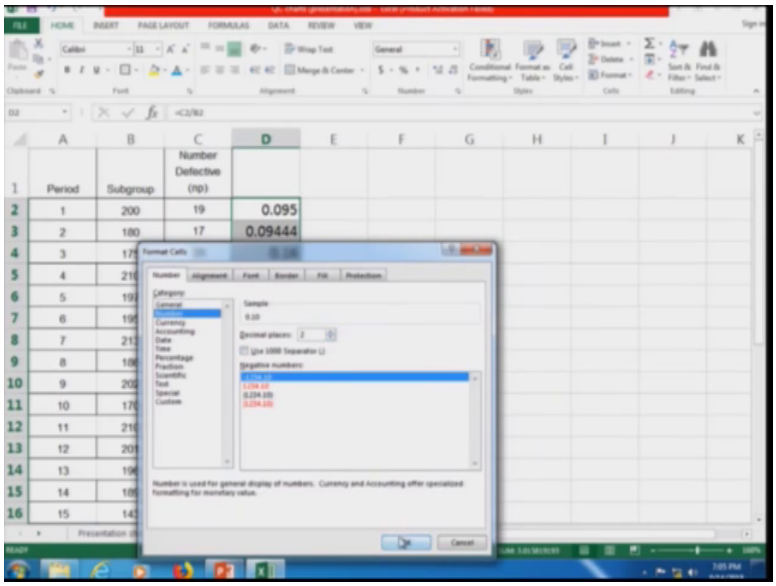

So, this is my proportion defective. So, I will call it percent defective ok. Now this is my percent defective. Now I can calculate the upper control limit and lower control limit using the formula, upper control limit is it was equal to percent defective plus z sigma ok. So, what was that formula? It is percent defective plus z sigma. So, sigma is under root P bar into 1 minus P bar by n; n is the size ok. So, value of z that I will pick is 3 here, I have to calculate the P bar first.

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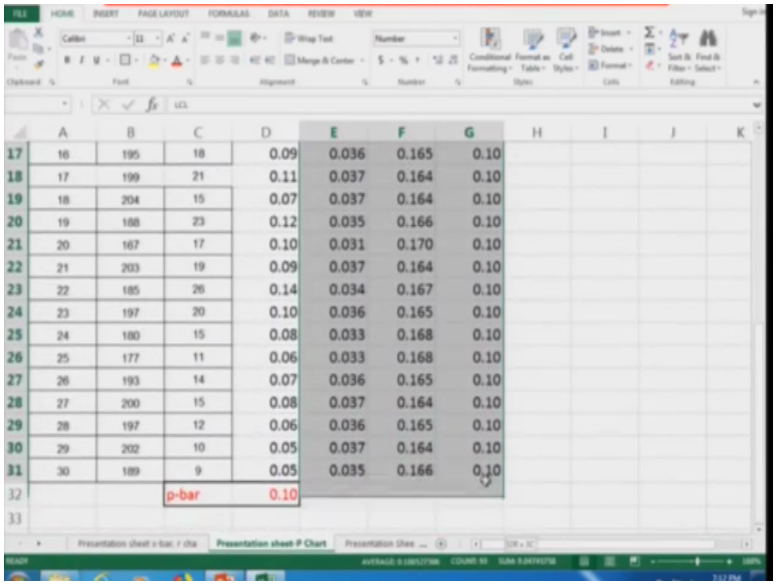

So, this is percent defective. So, what is p bar? Average of these values enter which is equal to 0.1 I will put it here. This is p bar, I will just include in a box and color it red to this p bar. So, this is equal to p bar this value ok. Then I am trying to put; I am trying to put here your just a second time to put a my lower control limit here first then I will put my upper control limit.

So, lower control limit is equal to p bar minus. So, the value of z which is kept 3 into p bar into 1 minus p bar is square root of that ok. I will just put p bar again this value p bar into 1 minus again value of p bar D 32 divided by n.

So, what is n here? N is my subgroup size ok, n is actually this value B 2 subgroup size. So, first let me close the bracket this is all complete course. I need to takes square root here as well. So, I will start the a basis here, then take square root of so I have to close this is and close this is I have to take it should be complete we founded typo, it is 0.36, so skip.

So, let me check it again it is p bar into 3 times square root of p bar into 1 minus p bar by n; n is the subgroup size ok. So, this formula is complete. So, let me put it dollar sign to lock the value of p bar; p bar is to be locked ok. This B 2 subgroup size will vary this is fixed now and I would also like to format it were bring it back to the second place of or I can put it to the third place of decimals. You know I am where working with the proportion defective this time and the proportion might need another 0, is significant 0 after the decimal.

So, this is the value and I am dragging this. So, I will get my lower control limit for these values, I can even find my upper control limit the only difference I am just dragging this formula here again ok. So, only thing is that I need to make sure that this B remains P subgroup is there. And in place of minus I will put plus enter, so this is the value ok.

Lower control limit upper control limit then centerline central line central line is nothing, but my value of p bar this is equal to dollar d, dollar across the d 32 ok. I will just pick it from there this value d 32 dollar enter. So, this is proportion defective. So, I am going to plot this using the line ok, insert line.

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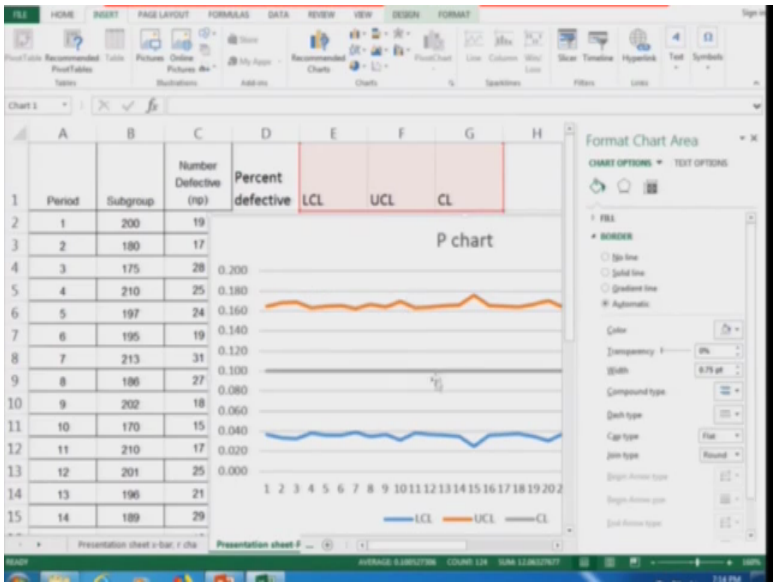

So, I have got this; this is my proportion defective chart or P chart ok. So, this is I will put the chart title here this is P chart ok. Lower control limit upper, control limit and centerline ok. Now I have got upper control limit lower control limit and central line I not need to put my process here process is percent defective. So, I will just plot it again, so I will pick my percent defective and my control limits and central line.

