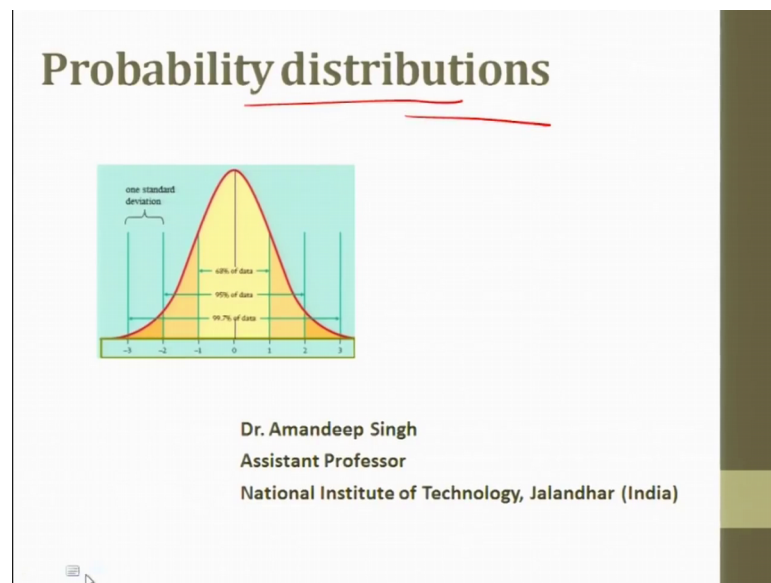


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Lecture - 43
Normal Distribution

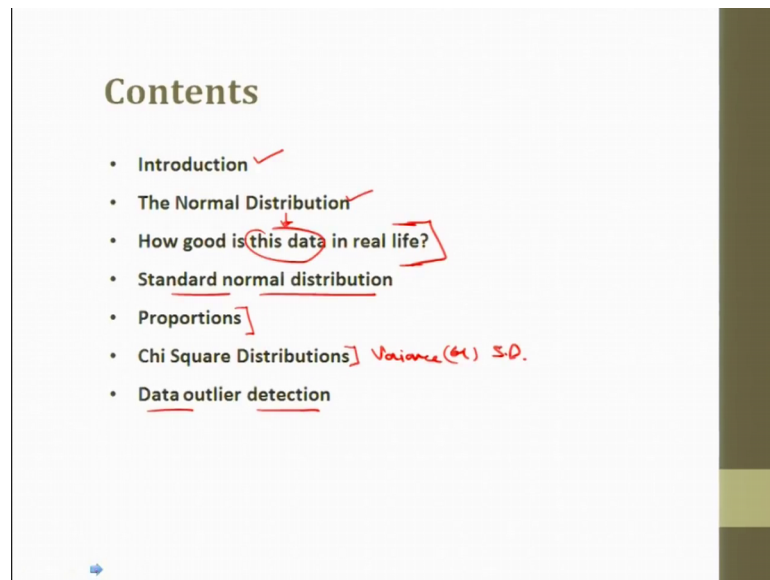
Good morning. Welcome back to the course Engineering Metrology. And I am taking statistics in metrology part in this course. So, this lecture, we will continue the probability distributions, and I will discuss more on the normal distributions some proportions.

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And chi square distribution are introduced and try to put some the light on the numericals on these as well.

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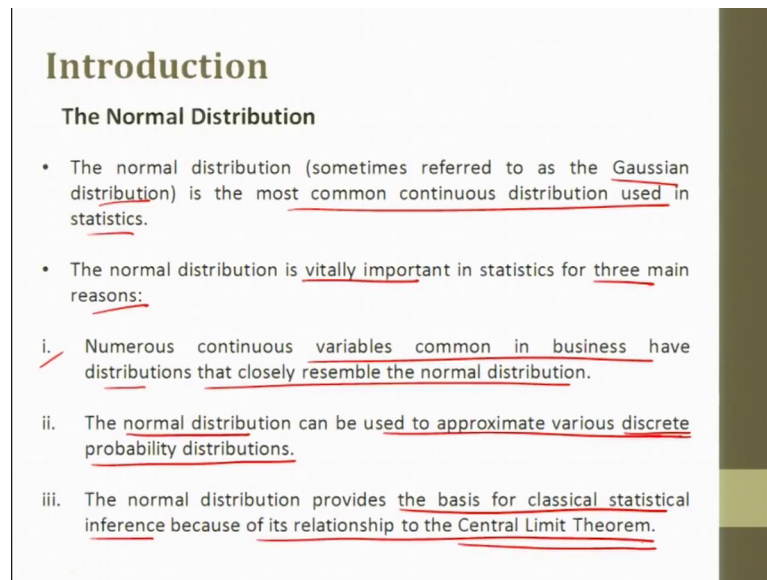
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- The Normal Distribution ✓
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- Standard normal distribution
- Proportions]
- Chi Square Distributions] Variance (σ^2) S.D.
- Data outlier detection

Now, contents will flow like this. First I will just introduce the normal distribution, and discuss in our distribution in detail. Then I will see the, how good of I will put and I will pick an example to explain that how good is this data or how good is not this here how good is normal distribution in real life. Then what is standard normal distribution. And I will discuss something about proportions, and I will try to solve an example on this.

