Engineering Metrology
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Lecture - 37
Statistics in Metrology, an introduction (Part 2 of 2)

Welcome back. So, we will continue the lecture in which we will discussing about measurements. So, we discussed about the measurement classification.

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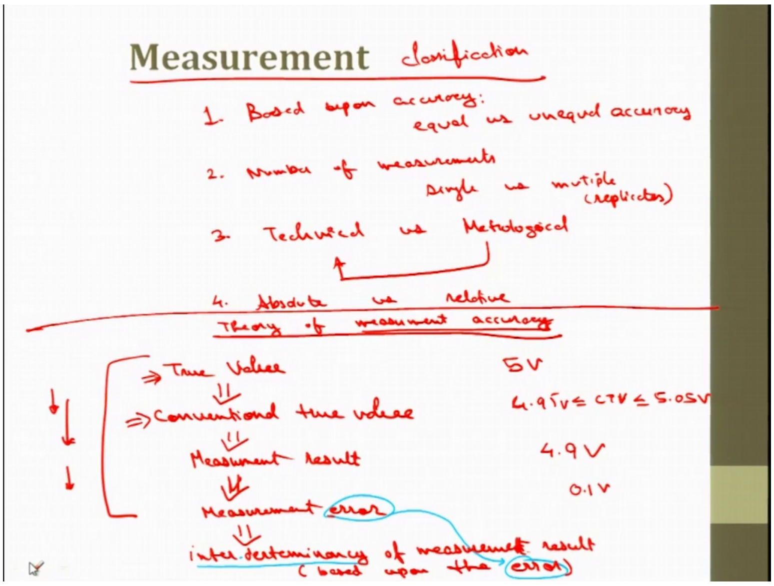

Recently and theory of measurement accuracy, how the theory is developed, how do we do the calculations; so, what are the various ways that the propagation of the calculations goes. So, it goes like this as we discussed true value conventional true value is obtained the measurement results. I have some error. So, error that is used to have inter determinancy or to obtain some inference so, there is a inter determinancy of measurement results based upon errors which gives us. Some interpretation about the results that are to what extent are our results correct.

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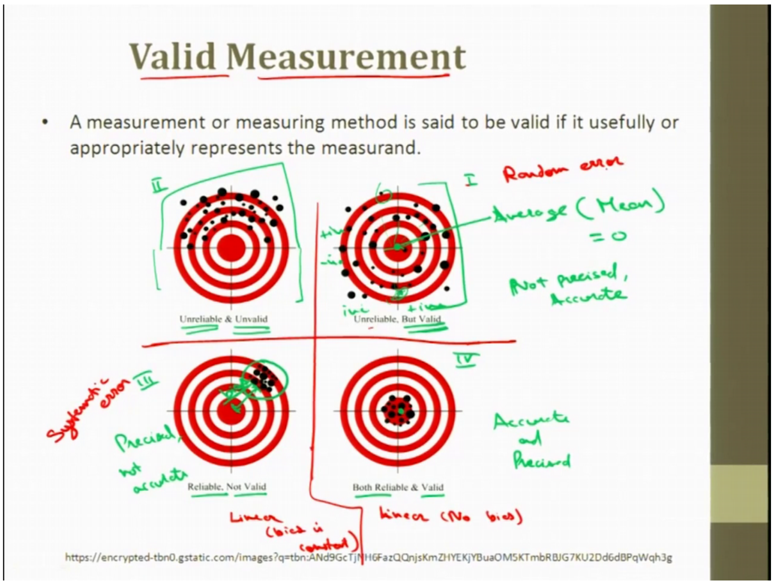

So, next is valid measurement. It is very important that the measurement is valid. So, this can be compared with a accuracy and precision like as been discussed by Doctor Ramkumar before. The data or the readings or the observations that we have gained or that we have obtained are to be valid to work on them. See the to major kinds of errors we will discuss upon on that as well, systematic error and random errors. I will first discuss before that what is a valid measurement.

We can see in this matrix there are 4 measurements. The perfect result should have been when it hit exactly at the centre. This is the perfect result, exactly at the centre I am at quadrant number 4. In the first quadrant, you can see this is the quarter number 1, 2, 3 and 4. In the first quadrant you can see the results are all scattered. So, it is not reliable; whatever the instrument we are using or whatever the observations we have taken those are not reliable, because those are very randomly scattered. So, it is a scatter which is a random, but it is within our acceptable it is within this target board. So, that is why this is valid ok.

So, in quadrant number 2, it is unreliable and un valid. See the thing is that you know if I take average of all this dots, all this (Refer Time: 03:00) distance is from the centre; that means, a distance is this distance is a b c d so on, if I take average of these distances, or distance from the centre ok, the final result would be here. This is average, average or in statistical term we call it a mean, the distance from centre. So, what would happen? You know what will happen if I take it as a positive direction, this as negative direction and so on from here to positive and negative. You know, this distance let me say this point would cancel almost this point, ok. This point would cancel this point approximately.