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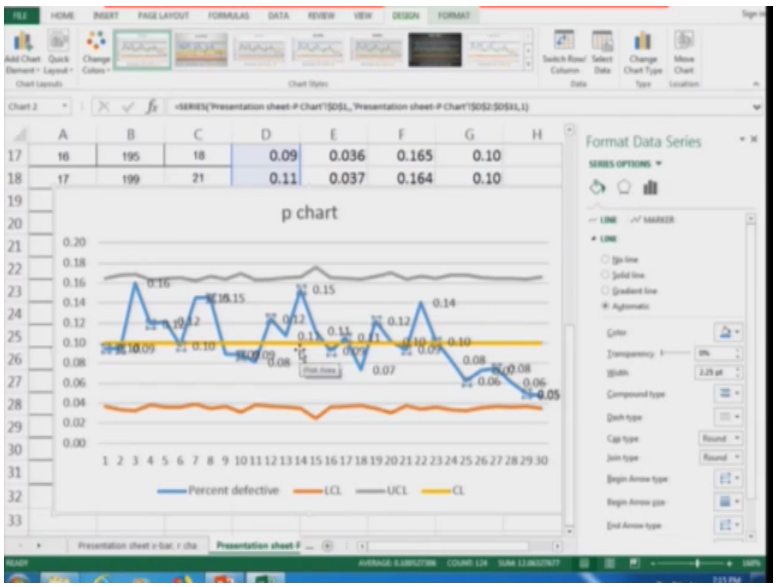

Then insert line diagram this is my p chart ok. So, this is you can see is the whole a process is in control ok. The qualities in control the proportion defectives are there, but if there are those are within this three sigma limits.

If I reduce my sigma limits it might get out of control this value it is the height highest value it might get out of control at help me at labels data labels ok, 0.16 is the maximum value and the minimum value is 0.05 ok. So, this was the p chart.

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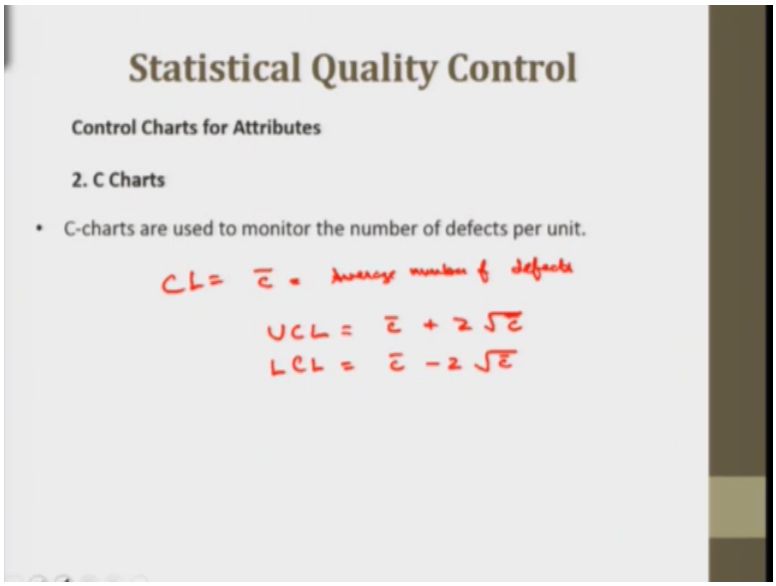

So, next similar to p charts we have np charts before that I will try to cover the C charts. What C charts? C charts due to monitor the defects per unit when we have the number of defects, or defects per unit we can use the C chart of the example you have can be the number of defects.

As I said when we manufacture a car the number of defectives in a specific section, or the number of defective pieces of the a bottles with perfect, or the number of a return meals in a restaurant, or of the number of trucks that exceed their weight limit in a month, or like this when number of defects are there, C charts are used.

So, the C charts at the average number of defects is denoted by C bar ok, C bar is average number of defects, and the upper control limit and lower control limit is equal to our C bar plus z into under root of C bar that is all ok, C bar minus z into under root of C bar I will pick these values as 3 and 3 here.

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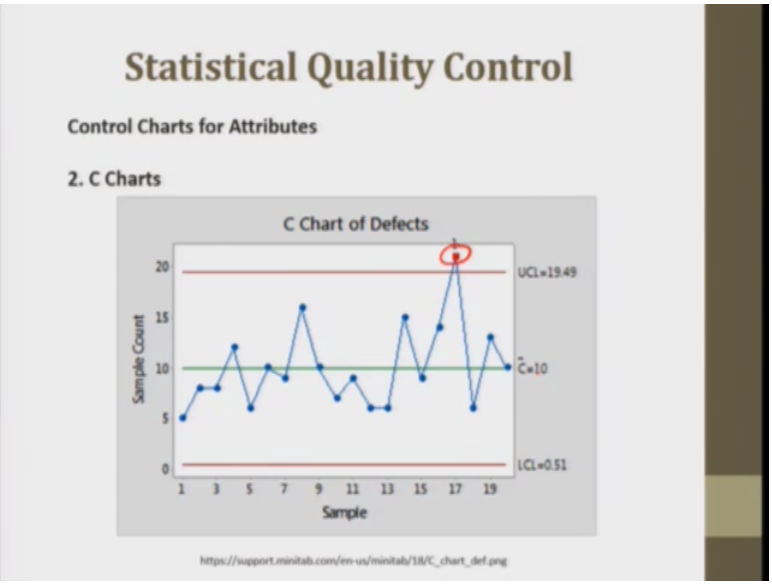

So, C charts one of the C charts is plotted here. So, it is as showing that one of the value is not in control. It is the just the number of defects the C bar is value that is that works as a central line also this put as this works as our central line. Central line is equal to this value and upper control limit lower control limit are there.