Then I will discuss the chi square distributions, chi square distributions are actually distributions of variance or standard deviations. So, then as I have told you that sometime data outlier is there, and the data some outliers are there data outlier detection there is a rule (Refer Time: 01:33) rule on this, so we will discuss that.

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Introduction

The Normal Distribution

- The normal distribution (sometimes referred to as the Gaussian distribution) is the most common continuous distribution used in statistics.
- The normal distribution is vitaly important in statistics for three main reasons:
 - i. Numerous continuous variables common in business have distributions that closely resemble the normal distribution.
 - ii. The normal distribution can be used to approximate various discrete probability distributions.
 - iii. The normal distribution provides the basis for classical statistical inference because of its relationship to the Central Limit Theorem.

First introduction, what is normal distribution? Normal distribution sometimes is also referred as Gaussian distribution, because gauss came up with distribution is the most common continuous distribution used in statistics. The normal distribution is vitally important statistics for three main reasons. Number 1, numerous continuous variables common is business have distribution that are that closely resemble the normal distribution. So, normal distribution can be fit into the maximum of the measurements that we take to the heights, to weights in metrology.

In business I am if I am talking, the wages of the customers the prices in the market for the specific products ok, this different distributions can happen with but I can call normal distribution as the mother of many distributions. However, there are many distributions like (Refer Time: 02:37) distribution, (Refer Time: 02:38) distribution is not a single distribution is, it is a family of distribution. Similarly, normal distribution is also a family of distributions..

So, normal distribution the 2nd reason here can be the normal distribution can be used to approximate the various discrete probability distributions. Now, we have various distributions like Poisson distribution, binomial distribution, (Refer Time: 03:01) distribution, Rayleigh distribution, and uniform distribution. We can have normal approximation of many distributions. What if we try to plot our data on normal distribution ok, it can be skewed it can be it can have some courtesies. But, how well is if

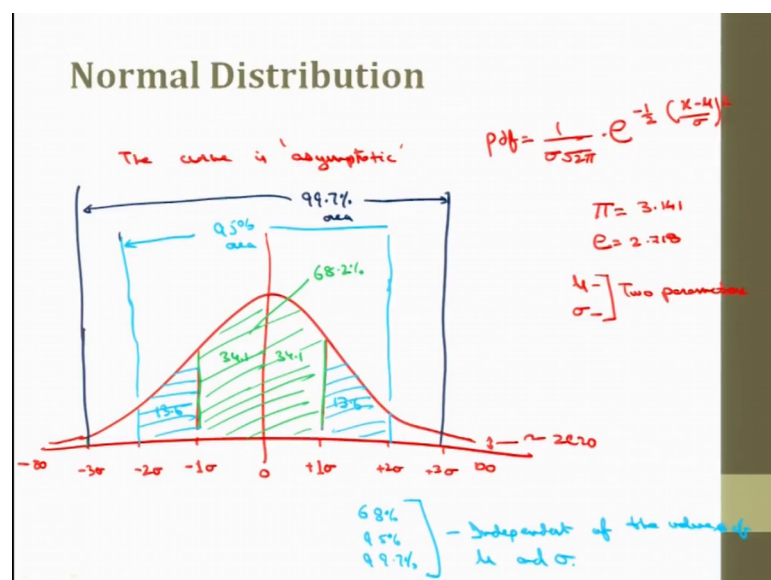
we consider distribution to be normal, whatever the other distribution is let me say normal to be normal or the Poisson distribution to be normal.

As I said when I increase the value of lambda in Poisson distribution in the previous lecture, it was approaching close to the normal distribution. So, normal distribution can be used in an approximation of various discrete and continuous distributions. So, a various discrete probability distribution is an important point here, because this could discrete data is able to produce less powerful analysis. So, it is important here.

So, normal distribution provides the basis for the classical statistical inference because of its relationship to central limit theorem, I did not discuss the central limit theorem. Central limit theorem means that there is a central value along, which the other values lies the true value.

True value is actually the actual value, this is a central value a central value, it can be mean here along which the other values lie. This is central limit you know we would, we need not to get into the details of this, but yes normal distribution is actually obeying the central limit (Refer Time: 04:31) theorem in a very good way. So, these are the few reasons for which normal distribution is used.

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Now, what is normal distribution? As I discussed in the previous slide, normal distribution has pdf this is equal to sigma under root 2 pi. And I took the standard

variance out and it becomes standard deviation. It was sigma square, before this another way to represent. Then e to the degree of minus 1 by $2 \times$ minus mu by sigma whole square. We can note that the value of pi and e are known here, pi is 3.141, and e is 2.718. So, the only variables here are sigma and mu ok. So, these are the two parameters statistical parameters that are need to be determined in normal distribution.

