Advanced Composites
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Lecture – 71
Theories of Stress Transfer (Part-II)

Hello, welcome to Advanced Composites. Today is the 5th lecture of the ongoing week and what we will do is we will continue our discussion short fiber composites, and understand how is Stress Transfer takes place in short fiber composites.

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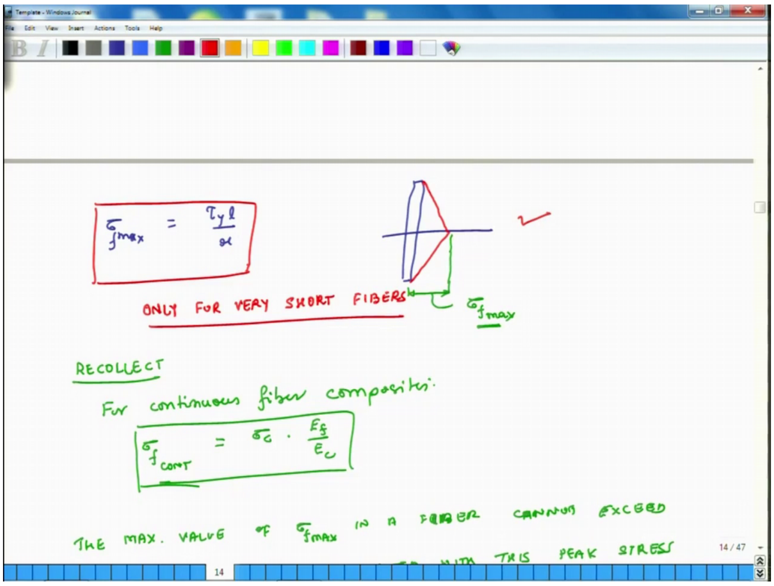

So, what we had shown earlier is based on principles of simple mechanics the value of stress sigma f x max is equal to tau y times l over r. And what does this mean it means that if there is a fiber of length l on that fiber. So, this is the value of stress in the length direction sigma f it will increase linearly, and it will pick at the middle position of the fiber. But this relation I had mentioned and I had not given any explanation for that is that this is only for very short fibers. And I had not explained what does very short mean, so only for very short fibers.

So now, we will try to understand what does the short fiber mean. Now if you remember that when we were discussing are continuous fiber composites, we had said that we have calculated the stress in the fiber. So, what was the stress (Refer time: 02:00) in the we had calculated, that for continuous fiber composites for continuous fiber composites we have calculated that sigma f continuous stress in the fiber was what. It was nothing but a stress in the composite times Young's modulus of fiber divided by Young's modulus of composite ok.

This is something we had explained and this is a very well tested relation so it is actually it works very well. So, in continuous fiber composites this is the stress. Now as I keep on making fiber my fiber longer and longer this sigma f x max; so this is sigma f this is sigma f max. So, as I keep on making fiber longer; slowly the behavior of the short fiber will approach that of the continuous fiber. And add some value of the length of the fiber this value of sigma f max will become will equal sigma f continuous. Above that it will not increase any further.

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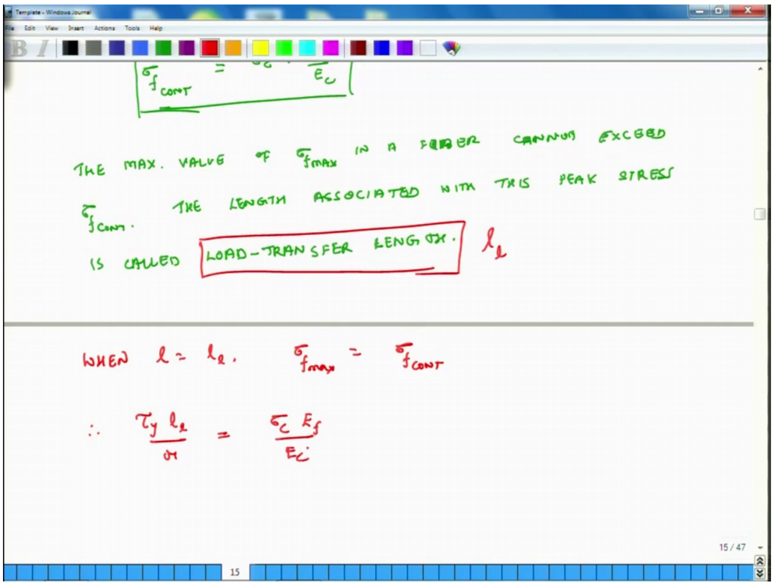

So, the maximum value the maximum value of sigma f in a fiber cannot exceed, it cannot exceed sigma f continuous. So, the length associated with this is stress; this peak stress is called load transfer length. Because we would like that are composites become as efficient as possible. So, we have to ensure that the stress in the fiber is as much as possible because fibers can take a lot of load.

