

Engineering Metrology
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Lecture - 24
Gears Metrology (Part 1 of 2)

Welcome back to the course on Metrology. So, the next topic of discussion is going to be on Gear.

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So, when somebody tells Mechanical Engineering. Immediately what comes to everybody's mind is only gears, because these gears have brought in big revolution in the industrial revolution days. So, in 1857 when industrial revolution happened or just before that so, what happened was people were trying to convert one RPM; that means, to say from a drive to another RPM, they wanted to increase decrease, they want increase the RPM, second thing they wanted to increase their torque. So, they were looking forward for several methods.

First they tried meshing to different wheels without teeth, then they are found out there is lot of slip. So, they wanted to stop the slip, then they had these tooth first initially they had only 3 or 4 tooth meshing with another wheel where they had 3 or 4 tooth. Then they

realise these 3 4 is not going to help us, then they started making all around the periphery teeth.

And, once they made all around the periphery of the wheel these teeth and then when they started meshing it they found out there is a positive drive. So, then people started realising and playing with the gear, gear profile, gear geometry, and then they realise that they can get many more outputs or they can modify to the gear design to meet out their requirements. So, gear plays a very very important role.

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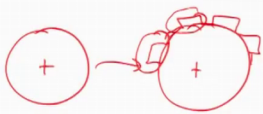


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•	Measurement of Gear Elements
•	Composite Method of Gear Inspection

So, in this lecture we will try to cover topics introduction, then gear terminology, error of spur gear, there are several gears. So, out of it is spur is the most common and simplest gear so, that is why we talk about spur gear. Then measurements of gear elements, then composite method of gear inspection, these are the topics which we will cover in this lecture.

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Introduction



- Gears are the main elements in a transmission system. It is needless to say that for efficient transfer of speed and power, gears should conform perfectly to the designed profile and dimensions.
- Misalignments and gear runout will result in vibrations, chatter, noise, and loss of power.
- On the other hand, threaded components should meet stringent quality requirements to satisfy the property of interchangeability.
- The most common forms of gear teeth are involute and cycloidal.
- The major gear types are spur, helical, bevel, spiral, and worm gears.

Gears are the main element in transmission system. As, I said from one point to another point you have to transmit power. So, then gear is one of the main element which is used for it. It is needless to say that the efficient transfer of speed and power gear should confirm perfectly to the designed profile and dimensions.

As, I told you in the beginning of wheel got converted into a gear, it got converted into a gear. So, when it got converted into a gear this is a gear tooth and this gear tooth placing at different locations and their geometry plays a very very important role. If there is any small discrimination or discrepancy from the drawing to manufacturing then, whatever was the efficiency will go down drastically misalignment and gear run out will result in vibration, chatter noise and loss of power.

Many a times when we see when we drive a bike or when we drive a car or any other system we try to get lot of sound. So, this sound are nothing, but vibration. So, it can be chatter, it can be noise. So, all these things can happen if there is a misalignment and gear run out which is there. On the other hand, threaded components should meet stringent quality requirements to satisfy property of interchangeability.

Today, we can go to the market and we can give certain specification and directly by gears. If these gears are made out of plastic these gears are made out of brass metals or like brass, it can be made out of steel.

So, you directly go to the market and say this is these are the specifications I need a gear. So, you can get those gears in the market so; that means, to say the concept of interchangeability is been well introduced in gears. The go most common form of gear tooth are involute and cycloid.

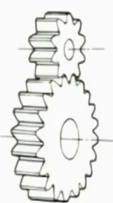
The major type of gear are spur gear, helical gear, bevel gear, spiral gear, and worm gears. These are the most common one and apart from that you have herringbone. So, many other gears are there, but for our discussion we will try to look into these 5 gears and more in particular towards spur gear, because this is the most common gear which is used. So, how are these gear tooth manufacture they follow involute and cycloid forms.

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Gear Terminology

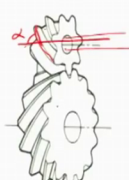
Spur gears

- This is a cylindrical shaped gear, in which the teeth are parallel to the axis.
- It is the most commonly used gear with a wide range of applications and is the easiest to manufacture.



Helical Gear

- This is a cylindrical shaped gear with helicoid teeth
- Helical gears can bear more load than spur gears, and work more quietly.
- They are widely used in industry.
- A disadvantage is the axial thrust force caused by the helix form.



When, we talk about spur gear this is a typical spur gear. This is a cylindrical shaped gear, in which the teeth are parallel to the axis. This is the axis, teeth are running parallel to the axis. This is the most commonly used gear with a wide range of application and it is easiest to manufacture. Lot of gear trains use spur gears right. So, this is the most common gear and lot of inter changeability concept can be followed in these gears.

The next one is a Helical Gear. Helical gear and Spur gear there is only one change, you see there is a change in the gear tooth profile angle with respect to the centre axis. So, this is a cylindrical shaped gear with helicoid teeth, helical gears can bare more load than the spur gear and works more quietly. So, compared to here this gear because it can come

in contact with another teeth for a longer time. So, it can take heavier loads. Second thing is it is not straight side is at an angle contact. So, it can work also very quietly. The difference between spur and helical is the angle with which the tooth is fabricated.


So, this is widely used in industry, wherever you want to have very heavy load transmission and a quiet working we used to go for helical gear. The main disadvantage of this gear is the axial thrust force caused by the helical form, there be axial thrust force. So, that will try to push this fellow out of the shaft. So, axial thrust force caused by the helical form makes it little difficult when to handle while it is an operation. So, when you are trying to make assembly you have to be more conscious and cautious as compare to that of spur gear.

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Gear Terminology

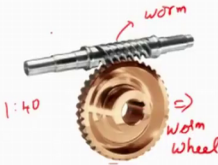
Herringbone gears

- This is a cylindrical shaped gear, in which the teeth are parallel to the axis.
- It is the most commonly used gear with a wide range of applications and is the easiest to manufacture.



Worm Gear

- Worm gear pair is the name for a meshed worm and worm wheel.
- An outstanding feature is that it offers a very large gear ratio in a single mesh. It also provides quiet and smooth action.
- However, transmission efficiency is poor.