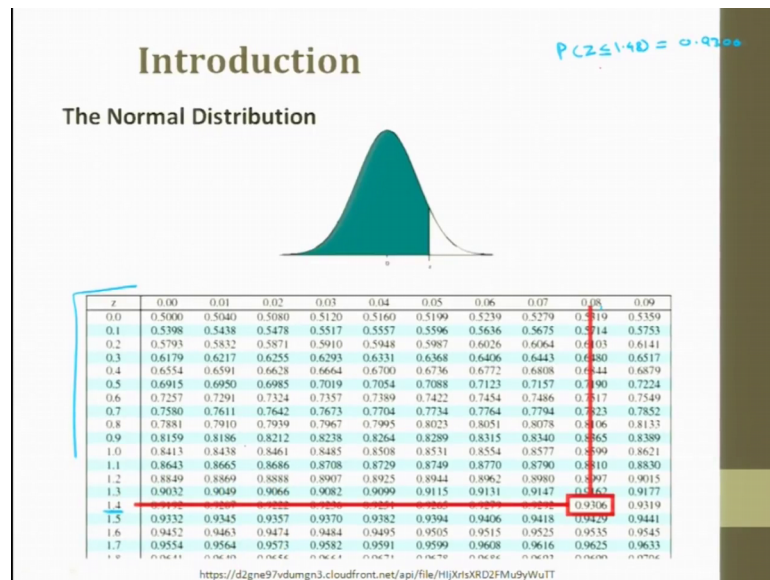
If I draw the normal curve, the curve is symmetrical around this central line. And it is asymptotic curve, I can put here the curve is asymptotic, by the word asymptotic. I mean that the curve the our asymptotic of a curve is a line such that the distance between the curve and the line approaches 0. As one or both of the x or y coordinates tends to infinity.

Even this tends to minus infinity, and this tends to plus infinity. This distance tends to 0 distance tends to 0 that means, the curve c curve is asymptotic here. Also we can divided into various parts based upon the standard deviation. I can put the value 0 have that means, the value 0 is here, this is minus 1 into standard deviation, plus 1 into standard deviation. This distance is same from minus 1 to plus 1, I can even have minus 2 sigma, minus 3 sigma, plus 2 sigma, plus 3 sigma.

Now, the area between minus 1 and plus 1 sigma is 68.2 percent this area, this is 68.2 percent or I can even say it is a 34.1 percent here, and 34.1 percent on this side this area. And in two sigma like either between two sigma that this is actually the beauty of the normal curve, whatever the kind of data is this probability in the basically this area that will be the probability this does not matter, this would always remain. We will call it 68 percent between once then 95 percent between this is 95 percent..

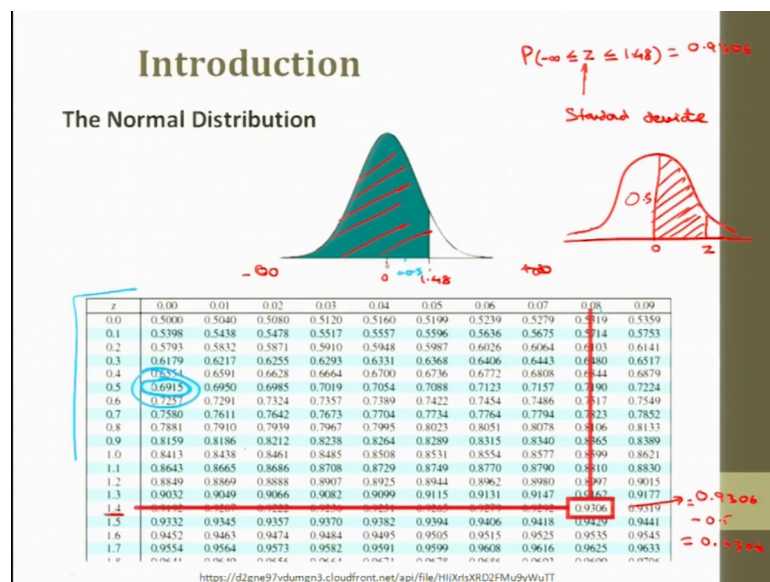
If I colour this, this total area this is actually 13.6 here and 13.6 here, which adds to this 68.2, it is actually 95 percent. And (Refer Time: 08:54) sigma we have 99.7 percent of area 99.7 percent ok. Area 95 percent area. So, whatever is the value of u or sigma this area, this probability 68 percent, 95 percent, call it 99.7 percent. This is independent of the values of mu and sigma, so that is why normal curve is most used in practical situations on the data that is having continuous distributions.

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But, this is the tabular form of the distribution. In this table, we are presenting the data. And if I see this value at 1.48 this is value at, so the probability at Z less than equal to 1.48 is equal to 0.9306 ok.

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Actually if I write it, so here the probability 0 is less than equal to Z is less than equal to 1.48, you can see 1.4 here, and point 0.8 here, this is equal to 0.9306. Now, this is the curve you can see this area is here, this value is 0 actually this is minus infinity here I am so sorry based upon this curve this should be you show minus infinity. This is minus

infinity to 0, then 0 to plus infinity, I have value 1.48 here ok. From minus infinity to 1.48 the value is nine ninety now point 0.9306. What is Z here, this is known as standard deviate. I will just put some more light on this final discussed standard normal distribution.

