Engineering Metrology

Prof. J. Ramkumar
Dr. Amandeep Singh Oberoi
Department of Mechanical Engineering & Design Programme
Department of Industrial & Production Engineering
Indian Institute of Technology, Kanpur
National Institute of Technology, Jalandhar

Lecture - 47
Quality Control, control charts for variables

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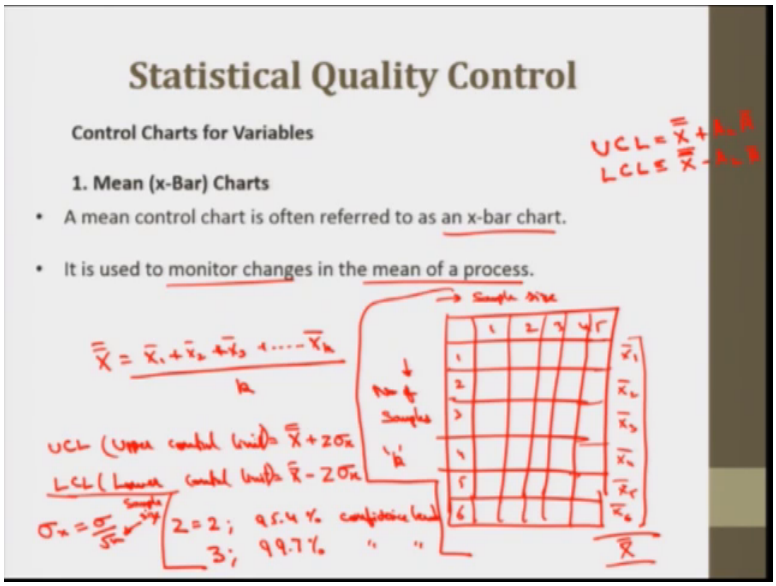

So, let me come to the very commonly used charts that is mean or x-Bar chart. So, a mean control chart is often referred to as an x-bar chart. It is used to monitor the changes in the mean of a process. Mean we can take only when the data is continuous at that I have said that. So, in the to plot the x-bar charts specifically. So, to construct a mean chart, we first need to construct a central line of the chart. To do this we take multiple samples, compute the means, usually these samples are small with about 4 or 5 observations. Then each sample has its own mean. So, these mean the if a central line of a chart is then computed as mean of all the k samples when the k is the number of samples. So, I will take an example to explain this.

So, what we do here, we have we will have a data something like this. This is this is sample size. Sample size a let me say this is 1 2 3 4 5 units per sample ok. And this is number of samples that I have said, if I have taken k samples I will take I will just

putting number 1 2 3 4 5 and let me say I have picked 6 samples and each sample is having 5 observations in it. So, I will take this mean x 1 bar, x 2 bar, x 3 bar, x 4 bar, x 5 bar and x 6 bar. This mean is taken then the overall mean which I call it x double bar is the mean of all the values or mean of these x-Bars ok. This might be put as a center line if it is not given.

So, in case of x-Bar chart. So, this x double bar is equal to x 1 bar plus x 2 bar plus x 3 bar we will have up till x k bar divided by k. So, this is our central line. So, to construct the upper control limit, the upper control limit that would be equal to x double bar as I said before plus z into sigma for x for the sample and lower control limit that is equal to x double bar minus normal deviate z into sigma of x. This value of z is generally taken as 3.

So, we can take any value of that. For instance, for we need to see that when, what is the confidence level, what is the significance level that we need to where we are trying to work on. For instance for z is equal to 2, what would the value z is equal to 2 implies; the 95.4 percent confidence level ok. Confidence level, then if this value is 3 this will equal to? What is the value 99.7 percent of confidence level right.

So, there sigma x is standard deviation of the distribution of sample means. Sigma x it can be calculated from the standard deviation of the population as sigma by under root n. So, if the process, if the population standard deviation is given, then we can calculate like this. Here, n is our sample size ok. So, we can calculate this. The upper control limit I will put it UCL ok. Then LCL and we can try to plot the chart.

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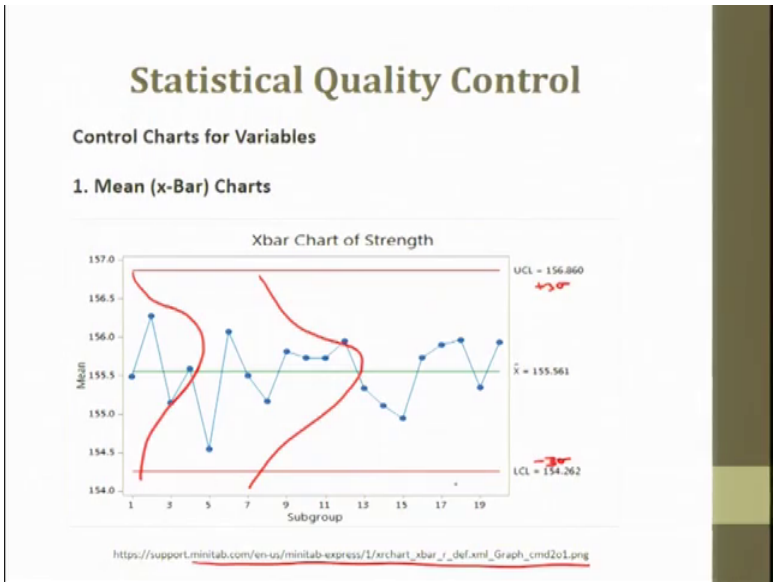