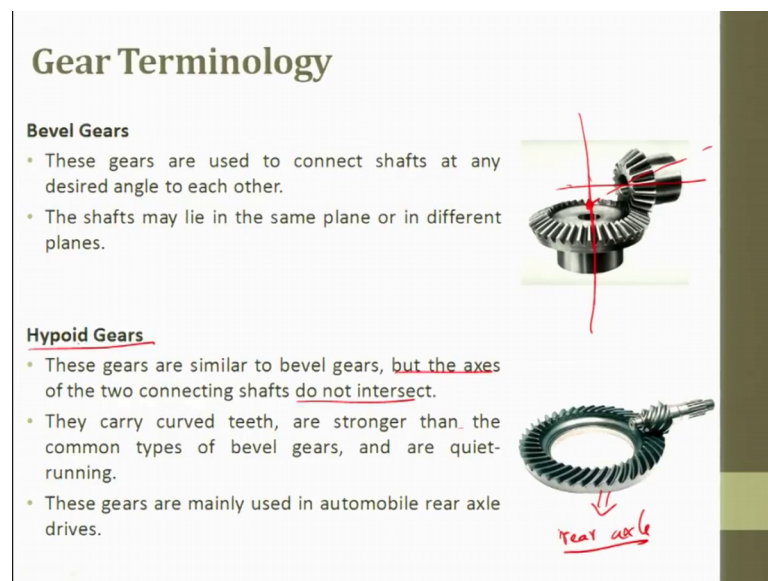
Herringbone gear; herringbone gear you see herringbone right this. So, this is like a fish bone right on both sides. This is a cylindrical shaped gear in which the teeth are parallel to the axis; the teeth are parallel to the axis. This is the most commonly used gear with a wide range of application and it is also easy to manufacture. Easy to manufacture please do not compare it with that of spur gear and helical gear it is little complex, but still herringbone gears are easy to manufacture and it has lot of applications. Where you want to transmit more torque we also go for this. Worm gear; worm gear this is this is a gear and here is a helix which is maintained. So, this helix should match with this helix. So, we try to transmit power from one axis and which is perpendicular to the other axis.

So, the worm gear pair is the name for a meshed worm and a worm wheel. This is called as a worm wheel and this called as a worm. An outstanding feature is that it offers a very large gear ratio in a single mesh, generally when we talk about is 1 is to 40.

So, if you rotate this 40 times maybe this gear rotate one time. So, an outstanding feature is that it offers a very large gear ratio in a single mesh. It could provide a quiet and a smooth operation. The transmission efficiency is relatively poor as compare to that of herringbone spur gear and helical gear.

And, people call also call it as a self-locking mechanism, because in a spur gear if you want to a make crane for example, so, you would start rotating in one direction, the crane pulls the weight up and then suppose you want to lock at one point. Then, we always prefer to go for worm gear rather than spur gear and helical gear.

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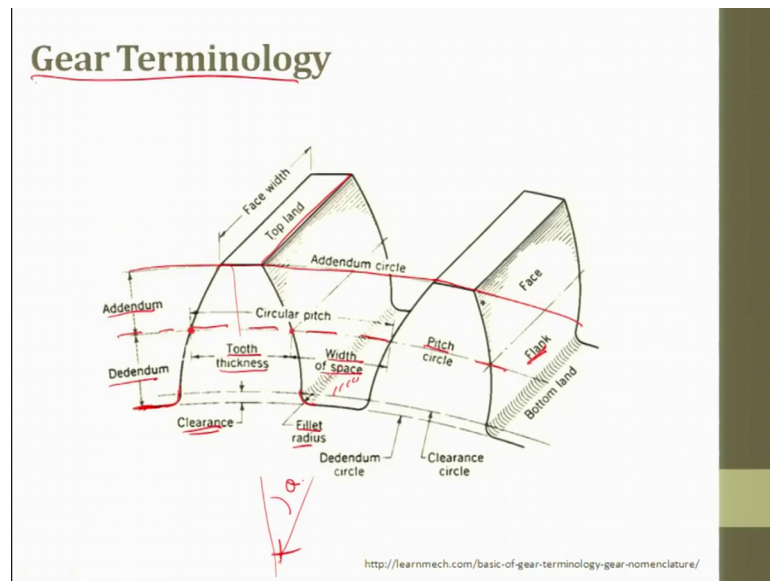
This is a bevel gear. So, bevel gear look at it so, here is an axis here is an axis right. These gears are usually to connect shafts at an desired angle to each other. So, here I have just put perpendicular, but if you see more closely this is certain angle, this is certain angle, but here I have just intentionally draw it as 90 degrees.

The shafts may lie in the same plane or in the different plane. The shafts may lie in the same plane or in the different plane, bevel gear. So, bevel gear is also used to transmit power or motion at an angle between these 2 axes. Hypoid gear, these gears are very

similar to bevel gear, but the axis of the 2 connecting shafts do not intersect, here it intersects for example, if I want to be more specific. So, it intersects here, but here it will never intersect. They carry curved teeth are stronger than the other common type of bevel gear and are quietly running. So, we have this Hypoid gears.

These gears are mainly found in rear axle of automobile; that means to say basically trucks. So, rear axle we will have this. So, it is called as Hypoid gears.

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So, if we start looking at Gear Terminology. So, the first thing which you should understand is the pitch circle, pitch circle is this which passes through which is almost in the mid of the tooth is pitch circle. Top of the pitch circle up to the face of the teeth, if you try to draw one so, that circle is called as addendum circle. So, this is a pitch circle which a circle which passes through the face, top tooth face right or the yeah tooth face. So, that is called as addendum.

And, below the pitch circle to the root to the base is called as addendum dedendum. So, this is called as the addendum circle, this is called as the dedendum circle. You cannot have a perfect flat teeth so, what we does we always try to have a curvature right. And, this curvature will also have this there is a curve and then you also have a curve at the base. So, if there are sharp corners these are going to be stress points. So, it will go it is going to a fail. So, this radius is called as fillet radius which is given at the bottom of the teeth.