So, if the loads transfer length is sufficient then the stress in the fiber will be same as that in the continuous fiber. So, this load transfer length we call is l l. So, when l is equal to l l l l sigma f max, so sigma f max equals sigma f continuous. So, I pull in the equations from both sides so therefore, tau by l l over are equals sigma c E f over E c.

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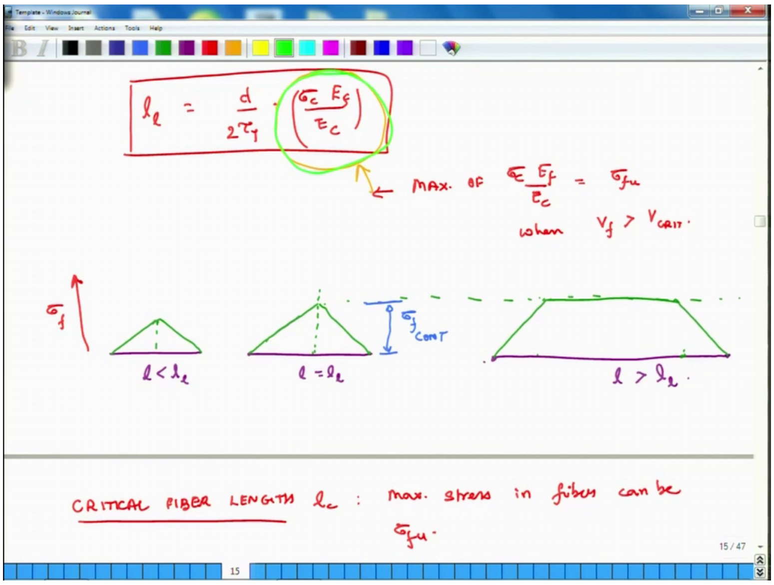

Or l l equals r over tau by times sigma c E f over E c. Or I can instead of r I can put d and I putted denominator here so this is the load transfer length ok. So, if the stress; if the length of the fiber is less than load transfer length then by stress in the fiber will be like this. If the load transfer length is longer if the load transfer length is if the length of the fiber is more than l l, then the stress in the fiber will peak at this value which is sigma f continuous and then it will remain constant.

So, what we will do is we will make some graphs so this is l is less than l l and it will peak here this is l equals l l. So, the slope of the line will be the same. So this is stress. So, on the x axis represents the z dimension and the y axis represent the stress value. So, y axis it represents sigma f stress in the fiber. And what is this value so this is stress value is sigma f continuous.

So, when load transfer length the length of the fiber is equal to load transfer length and the stress and the fiber is same as it would have been in the continuous fiber the maximum stress. And what happens when the length of the fiber is more than the load transfer length so that is my ok. So, in this case this is how the stress is going to be. So, this is the case when l is more than l l ok. So, this is the importance of load transfer length. Now the load transfer length we see it depends on d tau y and also this term which is there in the circle. So, it depends on sigma c E f over E c, E c.

Now what is sigma E c E f over E c; what is the maximum value so if this sigma c E E f over E c this will also change it can change. So, e f is constant e f is a material property e c is constant, but sigma c I can keep on increasing. And I can keep on increasing sigma c till the fiber will break right. So, this term the maximum value of sigma c E f over E c is equal to sigma f u which is the ultimate tensile strength of the fiber when this happens when volume fraction is more than V critical, right.

If you go back and check, if volume fraction is more than the critical volume fraction then the maximum stress in the fiber will be same as the breaking strength of the fiber it is. So, this is the situation we want to have any short fiber composite that it can take maximum possible amount of stress in the fiber. So, there is something called a critical fiber length and that is l c. And l c is associated when maximum stress in fiber can be sigma f u. So, how do I calculate l c essentially I look at this equation and I whatever is there in this green circle I replace it by sigma f u and in then I can calculate l c.