So, overall that average would be if I take positive negative the average mean would be 0. 0 means exactly at centre. That why this data is this kind of information is somewhat valid; however, in this it is not reliable in part 2, in quadrant 2 it is not reliable not valid. But in this part quadrant number 3, it is reliable. Because the data is too close, and in quadrant number 4 it is both reliable and valid. It is at the centre and also the all the observations are close.

Now, if I talk about precision and accuracy, you can very all determine or very well look into this based upon that, this is accurate. This is precised accurate and precised actually; this is precised and not accurate. So, this is not precised, but this is accurate. So, this is the actual definition accuracy means when we talk about accuracy we need to trace the overall data what is the main; if the scatter is there we can work on that. So, this is the kind of a random error. We are working on the random errors, the statistics whatever we will work on we will work on the random errors not a systematic error. This is in quadrant number 3, the kind of error that is there is a systematic error.

Systematic error whatever the values are there all on the right hand side. If someone is targeting this thing it has to just come below 45 degree towards left and it will come to the centre. So, this distance is our bias. But here we cannot work much in quadrant number 2, ok. This is not there these are actuate not précised ok.

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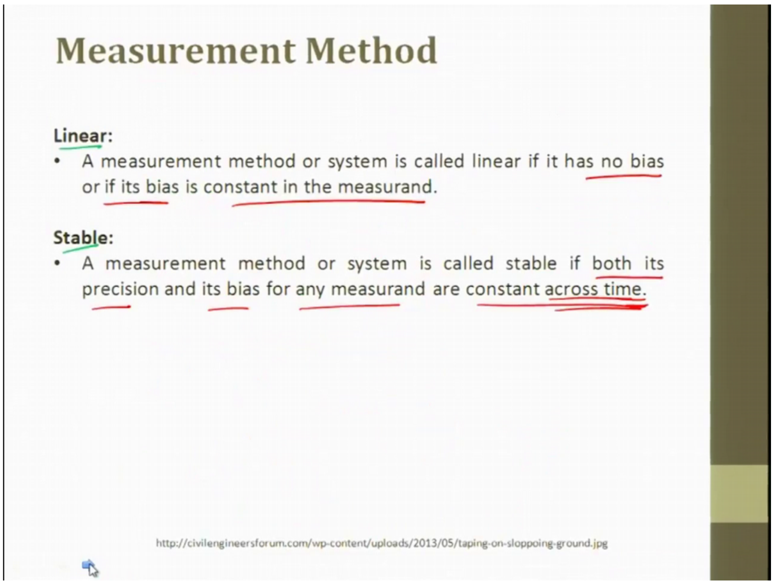

So, the data has to be valid. Next is another measurement method, I could have put in the various classification of measurement. This measurement is linear versus stable or linear not is not versus linear and stable are to different kinds of measurements. Linear measurement method is the measurement method of system is called linear, if it has no bias or if it is bias is constant in the measurand.

Next is stable a measurement method of system is called stable if both it is precision and it is bias for any measurand are constant across time, across time it is constant. So, linear and stable, could you please see which is the linear measurement here? You see the definition again; it has no bias, or if it is bias is constant in the measurand. I think this is linear and this is linear. Here it has no bias, here the bias is constant. So, mostly we will be working on this kind of data, the data which is unreliable, but valid. So, this kind of data is obtained that is in quadrant one, mostly and we work on this to bring some statistical inference.

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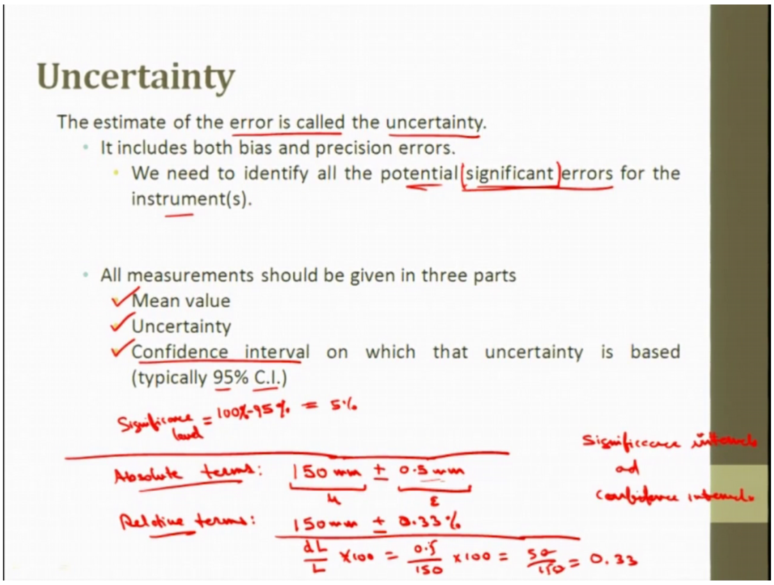

Next is uncertainty; uncertainty is equivalents to or similar to the errors. When we talk about errors in measurements, those are equal in statistics we call it uncertainty, or we can also call it error (Refer Time: 08:29) I can use this interchangeably here; however, there is a difference between errors and a certainty uncertainties. Sometimes errors is the term that is also used for the mistakes there were made. But in this context in this specific module in which we were working, when we will talk about error, the errors are not mistakes, errors is just uncertainty that the results are not certain and we just need to see what is the reason for that. Could we correct that somehow; we will try to see that.