So, this value 1 sigma, 2 sigma, 3 sigma, the value 1, 2, and 3. This is the value of the normal deviate. The normal deviate that is contributing to my the spread of the data. What spread we are trying to keep our, what spread we are trying to look at ok. So, the curve or the table can be given this tabular form can be given in two ways. This is one way, we have complete data. Another way can be, it can be presented in this way as well sometimes.

This illustration would always be there that will be presenting what kind of data do we have. So, if it is like this that means, that table represent from values from 0 to z. In this case, this value will would be actually 0.9306 minus this area, this is 0.5. 50 percent on left side, 50 percent on right side. So, 50 percent minus 0.5, this is equal to 0.4306. So, when we see the normal curve, and we need to see the normal table, we need to be careful, but kind of table what kind of data we have what is the limit of the data for where the value starting is it from minus infinity or is it from 0, we need to see this carefully ok.

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Standard Normal Distribution

All normal distributions can be converted into the standard normal curve by subtracting the mean and dividing by the standard deviation:

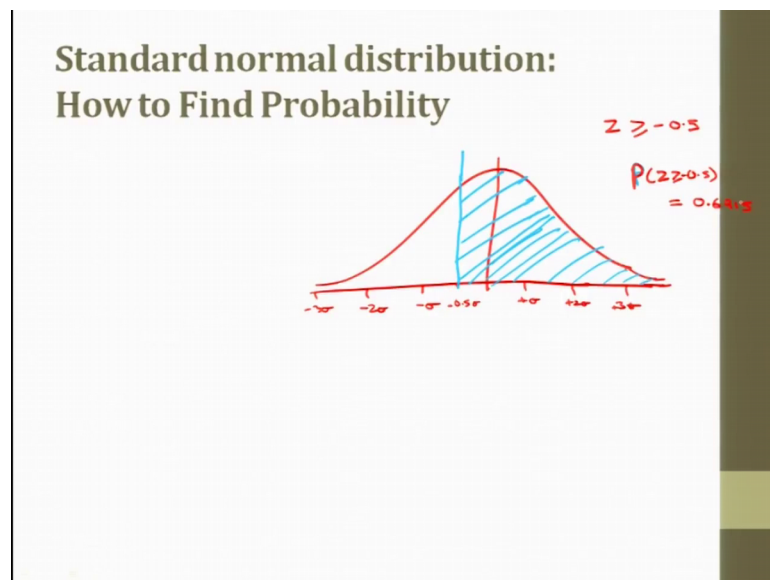
$$Z = \frac{X - \mu}{\sigma}$$

The diagram shows the formula $Z = \frac{X - \mu}{\sigma}$ with handwritten red annotations. An arrow points from 'Random variable' to 'X', another from 'Mean' to ' μ ', a third from 'Standard deviation' to ' σ ', and a fourth from 'Normal deviate' to 'Z'.

So, next is standard normal distribution. All normal distributions can be converted into standard normal curve by subtracting the mean and dividing by the standard deviation. Now, this is my random variable X , if I subtract the mean from it, and then divide it by standard deviation, I get my zender normal distribution.

Now, three somebody calculated all the integrals position normal, and put them in a table. So, we never have to integrate air like even better now computers are available to do the integration for this n normal distribution. This Z this value Z is known as normal deviate ok. This is our random variable we can just call it variable as well, this is mean, this is standard deviation. So, n normal distribution is used for the calculations.

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Now, how to find the probability, how to find the probability in the standard normal distribution? First, we just draw a bell curve, I shared it in the area that is asking the question. For instance, if this is my curve, and I get this curve here, and I need area let me say they say calculate Z that is greater than or equal to minus 0.5. Please be mindful here the value of Z is 0.5, which is not sigma..

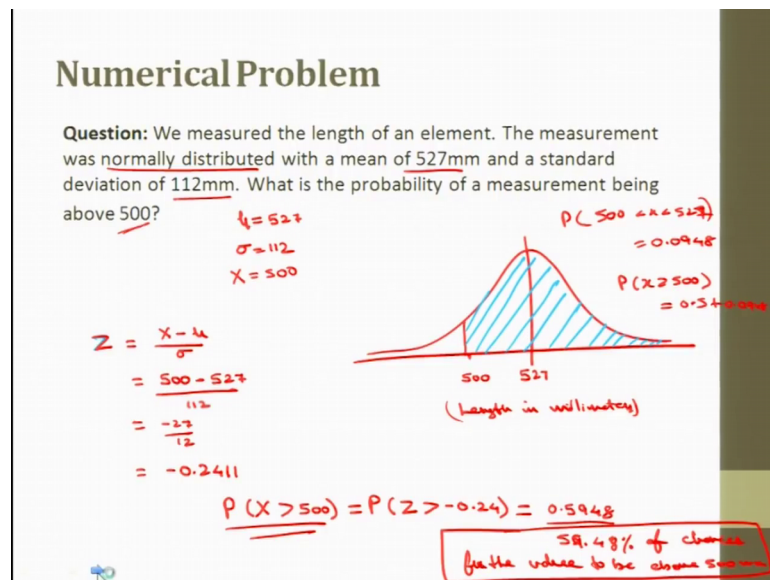
So, it is actually if this is minus 2 sigma, minus sigma, this is plus sigma, plus 2 sigma, plus 3 times standard deviations. So, it is asking the value of Z greater than equal to minus 0.5 value of actually, it is asking the probability that is for the value of Z greater than minus equal 5. I will just mark the value 0.5 sigma minus ok, but this is the line I need to mark here, I will shade the area, now I need this value..