So, this is one of the charts that I have picked from the Minitab software. Minitab data of the help files are there and I have just picked the chart to just show that how the x-Bar chart is plotted, how it looks like.

So, we have the central line, upper control limit, a lower control limit, where they have picked the values has this values as plus 3 sigma and this is minus 3 sigma ok, but we will talk about the process capability. We can even pick some other values. In actually, in actual working these limits sometimes these limits are not specifically 3 sigma. This can be 3.14 sigma minus 3.25 sigma. So, this limits are defined all you know it is a kind of a normal distribution here, is a kind of normal distribution here. So, I have put some skewed distribution here. So, it is kind of a distribution here ok. So, when we will talk about process capability we will have more details on this.

So, this is the process we can see the whole process. All the 19 samples that we have taken they are within control. Ok they are within control limit. That is, the upper control limit or lower control limit.

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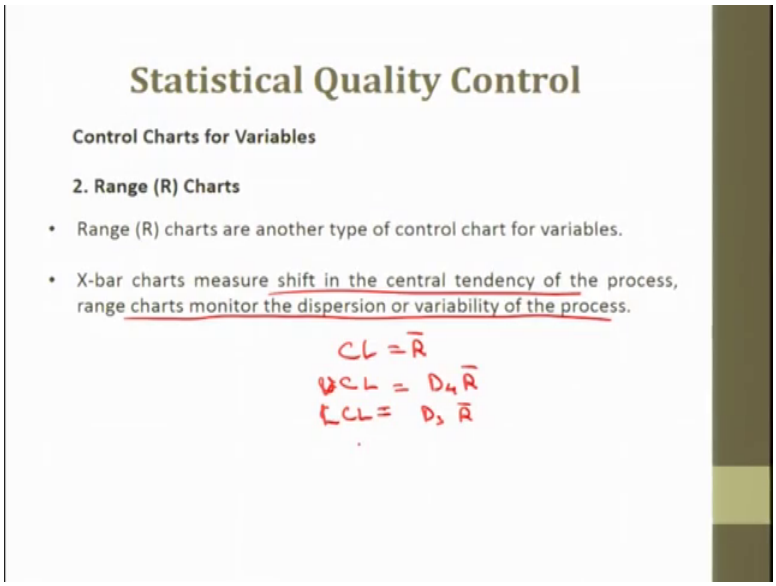

So, next is the range chart. I will now explain the range chart then I will pick a data and try to plot both the x-bar chart and the range charts using excel sheets. Range charts another type of control charts for variables. x-bar chart measure the shift in the central tendency of the process. Range charts monitor the dispersion or variability of the process ok. As I said before, so the method of developing and using R chart is same as that of x-bar chart. The central line of the control chart is the average range and the upper and lower control limits are computed using tabulated constants and the central control limit is just the R bar ok.

So, upper control limit and lower control limit is equal to D4 into R bar and D 3 into R bar. So, what are these D3 and D4? These are the values which are obtained from the table of control charts.

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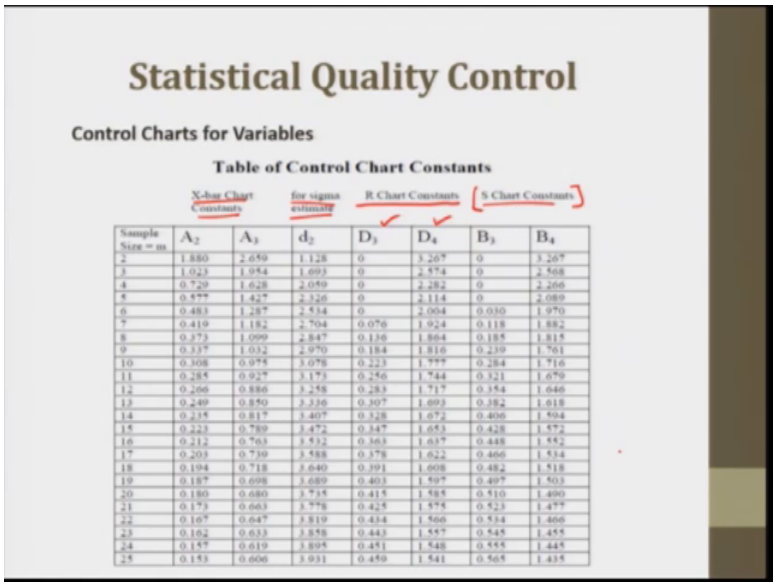