If, you try to draw a circle between the start point of the radius and the base that is called as clearance so that is, an every teeth has it is thickness. So, that is called as a tooth thickness. So, tooth thickness is when the tooth this is a flank. So, the flank when it passes through the pitch circle diameter between the between that the tooth flank from one face flank from the other face that thickness is called as the tooth thickness. So, then the flat portion on the tooth is called as the top land and the width is called as the face width.

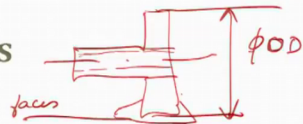
Whatever, I called it as the flank it is this is the flank which is there and this is a face which is there. So, bottom land, this is bottom land; bottom land is this portion this is called as bottom land this is called as top land. So, on top of the tooth or bottom of the tooth, then this is the tooth. So, the between 2 teeth there is a spacing that is called as the width of space, that is called the width of space of from here to here this is called a width space. So, I have covered all so, clearance circle, dedendum dedendum circle, addendum addendum circle, pitch circle right this is a pitch circle so, then tooth thickness, width of their shape, face width, top land and bottom land.

And the face of the tooth is called as the flank. These are the terminologies. Now, you if you look it very closely you see there is lot of complex geometry which is involved. And, generating this geometry through machining, we always try to use the concept of involutes which we study in the engineering drawing. So, that is used when 2 meshing happens between the tool and work piece we try to generate or you can try to generate it by milling the cutting these slots in between. So, the gear terminology you have to understand all this terminology this is very very important.

So, when you look at here there is lot of complexity in the geometry lot of terminologies. So, and this gears are mechanical in contact. So, when there are 2 objects are mechanical in contact than over a period of time there is going to be warrantor. So, this warrantor leads to error.

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Errors in Spur Gears

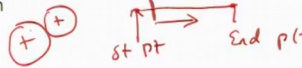


Gear Blank runout errors

- Gear machining is done on the gear blank, which may be a cast or a forged part.
- The blank would have undergone preliminary machining on its outside diameter (OD) and the two faces.
- The blank may have radial runout on its OD surface due to errors in the preliminary machining.

Gear tooth profile errors

- These errors are caused by the deviation of the actual tooth profile from the ideal tooth profile.
- Excessive profile error will result in either friction between the mating teeth or backlash



So, the error which happened in a spur gear, by the way if you go back and see this the terminologies I have drawn with respect to only spur gear. If, there is a small tilt with respect to the centre axis, if there is a small tilt or small angle between the centre axis and the tooth, you that is called for example, if it is at an angle then this angle is called as helix angle.

At this helix angle then you will have bevel gear. So, in bevel gear the geometry slightly complex, herringbone much more complex, hypoid gears much more complex. So, the gear terminologies for each and everything is the same, but it varies with respect to the angle, and you have to always have various angles with respect the normal one.

So, the error which generally happens or gear blank run out error, what is a blank, before we manufacture a gear, before we manufacture any gear the raw material is cut into a cylindrical wheel or a cylinder it is cut as a cylinder. So, that cylinder is called blank, which will be later converted into a gear during machining operation. Again, manufacturing gear can be done, one as a back size can be done many as back size.

When we do many we have processes like gear, honing and gear honing we have and then we have many more processes, when it as a single one you can do it by milling machine. In milling machine what we do is we try to use an indexing mechanism. So, that the gear blank indexes at an angle and stops and then you just use a horizontal miller milling cutter to cut and generate this teeth. So, if there is a run out in the blank; that

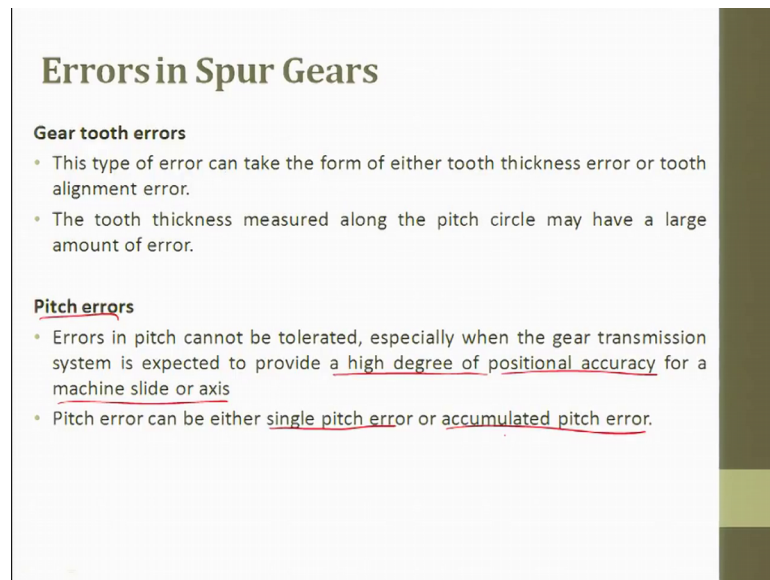
means, to say base so, then that you also leads to error. Gear machining is done on the gear blank, which maybe cast or forged part or after casting it is generally machines. So, that you get it to the perfect dimension. The blank would have underground preliminary machining on it is outside diameter O D and the two faces.

So, when you try to talk about it. So, you have something like a, this is a blank. So, what we try first we try to maintain this diameter, outer dia O D and then we try to also maintain these are the faces; these are the faces where the gears are there. Sometimes, that is on this on the face then that also has to be done. The blank may have radial run out on it is O D surface, due to the error in the preliminary machining. So, this can lead to error when you machine a gear. The gear tooth profile error, these errors are caused by deviation of the actual tooth profile from the ideal tooth profile.

Maybe a problem with the indexing or maybe a problem with the cutter, excessive profile error will result in either friction between the mating teeth or backlash. What is backlash? When 2 gears mate with each other or mesh with each other. They try to rotate, when you rotate in the forward direction they will reach a destination point and then we have retract it this is the start point, and this is the end point, this is onward.

So, from here to here it goes while return suppose it stops here then; that means to say there is a backlash. This can happen because of error between the 2 machine gears. It can be either friction between the mating gears or can be backlash which will lead to gear tooth profile errors.