I will just do it taking some problems taking some numerical problems. Now, that means we are looking for the probability for the value of Z or Z that is greater than minus 0.5. And we need to draw the vertical line at this, like I have drawn this, this vertical line here, vertical line at this, vertical line has to be drawn here.

This vertical line, I will draw the vertical line here at the point, where the value is required and standard deviations from mean and we shade everything that is greater than this number that is we need this area, whatever the shaded area is whatever the shaded area is we need this value what is the this for area, this area would be the probability.

Now, we need to visit the normal probability distribution index, and find a picture that looks like our graph. Like we had this picture before, yes this is a kind of the picture that we had in our graph the only difference is that the shaded areas on the left side, we can always take it. If it is minus 0.5, we can take the value at plus 0.5 as well. So, at plus 0.5 the value is 69.15 this is the value 69.15 (Refer Time: 17:28) double circles over it. So, this value we will see, so this for P greater than for P of Z greater than equal to minus 0.5. The value is 0.69 something equals 6915 0.6915 ok, this is the way..

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Now, let us try to solve a numerical problem. Now, the question tells that we measured the length of an element. The measurement was normally distributed normally distributed with a mean seven 27 mm, and the standard deviation of 112 mm. What is the probability of measurement being above 500?

Now, a test note on the data mean is equal to 527 and standard deviation is equal to 112. So, if I try to draw the curve for this, this is 527 this is length in mm length in millimeters 527 here. Now, I just draw a bell shaped curve here ok, which is symmetrical or the central value here. So, he needs the value that is above 500. Now, X is equal to 500 here, we have above 500, 500 means somewhere here, if we have 500, so can we find this value. Let me shade this area here. I need this value ok.

Now, the way to do this is we know that standard normal deviate Z or Z is equal to X minus mu X minus mu by sigma, which is equal to 500 minus 527 by 112. So, this comes down to minus 27 by 12, which is equal to minus 0.2411 ok. Now, probability for the value of the random variable X greater than it is not saying it an equal to just greater than 500, this is same as probability for the value of Z greater than what, I need to see the value of Z here minus 0.24, which is equal to now where do you find this probability, we need to see the probability distribution table or the probability distribution area that we have.

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z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0000	0040	0080	0120	0160	0199	0239	0279	0319	0359
0.1	0398	0438	0478	0517	0557	0596	0636	0675	0714	0753
0.2	0793	0833	0871	0910	0948	0987	1026	1064	1103	1141
0.3	1179	1217	1255	1293	1331	1368	1406	1443	1480	1517
0.4	1554	1591	1628	1664	1700	1736	1772	1808	1844	1879
0.5	1915	1950	1985	2019	2054	2088	2123	2157	2190	2224
0.6	2257	2291	2324	2357	2389	2422	2454	2486	2517	2549
0.7	2580	2611	2642	2673	2704	2734	2764	2794	2823	2852
0.8	2881	2910	2939	2967	2995	3023	3051	3078	3106	3133
0.9	3159	3186	3212	3238	3264	3289	3315	3340	3365	3389
1.0	3413	3438	3461	3485	3508	3531	3554	3577	3599	3621
1.1	3643	3665	3686	3708	3729	3749	3770	3790	3810	3830
1.2	3849	3869	3888	3907	3925	3944	3962	3980	3997	4015
1.3	4032	4049	4066	4082	4099	4115	4131	4147	4162	4177
1.4	4192	4207	4222	4236	4251	4265	4279	4292	4306	4319
1.5	4332	4345	4357	4370	4382	4394	4406	4418	4429	4441
1.6	4452	4463	4474	4484	4495	4505	4515	4525	4535	4545
1.7	4554	4564	4573	4582	4591	4599	4608	4616	4625	4633
1.8	4641	4649	4656	4664	4671	4678	4686	4693	4699	4706
1.9	4713	4719	4726	4732	4738	4744	4750	4756	4761	4767
2.0	4772	4778	4783	4788	4793	4798	4803	4808	4812	4817
2.1	4821	4826	4830	4834	4838	4842	4846	4850	4854	4857
2.2	4861	4864	4868	4871	4875	4878	4881	4884	4887	4890
2.3	4893	4896	4898	4901	4904	4906	4909	4911	4913	4916
2.4	4918	4920	4922	4925	4927	4929	4931	4932	4934	4936
2.5	4938	4940	4941	4943	4945	4946	4948	4949	4951	4952
2.6	4953	4955	4956	4957	4959	4960	4961	4962	4963	4964
2.7	4965	4966	4967	4968	4969	4970	4971	4972	4973	4974
2.8	4974	4975	4976	4977	4977	4978	4979	4979	4980	4981
2.9	4981	4982	4982	4983	4984	4984	4985	4985	4986	4986
3.0	4987	4987	4987	4988	4988	4989	4989	4989	4990	4990
3.1	4990	4990	4991	4991	4991	4991	4992	4992	4992	4992
3.2	4993	4993	4993	4993	4993	4994	4994	4994	4994	4994
3.3	4995	4995	4995	4995	4995	4996	4996	4996	4996	4996
3.4	4996	4996	4996	4997	4997	4997	4997	4997	4997	4997
3.5	4997	4997	4997	4997	4998	4998	4998	4998	4998	4998
3.6	4998	4998	4998	4998	4998	4998	4998	4998	4998	4998
3.7	4998	4999	4999	4999	4999	4999	4999	4999	4999	4999
3.8	4999	4999	4999	4999	4999	4999	4999	4999	4999	4999
3.9	4999	4999	4999	4999	4999	4999	4999	4999	4999	4999