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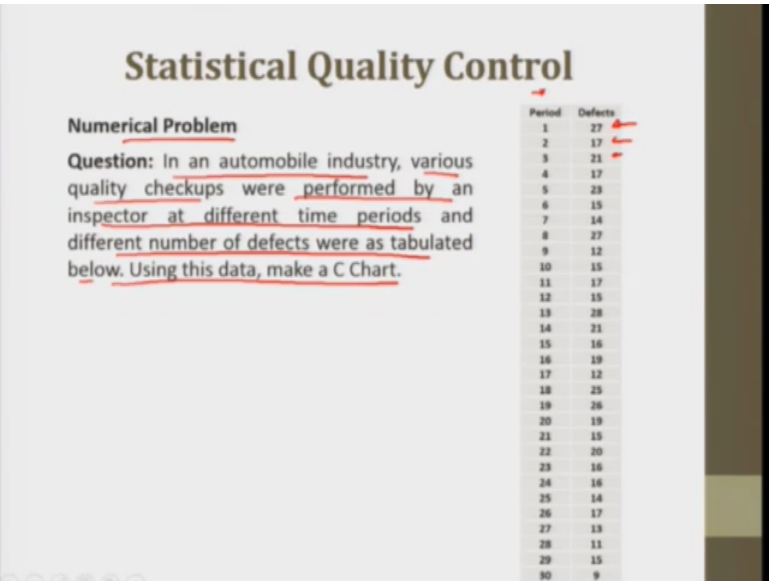

So, next I will taken a numerical problem to elaborate my C chart and in the question we have in an automobile industry various quality checkups were performed by an inspector different time periods, and different number of defects were found which are tabulated

below which as tabulated here. And the using this data, try to make a C chart and see whether the processes in control or not.

So, these are the number of defects 27 defects per automobile. This automobile industry just put automobile, it can be motorbike, it can be a car, or truck, bus whatever it is. So, number of defects in the final inspection those are pound is 27 ok, so it is 17 here, 21 so on. So, I will take this data to the excel sheet inside to plot these C chart.

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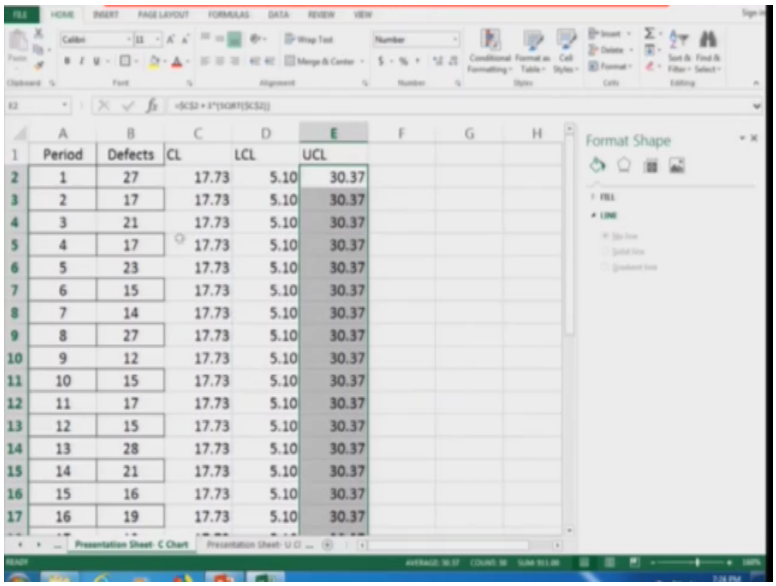

Period	Defects	CL	LCL	UCL
1	27	17.73	5.10	30.37
2	17	17.73	5.10	30.37
3	21	17.73	5.10	30.37
4	17	17.73	5.10	30.37
5	23	17.73	5.10	30.37
6	15	17.73	5.10	30.37
7	14	17.73	5.10	30.37
8	27	17.73	5.10	30.37
9	12	17.73	5.10	30.37
10	15	17.73	5.10	30.37
11	17	17.73	5.10	30.37
12	15	17.73	5.10	30.37
13	28	17.73	5.10	30.37
14	21	17.73	5.10	30.37
15	16	17.73	5.10	30.37
16	19	17.73	5.10	30.37

So, this is number of defects, first thing I need to calculate is C bar, C bar is the average number of defects ok.

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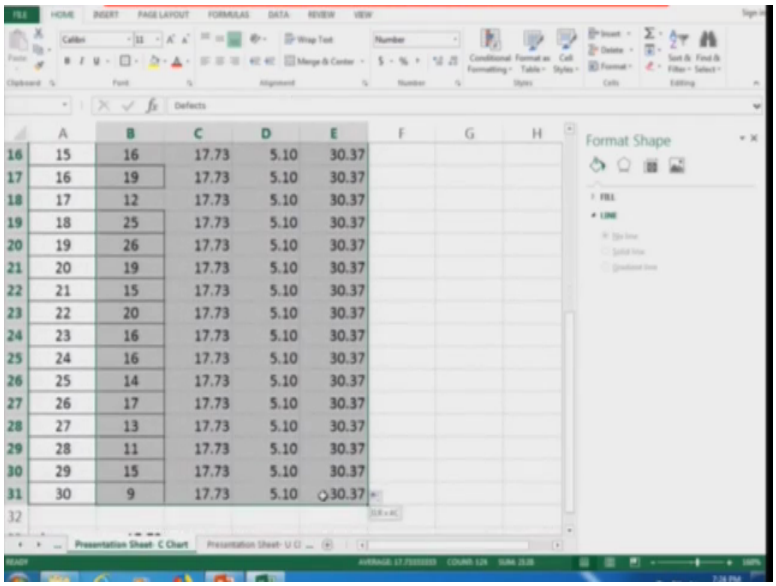

So, C bar I can put here, C bar this is equal to average of these numbers ok, enter it is 18. So, let me (Refer Time: 13:54) wanted to do 2 decimal places 17.73. Another way to do it is in place of average I would better put this is equal to sum of these numbers divided by count of these numbers actually which is a one of the same thing.

But why I am doing it because in the U chart I will tell you that sometimes the number is different. We are just taking one automobile this divided by or even just learning another formula here count of these numbers ok. The count would be again 30 only, so this is C bar ok.

Then I can have just put my central line that is equal to C bar then lower control limit, then I will put my upper control limit ok. Central line is equal to C bar enter let me lock it dollar and dollar ok. Lower control limit is C bar minus 3 into under root of C bar this is equal to C bar this value minus 3; 3 is a value of z that I have selected into square root of square root of this C bar (Refer Time: 15:33) this is here, and enter yes. So, it is 5.1.