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Errors in Spur Gears

Gear tooth errors

- This type of error can take the form of either tooth thickness error or tooth alignment error.
- The tooth thickness measured along the pitch circle may have a large amount of error.

Pitch errors

- Errors in pitch cannot be tolerated, especially when the gear transmission system is expected to provide a high degree of positional accuracy for a machine slide or axis
- Pitch error can be either single pitch error or accumulated pitch error.

Then gear tooth error, this type of error can take the form of either tooth thickness error or tooth alignment error, tooth single teeth, the single geometry in a gear is called as the tooth several tooth put together makes a teeth gear teeth. The tooth thickness measured along the pitch circle may have a large amount of error.

So, where is a pitch circle if you go back, this is a pitch circle when you measure it with respect to pitch circle there would be an error. Then pitch error; error in the pitch cannot be tolerated, what is pitch, the distance between one teeth to the next teeth is a pitch right. The error in the pitch cannot be tolerated especially when the gear transmission system is expected to provide a very high degree of positional accuracy for machine slides or axes. The pitch error can either be single pitch error or accumulated pitch error; that means to say several of the teeth right.

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Numerical Problem

Question: Calculate the setting for a straight spur gear having 40 teeth of module 3 pitches.

Solution

Setting of a Vernier for a gear of 40 teeth of module 3 pitches

$$w = Tm \sin\left(\frac{90}{T}\right)$$

$w = 40 \times 3 \times \sin\left(\frac{90}{40}\right)$
 $= 120 \sin 2^{\circ}15'$

Depth of tooth, $h = m + \frac{Tm}{2} \left[1 - \cos\left(\frac{90}{T}\right) \right]$

$$= 3 + \frac{40 \times 3}{2} \left[1 - \cos\left(\frac{90}{40}\right) \right]$$

$$= 3.042 \text{ mm.}$$

Legend:
 w = width of teeth
 T = no. of teeth
 m = module.

So, let us try to now solve a simple problem and let us see how do we do it? So, calculate the setting for a straight spur gear having 40 teeth of module 3 pitches. So, let us try to solution setting of a vernier for a gear of 40 teeth. So, it is the teeth around the spur gear of module 3 pitches, so Tm so w equal to $Tm \sin 90$ by T . So, this is w can be expressed as 40 teeth module is 3 and since it is per gear it is $90 \sin 90$ by number of teeth T .

So, which is nothing, but $120 \sin 2$ degrees and 15 minutes, this is the width of the teeth. So, the next is depth. Depth of tooth is given by the formula h equal to m plus Tm by 2 1 minus $\cos 90$ by T , which is 3 plus Tm 40 into 3 divided by 2 1 minus $\cos 90$ by 40, which is equal to 3.042 millimetre. So, this is the formula. So, you have to this is a formula and here this is also a formula, where w equal to width is equal to T , teeth into pitch divided by 7. So, this is also a formula.

Where w is nothing, but width of teeth w T is nothing, but number of teeth and m is nothing, but the module. So, this will be the depth of tooth and this will be the width of tooth. So, width and depth so, if you go back and see what is your width? This is the depth from here to here this is the width.

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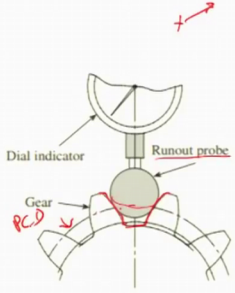
Measurement of Gear Elements

Measurement of Runout

- In case of a gear, runout is the resultant of the radial throw of the axis of a gear due to the out of roundness of the gear profile. Runout tolerance is the total allowable runout.

Single-Probe Check

- It uses an indicator with a single probe whose diameter makes contact with the flanks of adjacent teeth in the area of the pitch circle.



The diagram shows a cross-section of a gear with a dial indicator positioned above it. The dial indicator has a spherical 'Runout probe' in contact with the flank of a gear tooth. A red arrow points to the probe. Below the gear, the 'Pitch Circle Diameter' (PCD) is marked with a red arrow. A red arrow also points to the gear's axis.

So, what are the different ways of measuring the gear elements? So, first we are trying to measure the run out. So, in case of gear run out is a resultant of a radial through of the axis of a gear due to out of roundness of the gear profile. So, there is a centre. So, there is a shift in the centre. So, a radial throw of the axis of a gear, due to the out of roundness of a gear profile, the run out tolerance is the total allowable run out.

So, run out also when we try to specify, we also try to specify in the tolerance run out tolerance also, when we talk about assembly we talked about run out tolerance also. So, how do we do that? We have a dial gauge with a set on top of a probe. So, this is called as a run out probe which is nothing, but a ball. So, with the ball or a wire cylinder. So, this is the resting on top of it with this we will try to find out the deflection on the dial indicator and find out what is the run out. A single probe check it uses an indicator with a single probe whose diameter makes in contact with the flank. So, this is a flank.

So, whenever you do this gear measurement or thread measurement, you always should make sure that the known reference whatever you have must rest on both sides, makes in contact with the flank of the adjacent teeth in the area of the pitch circle diameter. So, it should not be in such a way that it does meeting somewhere here. It has to meet at the in the area of the pitch circle; that means, to say in the pitch circle this is the pitch circle diameter P C D.

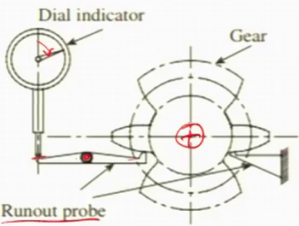
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Measurement of Gear Elements

Measurement of Runout

Two-Probe Check

- In this one fixed and one free-moving probe, are positioned on diametrically opposite sides of the gear and make contact with identically located elements of the tooth profile.
- The range of indication is twice the amount resulting from single-probe check.



You can use 2 probe also, the 2 probe method is we try to have a dial indicator, which is attached to an arm. So, this arm in is pivoted in between one side comes in contact with the teeth of a gear, the other side comes in contact with the dial bottom of the dial gauge which rests on it. So, in this one fixed this is one fixed end. So, this is one free end the other just to opposite to that teeth we will have a tooth profile with again there is a probe which is fixed now. So, this is the fixed run out probe, this 2 are fixed run out probe one is fixed, the other one is free and when the gear comes in contact it slides over this or cribs over this and the deflections are read.