Let me go to my distribution table here. This is the probability distribution table, you can note one thing, I have intentionally put this table here for value for the 1.48 the value that we selected was 9.43, here value is 0.43 0.4306. So, I have not put the curve here, which kind of table is it. So, I will draw two curves please let me know, which kind of table this

is it this table or this table ok. Please take a few seconds to see whether case 1 or case 2, which kind of table is this. Yes, I think you got it right, it is not the case 1, it is the case 2.

So, this is not selected, this is selected, because the value is the only on the right hand side from 0 ok. It was 0.9306, it was 0.9306 had it been this table this curve here. So, it is case 2 here. So, we need the value of minus 0.24 0.24 let me see the value 0.2. 0.2 is here and 0.2, and 4 ok. So, the I have got the value 0.948 this value is here 0.948.

But, what do we need in a question, we need this complete area. Now, this area now from 500 to 527 actually from 500 to 527 the area is so this is 0.0948. So, the probability for X greater than 500 will be equal to the other half of the area that is 0.5 plus this 0.0948. So, this is equal to 0.5948 ok. So, this probability for the measurement being above 500 is equal to 0.5948 that is if I say percent it is 59.48 percent of chances for the value to be above 500 mm, this is my inference here ok.

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Numerical Problem

Question: The length of a horizontal bar is normally distributed with mean 266 mm and a standard deviation of 16 mm. What proportion of all measurements will last between 240 and 270 mm?

$\mu = 266 \text{ mm}$
 $\sigma = 16 \text{ mm}$
 $Z = \frac{X - \mu}{\sigma}$
 $Z = \frac{240 - 266}{16} = -1.625$
 $Z = \frac{270 - 266}{16} = +0.25$

$P(240 < X < 270) = P(-1.63 < Z < 0.25)$
 $= P(Z < 0.25) - P(Z < -1.63)$
 $= (0.5 + 0.0987) - (0.5 - 0.4494)$
 $= 0.5987 - 0.0516$
 $= 0.5471$

54.7% of chances that the values lie between 240 mm and 270 mm

So, let me take another question here. A length of horizontal bar is normally distributed with mean 266 mm and standard deviation of 16 mm. What proportion of the measurement will last between 240 and 270 mm. So, I just picked one value greater than 500 in the first case, just one value and try to solve that. Now, I am taking an interval 240 to 270, but within this interval, how to calculate.

Now, I will follow the same steps, first put out put the data here, the mu is mean is 266 mm, and value of standard deviation is 16 mm. So, let me draw the normal curve here with mean is equal to 266, we need to see the value between 240 and 270 240 would be little this side values between 240 and 260. Now, again the value of normal deviate Z is equal to 240 minus 266, I am using the same formula $X - \mu$ by σ Z is equal to by 16, which is equal to minus 1.625 the value of normal deviate. And for value of Z for 270, it is 270 minus 266 by 16, 4 by 16 is 0.25 ok, this is plus.

So, I need the probability from 240 is less than x is less than 270, which is equal to probability for the value of the normal deviate from minus 1.6, I have valuable up to two places of decimal, so I will round it off to 1.63 is less than Z is less than 0.25 ok. So, there are ways to deal with this, one way is I calculate this area separately, and this area separately, and then sum them up. For that, I would have to separately see the values for 1.63 and 0.25 like considering that video (Refer Time: 28:12) only one side is required, and sum them up ok.

The another way here is I will just put this as in a line here, I will put this in a line, and see that this probability is 0, and this probability is 0.25, and this probability is minus 1.63 ok. What will that make me to do, I am just interested in this value this value, so what can I do. If I subtract from this value 0.25 the value is minus 1.63, I can get this thing. So, one way to do this is, I can take it as this is equal to probability for Z less than 0.25 minus probability for Z less than minus 1.63. So, this whole area is subtracted from this, you got my point.

