Engineering Metrology
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Lecture – 44
Statistics for proportions

Good morning. Welcome back to the course Engineering Metrology and I am taking Statistics in Metrology part in this course. So, this lecture we will continue the probability distributions.

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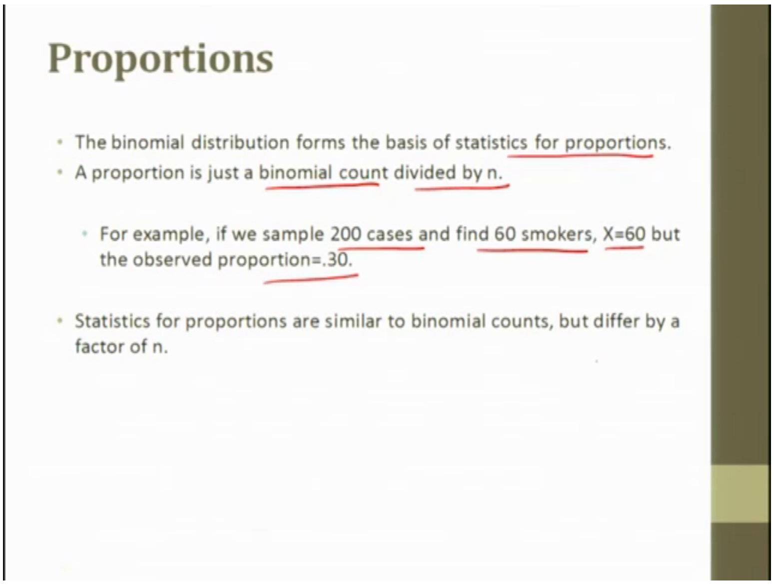

Next I will move to the proportions. Now, the binomial distribution forms the basis of statistics of proportion statistics for proportions. Proportion is just a binomial count divided by n as I discussed about proportions. When I discuss the binomial distributions in the previous lecture now for example, we have 200 cases and we find 60 smokers. So, X is equal to 60 but the observed portion is 0.3 not in this case 60 by 200 is 0.3 X is equal to 60. Now, how to do statistics here? I will just discuss this. How to use binomial distribution for that, statistics for proportions are similar to binomial counts but differ by a factor of n.

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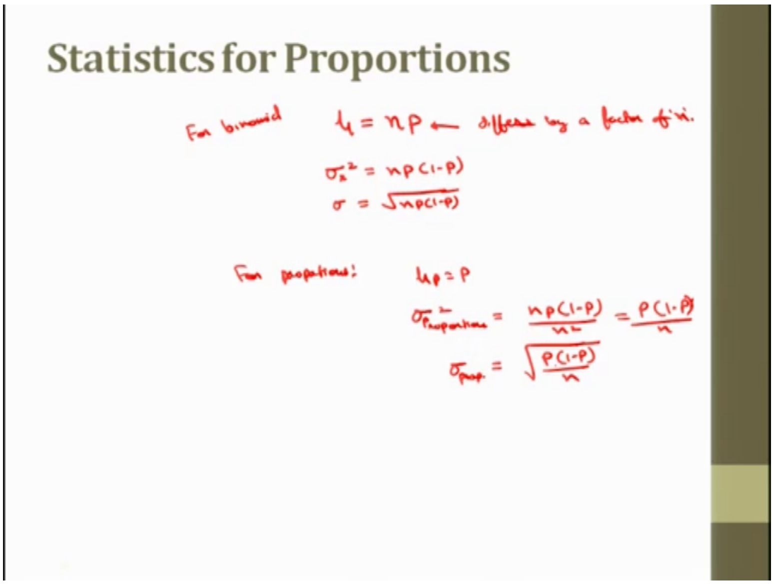

So, statistics for proportion can be written as the mu the value of mean is equal to n into p, and this actually differs by, it means it differs by a factor of n. And for this sample I will put it sample the standard deviation is equal to n p into 1 minus p. I can even use n p q; n p q, q is actually 1 minus p only. So, standardize this is actually variance, variance is this value and standard deviation is under root of this so that becomes n p into 1 minus p that is allegation. Now, this also differs by a factor of n. This is for binomial actually, ok.