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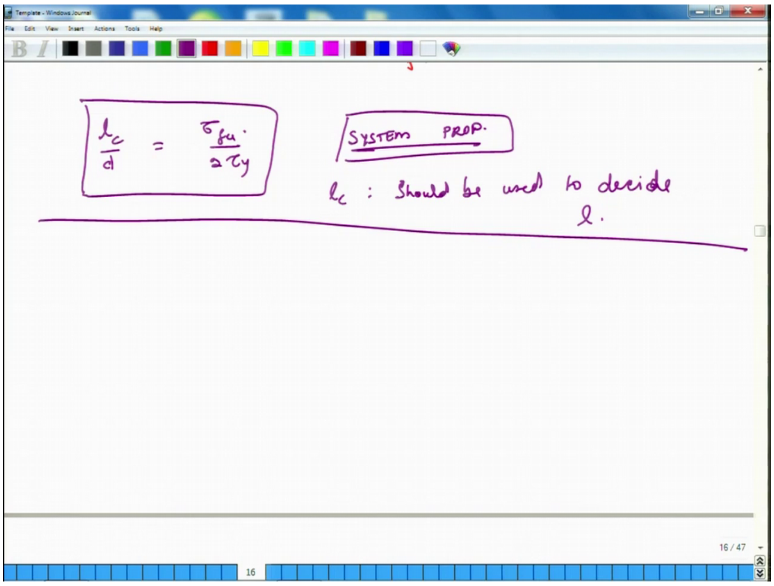

So, using that approach l c equals; so l c over d equals sigma f u over 2 tau y ok. Now this is a system property ok, it does not depend on the load which you are applying. Because, sigma f u is the material property of the fiber, tau y is the material property in the matrix, d is the diameter of the fiber. So, if my fibers are longer than this critical fiber length, then when they fail they will fail at the fiber fracture breaking stress. Otherwise, they will get stressed, but when they fail the failure will not be of this type ok.

So, my material will get maxima utilize to the maximum possible extend if l c x E or if the length of the fiber exceeds l c. So, if I designing short fiber composite I have to make sure that the length of the fiber should exceed l c. Then I will be optimally or utilizing my composite materials in a very efficient way, no. E f upon E c times sigma c will be sigma f u; sigma c is the stress on the composite, stress in the fiber is sigma c times E f over E c. And that when it becomes sigma f u then I get this load I mean the critical length.

So, this is an extremely important parameter and it is a property of the material system what I mean is that it will property of the material and the it depends on E f, it depends on E c, it depends on E m, it depends on volume fraction, it depends on tau y. So, it does not depend on the load it does not depend on the load. So, this term this parameter l c should be used to decide l l should always be more than l c this is an important point to me. Now all this discussion has happened in the context that the material is the matrix material is rigidly plastic the material matrix material is rigidly plastic.

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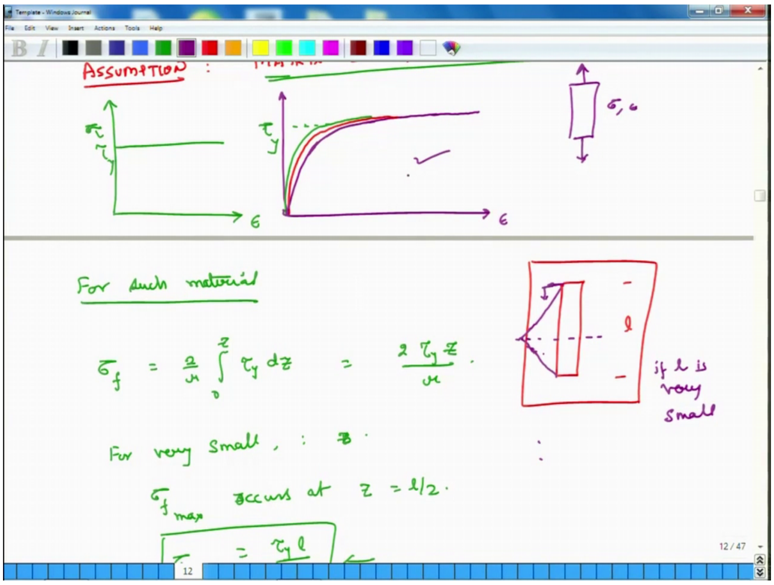