Sample Size = n	A_2	A_3	d_2	D_3	D_4	B_3	B_4
2	1.880	2.659	1.128	0	3.267	0	3.267
3	1.023	1.954	1.693	0	2.574	0	2.568
4	0.729	1.628	2.059	0	2.282	0	2.266
5	0.577	1.427	2.326	0	2.114	0	2.089
6	0.483	1.287	2.534	0	2.004	0.030	1.970
7	0.419	1.182	2.704	0.076	1.924	0.118	1.882
8	0.373	1.099	2.847	0.136	1.864	0.185	1.815
9	0.337	1.032	2.970	0.184	1.816	0.239	1.761
10	0.308	0.975	3.078	0.223	1.777	0.284	1.716
11	0.285	0.927	3.173	0.256	1.744	0.321	1.679
12	0.266	0.886	3.258	0.283	1.717	0.354	1.646
13	0.249	0.850	3.336	0.307	1.693	0.382	1.618
14	0.235	0.817	3.407	0.328	1.672	0.406	1.594
15	0.223	0.789	3.472	0.347	1.653	0.428	1.572
16	0.212	0.763	3.532	0.363	1.637	0.448	1.552
17	0.203	0.739	3.588	0.378	1.622	0.466	1.534
18	0.194	0.718	3.640	0.391	1.608	0.482	1.518
19	0.187	0.698	3.689	0.403	1.597	0.497	1.503
20	0.180	0.680	3.735	0.415	1.585	0.510	1.490
21	0.173	0.663	3.778	0.425	1.575	0.523	1.477
22	0.167	0.647	3.819	0.434	1.566	0.534	1.466
23	0.162	0.633	3.858	0.443	1.557	0.545	1.455
24	0.157	0.619	3.895	0.451	1.548	0.555	1.445
25	0.153	0.606	3.931	0.459	1.541	0.565	1.435

The table control chart that I have also put it here. So, this is the value of D3 and R chart this is the R charts constants. This is for sigma estimate I will just let you know what is this thing. Also we can use the A2 and A3 like a plus x double bar plus A2 as double bar minus A2, find the control limits. So, these are the standard, some standard values which are obtained. So, these constants are often used in practice to calculate the value of the standard deviation for the sample.

So, this help to speed up the process. So, we do not have to calculate. So, this constants are developed and we can just speed up the process.

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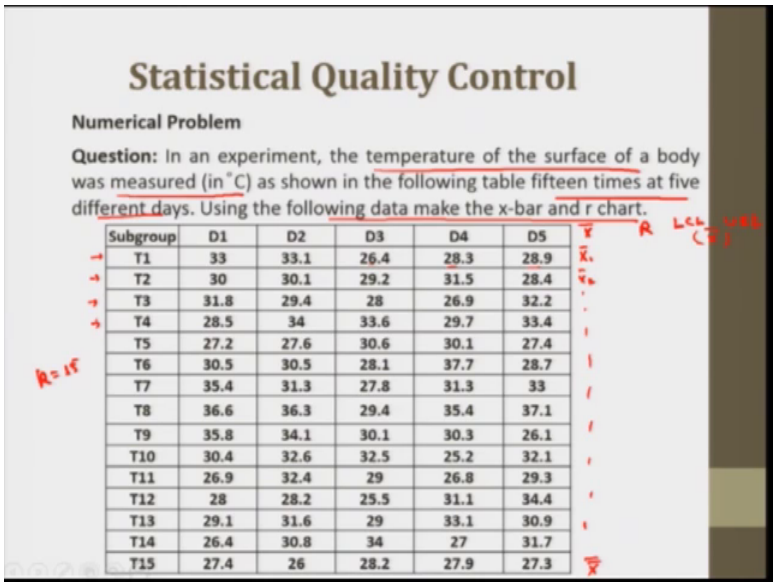

So, I have this numerical problem here. In an experiment the temperatures of the surface of a body measured in degree centigrade as shown in the following table 15 times at 5 different days. Using the following data make the x-bar and R chart. So, this is the temperature. Temperature as I said is a continuous data in this case. So, we can this is a control chart for variables that will construct here and this is taken for 15 times that is, our k is equal to 15 and sample size at 5 different days. 5 different date is taken and sample size is 5 ok. How could we proceed to do the calculations? First of all, we will calculate the x-bar. We will calculate X1 bar X2 bar, so on; the average of these values, 33, 33.1, 26.4, 28.3, 28.9.

So, this x-bar would give me x double bar. That can be taken as the central line ok. So, also I will construct both x-bar and R chart. For R chart I will calculate the range ok. The lower control limit then upper control limit for the x-Bar chart; and lower and upper control limits for the R chart and the central lines. So, I will take this data to the excel sheet and try to plot it.