So, let me just put dollar sign here enter and up to two places of decimal ok. So, this is my lower control limit ok, similarly upper control limit would be very similar to this. But I would have C bar plus 3 times or this is z times of square root of C bar enter. So, this is my upper control limit. So, I can just plot this.

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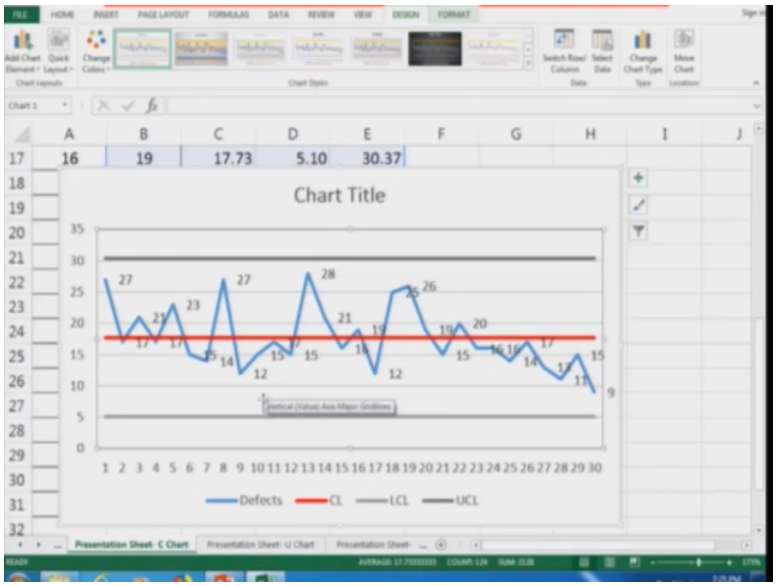

Insert a line diagram, so this is my number of defects due line, control limit, lower control limit and upper control limit ok. So, if you like to change this color we can change this color to the to gray only ok, and this can be colored maybe red ok.

So, lower control limit upper control limit are there I can put the labels here data labels not like this. I will just put data labels here, so it is showing my C chart here ok. So, C chart is used when we have number of defects, number of defects is fixed.

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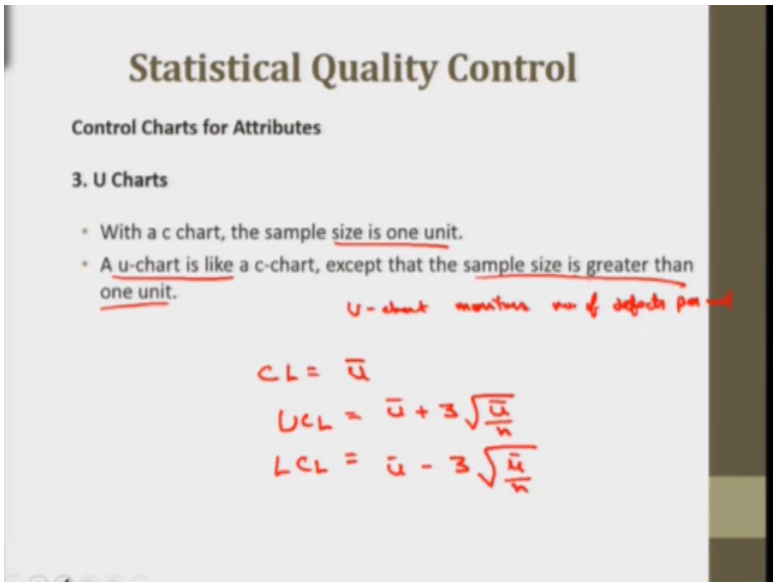

Similar to C chart we have U chart with a c chart is sample size is one unit that is we had the number of defects in an automobile in a in one unit. But when the number of units are different u chart is like a C chart except the sample size is greater than one unit.

So, with a c chart the sample size is one unit, but when the number of units is more than one are sample size is greater than one u chart is used, but it is similar to c chart only thing is that the number of units are there.

So, we take the proportion of that. So, here as like you can say that c chart monitors the number defects in one unit, but u chart the monitors, or track the number defects per unit, u chart monitors number of defects per unit ok. So, we divide just it the number of defects by n.

So, centre line center line is u bar average here. And upper control limit and lower control limit are given by u bar plus z I will just put the value 3 here directly which I am using into u bar by n. Where n is the subgroup size or the sample size when it is greater than 1. So, similarly lower control limit is u bar I will put small u minus three times of u bar by n ok.

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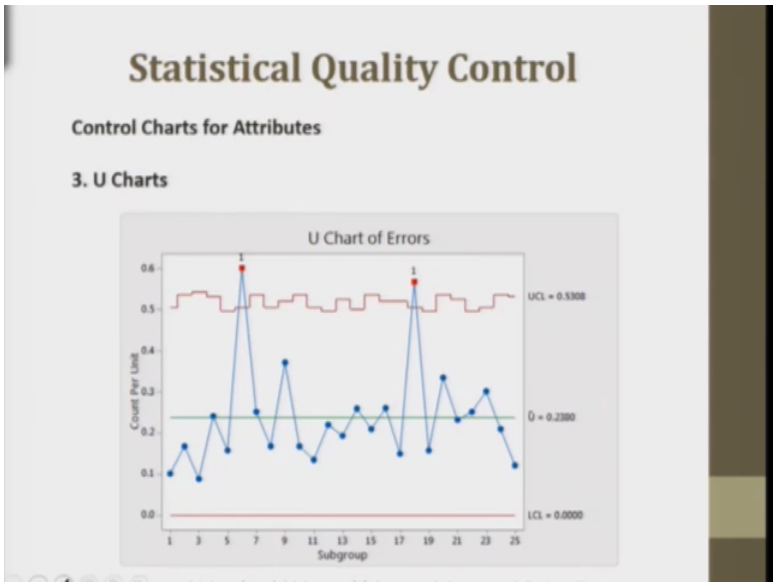