So, uncertainty it is the estimate of the error. Estimate of the error is known as uncertainty. So, it includes both bias and precision errors. We need to identify all the potential significant errors for the instruments. All the potential significant errors, the word significant is being used again and again here. I am also saying the word what is significant, significant little meaning of significant is, is it having some influence? Is it having some effect? Is it bringing some change, actually is it significant or not. If the errors are significant, we need to work on that, if there were not significant we might ignore that. Or they might be some error which are not due to the instruments, those are called noise we have discussed that we have signal to noise ratio all those things. Those are the external factor the environmental factors, that we cannot control the factors which we can control within our environment, within our experimental setup, those are the sound factors sound to noise ratio is taken, those are the factors those are the errors which will work on.

So, uncertainty is the estimate of the error and it is to be identified for all the potential significant errors for the instruments. All the measurements should be given in 3 parts. Number 1, mean value; number 2 uncertainty number is a confidence interval on which this uncertainty is based. We will discuss about significance and confidence intervals.

So, a statistics there are certain assumptions. So, here 95 percent confidence interval means, the significance level is 100 minus 100 plus this actually minus 5 percent is equal to 500 minus 95 percent is equal to 5 percent. This is significance level. So, the broader meaning the exact meaning we will discuss when I will discuss the probability distribution and specifically normal distribution I will talk about the confidence intervals, significance levels and others parameter of the statistic that are important.

So, broadly confidence interval is the confidence or the assurance that whatever we are telling, that this is the main and this is our uncertainty, we are telling this with 95 percent confidence 100 percent confidence is never there. This is the characteristic of uncertainty or this is the characteristic of statistics as well that we have such as confidence level. In mechanical measurements 99 per 99 percent confidence level could also be there. And in a such an maybe micro manufacturing on and manufacturing or nano manufacturing sometime the confidence intervals are much greater it is 99.997 those are known as 6 sigma we will discuss those.

So, we have to tell that what is our confidence interval in which we are working overall. So, if you are 95 percent confidence, if you are 95 percent confident, what is the influence? If you are 90 percent confident what is the influence, if you are 99 percent what is the influence. So, what we can say sometime it happens that it is significant at 90 percent, but not 99 percent. 90 percent confidence interval we are 990 percent confident that the data is significant. At 99 percent it cannot be, we will discuss this when we will draw the normal curve.

So, uncertainty has certain characteristics it can be given in absolute terms absolute terms per relative terms. So, it can be expressed in both the ways absolute terms means, uncertainty it is actually the mean value plus something is that something that we adds to mean value add or subtract to mean value is uncertainty. So, this uncertainty by the sense if my value. So, it is influence the height IG, I calculate the height of some instrument let me say height of this pen ok. This is about 150 mm ok; this is actually stylus. So, this is about 150 mm. So, it can be plus minus something it is let me say 150 mm plus or minus 0.5 mm.

So, this is the mean value. This is our uncertainty I put epsilon. In relative terms, what could be relative terms relatively it could be the I could pick percentage here. So, it can be 150 mm plus minus. So, what is 0.5 of 150? So, this is actually delta L by L. That is length is this is L by this here. I am actually calculating delta L by L into 100 this is percentage delta L is 0.5 by 150 into 100 is come from to 50 by 150 and I cross this is equal to 0.33, plus minus 0.33 percent.

So, you can see in absolute terms, the uncertainty is in the same units in relative terms it is percentage or it might be some other quantile or quartile of this something like that, ok.

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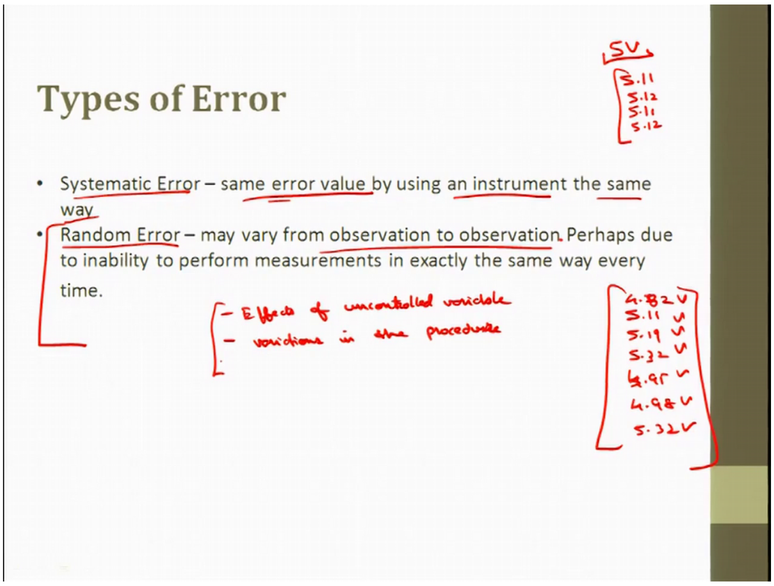

So, types of error just to recall the 2 major types of error systematic error and random error in systematic errors, same error value by using an instrument the same way. Same error value comes like this was the bias which I just showed, this is a systematic error ok, and this is the random error.