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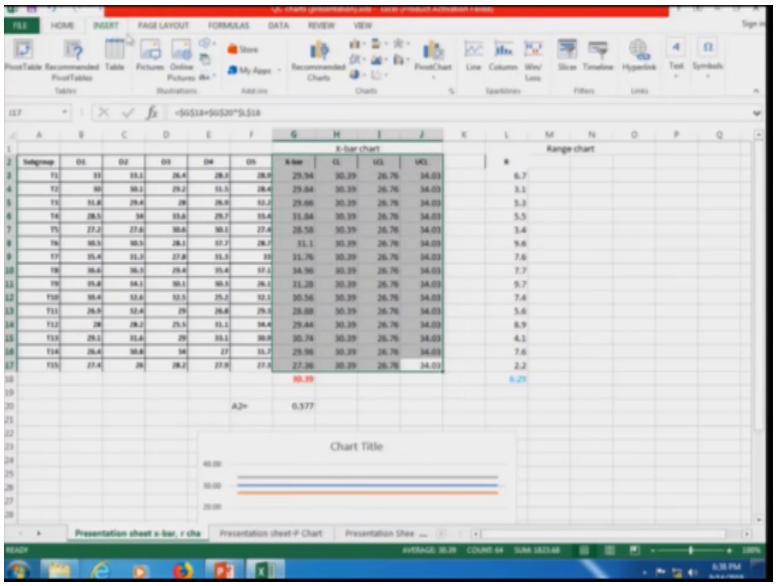

So, this data is produced here in the excel sheet. So, we have this temperatures at different times, the days. So, first of all I will try to calculate the x-bar, the average of these values.

So, this is equal to average. Average of I can put I can just pick these values like this from B2 to F2, or I can separately even put average B2 comma C2 comma T2 comma E2 and F2, just telling you another way ok. It will give me the average. So, average of these five values is this. Now, I can drag this formula, if you remember this plus sign, the small plus sign you know they were three plus signs actually. The small plus sign is the selection of the formula. If I drag I will drag the formula ok. This formula is dragged and I had I have got the x-bars. This is X1 bar, X2 bar, 3 bar, 4 bar so on ok. These are x-bars. Let me calculate x double bar which is equal to average of all these values. So, this is equal to average of all these values, enter.

So, the overall average which is equal to x double bar that is equal to 30.39.

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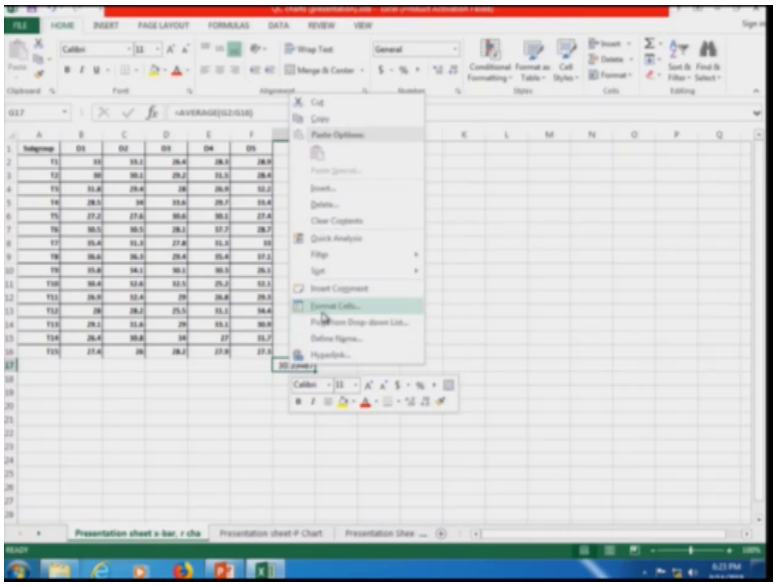

I will just format.

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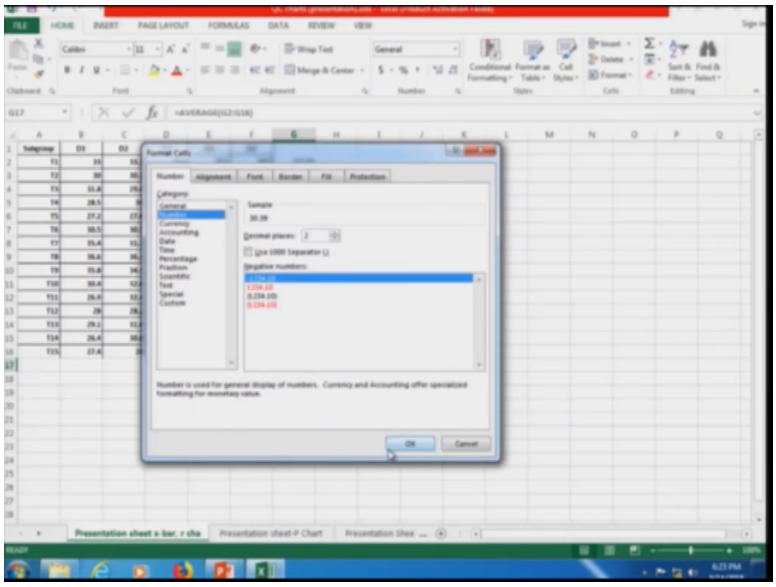