In this one fixed and one free move in probe are positioned on diametrically opposite side of the gear and make the contact with identical located elements of the tooth profile. The range of indication is twice, because 2 probe is twice the amount resulting from a single probe check. The range of indication is twice, this is an indicator is twice that of a single probe this is 2 probe the first one what we saw was a single probe both these things are used to find out the run out, if there is a run out then what will happen is you will not be able to transmit power efficiently.

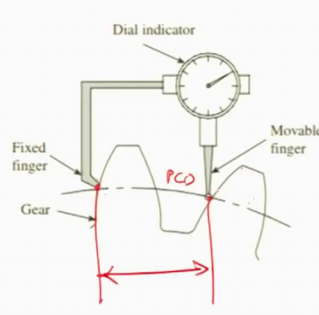
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Measurement of Gear Elements

Measurement of Pitch

Pitch Measurement

- These instruments enable the measurement of chordal pitch between successive pairs of teeth.
- The instrument comprises a fixed finger and a movable finger, which can be set to two identical points on adjacent teeth along the pitch circle.



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The next one is how do you measure the pitch? What is pitch? Pitch is nothing, but the distance between the 2 for example, from here to here is called as a pitch. This circle is called pitch circle diameter. Here, what happens these instruments enables to measure the chord, chordal pitch with successive pairs of teeth between two. So, here is a fixed finger. So, that rests and then you have a dial gauge. So, this dial gauge is resting on the moving finger, the instrument comprises a fixed finger and the moving finger. So, this can be set to two identical pointer of adjacent teeth along the P C D can be measured.

So, with this what we do if we try to measure the pitch, this is the pitch circle diameter. And, whenever you try to touch it is better that you try to fix the arm on the P C D and this dial on the P C D so, that we can try to measure these 2.

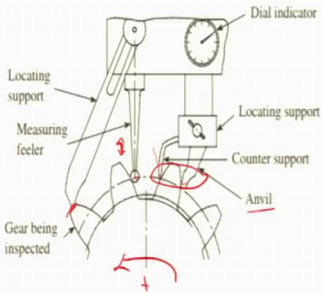
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Measurement of Gear Elements

Measurement of Pitch

Pitch Checking

- A pitch-checking instrument is essentially a dividing head that can be used to measure pitch variations.
- It has two probes— one fixed, called the anvil, and the other movable, called the measuring feeler. The latter is connected to a dial indicator through levers.
- The instrument is located by two adjacent supports resting on the crests of the teeth.
- A tooth flank is butted against the fixed anvil and locating supports.

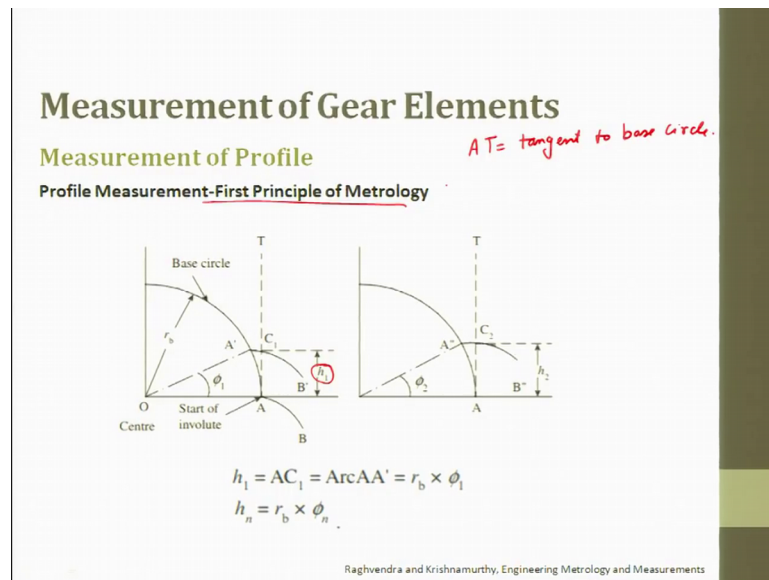


Then, the other check we have this pitch checking, a pitch checking instrument is essentially a dividing head dial gauge it has a dividing head that can be used to measure pitch variation. So, here you will have a measuring feeler and then here you will have a locating pin which is a pin, which locates the gear being inspected it locates here. So, then every time what we do if we try to index, when with this index we try to take gear after gear after gear we try to check the pitch.

So, this is an anvil and this is a counter which is there a locating. So, this tries to measure the deflection in the dial gauge. So, this is fixed this is moving. So, because of that here there will be a motion in the dial gauge you try to check it. So, the pitch checking instrument is essentially a dividing head that can be used to measure the pitch variation. In it has 2 probes; one is called the fixed which is with the anvil, the other one is called as the moving. One fixed call the anvil and the other movable probe call the mechanical feeler. So, this is a moving one up and down feeler.

The latter is connected to the dial gauge the moving is connected to the dial gauge and the dial gauge reach the reading. The instrument is located by 2 adjacent supports resting on the crest of the teeth; crest of the teeth. The 2 flanks is butted against the fixed anvil and the locating support. So, it is one locating support. So, one is fixed and the other one is moving.

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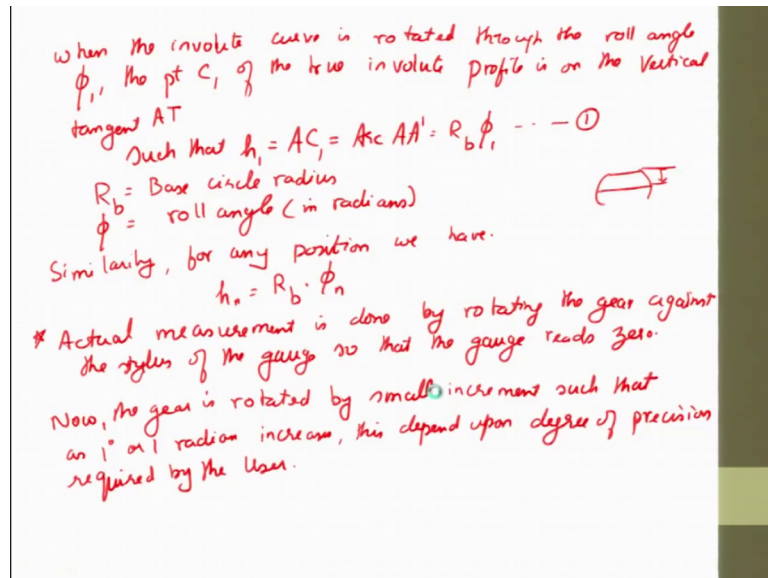
So, measurement of gear element so, we wanted to measure the gear profile. So, we want to measure it by using the first principles of metrology. So, in this figure let us first try to understand. So, this is the base circle, this is the base circle the radius of the base circle is going to be r_b right.