So, here we are telling it is same error value by using an instrument in the same way. So, why did? The thing is that if it is provided that all other conditions remains same. The instrument is the same, maybe the experimental is the same, and the environmental conditions are same. So, it is if the error value here the error value is same ok. So, error value is same at whatever error value is so, instance 5 volt has to be final true value if results of 5.1, 5.1, 5.1, 5.12, 11 so on.

So, this is a systematic error ok. Random error, it may vary from observation to observation full. Perhaps due to inability to perform measurements in exactly the same way every time this error might be due to the change in environment, due to the change in instrument, due to the change in the condition due to climatic condition, due to the change in an experiment, there might be any change, but the thing is that the error is random, it is scattered. So, it is something like that, it is something like 5.1 5.11 ok, 5.19, 5.32 ok. We can have 4.82. May be you can be 4.95, 4.98, 5.32, this is random error, ok. This is this is the voltage.

So, the error that causes readings to take random like values about mean value is known as random error. These are the effects of uncontrolled variables ok, or the variations in the procedure. There might be certain reasons like this.

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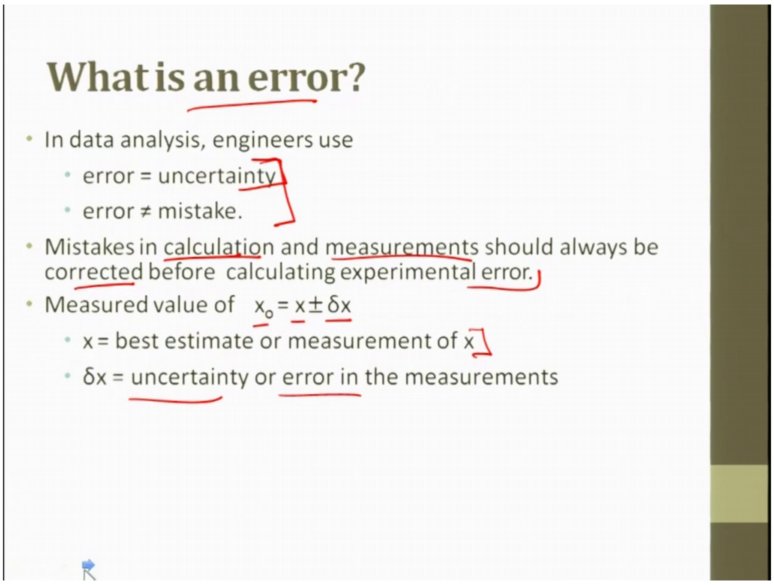

So, next again what is an error, what is an error is the thing which I just said before; that error is equal to uncertainty it is not equal to the mistake. The mistake is sometime like something an ignorance, carelessness or environmental conditions are different some experiment using, one instrument another experiment using, second instrument for instance you might be using vernier calliper of one company for taking one reading, and vernier calliper which is having different type of scale for taking another reading. These are a some kind of I can say these are kind of some kind of carelessness, if not exactly mistakes.

So, if we are going to work if we are going to generate data on which we have to apply statistical analysis, it is important that the data is as accurate as possible. So, errors is equal to uncertainty that we are working on, mistakes in calculation or in measurements as I just said should always be corrected before calculating the experimental error. So, measured value is equal to the mean value plus error ok. This mean value is the best estimate of the measurement of x, and delta x is the uncertainty or error in the measurements.

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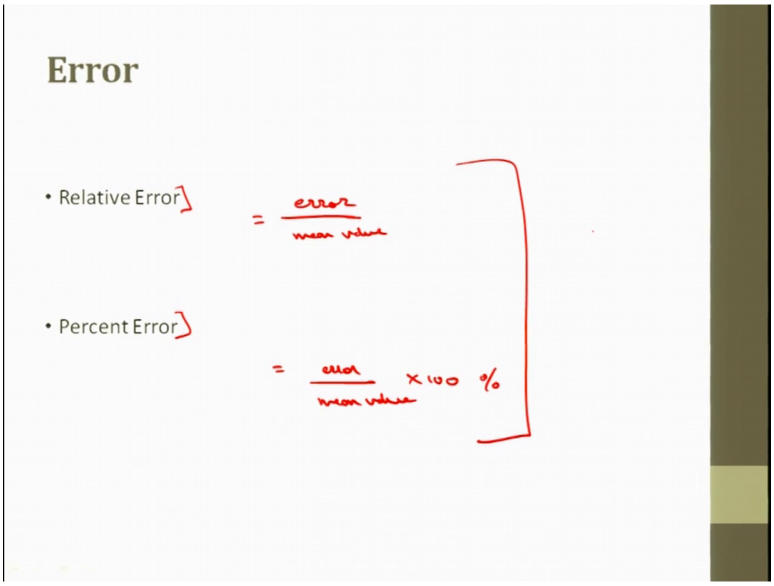

So, the error can be relative or percent error we are discussed other types of errors as well, like we discussed the systematic or random error we have we will be mostly working on the random error here only. Systematic error might be due to the faulty calibration, the incorrect calibration or incorrect use of instrument change in condition for instance temperature rise may be the change of experiment and all those things. So, random error is due to the statistical variation, statistical variation which we work on.