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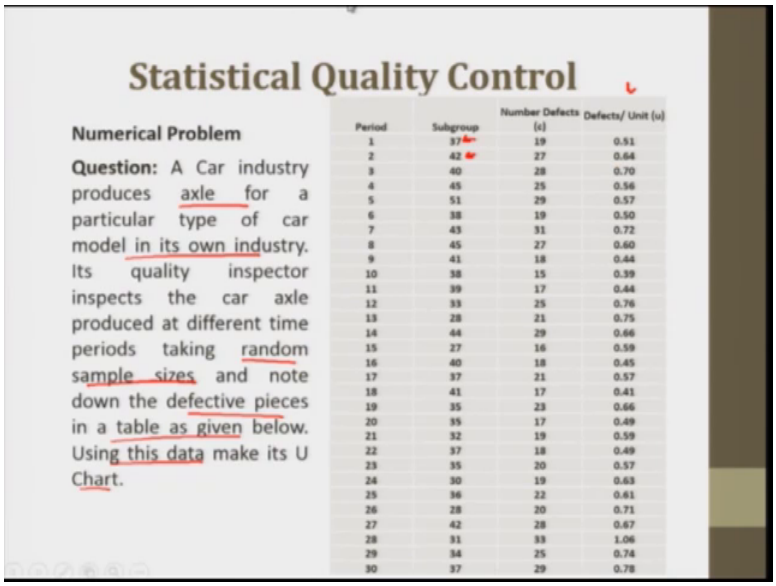

So, this is one of the u charts that is plotted here. So, I will try to plot this as was, so I have numerical problem here. So, similar to C chart, the U chart is choosing it the chart are the total number of defects per unit. So, using this numerical what is the problem here? A car industry produces axle for a particular type of car model ok, in its own industry. Its quality inspector inspects the car axle produce a different time periods taking random sample sizes, random sample sizes. And note on the defective pieces in a table given below so it is tabulated here. This is a given data make a U chart.

So, what they are telling that number of axles are produced it just pick the random size of axles. In the previous c chart we had a one full automobile and we were try to look at defects the total defects which are noted down at the end while dispatching that.

So, here what we has he does he first pick 37 axles, then you fix 42 axles and out of 37 you find that there 19 defects are there. And out of 42, they find that 27 defects are there.

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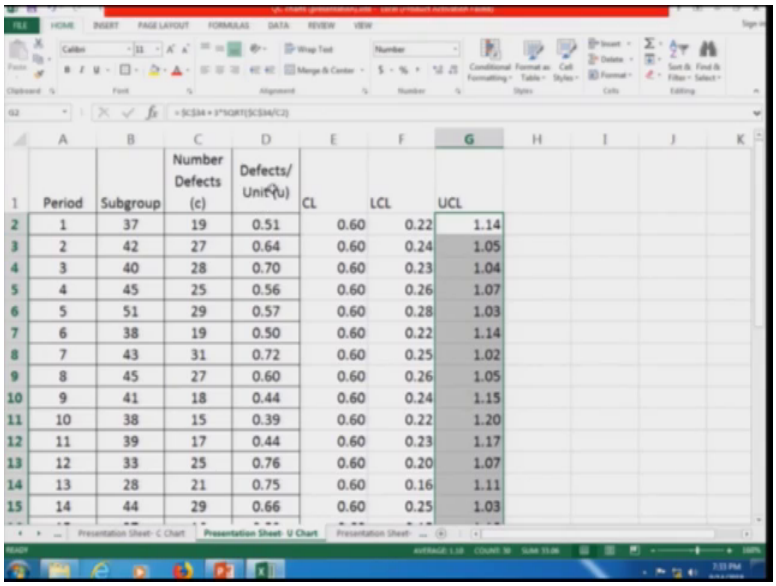

	A	B	C	D	E	F	G
1	Period	Subgroup	Number Defects (c)	Defects/ Unit (u)	CL	LCL	UCL
2	1	37	19	0.51	0.60	0.22	1.14
3	2	42	27	0.64	0.60	0.24	1.05
4	3	40	28	0.70	0.60	0.23	1.04
5	4	45	25	0.56	0.60	0.26	1.07
6	5	51	29	0.57	0.60	0.28	1.03
7	6	38	19	0.50	0.60	0.22	1.14
8	7	43	31	0.72	0.60	0.25	1.02
9	8	45	27	0.60	0.60	0.26	1.05
10	9	41	18	0.44	0.60	0.24	1.15
11	10	38	15	0.39	0.60	0.22	1.20
12	11	39	17	0.44	0.60	0.23	1.17
13	12	33	25	0.76	0.60	0.20	1.07
14	13	28	21	0.75	0.60	0.16	1.11
15	14	44	29	0.66	0.60	0.25	1.03

So, we calculate the defects per unit which is nothing, but 19 bar 37 ok. So, 0.51 is 19 bar 27; 27 a bar 42 comes down to 0.164. So, I will try to plot the u chart as well, here u chart. So, we have calculated the number of defects number of defects is you can see the formula it is C 2 by B 2; C 2 means 19 bar 37 to defects per unit. So, to calculate the lower and the upper control limits I need to calculate the u bar. So, u bar is the total number of defects by total number of pieces was inspected.

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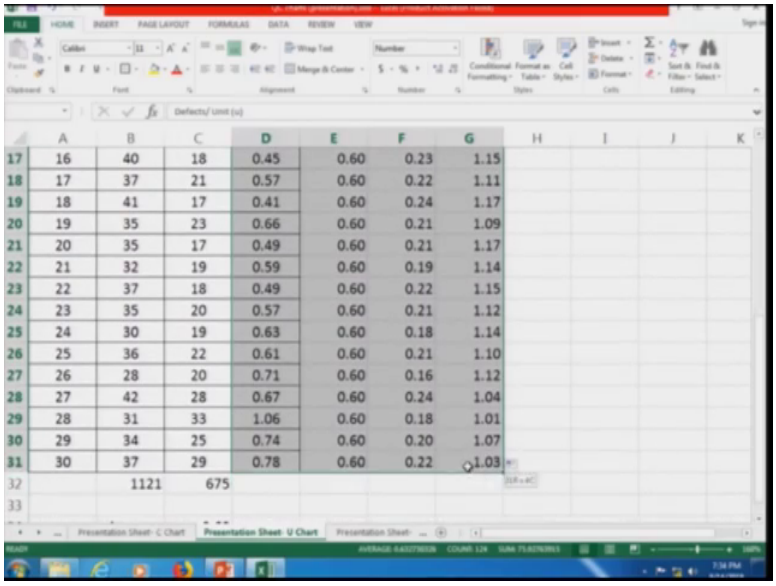

	A	B	C	D	E	F	G
17	16	40	18	0.45	0.60	0.23	1.15
18	17	37	21	0.57	0.60	0.22	1.11
19	18	41	17	0.41	0.60	0.24	1.17
20	19	35	23	0.66	0.60	0.21	1.09
21	20	35	17	0.49	0.60	0.21	1.17
22	21	32	19	0.59	0.60	0.19	1.14
23	22	37	18	0.49	0.60	0.22	1.15
24	23	35	20	0.57	0.60	0.21	1.12
25	24	30	19	0.63	0.60	0.18	1.14
26	25	36	22	0.61	0.60	0.21	1.10
27	26	28	20	0.71	0.60	0.16	1.12
28	27	42	28	0.67	0.60	0.24	1.04
29	28	31	33	1.06	0.60	0.18	1.01
30	29	34	25	0.74	0.60	0.20	1.07
31	30	37	29	0.78	0.60	0.22	1.03
32		1121	675				
33							

So, this is equal to sum of total number of defects, and similar to this we have total number of species which are inspected. And we have u bar this is equal to total number of defects divided by total number of pieces which are inspected and try this is 0.6, double the value is while just bring the two second place of decimal ok. So, we have the value u bar here.