I am having this curve here, I will just I just need this area. So, what I will do, I will pick this whole area on the left hand side, this whole area left hand side that is from 0.25, and I will subtract this area from it. From the whole red area from this whole red area ok, I am subtracting this blue area ok, this is one way to do it. So, let me follow this way.

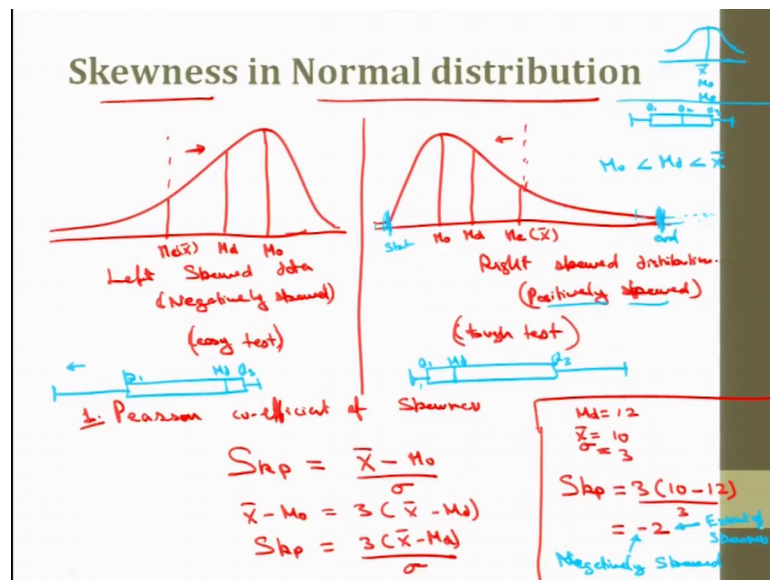
So, here the value of Z at 0.25, I need to see the value and 1.63 0.25, the value is 0.0987, so 0.5 would be added here, this plus 0.5 in my case, because I need the whole probability. Then for minus 1.63 1.6 is here, 1.6 and 1.63, it is 4484. And I need the other side of value, I will actually subtract 0.5 from this 4484 and zero point nine eight seven 0.0987.

So, this is equal to I will put it in a bracket separately 0.5 plus 0.0987 minus, so this value next value is 0.4484, but this would be subtracted from 0.5. So, I think you can now make it clear that why I am adding 0.5 to this, because I need in this case 2 here like now case 2 actually figure number 2, I have this is figure number 1 ok, this is figure number 2.

In figure number 2 here the red area red shaded area, we need to add 0.5 here; we need to add this 0.5 here in the red area. And for the blue shaded area, only this area is required blue shaded one, I need to subtract this subtract the value that I have got from 0.5. So, this is 0.5987 minus 0.6150 0.0516, this is equal to 1745 0.5. Now, this means that 54.7 percent of chances that the values lie between 240 mm and 270 mm.

So, I think with these two examples, the normal distribution how do we use normal distribution, and the normal distribution is actually population distribution. How from the information of the sample, we can calculate the probabilities, we can calculate the 12 points at between which our values lie or above or below which our value or measurement could lie ok.

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So, next I will try to discuss this skewness in normal distribution. Sometimes, as I discuss (Refer Time: 33:42) distributions had been telling that a distribution is skewed towards right. What is this skewness, they can be two kinds of skewness is here, data can be left skewed or right skewed. Generally, I have found in my in my past that the

students generally consider the peakness of the curve as the skewness, they think that the curve is peaked to other rights, they call it right skewed generally this (Refer Time: 34:13) but, the skewness is attributed to the tail of the curve the tail.

If the tail is towards left, this is known as left skewed left skewed data or data or distribution whatever you call. This is right skewed distribution ok. So, in left skewed, data this is also known as negatively skewed negatively skewed; this is positively skewed data. So, the curves that we have been discussing log-normal, Poisson, those were a right skewed most of the them.

So, what happens what does this mean, it implies this is actually peak. Peak means this is mode, I call mode as M_o , mode remains at the peak mode is the maximum frequency of some value mode remains at the peak. And with the mode, and I will first talk about the right skewed distribution, I have median and mean here; this is median, and this is I will call it M_e mean just giving a notation here, median I will call it M_e mean ok.

So, an example here can be put as like for instance if I give tests to the students, if the test is too easy, if the test it too easy, most of the students would be getting the marks higher. So, the data would be because let us consider would it be a left skewed or right skewed, most of our students would be getting marks higher more than the more than the what is average ok, data would be left skewed in that case.

So, this is easy test, and this is a tough test. Tough test means this is not mean, and most of the students are having marks less than mean less than this mean, and most of students having marks greater than this mean in this direction, in this direction. This can be an example to just to explain this skewness ok.