So, errors can be represented in these ways, relative error, percent error, the relative error is (Refer Time: 21:44) I just calculated is the error per unit mean value in percent error is the same relative error into 100. It is error per unit mean value into 100, this is actually percentage. So, what we have actually do not have the true value, we just know that is some true value, and what we are generating the data which we were generating is the estimate; is the estimate of the true value. So, those that estimate we need to work on.

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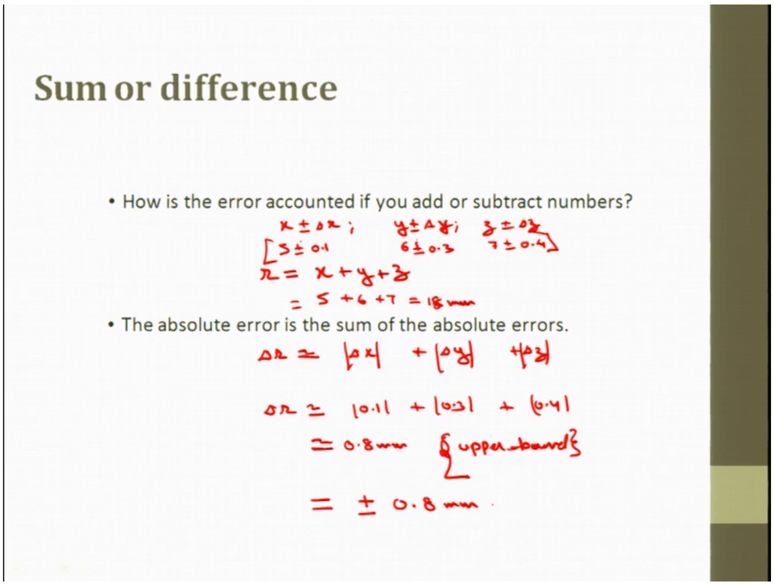

So, error has such an properties the sum or difference for instance if there are 3 values for which the error is such as, for x the error is delta x, for y the error is delta y, and for z the error is delta z. So, what happens? When the final outcome that is the I will put the resultant here, the resultant is equal to x plus y plus z. What you think? How would the errors behave in this case? This is the property that we need to see that what could be the maximum error. You know, there is a plus minus sign with delta x, with delta y and delta z.

So, to see the maximum value to see the upper bound, the errors are always added and moreover the absolute values are added. So, I can say delta r for the overall error, here would be equivalent to I am not putting equal to it is equivalent to delta x plus delta y plus delta z modulus. Even if the value of r, let me say if the value is 5 plus minus 0.16 plus minus 0.3, it is 7 plus minus 0.4, if these are the values, and the value of r would be the mean value would be 5 plus 6 plus 7, this equal to 18.

You call it mm whatever you like to call you call it some length sometimes ok, the temperature we do not had. It is a continuous I will discuss what is different kinds of scales as well. You know, I just kind of need an error or a mistake while calling it a temperature we never add the temperatures. Temperature is not in a ratio scale, it is in a interval scale.

We cannot say that if the temperature of one body is maybe 25 degree another body is 10 degree if we keep them together the temperature will be 25, plus 10 it would be 35 degrees. So, it is not possible, it is in an interval scale, but the length; that means, if it is the height, yes, height if we keep on adding them yes that can be added ok.

So, what can be the maximum error? Maximum error here would be 0.1 plus 0.3 plus 0.4; which is equal to 0.8 mm. Please remember this is an upper bound, ok. So, this is delta r, if I need to make it more clear, I can even put it as, I put it equivalent to you put it as equal to plus minus 0.8 mm.

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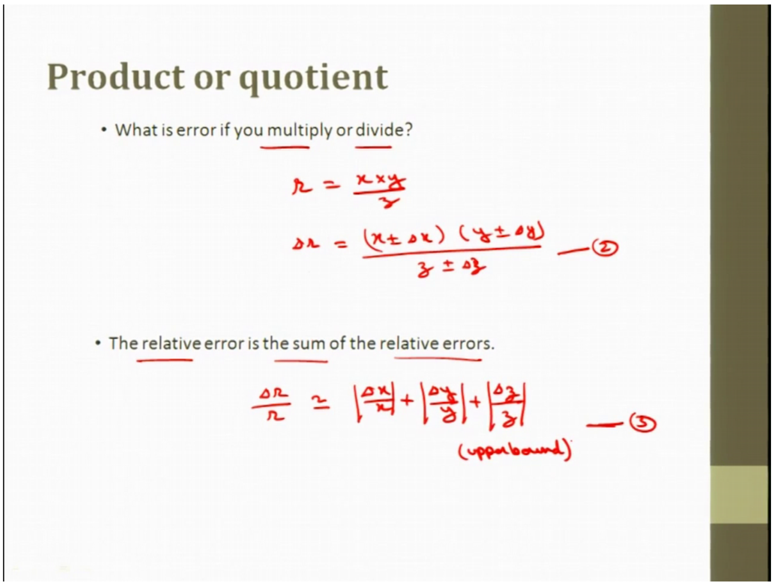

So, next is product quotient, what if you multiply or divide the error term. I consider the same values, x plus delta x y plus delta y z plus delta z. So, the resultant if it is x into y by z. So, here the delta r could be equal to x plus minus delta x or y plus minus delta y over z plus minus delta z. If I talk about the relative error, that is the sum of the relative errors again. So, relative error is what? Delta r by r, this is again the sum of the relative error in the similar fashion like this, I call it equation 1. This is equation 2 equation 3 would be delta x by x delta y by y delta z by z, absolute values and sum of this.