I will just bring it down to the second decimal place, decimals ok, and let me give it a different colour. So, this is my average. Now, this is my I will put x-bar. In the next column, I will try to calculate the lower control limit or first I will put the central line then lower control limit then upper control limit. So, I will try to put another line here just above this line. Insert a line, and I will merge these and this is my x-bar chart ok. So,

central line would be this value, x double bar 30.39. So, I will put this value as, this is equal to enter, so, this value.

Now, if I drag this directly. It will try to pick the values from the elements. So, they form the boxes which are below this. You can see it is trying to pick the value from here. For that, for that purpose I need to lock these values because I have I am just picking this one value, 30.39. So, this one value I will just lock the row and the column by putting a dollar sign before the column and the row number of the cell. Now this cell is locked. Now if I drag this it will just show the same value, the cell G18. You can see the cell G18 is locked, ok. Here also, the cell G. So, this is central line.

So, next I will try to pick my lower control limit. To pick the lower control limit I can calculate the sigma value, another way is I can use the values given in the table, the table that I discussed, in this table ok. So, in this case I like to put another formula that is a quick formula for the x-bar chart. The upper control limit and lower control limit here. The upper control limit and the lower control limit can you be calculated using the constants which were tabulated. So, the upper control limit here can be given as x double bar plus A2 into R bar. So, because I need to plot both the charts, I will calculate the R value and then calculate these values. So, x double bar lower control is minus A2 into R bar.

So, this constants are connecting the x-bar chart to the R chart. So, for that I can first calculate the values for the R chart as well. So, let me calculate or let me pick these columns, merge, I will put here range chart. So, value of R, value of R is maximum value minus minimum value. So, this is equal to I can because these are 5 samples I can just see this is a maximum value out of the 33, 33.1, 26.4, 28.3 and 28.9, 23 point this is maximum value and the minimum value is 26.4, enter. The range is 6.5.

So, I am just observing it manually, but the right way to do it is I have; can put the formula. I will just put the formula as it is maximum of this cell would be maximum of these numbers ok. Close the braces minus minimum of these values again. Close the braces and enter. So, it has returned back to this value. So, you can see it is maximum of these values B3 to F3 minus minimum of these values B3 to F3. So, if you have number of values. For instance if you have 20 values 100 values, we need not to see the things

manually. So, excel is just helpful in just calculating with the values. So, the competition is lowered here.

So, I can just put the enter and if I drag this I will get the range is. I will just drag it from any point because the formula is same. So, the ranges have come down to these values. Maximum of this is this is 0. This is 0 means two values should be same here. 27.4 and 27.3 oh sorry, maximum is 28.2. No w it should not be 0, escape, oh if this some error, I am picking the formula it has picked the formula for the next row. So, I will just drag the formula from the beginning. Actually I have started working in a row in this row, row two here. Row two here. So, I was working in row two. I will just now move this data. So, this is the moved sign. I move this data here with this is my value R not range chart this is my value. R.

Now, I know the my value of the range and lower control limit as I know I can pick the formula. Lower control limit is equal to x double bar plus A2 into the range ok. R bar, R bar would be the average of these values. So, let me put it as this is equal to average of, this is average of these values enter. So, format cells reduce it to the two places of decimal done ok. 6.2 I will just colour it maybe blue ok. So, this is my R bar only ok. So, this is my R bar. So, my lower control limit would be equal to as had put in the formula. This is equal to x double bar plus A2. What is the value of A2? Now let me come back to this table here. The value of A2, the sample size D1, D2, D3, D4 sample size is 5.

So, the value of A2 would be taken for 5 sample size. The value of A2 here is 0.577. If you see, this is sample size is 5. This value is 0.577 through I will put the value of A2 in my excel sheet. Escape. Let me put A2, A2 this is equal to 0.577 ok. So, this value lower control limit is equal to x double bar minus A2 times A2 times R. R bar sorry. R bar value is this value. Enter. So, this is the lower control limit. So, this lower control limit remains the same. X double bar is fixed. So, I will just lock this value, I am putting dollar signs ok. The value of A2 is fixed, the constant A2. So, I am putting dollar signs here. The value of R bar is fixed. I am putting dollar signs again. Enter.