And for proportions this value mu p is equal to p only and this is for p, p is for proportions it is not for population portions and variance proportions is equal to n p into 1 minus p by n square which can be put as p into 1 minus p by n. So that means, standard deviation for proportions is equal to under root of p into 1 minus p by n, where p is the probability of success, ok.

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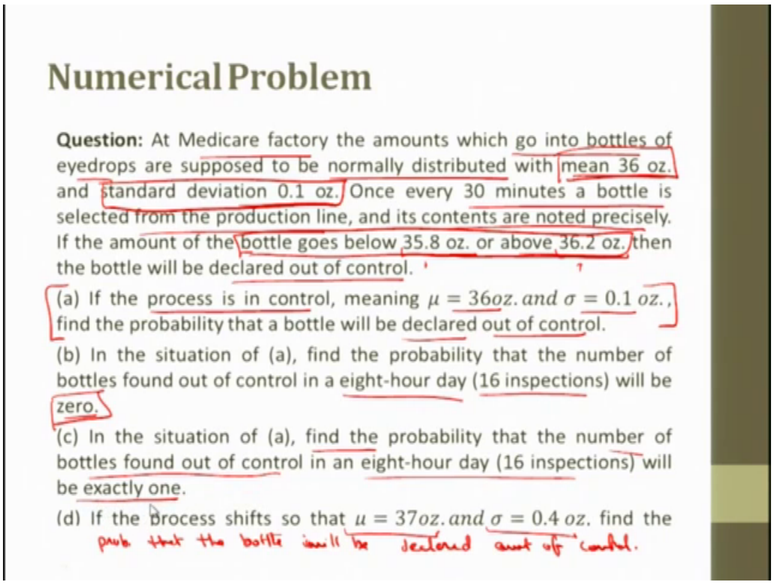

Now, I have a problem here numerical problem here. At medical factory that amounts medical factories are interested in to find the amounts which go into the bottles of eye drops which are supposed to be normally distributed with the mean 36 ounce and standard deviation 0.1 ounce. Once every 30 minutes a bottle is selected from the production line and its contents are noted precisely. If the amount of bottle goes below 35.8 ounce or above 36.2 ounce then the bottle will be declared out of control; so, you see the figures mean is here, standard deviation is here, now this bottle will be taken considered out of control if it is between 35.8 and 36.2.

Now, first question is if the process is in control that is mean is equal to this and sigma is equal to this find the probability that bottle will be declared out of control. Now, you know it the important aspect is the process is in control the process of filling the bottles is in control. Now, we need to see that the bottles that we have produced are they in controller out of control. So, to find this, let me refer read out of the questions here. In this situation of problem number a when the problem is out of control find the probability that the number of bottles found out of control in a eight hour day that is 16 inspections will be zero, ok, in eight hour day where 16 inspections happen that will be zero, ok.

In the situation of the problem a, when it is out of control find the probability that the number of bottles found out of control in an eight hour day will be exactly equal to one, ok. If the process shifts so that mu changes and standard deviation changes and find the probability that the bottle will be declared out of control, find the probability that the bottle is or will be declared out of control, ok. Now, first part is just the normal distribution as we did mu is this, sigma is this, we have a lower bound, we have a upper bound, ok.