And. So, here is a tooth which we are going to generate the involute. So, this r_b meets at up to the base at a point A. So, now, you draw perpendicular AT. So, the A dash is a point where it is on the r_b base circle. So, it meets at a point A dash. Now, project A dash C dash right, the A dash continuous when you do the involute it continues to be dash.

So, now, you have found out what is C 1 you know what is T which is drawn perpendicular to it and the distance between the intersecting point C dash to A is called h_1 . The, h_1 can be expressed as AC dash which is nothing, but arc of A dash, which is radius into this phi.

So, let us write it down. So, this is for the next teeth. So, it is teeth be A phi 2. So, you have a double dash the same thing. So, the involute changes so, it is h_2 . So, let us write it down. So, AT is nothing, but the tangent to base circle.

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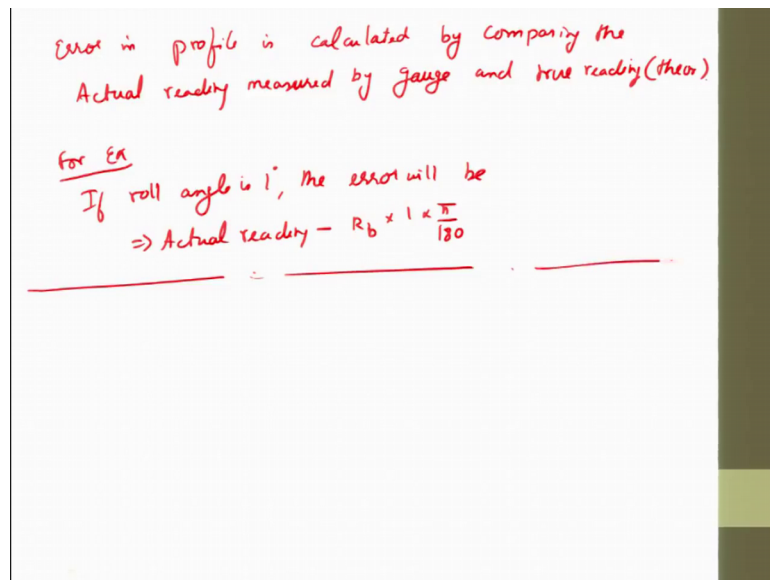
So, when the involute curve is rotated through the roll angle ϕ_1 , the point C_1 of the true involute profile is on the vertical tangent $A T$ such that, what are they have h_1 equal to $A C_1$, which is equal to arc $A A'$. So, it is nothing, but $R \theta$. So, it is $R_b \phi_1$. So, let us take it as question 1. So, where R_b is the base circle radius and ϕ is the roll angle in radians.

Similarly, for any given position we have h_n equal to R_b into ϕ_n . So, for any given position h_n is equal to. So, if we try to take the actual measurement is done by rotating the gear against the stylus of the gauge so, that the gauge reads zero. Now, so, I am trying to fully write it down because once you understand and appreciate, then only you can try to do gear measurement, as far as a gear is concerned there are 2 things; one is you have to find out the gear teeth thickness and the other one is the height.

So, these 2 are the very important things so, that is what. So, here what happen theoretically they find out and experimentally they are find out find out whether there is an error. Now, the gear is rotated by a small angle or increment such that as 1 degree or 1 radian increases.

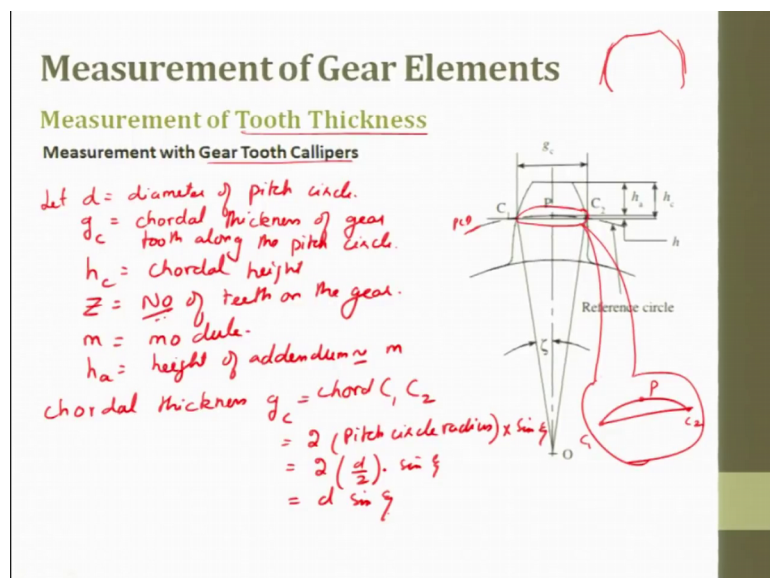
These depend upon degree of precision requirement by the user. So, this is not a very important statement, but still you should understand when the gear is rotated by a small increment, such that 1 degree or 1 radian increase, this depends on the precesion of the user.

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So, the error in profile is calculated by comparing the actual reading measured by gauge and true reading; true reading is theoretical calculation. For example, if roll angle is 1 degree, the error will be actual reading minus R_b actual reading R_b into 1 into pi by 180 degrees 180 as measured in the gauge. So, this is how you get the value.