This is again our upper bound. So, what if we multiply the errors with a constant?

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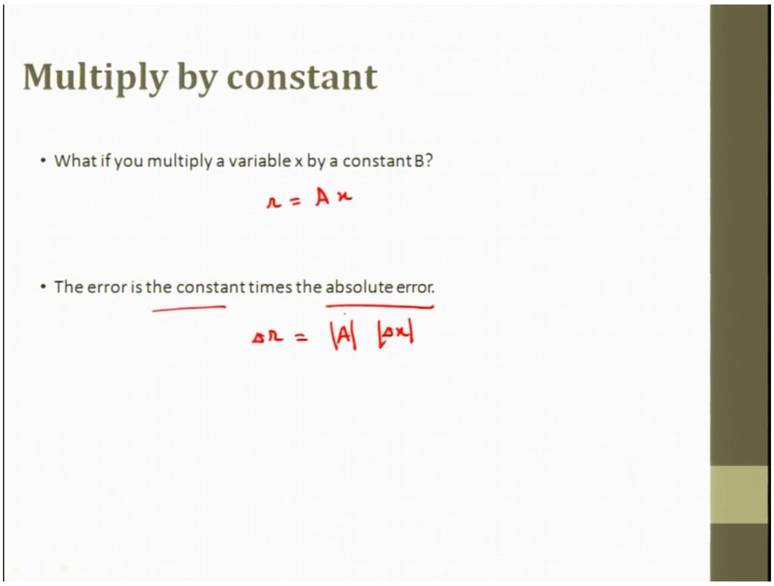

If result is equal to sum constant value A into x, the error of the, is the constant times the absolute error. So, then the absolute error here is equal to a times delta x, is it ok? Now when we talk about the error, errors are absolute terms mostly. So, we will put absolute value of the constant here as well, irrespective of it is original value.

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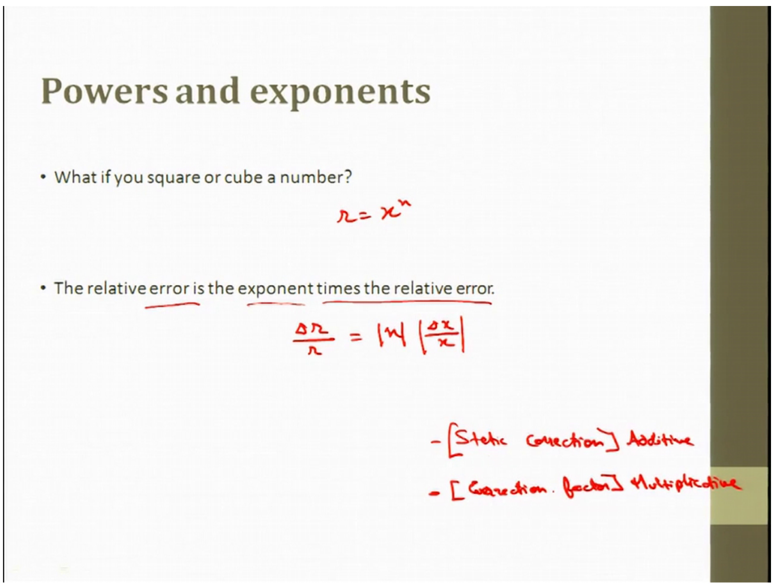

So, what if you square or cube or put some degree to a number, for instance if r is equal to x power n, the relative error is the exponents times the relative error.

So, this becomes, here we will just take the relative errors only. You know, we are working in a kind of a taking log of this. So, we will use delta r by r is equal to n times delta x by x absolute values. What we need to do? We need to calculate the error and we need to correct an instrument. So, this kind of correction that is inculcated to the instrument is known as static correction, static correction.

Or if it is just added or subtracted, it is additive in nature, additive; if it is multiplicative we call it correction factor, this is correction factor, most of the times instruments which we are using are linear in nature. So, we will use static correction, not correction factor ok. Correction factor is multiplied and static correction is added might be it includes the sign; it might be added or subtracted both ok.