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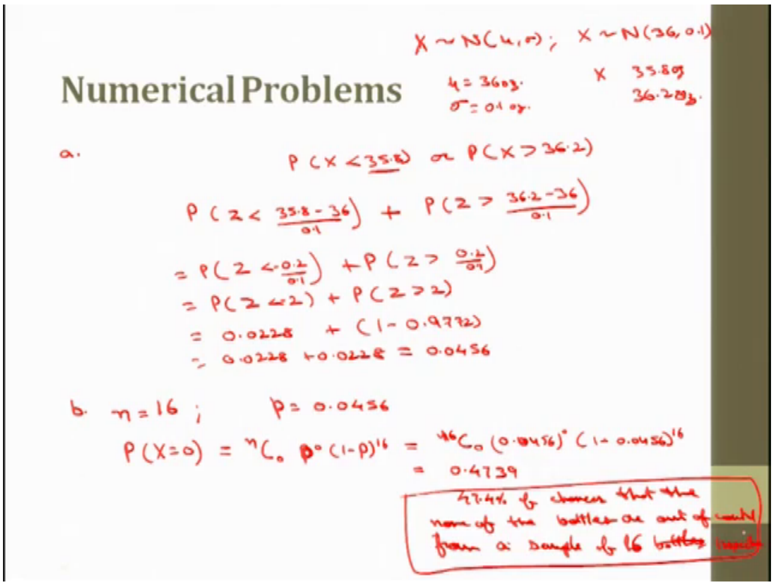

So, let me try to put these values. So, I have the lower bound as first of all I have mean as 36 ounce and standard deviation as a standard deviation as 0.1 ounce. I have values of X the lower bound and upper bound as 35.8 ounce and 36.2 ounce, ok. So, I need to find the probability that it is out of control that means, whether X is out of control, whether X is less than 35.8 or probability whether X is greater than 36.2, ok. This probability plus this probability this is part a.

When we try to compute this thing we donate normal distribution as random variable X is normal with mu and sigma, which means that this is this is a normal distribution with 36 and 0.1. Now, for this I need to calculate the value of z if I see the value of z, for this it is equal to z less than I will just put the value of z for 35.8 X minus mu that is 35.8 minus mu that is 36 by sigma 0.1, ok. Plus probability of z value of normal deviate greater than 36.2 minus 36 by 0.1, this is equal to probability of z this is actually 0 point less than 0.2 by 0.1 plus probability of z greater than again 0.2 by 0.1. This is equal to probability z less than 2 plus less than actually minus 2 here plus probability z greater than 2.

So, this value for minus 2 and 2 we need to see, and if I go to the table and try to find this value this value would be 0.0228 plus this value would be 1 minus 0.9772 which means 0.0228 plus 0.0228 this is equal to 0.0456, ok.

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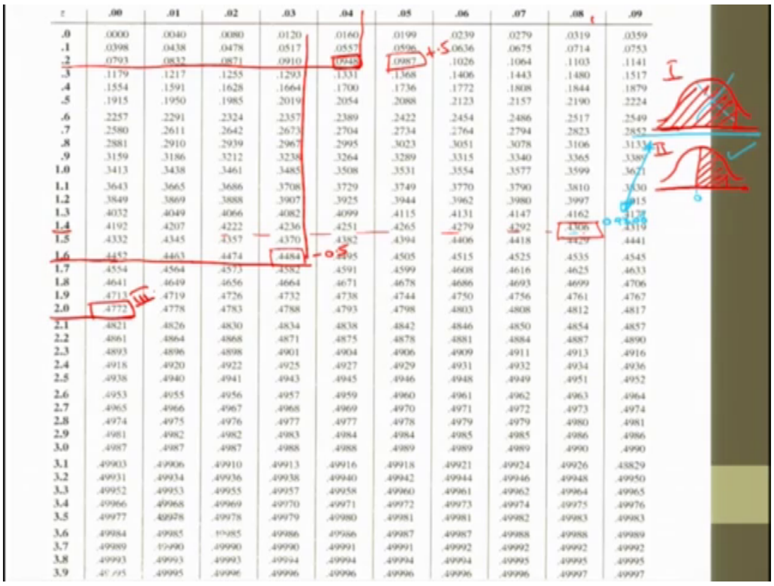