So, we have central line, lower control limit, and upper control limit. Centre line is my u bar this is equal to u bar enter lock the value ok. The lower control limit is this is equal to u bar minus 3 times of square root of take it star u bar by n, u bar divided by n. What is n here? N here is my number of species which are respected that is subgroup that is 37 here that is B 2, so bracket close enter.

So, 3 into would not accept unless we complete the formula. So 0.219 so let me bring it back to the second place of decimal ok, lower control limit is there. So, I will lock the u bar enter and track. So, upper control limit we were similar to this only difference would be in place of minus I would have plus 3 times of the valuable that is why it is plus. That is all it, so I will just plot this defects per unit and this.

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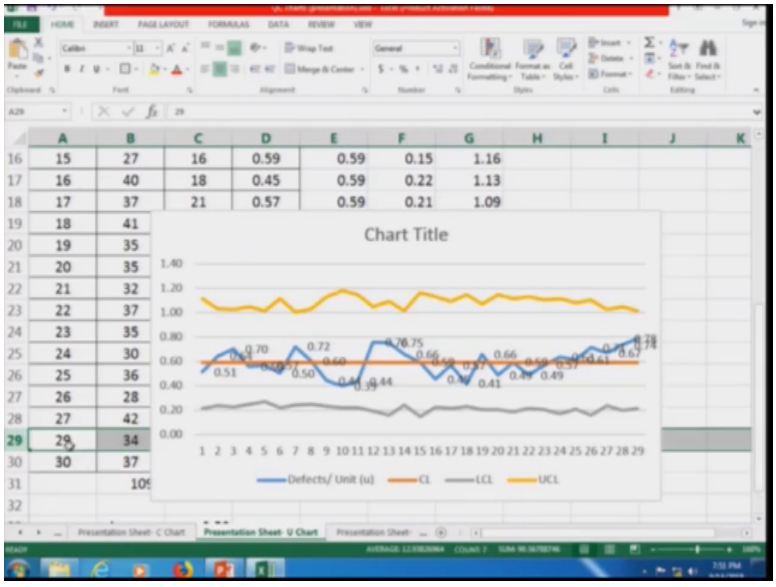

And I will directly get my u charts. So, it is lower control limit upper control limit central line and I have this u chart. So, we can see that one of the observation here is out of control. First I will at the data labels here, the data labels this value is 1.06 and if I go to

this value it is. So, if the series is defects per unit and the reading number it also say ok. The 0.28 data label ok.

So, point number 28 is here, here I am having point number 28. So, let me try to see you what is if I limit at this point. Let me try to just delete cell completely if I leave at this point we are left with total 29 points actually, but the process comes in control. So, this is also not to worry. So, I will just undo it to keep the whole data intact here. So, this is my if it is chart title, this is my the u chart ok.

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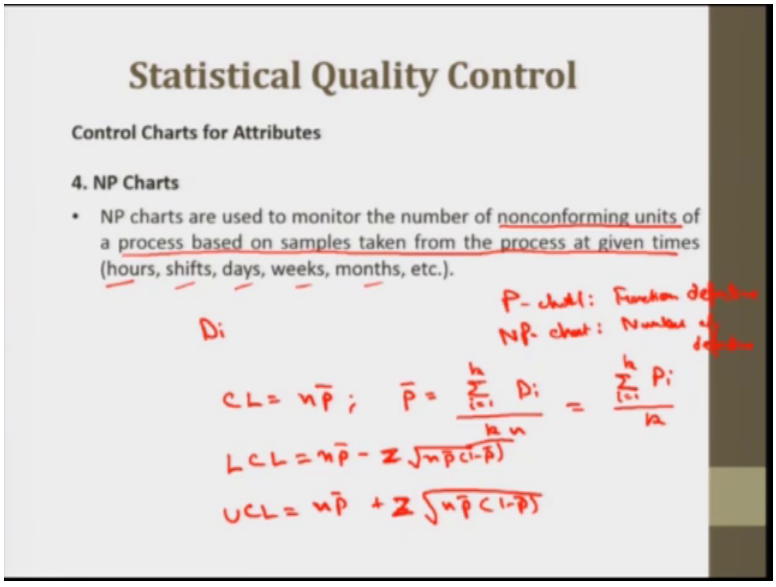

So, next comes NP chart, the u chart will discuss the NP chart NP chart is similar to the P charts. NP charts is used to monitor the number of nonconforming units of a process based on sample taken from the processes at a given time maybe at specific hours, shifts, days, weeks, months, etcetera.

So, typically an initial series of samples is used to estimate the average number of nonconforming units per sample. Then the estimated average is then used to produce control limit for the number of nonconforming units. So, during this initial phase the process should be in control.

If points are out of control during the initial phase that is during the initial estimation only, the assignable cause should be determine at a sample should be removed from at estimation that we have been doing.

So, the P chart is one that shows the fraction defective, whereas NP chart shows in number of defectives, I will put it theory here P chart ok. It shows the fraction defective and NP chart it shows the number of defectives. As you can see the NP chart is there, P was the fraction and N is the number. If I multiply the number to the fraction it will show the number of defectives ok.

So, for the NP charts each point in the chart is given by a value node denoted by D i which is the number of nonconforming units maybe of the I th sample ok. If NP chart produce the central line proportion maybe input directly, or it may be estimated from the series of samples. If it is estimated from samples of formula the central the central line is n p bar this central line.

And if where and the sample size be calculated here where this p bar is equal to summation i is equal to 1 to k that is number of samples D i by k into n to put it in a simple form it is just summation of i is equal to 1 to k, p i by k, this is p bar ok.