So, let me convert ok. This is format cells two places of decimals. So, now, I will, if I drag it here the low control limit is this one ok. Similarly, upper control limit would be very similar to this. I will just drag this formula and in place of minus I will put plus. But this is the upper control limit ok. Now I have got my central line, my lower control limit

and upper control limit for the x bar chart. Now, I can plot my x bar chart using the line diagram that I did in the previous lecture. So, during the line diagram I can just plot this three lines.

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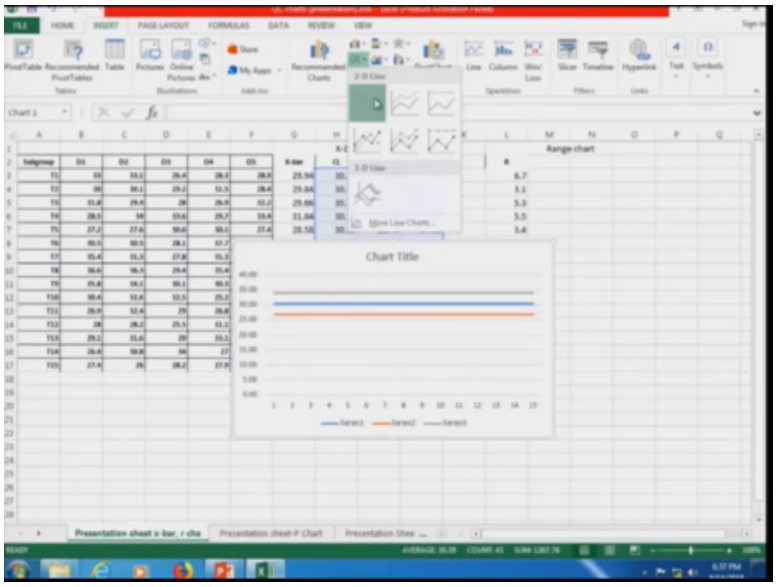

So, I will just pick these three lines and insert a line diagram. So, it is showing these line diagram, these lower control limit, upper control limit and the central line.

Now, I need to plot my data in here as well. What is the data points? Data points are the X-Bar. So, if I need to plot these values as well, I will just pick these four columns ok.

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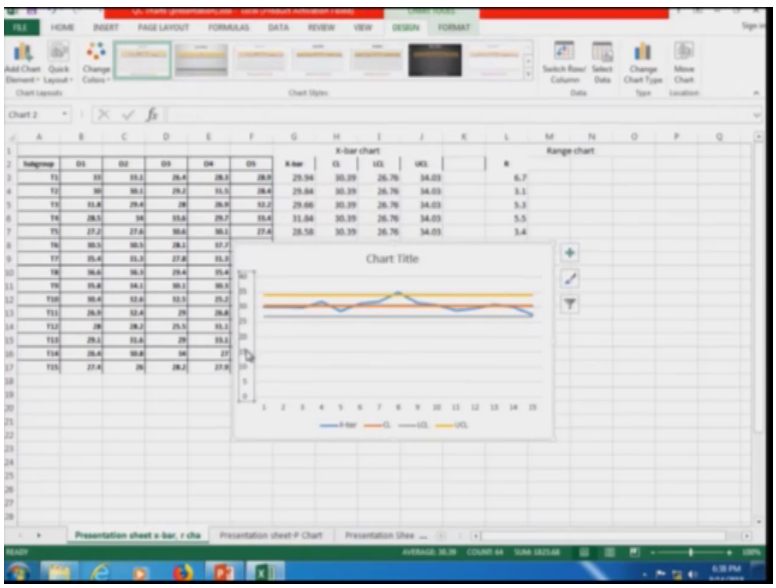

I will also pick these title, insert it has made this control chart. Let me delete the previous one. So, now, I can edit this axis to make it look more clear.

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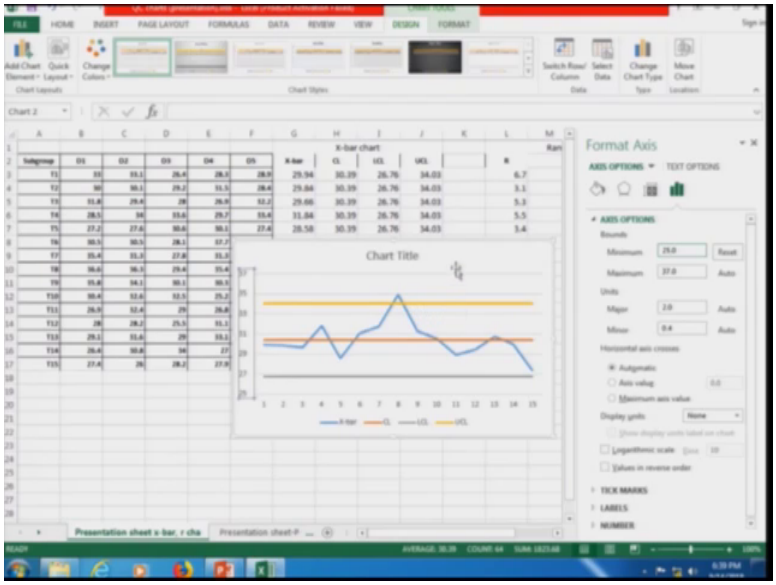

Format axis it starts from it is from let me say 25 to 37. It is a good range. So, this is ok, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, ok. More the temperature at that point, temperature is taken. So, X-bar is given in blue colour which is my variation in the process and my control limit is in orange colour. Lower control limit is in the grey colour. Upper control limit is in the yellow colour.