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.49903	.49906	.49910	.49913	.49916	.49918	.49921	.49924	.49926	.48829
3.2	.49931	.49934	.49936	.49938	.49940	.49942	.49944	.49946	.49948	.49950
3.3	.49952	.49953	.49955	.49957	.49958	.49960	.49961	.49962	.49964	.49965
3.4	.49966	.49968	.49969	.49970	.49971	.49972	.49973	.49974	.49975	.49976
3.5	.49977	.49978	.49978	.49979	.49980	.49981	.49981	.49982	.49983	.49983
3.6	.49984	.49985	.49985	.49986	.49986	.49987	.49987	.49988	.49988	.49989
3.7	.49989	.49990	.49990	.49990	.49991	.49991	.49992	.49992	.49992	.49992
3.8	.49993	.49993	.49993	.49994	.49994	.49994	.49994	.49995	.49995	.49995
3.9	.49995	.49995	.49996	.49996	.49996	.49996	.49996	.49996	.49997	.49997

You can see the value of at 2 here because that is equal 2 is equal 0.4772, for the other table it would be 0.5 minus this value which is equal to 0.228, ok. This is for question number 3, part a. Now part b, the situation of a when the process is out of control now we have got a probability here, which is equal to 0.045 will may be 4 percent 4.5 percent of chances are there. Now, if the probability that the number of bottles found out of control in a eight hour day 16 inspections would be 0. Now, what we have here is when b we have 16 inspections what is this value this is value of n in case of binomial distribution and the probability of success actually a probability of out of control is equal to 0.0456.

So, now, you see that from normal distribution first we find the probability, now this probability would be used as proportions or maybe as a probability of success in binomial distribution then the question is progressing further. Now, this p is equal to this. Now, what is the question asking? It is asking the probability of out of control that the probability that it is out of control and bottle found out of control will be 0, bottle found out of control will be 0.

So, let us see here carefully that out of control this p is success and they need when none of the bottle is out of control that is p for X is equal to 0, for X is equal to 0 the relation that I have it would be n c 0 then probability of out of control n c 0 p power 0 and 1 minus p power 16, which is equal to n that is 16 c 0.0456 power 0 and 1 minus 0.0456 power 16, ok. Now, this value if I calculate this value comes down to 0.4739, that is we can say 47.4 percent of chances that the none of the bottles are out of control from a sample of 16 bottles, ok.

Now, let us try to do part c. Now, the part c is the similar to part b but the only difference is that he says that bottle out of control will be exactly 1. Now, we need to find the value of X exactly equal to 1, part c when probability of X exactly equal to 1.

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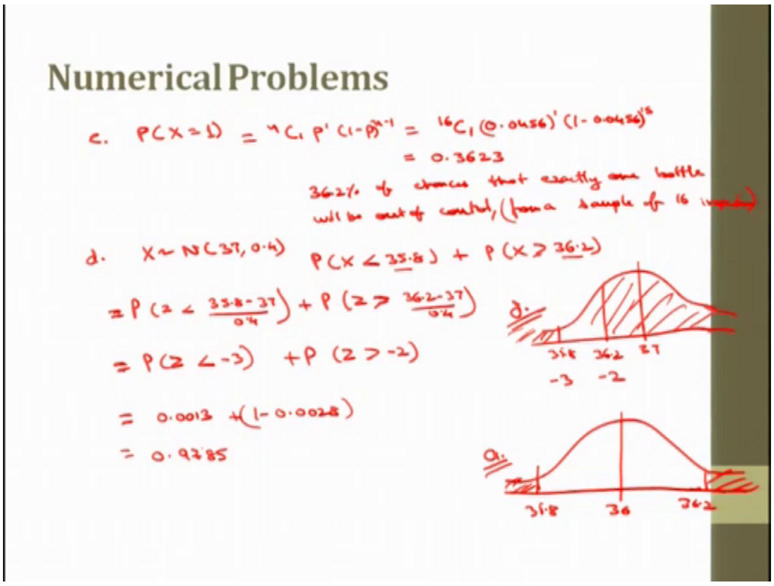

This is again we can put it this is equal to n c 1 p 1, 1 minus p power n minus 1, ok. So, this is equal to 16 c 1 0 0.0456 power 1 and to 1 minus 0 0.0456 power 16 minus 1 that is 15, ok. If I calculate this value will come 0 0.3623 that means, 36.2 percent of chances that exactly one, exactly one bottle will be out of control in a sample of out of control in a sample of 16 bottles or inferred form a sample of inferred from a sample of 16 bottles. So, 16 inspections sorry inspections, here also it will be inspections; so, sorry for this 16 inspections.