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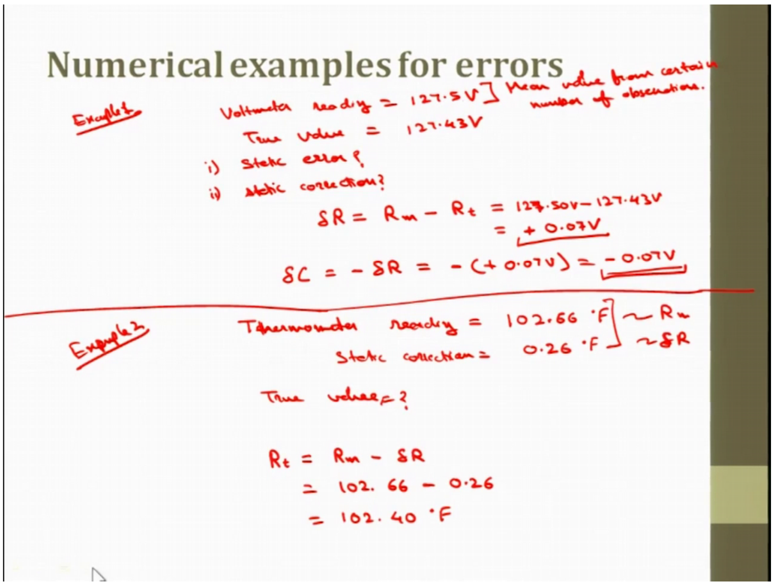

Next I will try to take some examples for instance if a meter reads a voltmeter, example 1 voltmeter reading is equal to 127.5 volt. And true value is equal to 127.43 volt. So, the question is number 1, what is static error; number 2, what is static correction. Please remember, static connection is for the instrument. What is the static correction that we should adds to the instrument that should be inculcated every time we use this instrument? So, sometimes the static correction or the correction, factor is given in the like in the software which I used to calculate the force which is applied during machining.

The force is converted into electrical signal, and that an electrical signal is electrical signals in the voltage, that voltage is then converted into Newton's. There the correction factor is given, that for this range in the software is equal to correction factor is given ok. For this range you please apply 1.5, one point whatever the correction factor is.

So, here it is static correction, because these are additive in nature, we need to find the static correction. So, what is the static error? Think it is very easy. So, if I take third reading or result as r ok, the delta R is equal to the reading that is from the instrument that is the mean value minus reading that is true value. So, this equal to 127.5 minus 127.43 I will put it 0 and volt and volt, this is equal to 0.07 volt.

So, this is a positive value and when we need to add the correction factor; the correction factor is whatever the value comes. So, when we need to work on the static correction static correction is subtracted from the mean value, because you know if it is a positive sign it is subtracted, you can see that voltmeter reading is greater than the true value.

So, here I have put positive sign here. So, the static correction which is denoted by delta; C is equal to minus times delta R which is equal to minus times plus of 0.07V is equal to minus 0.07V. So, the static correction value here is minus 0.007, and the static error is plus 0.07 ok. Whenever the voltmeter is used, this is actually the mean value; this is a mean value from certain number of observations.

So, we know the true value is there, the main value is this much. So, we took the difference to find the static error, in the static error the negative of the static error is f static correction, this was example 1. So, I will pick another example. So, let me take a temperature you consider thermometer reading. Let me take the thermometer reading is 102.66 degrees Fahrenheit. And the static correction is given, static correction is equal to 0.26 degree Fahrenheit.

So, we need to find the true value ok. So, what we have? This is our thermometer reading which is equal to R m this is equal to delta r. So, we can find the true value as, true value that is equal to question mark, now the true value R t written as a true value is equal to R m minus delta R is equal to 102.66 minus 0.26 is equal to 102.40 degree Fahrenheit. So, it can work in either way. So, multiple examples can be solved here. So, we will put some questions in the quiz as well. So, similar examples can be solved using this relation and relations for relative error, relation for absolute errors similar examples can solved, I will better more focus more on the theoretical part in this course.

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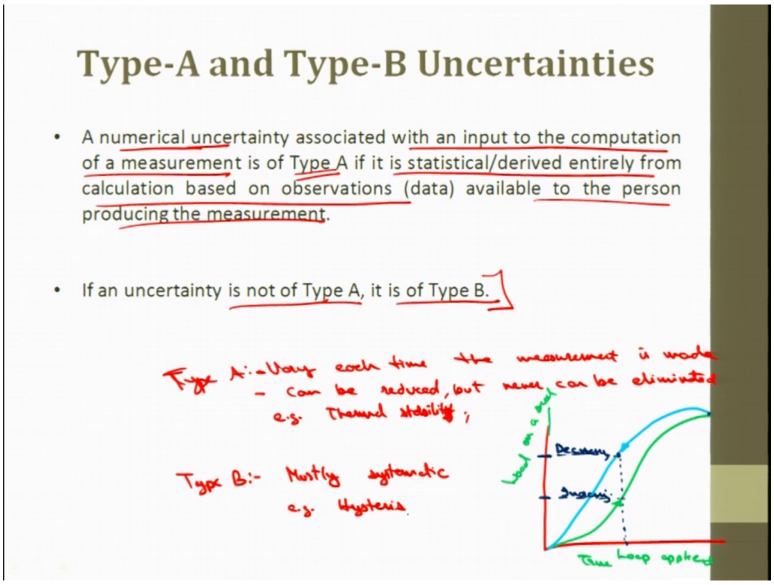

Next is Type-A and Type-B uncertainties. So, it is important to know what are the type-A and type-B. The type-A uncertainty is the numerical uncertainty that is associated with an input to the computation of a measurement. If it is derived from the calculation based upon the observation data available to the person producing the measurement.