And the lower and upper control limits lower control limit and upper control limits, the lower control limit can be given by n p bar minus z times n p bar into 1 minus p bar you can see we have just multiplied n with the p chart. It was p bar minus z into p bar n minus 1 minus p bar by n here, if you multiply n here, so the n goes inside it becomes n squares, so 1 n is cut.

So, we get this value and the upper control limit would be n p bar plus z times n p bar into 1 minus p bar this is z small z. So, where this multiply the value of z is generally chosen? So, chart would 3 ok, so the false alarm sort out of control signals where I have a given when the process is not in control.

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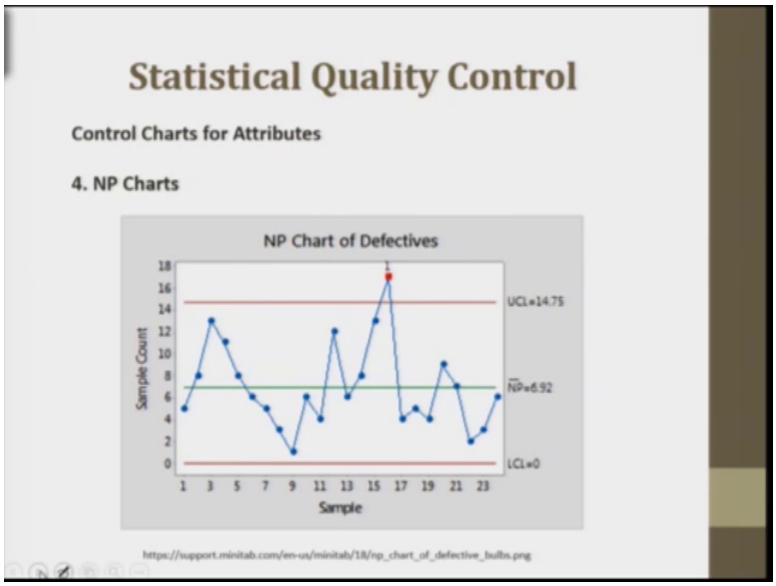

So, this is an example of NP chart.

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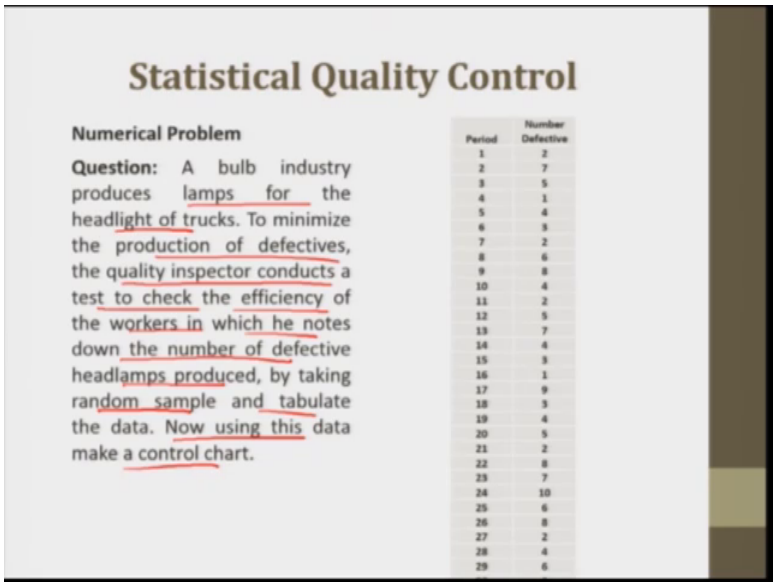

So, we will provide you this in the slides, so this is a problem here. So, in the numerical problem we have in this question a bulb industry produces lamps for the headlights of trucks. To minimize the production of defective the quality inspector conducts a test to check the efficiency of the workers in which he note down the number of defective headlamps produced, by taking a random sample and tabulate the data. Now using the

data make they make it a control chart the control chart that will have will make here is NP chart. So, number defectives are given and the period is given many took the sample.

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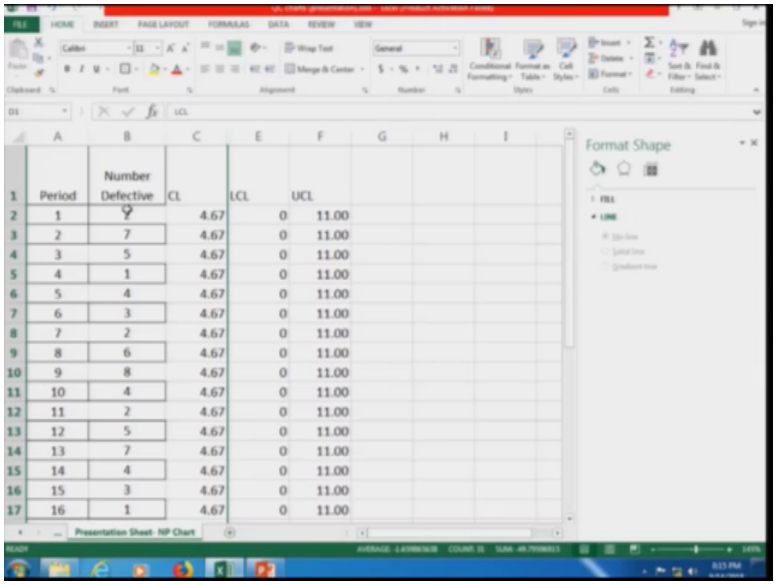

Period	Number Defective	CL	LCL	UCL
1	7	4.67	0	11.00
2	7	4.67	0	11.00
3	5	4.67	0	11.00
4	1	4.67	0	11.00
5	4	4.67	0	11.00
6	3	4.67	0	11.00
7	2	4.67	0	11.00
8	6	4.67	0	11.00
9	8	4.67	0	11.00
10	4	4.67	0	11.00
11	2	4.67	0	11.00
12	5	4.67	0	11.00
13	7	4.67	0	11.00
14	4	4.67	0	11.00
15	3	4.67	0	11.00
16	1	4.67	0	11.00

So, I will just spot these to my excel sheet. And we have number of defectives and the period when the reading is taken ok. For the NP chart what we need to do? We can need to calculate the value of n p bar n p bar is here can be just I will put n into p bar n into p bar ok.