So, I have constructed the x-bar chart. It is interesting to note here that one of the readings is not in control. So, what do we do here, whether the reason for this is common or normal, whether this is reason is common or whether is the reason is assignable. We can see that. So, to see that we can just once try to eliminate that reading and then plot the chart again. Let me try to do this.

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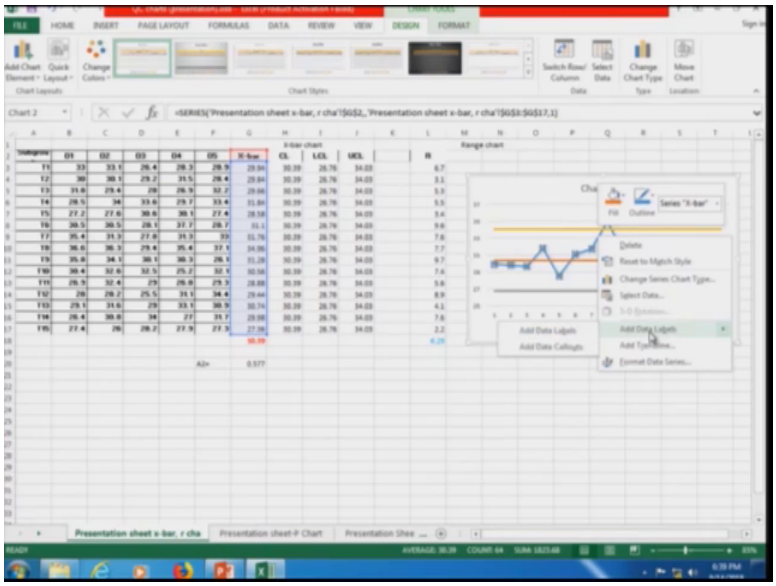

So, before that I will try to put the label here. No, the lag end is already there. Sorry, I will I will put the labels here. The data labels, add data labels.

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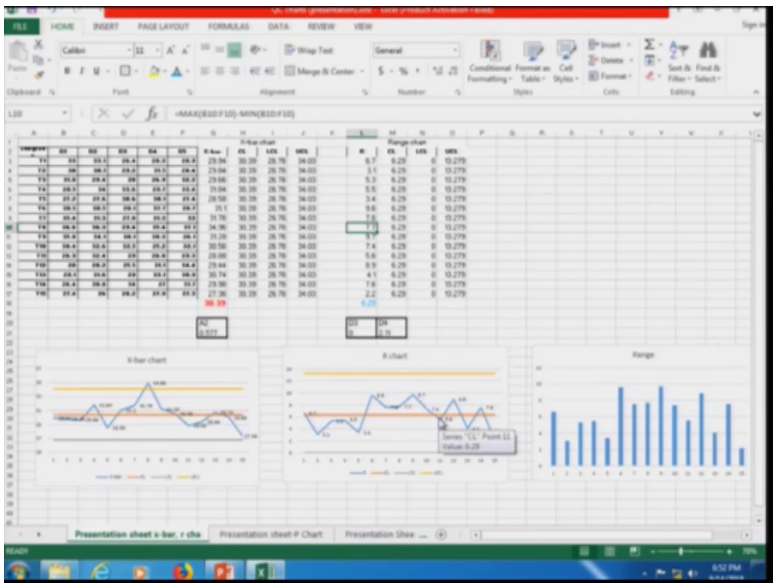

So, this value which is out of control is 34.96 which is 8th value. So, 8th value this is 34.96. 9 6, this is out of control if even if I click here, if we show this value. So, let me try to eliminate this row and then check whether the process is in control because you know, if I eliminate one reading, would x double bar would change. Overall R bar would

change, the values would change ok, but the sample size remain same that therefore, the value of A2 remain same. So, x double bar and R bar would change overall.