But in reality as I said the material behavior is something like this as shown here. So, if the material is like this which is most of the cases what do we do. If we really want a good solution we really want a good solution more accurate solution in terms of estimating l c and things like that. What we have to do? Is we have to put this kind of a material property in a model we construct a finite element model in that model we put this kind of a material property and do the stress analysis so we put one fiber and matrix.

And we see how stress and the fiber is changing and we keep on changing its length. And based on F e analysis we can calculate a better estimate on l c. But if we do not have access to various sophisticated computer tools then this gives us a reasonably answer for starters. So, this is something important to understand. So, this is the discussion which I wanted to have.

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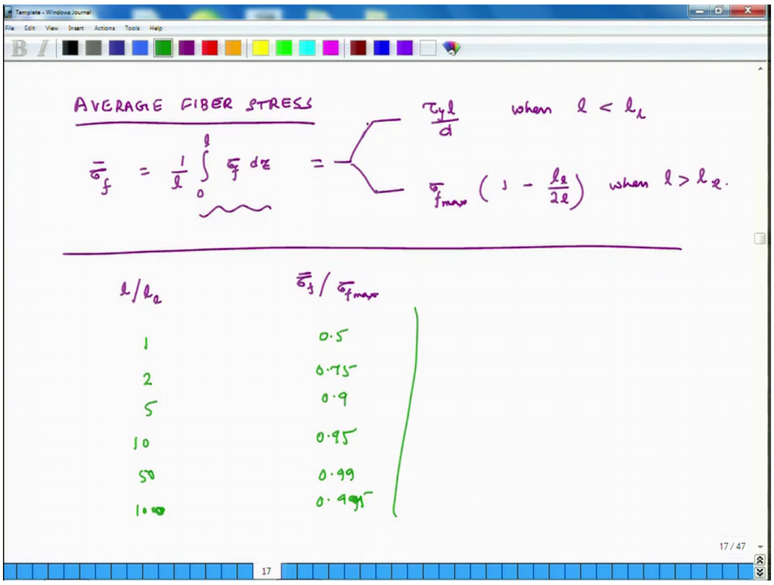

The other thing I would like to say is that there is something also known as average fiber stress. So, what is the average fiber stress basically I am trying to calculate; what is the average stress in the fiber over its length. So, this is sigma f bar and how do I calculate it? I integrate sigma f times d z from 0 to l which is the length of the thing and I divided by l this is how I calculate average of anything.

Now this has two possible values one is tau y times l over d and this is when l is less than load transfer length. And if l is more than load transfer length then it is equal to sigma f max into 1 minus l l divided by 2 l. This is when l is more than load transfer length how did I get these relations. Essentially if you do this integration for both the situations you will get these equations. And the integration process very straight forward so I am not going to discuss this in more detail.

So, based on this we will just very quickly look at a table. So, the first column is the ratio of l over l l, and the second column is normalized average stress. So, how do I normalize it I just sigma f bar divided by sigma f max you take the ratio. So, I have different values 1 2 5 10 50 100. So, if the load transfer length is same as length of the fiber then this ratio is about 0.5 in this case it is 0.75, in the third case it is 0.9. If the length is 10 times the load transfer length then it is about 0.95.

If it is 50 times then it is about 99 percent of the maximum possible stress in the fiber. And if it is 100 so this is 100 then it is 0.995 ok. So, what we would like to have is that the average stress also in the system should be fairly close to the maximum possible stress. So, we would like the fibers so one parameter is l critical and the other parameter is this average fiber stress we would like it to be as close as to the maximum stress in the fiber.

So, we like that the length of the fiber should be several times more than the load transfer length. And the maximum possible value of load transfer length is critical length; so if this l over l c is fairly large. Then even though composites are short fiber composite will behave as continuous fiber composites. So, we will not lose any functionality of that.

So, that is the closure of the discussion for today. Tomorrow we will continue this discussion further and bring this topic also to a close. So, till then have a good time. Bye.