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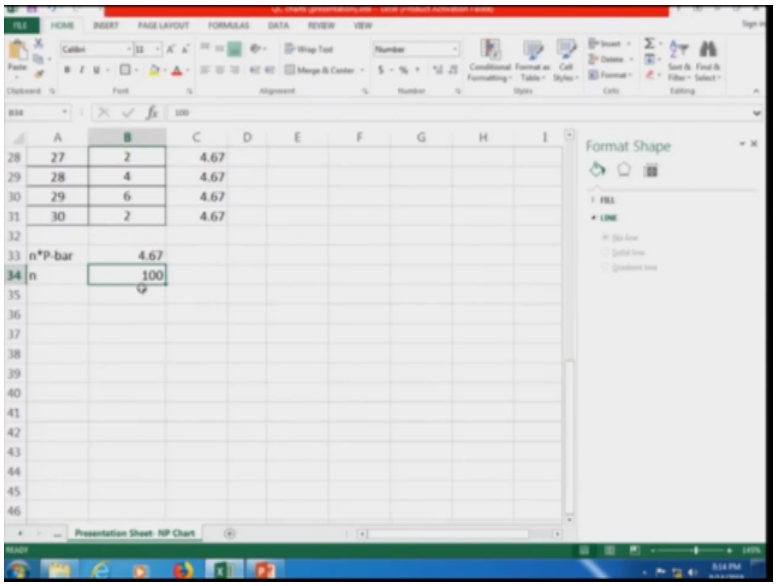

27	2	4.67
28	4	4.67
29	6	4.67
30	2	4.67
n*P-bar	4.67	
n	100	

So, this is equal to the average of the defects is equal to average of these values enter. So, I will just give this where things this have to this two places of decimal, so this is n P bar.

So, we have this centre line as our n P bar and lower control limit and upper control limit central line is our this value n P bar enter. I will just lock the values dollar and dollar and drag it is my central line. So, lower control limit this is equal to n P bar it is this value now minus 3 times 3 into three is my value of zee, or z square root of n P bar into 1 minus P bar.

So, it is square root of n P bar n P bar is same number into 1 minus P bar ok. I have to calculate P bar first to P bar the value of n here is because you are talking about the percent defective ok. I mean just put this later I will just put here n is equal to 100, the value of n here is 100.

So, n is a constant value I will better put it is a question as well the value of n is equal to 100, n is equal to 100 that is out of the 100 samples taking this see. So, we have which obtain the central line and the lower control limit is equal to n P bar I will just put this value system directly n P bar value is 4.67 P bar minus 3 times of this square root of this square root of n P bar which is 4.67 into 1 minus P bar 1 minus P bar would be n P bar by n; n P bar is 4.67 divided by n, the value of n is 100 ok, so let me close the bracket ok. So, the value of n is 100.

So, I have putted in the equation as well in the question as well that the value of n is 100 ok, he is pick it from the because n is constant in case of n P charts. So, the value is minus here, so we can put the value 0 let me see ok. This n P and P bar would all remains same here, so the value is minus. So, LCL would actually we put as 0 now, so I can even put the if loop here, you can even put the if loop that if this value is less than 0, then take it 0.

So, LCL is taken as L C L because we cannot have negative values L C L is 0 ok. And the upper control limit that we will we can have we can just put this plus here. This value is there this value is 0, and this value I can this bring it to the second place of decimal ok. This value is 11.00 and it is here ok. Now I will hide this.

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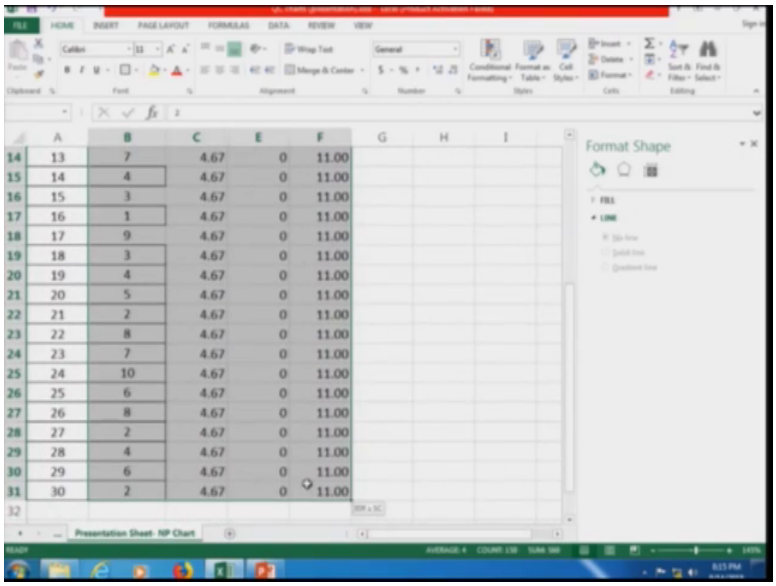

And now I will try to plot these values.

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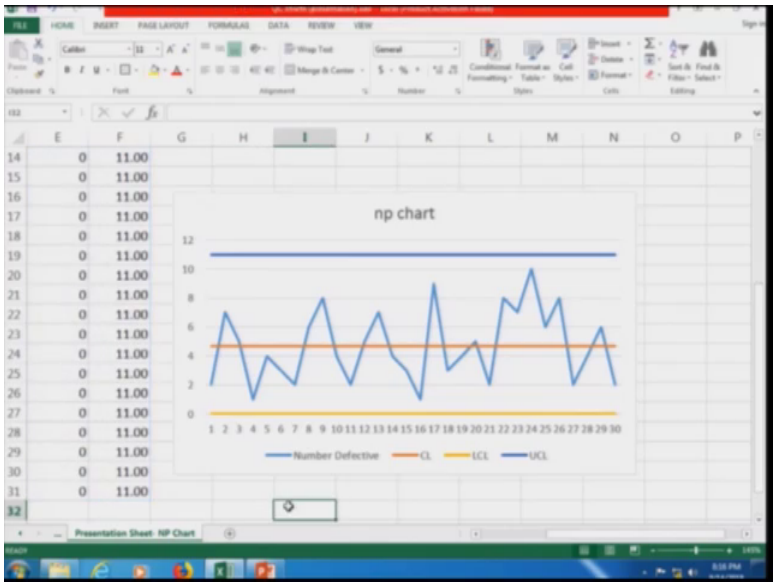

I have not picked the series name number defective central limit ok. This number defective this is the np chart I can put the chart title as np chart ok, save ok, also it is keep this thing. So, this is our np chart, so the whole process is in control within the control limits this was about the control charts. So, we will meet in the next lecture.

Thank you.