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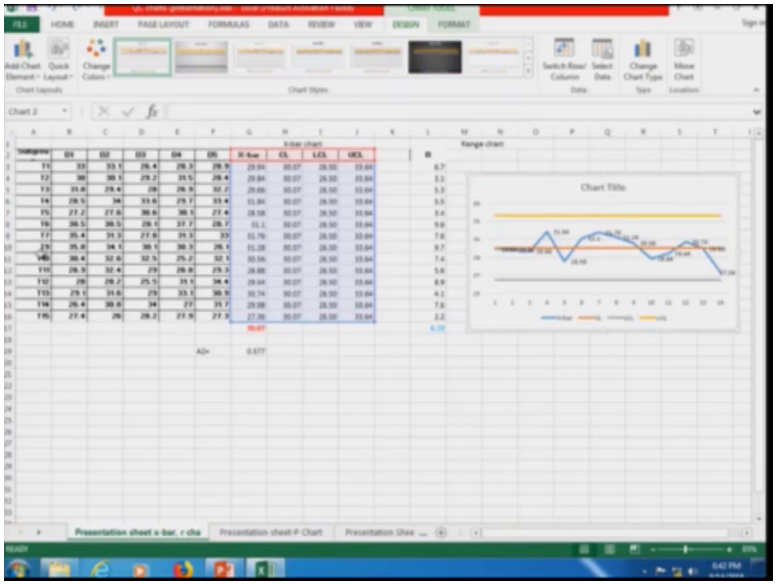

So, if I eliminate this value, delete. We have seen that the value is changed to 30.07 and this value is also changed to 6.19. So, these values of x double bar and R bar are changed. So, I have deleted this row.

So, now the process if you can see, the process is in within the limits. So, here we can say that this is not a very serious issue. So, we need not worry and just by elimination of that one value. So, there is some error that has happened just because the machine might have not, might not have worked properly or some temperatures not taken properly or reading is not taken by the observer properly or noted properly. Some error might have accounts. So, the overall process is in control ok. So, I will just undo this, so, as to keep all the values this T8 as this is my x-bar chart.

Now, let me try to plot the range chart as well. So, what was the formula for range? It was D4 upper control limit is D4 into R bar, lower control limit is D3 into R bar. So, the value of D 4 we need to see for 5 sample size ok, value of D4 and D3. For 5, for sample size 5, the value is for D3. It is 0 for D3. It is 2.11. So, it is 2.11. Let me put it here, D3 and D4. D4 is 2.11 and D3 is 0. I will also put value of A2 here only ok, just a second. Please let me try to show this. This is one block, this is one block and this is one block.

So, the R bar value is there. That is our central line ok. Then we have upper control limit and lower control limit. Central line is nothing, but R bar. This is equal to this cell enter. I lock it, this in dollar sign and dollar sign ok. Then upper control limit and better put lower control limit. First I will put upper control limit here. So, lower control limit is D3 value of D3 is this value is 0. This is equal to 0 into the R bar. Enter ok.

So, this value I will lock everything. Actually the value is 0 only. So, it could not matter much. So, all the values would be 0 here only, but just to put the formula, I am putting it here. I have locked everything. So, this lower control limit is 0 because the value of D3 is 0. The upper control limit is equal to D4. D4 is this value multiplied by the R value R bar value ok. So, I will put it dollar signs here as well.

So, I have locked all the rows and columns enter. So, we have got lower control limit, upper control limit and the values of the central line. So, let me try to plot this. Insert line ok. So, this is my R chart and we can see that ok. It is, it has not pick the name of the series. So, let me undo and pick it from here. Now it will show the lag end and show the name of the series as well. Insert line.

So, this is my R chart. So, I can put here. R chart, the name of the chart is R chart and this is my x Bar chart. So, I have plotted the R chart, but as I said before actually, just a second now. Now, I will add the data labels as well here. It is showing here. I have put the R charts just like this only. It will just show whether it is within control or not, but you know R is the range between within the subgroup or within this sample. If we selected ok, sample sizes of within subgroup it is the range, it is better to plot the bar charts to see which bar is the larger. So, line the connection of the line because it is the information that is within subgroup. It is subgroup 1, subgroup 2, subgroup 3, subgroup 4. Connection between subgroup 1 and subgroup 2 does not mean a great thing here.

So, to see the actual range, it is better to plot the bar charts. I will plot the bar chart to see this is range. Now, it is showing that range is minimum in the 15th subgroup. That is the last one and range is maximum or is a very close to the two. Two can be comparable is the 6th and the 10th you know. This chart, that is why I was telling that in data visualization, it is very important to select the chart properly. Which chart we need to plot, which chart we need to select to plot the data because we need to see the maximum

and the minimum range within the subgroup. So, this bar chart is trying as the size of the length of the bar is giving us the field that which is having a big range.

So, there might be we can work on this as well ok. These two points that these two subgroup 6 and 9. The values for the T6 is 9.6 and value for T9 is 9.7. The rage is big here. So, this was the control charts for variables. Next I will try to continue this lecture. I will just have a break here and I will like to continue this lecture. I will try to plot the control charts for attributes in the next part, ok. Using the excel sheet only so, let us meet in the next part.

Thank you.