Skewness can also or skewness or the coefficient of skewness can also be quantified that can be calculated using some relation, the very commonly used a method here is Pearson coefficient of skewness Pearson coefficient of skewness. This is also known as mode coefficient of a Pearson mode skewness.

Pearson mode skewness implies that skewness coefficient we call it, I will just denote it skewness S_k and I will put p here as Pearson coefficient of skewness. This is actually equal to the difference from the mode the difference from the mode of mean actually I

put mean M here, I can put \bar{X} here as well, this is mean \bar{X} , this (Refer Time: 38:08) best standard deviation.

This can also be taken to the alternative formula can be the median, or a Pearson median skewness, Pearson mode skewness, because we know that \bar{X} minus mode is generally taken as 3 times of \bar{X} minus median. So, this skewness coefficient can be written as 3 times of \bar{X} minus median by σ . So, I am more interested in median, because we are talking about the values, those lies to left or write in my curve. So, the central tendency or the location point here is the median, I can also (Refer Time: 38:56) so will just try to draw the box and whisker plot as well to explain this skewness further.

First, let me try to put the various methods to calculate the skewness. Go for the box and whisker plot, I can see the values here, here actually mode is less than median is less than mean. So, if I draw a box and whisker plot here, actually in normal distribution in a regular normal distribution, we have all these values aligning at this point. It is \bar{x} , and mode and median in a symmetrical normal distribution. And the box plot for this kind of distribution would be very symmetrical ok, this will be very symmetrical like this. So, this is my Q_1 quartile 1, quartile 2 and quartile 3.

But, in this case because mode is less than median is less than \bar{X} , the data we can see that most of the data would be lying towards the right side ok. So, here it would be somewhat like this, so that means, it is positively skewed. And most of a data is on this side, and this is my median, this is my median, this is my Q_1 , and this is my Q_3 . And in this case, it is the other way round. In this case, this is my Q_1 , this is my median, and this is my Q_3 .

We can also see that this is a long whisker here on the right hand side, because this line is some (Refer Time: 41:04) of point will be at very far state here, and the points are near here, the whisker is very small here. In this case in case of positively skewed data, the whisker here at this point some like the end point would be somewhat here ok. So, I can call it start and some. So, this is a long whisker. In this case, the long whisker is on this direction. So, this is another way to represent this skewness. So, Pearson coefficient of skewness is one criteria, there are some other (Refer Time: 41:37).

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Skewness in Normal distribution

2. Quartile coefficient of Skewness.

$$Sk_q = \frac{(Q_3 - M_d) - (M_d - Q_1)}{Q_3 - Q_1}$$

3. Percentile Skewness coefficient.

$$Sk_p = \frac{(P_{95} - M_d) - (M_d - P_5)}{P_{95} - P_5}$$

So, like quartile or quartile coefficient based upon this also, I can put a skewness coefficient here, then we have quartile coefficient quartile coefficient of skewness. Quartile coefficient of skewness, I can say S skewness Q 2 quartiles, this is equal to Q 3 minus median minus median minus Q 1 over Q 3 minus Q 1.

Now, when we will have the skewness coefficient, also we can have the percentile the skewness coefficient percentile skewness coefficient. Then skewness percentile can be something if I take let me say a from 90 percent 95 percent, it can be P 95 minus median minus median minus 5 5th percentile ok, then 95th percentile minus 5th percentile. I can pick any percentile rather any quartile to find the coefficient of skewness.

The thing is that I just need a coefficient; the coefficient would have a sign and a magnitude. Sign would represent whether it is negative or positively skewed. And the amplitude with represent the extent of skewness, how it is skewed is it highly skewed, or it is just a moderately skewed, or very lightly skewed that can be determined ok. Let me just pick some values, and try to calculate this coefficient of skewness.

I will just do some calculations here. Let me pick the value of standard deviation, mode, and X bar ok. Let me say if X bar I will X bar is let me say 50 or let me to make it simple, I will say just X bar is equal to 10 and mode or I will better so, put median here, median is equal to 12, and standard deviation is equal to 3.

Now, skewness coefficient here would be coefficient of skewness or Pearson coefficient of skewness, Pearson mode coefficient skewness would be 3 times of \bar{X} minus median divided by standard deviation. So, this value comes out to be minus 2. Now, they are two values here, minus 2 implies it is negatively skewed negatively ok. And this is extent of skewness. Negatively skewed like I said in negatively skewed, here we have mean or \bar{X} less than median, and less than mode, we can see that \bar{X} was less than median in this case, so it is negatively skewed.

This amplitude of skewness there is there are this different ah acceptability level for this, amplitude of skewness 2 is actually too high. So, sometimes depending upon the application, we can (Refer Time: 46:03) 2 as well sometimes. But, in case of normal distribution, the general value is less than 1 ok, but depending upon the kind of data we have, and the kind of distribution we have selecting for that.

And we can such have the feel of the data whether how is it is skewed, to what extent the skewness has happened. So, we can also see this, we can just find the coefficient of skewness. So, we will meet in the next lecture, we will discuss the statistical parts in metrology further.

Thank you.