So, type-A uncertainty is due to the experiments which we are conducting due to the data that is obtained based one certain instruments, if uncertainty is not type-A it is clearly mentioned it could be type-B; which we are not considering, which are not a factors which we are controlling, and we are not working much on type-B uncertainty; however, type-A uncertainty is important.

So, if the any statistical analysis has to be applied, the uncertainty has to be type-A uncertainty. Type-A uncertainty may be due to the instrument again due to the observer or due to the experimental condition experiments the way experiments are conducted. So, we are working on the type-A uncertainties. So, most random uncertainties are type-A uncertainties. So, they vary much time a measurement is made type-A; type-A they vary I am putting the characteristics, vary each time the measurement is made. So, the this uncertainty cab be reduced by average in lots of measurements, but it can never can be totally eliminated. So, this can be reduced, but never can be eliminated. So, it can never be eliminated, that is why we use the word confidence intervals, because 100 percent confidence would never come in random kind of errors.

So, the type-B uncertainties are the uncertainties which are not random which may or may not repeat each time. So, these are some time systematic, it is a kind of the systematic error only. So, these are systematic uncertainties. So, these are see both the type-A and type-B uncertainties can be reduced by having a better quality of the instrument.

Calibrations can a better calibration can quantify type-A uncertainty, and calibration can reduce the level of type-B uncertainty as well. But there will always be a small amount of uncertainty that would be left in the instruments. So, this are mostly type-B uncertainty are mostly systematic if not always.

A few examples where can be like thermal stability. If I pick a specific case, let me say there are many system both mechanical and electrical that can be sensitive to temperature, but for instance if I pick a strain gauge that depends upon the resistance of the metal foil, that makes the gauge the wire resistance depends upon the temperature. And if a gauge is used in a changing temperature environment, the output caused by the temperature change can wrongly be attributed to changing strain. So, the gauge the strain the wire we are talking about the instrument here.

So, here the effect of large temperature changes can be catered by using certain techniques. The techniques maybe temperature compensation so, there will be some residual non compensated effect. This kind of uncertainty is type-A uncertainty. So, the examples for the type-B uncertainty can be anything that we are not trying to control ok. I can put an example for hysteresis.

When we load or unload a spring when will load the spring; when the load is increasing the hysteresis is something like this. So, here actually the load is applied load applied. I will put the direction here it is moving in this direction ok. The load is applied and it is increasing like this, then load it is load indicated on the scale this is the load. On a scale it is true applied load, load applied (Refer Time: 43:22) applied.

So, when we unload this I will pick another colour for you to appreciate in a better way, when we unload this it becomes something like this. So, what happens the, this is increasing, this is decreasing the blue line is decreasing. If I put a line here, mechanical people would appreciate it in a better if I put a line here, and if I see the values at the true load applied.

So, the load increasing and decreasing these values are different, this is for decreasing and this is for increasing. So, this kind of error when the loaded or loading or unloading of spring when the measurement is taken at this point this kind of error is type-B error. This would always exist; this is not random this should always exist. And it is a kind of a systematic error. So, this is a an example of spring balance. So, type-A and type-B errors are the classifications.

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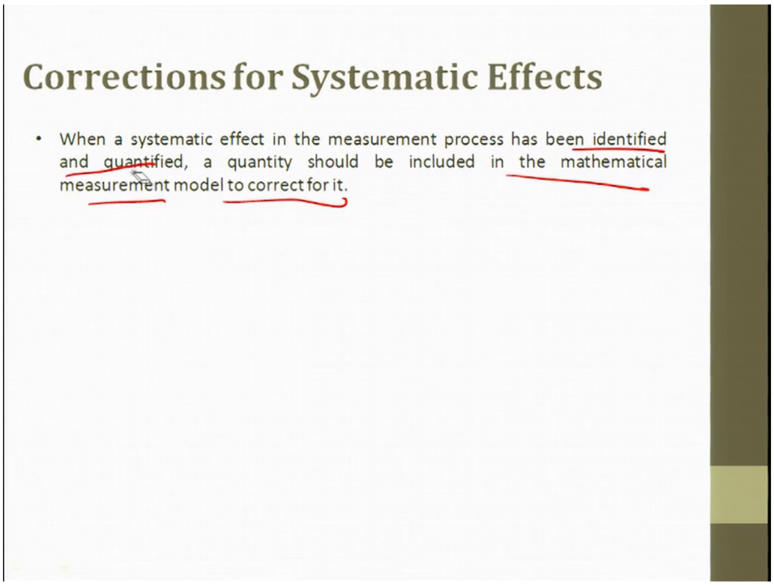

So, corrections of systematic effects; when the systematic effect in the measurement process has been identified and quantified, a quantity should be included in the mathematical measurement model to correct for it. So, let us meet in the next part of this lecture.

Thank you.