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So, the next topic will be measurement with gear tooth calliper, what are we trying to do we are trying to measure the tooth thickness. So, if you see here, the tooth will be approximately something like this. So, you see here it is varying, it is varying. So, it is

very difficult for us to directly measure the tooth thickness. So, what we do is we try to use a gear tooth calliper to measure it. For that also we have to understand little bit of terminologies and find and come out with the methodology, how do you find out the tooth thickness. So, what is tooth thickness here? The tooth thickness is going to be $C_1 P C_2$.

And, here if you look at this figure alone, let me zoom it down and then see you see here. So, this will be something like this and this is here. So, this is your P, this is your C_1 and this is your C_2 . So, now, let us start the deriving it let d be the diameter of pitch circle. So, what is pitch circle? This is the pitch circle right. Pitch circle $P C D$ right then let g_c be the chordal thickness of gear tooth along the pitch circle, h_c is the chordal height for this is h_c this is the chordal height.

What is Z ? Z is the number of teeth on the gear, then m will be the module will see that later, and h_a will be the addendum height which is approximately equal to module. So, now, first let us try to find out what is chordal thickness g_c which is nothing, but chord $C_1 C_2$, which is nothing, but 2 times into pitch circle radius into $\sin \psi$. So, this can be written as 2 times pitch circle radius is d by 2 into sine ψ this is $d \sin \psi$.

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Measurement of Gear Elements

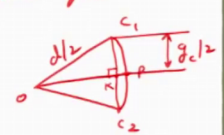
Measurement of Tooth Thickness

Measurement with Gear Tooth Callipers

Now, Arc $C_1 P C_2 = \frac{d}{2} \times 2\psi$ (Value of ψ is in radians)
 $= d \times \psi = \frac{\pi d}{2Z}$ [Z teeth in 2π radians]

Therefore, $\psi = \frac{\pi}{2Z}$
 $g_c = d \sin\left(\frac{\pi}{2Z}\right)$, where $\frac{\pi}{2Z}$ is in radians
 $g_c = d \sin\left(\frac{90}{2Z}\right)$, argument of \sin is in degree.

chordal height
 $h_c = h_a + h = m + h$
 length $OK = \frac{d}{2} - h$
 Now $\Delta O C_1 K$
 By Pythagoras theorem
 $\left(\frac{d}{2}\right)^2 = \left(\frac{d}{2} - h\right)^2 + \left(\frac{g_c}{2}\right)^2$



Now, arc $C_1 P C_2$ is nothing, but d by 2 into 2ψ value of ψ is always in radians, if now I cut down these two. So, this is nothing, but πd by $2Z$ and Z is the number of teeth in radians Z is the teeth in 2π radians. Around the circle what how many number

of teeth? Therefore, ψ equal to π divided by $2Z$. So, now, g_c is nothing, but $d \sin \frac{\pi}{2Z}$, where $\frac{\pi}{2Z}$ is in radians. So, g_c is equal to $d \sin 90$ by $2Z$. So, the argument of \sin is in degrees. So, now, chordal height, we are found out g_c . Now, we are trying to find out chordal height; chordal height is nothing, but h_c equal to h_a plus h , which can be written as m plus h . So, now, let me draw.

So, this is O this is C 1, this is P, this is C 2 and this is 90 and here is a k component and this is d by 2 and this is nothing, but g_c by 2. So, length from this figure length O K is equal to d by 2 minus h . Now, triangle O C 1 K by using Pythagoras theorem the d by 2 the whole square equal to d by 2 minus h the whole square plus g_c by 2 the whole square.

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Measurement of Gear Elements

Measurement of Tooth Thickness

Measurement with Gear Tooth Callipers

$$\frac{d^2}{4} = \frac{d^2}{4} + h^2 - dh + \frac{g_c^2}{4}$$

$$dh = h^2 + \frac{g_c^2}{4} \quad [h^2 \text{ is neglected}]$$

$$h = \frac{g_c^2}{4d}$$

So chordal thickness $g_c = d \sin \left(\frac{90^\circ}{Z} \right)$
 and chordal height $h = \frac{g_c^2}{4d}$

Also, $h_c = m + \frac{g_c^2}{4d}$

So, now we can make it as d by d square by 4 equal to d square by 4 plus h square minus $d h$ plus g_c square by 4. So, $d c$ square by $d c$ square by 4 is gone. So, what we have is we have $d h$ equal to h square plus g_c square by 4 and here h square is neglected.

So, finally, what we get is h equal to g_c square by $4 d$. So, chordal thickness is g_c equal to d times $\sin 90$ by Z . And chordal height h is equal to g_c square by $4 d$ also please make a note, h_c equal to m plus g_c square by $4 d$.

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Measurement of Gear Elements A $\xrightarrow{B} C$

Measurement of Tooth Thickness

Measurement with Tooth Span Micrometers

- Tooth thickness is measured by measuring the chordal distance over a number of teeth by using a tooth span micrometer, also called a flange micrometer.
- The measurement is based on the base tangent method.

In this case, tooth thickness is measured by measuring the chordal distance over a no. of teeth

We can see that if a straight generator ABC is being rolled along a base circle then its ends trace an involute profile ($A_2 A A_1$) and ($C_2 C C_1$)
 $AC = A_1 C_2 = A_2 C_1 = A_0 C_0$
 $A_0 C_0 \Rightarrow$ Arc length of the base circle between the origins of involutes.

Till now what we were doing is we were measuring it for a single teeth. Now, we will try to measure for a set of teeth. So, for which we use tooth span micrometre. Till now we have did only one tooth, now will take some 3 4 tooth and then try to measure tooth thickness is measured by measuring the chordal distance over a number of teeth by using the tooth span micrometre also called as flange micrometre.

So, the measurement is based on the base tangent method. So, now, let us see a base tangent method. In this case tooth thickness is measured by measuring the chordal distance over a number of teeth. So, we can see that if a straight generators line A B C is being rolled. So, this is a circle this is A B C A B C is rolled along a base circle, then its ends trace an involute profile which is nothing, but A 2 A A 1 and C 2 C C 1. So, we can say A C equal to A 1 C 2, which is equal to A 2 C 1, which is equal to A naught C naught. A naught C naught is nothing but the arc length of the base circle between the origin of involute.