So, next if the process shifts so that mu is changed, sigma is changed, now find the probability that bottle will be declared out of control. Now, mu and sigma are changed now we have a new normal distribution that is a normal distribution is n with a changed value of mu that is 37 and changed value of mu that is equal to 0.4, 0.4. We need to check the probability that whether it is out to control or not we I think we just need to do the calculation similar to question number a. We just only the value of mu and sigma are changed and we can find the probability in a similar fashion.

Now, for this we again need to find whether probability for X is less than 35.8 plus probability of X is greater than 36.2, ok. In this case you can see that both the lower and upper bound 35.8 and 36.2 are less than the mean value 37. So, upper bound is also lower. So, we will be lying somewhere here in the new probability distribution this is 37. So, it is 35 and 36, 35.8 and 36.2, ok.

Now, we are on this side probability less than this, I greaten that this, but the essential part would be this will be some minus value this mu some minus value the value of z this will be negative values. So, we can say it this is equal to probability for z less than 35.8 minus 37 by 0.4 plus probability of z greater than 36.2 minus 37 by 0.4. If I calculate these, this is the probability of z less than minus 3 plus probability of z greater than minus 2 this value is minus 3 this is minus 2. So, if I locate this probability or traces probabilities on the probability distribution table of normal distribution I will find that this probability is 0.0013 and this probability is 1 minus 0.0028, this total probability is 0.9785.

So, when the mean is shifted, when the upper bound is even lesser than mean we find that their probability of the process that would be out of control is very large 97.85 percent in the case a the probability was 4.5 percent. So, why is this big shift? If I see the previous this is actually part d, if I draw the plot for part a it was like this 36, then 35.8 and 36.2. Now, we see we are interested in the area that is before 35.8 and after 36.2. This area was too small here this area and the part d is very big. So, you can see when the shift of the process is there. So, the probabilities can change dramatically.

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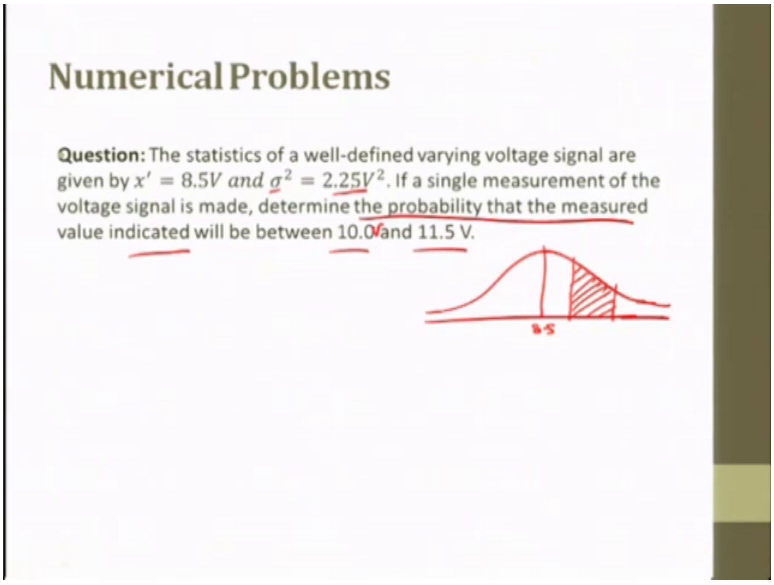

So, next I have another question the statistic of a well defined varying voltage signal are given by x dash is equal to this and variance is given. If a signal measurement of the voltage signal is made determine the probability that the measured value indicated will be between this and this, ok, this volt and this volt, 10 volt and 11.5 volt. So, now, I will just draw the curve here x dash is 8.5 and this is 10 and 11. I need this probability.

So, I will leave this question for you and will have the questions in the quiz based upon this question. Just be mindful that this is sigma square, this is variance we need to take under root to find the standard deviation, this is that for you. So, we will meet in the next lecture, we will discuss the statistical parts in metrology further.

Thank you.