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Measurement of Gear Elements

Measurement of Tooth Thickness

Measurement with Tooth Span Micrometers

Now, if we have a gear with (N) no. of teeth and the length (AC) on the pitch circle corresponds to 'S' number of teeth (called tooth span).
 Then, $AC = (S - \frac{1}{2}) \text{ pitches}$.
 Therefore angle subtended by AC = $(S - \frac{1}{2}) \times \frac{2\pi}{N} \text{ rad}$.
 Here δ is an involute function of pressure angle ϕ .
 Arc BD subtends an angle = $(S - \frac{1}{2}) \times \frac{2\pi}{N} + 2\delta$.
 $BD = (S - \frac{1}{2}) \frac{2\pi}{N} + 2(\tan \phi - \phi) \times R_b$
 $BD = (S - \frac{1}{2}) \frac{2\pi}{N} + 2(\tan \phi - \phi) \times R_p \cos \phi$
 $R_b = R_p \cos \phi$
 $R_p = \frac{mN}{2}$
 $\text{Arc length BD} = \frac{N m}{2} \cos \phi \left[(S - \frac{1}{2}) \frac{2\pi}{N} + 2(\tan \phi - \phi) \right]$

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Now, if we have a gear with N number of teeth and the length AC on the pitch; AC on this pitch, pitch circle which corresponds, corresponds to s number of teeth called tooth span as shown in the figure. Then AC is equal to S minus halftimes pitch AC is nothing, but half times pitch. Therefore, angle subtended by AC is equal to S minus half into 2 pi by N, by the way S minus half.

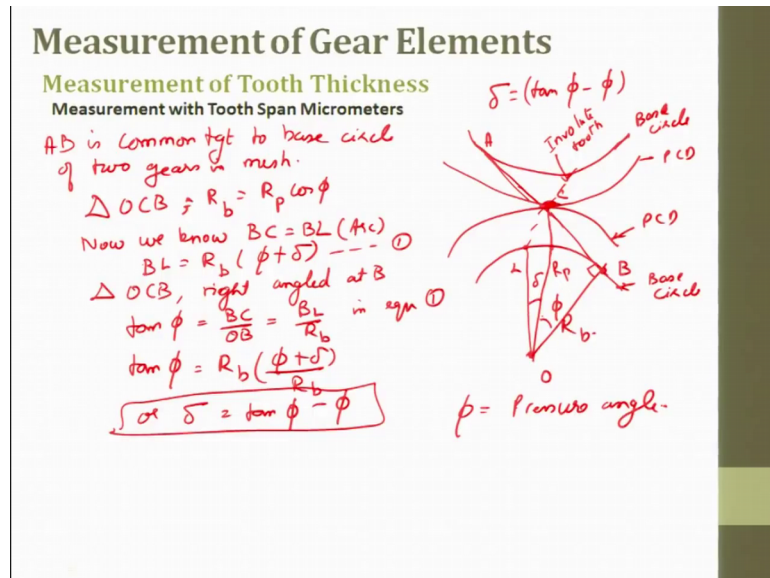
Half is nothing, but suppose you try to take a diameter and then you try to take a next one. So, we are trying to calculate from here to here. So, 1 pitch half pitch right. 2 pi by N radians. So, here delta is an involute function of pressure angle, which is phi. Arc BD subtends an angle, which is equal to S minus half into 2 pi by N plus 2 dell. So, which is equal to S minus half, 2 pi by N plus 2 times tan phi minus phi.

This is tan phi is dell. So, this is a separate derivation, how do you define dell converted into a pressure angle. So, then we will see that later. So, BD is now substituted like this. Thus BD is equal to S minus half 2 pi by N plus 2 times tan phi minus phi into R b, radius into theta R b. So, now, this R b can be written in terms of R b is now written in terms of R b into cos of phi. So, this is s minus half 2 pi by N plus 2 tan phi minus phi into R p cos phi.

R p is nothing, but m N by 2. So, now, the Arc length finally, of BD is equal to N m by 2 cos phi S minus 1 by 2 2 pi by N plus 2 tan phi minus phi. So, this will be the arc length

of B D from here we try to find out the chordal measuring error. So, we are left with one terminology, I have to tell you how did we get this $\tan \phi$ by ϕ from dell?

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The objective of this explanation is how do we get dell in terms of $\tan \phi$ minus ϕ ? So, what we do is we try to take a gear; try to take one more gear so, this is involute. So, this is the other thing. So, we draw up a tangent which passes through sorry, which passes to C which goes down. So, this point is called as C and this is called as A and this touches A circle call B.

And this is O this is R_p and this is the involute tooth profile, which meets at L this is centre A C. So, now, I join this which is nothing, but dell. And, I draw a perpendicular here which is nothing, but ϕ pressure angle ϕ . So, this is the pitch circle P C D this is the base circle and same way this is the P C D and this is the base circle. I am taking 2 base circle I am taking 2 different gears I am trying to mesh it. So, I have drawn a tangent. So, tangent is nothing, but A C B which is drawn perpendicular and then I tried to draw this is nothing, but the involute of a tooth.

So, that is C which passes through C and then which meets the base circle at L. So, this angle which is subtended is called as dell. The radius whichever this radius is called as sorry this is radius is called as R_b and this radius is called as R_p up to pitch circle. So, now, A B is common tangent to base circle of 2 gears in mesh right. So, triangle O C B is nothing R_b equal to $R_p \cos \phi$. Now, we know B C equal to B L, which is the Arc.

And, this derivation we have seen in the first one, we have seen in the first figure what we have explain to you how do you find out the thickness.

So, $B L$ is nothing, but $R b$ which is ϕ plus δ . So, from the triangle $O C B$ right angled at B , we have $\tan \phi$ is equal to $B C$ by $O B$, which is nothing, but $B L$ by $B L$. So, now, when you put it back into this equation into this equation 1. So, in equation 1 we get $\tan \phi$ equal to $R b \phi$ plus δ by $R b$ and this can be written as δ equal to $\tan \phi$ minus ϕ . This is what is a relationship, which is given and what is ϕ ? ϕ we know ϕ because ϕ is a pressure angle in a gear.

Thank you.